Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/20-1.1.2.3-a+b-x^2-^p-c+d-x^2-^q

Nasser M. Abbasi

September 5, 2023 Compiled on September 5, 2023 at 9:09pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [349]. This is test number [20].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (349)	0.00 (0)
Mathematica	100.00 (349)	0.00 (0)
Fricas	79.37 (277)	20.63 (72)
Maple	75.64 (264)	24.36 (85)
Giac	30.37 (106)	69.63 (243)
Sympy	29.51 (103)	70.49 (246)
Maxima	22.64 (79)	77.36 (270)
Mupad	18.91 (66)	81.09 (283)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

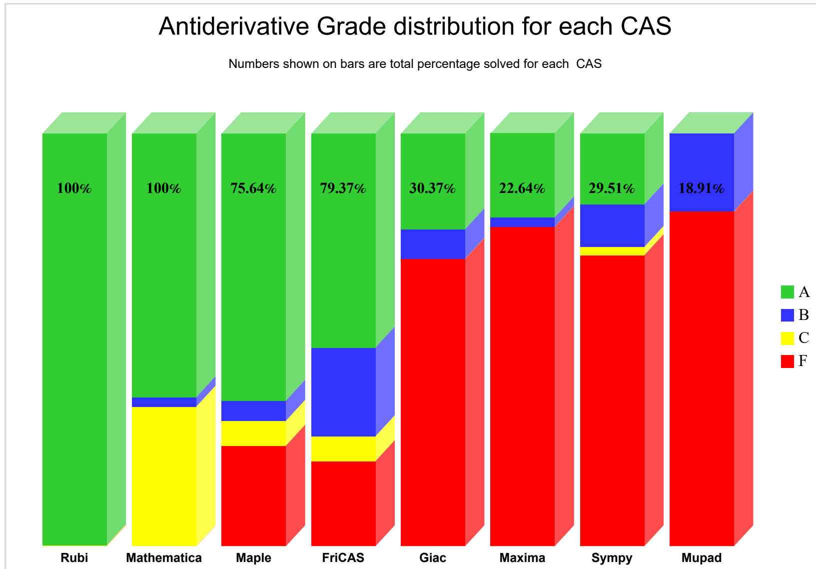
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

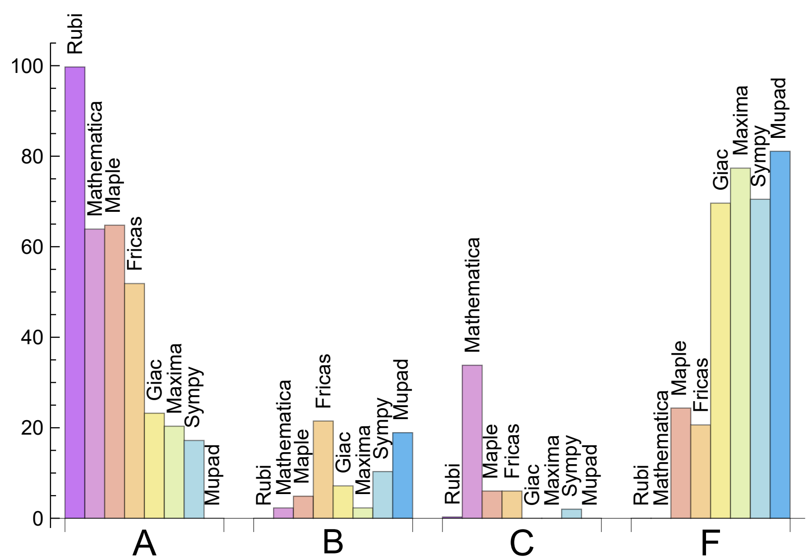
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.713	0.000	0.287	0.000
Maple	64.756	4.871	6.017	24.355
Mathematica	63.897	2.292	33.811	0.000
Fricas	51.862	21.490	6.017	20.630
Giac	23.209	7.163	0.000	69.628
Maxima	20.344	2.292	0.000	77.364
Sympy	17.192	10.315	2.006	70.487
Mupad	0.000	18.911	0.000	81.089

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	72	36.11	63.89	0.00
Maple	85	100.00	0.00	0.00
Giac	243	97.53	0.00	2.47
Sympy	246	91.46	8.54	0.00
Maxima	270	100.00	0.00	0.00
Mupad	283	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.09
Maxima	0.24
Giac	0.49
Sympy	1.91
Mathematica	2.70
Mupad	3.51
Fricas	3.55
Maple	4.19

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	127.63	0.98	93.00	0.99
Sympy	149.86	1.83	94.00	1.63
Maxima	157.72	1.24	116.00	1.16
Maple	165.37	1.44	104.00	1.00
Rubi	178.99	1.00	116.00	1.00
Giac	229.62	1.64	129.50	1.14
Fricas	348.17	2.75	133.00	1.70
Mupad	912.58	5.52	87.50	1.12

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

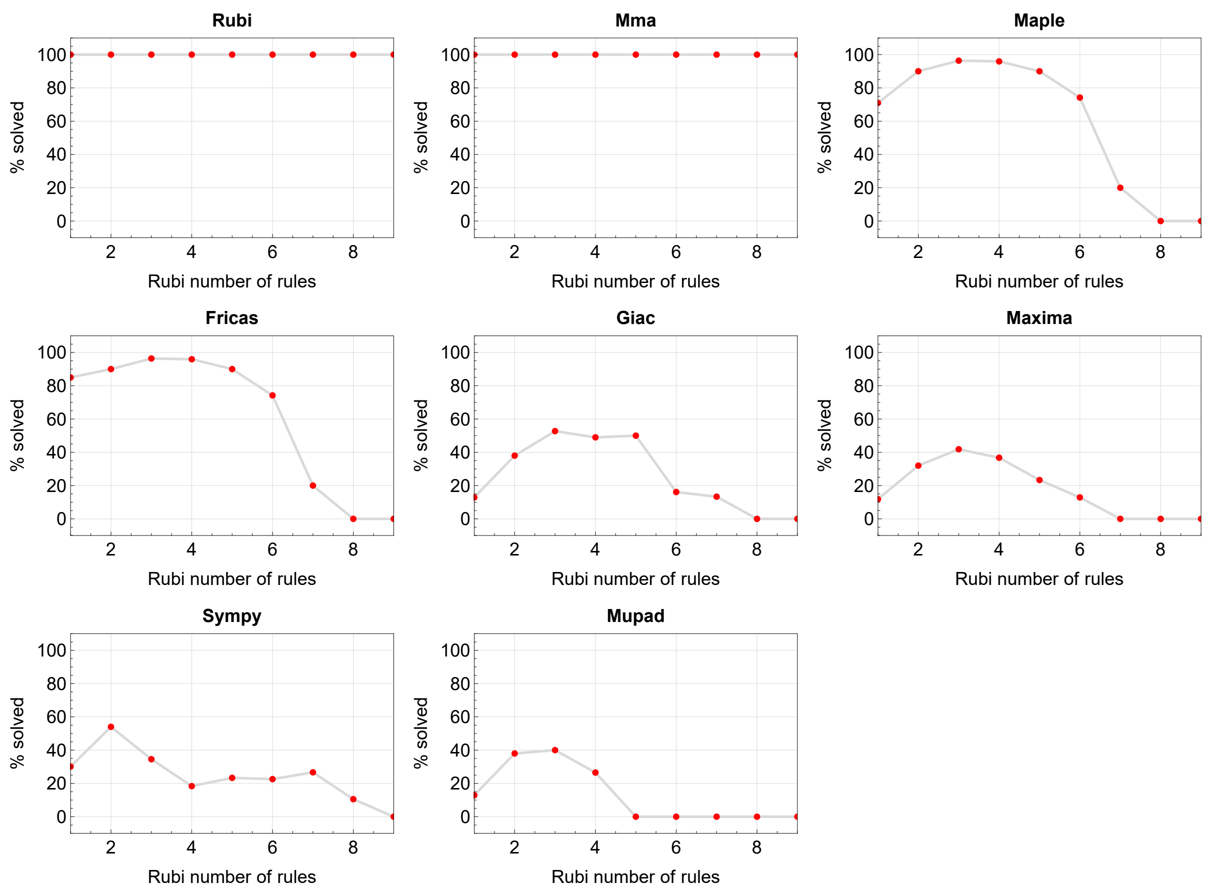


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

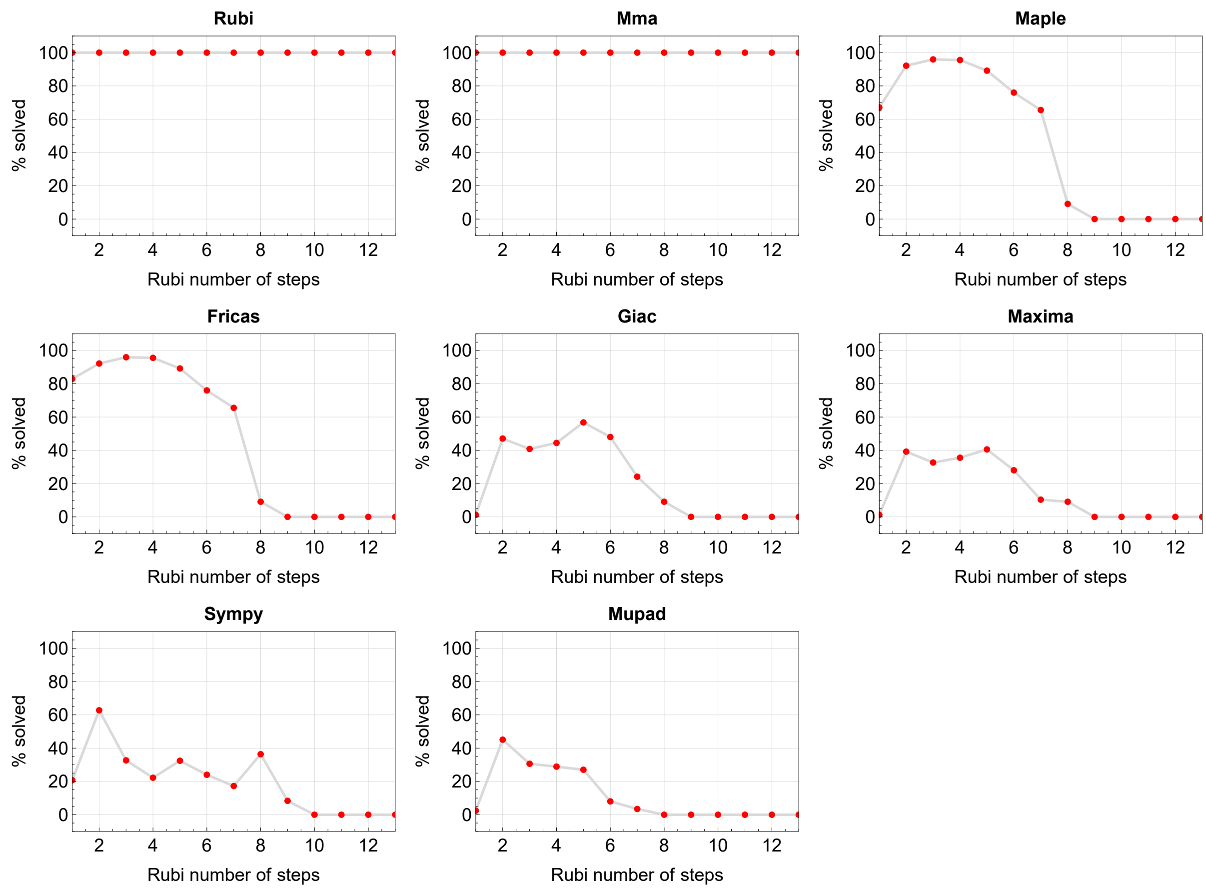


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

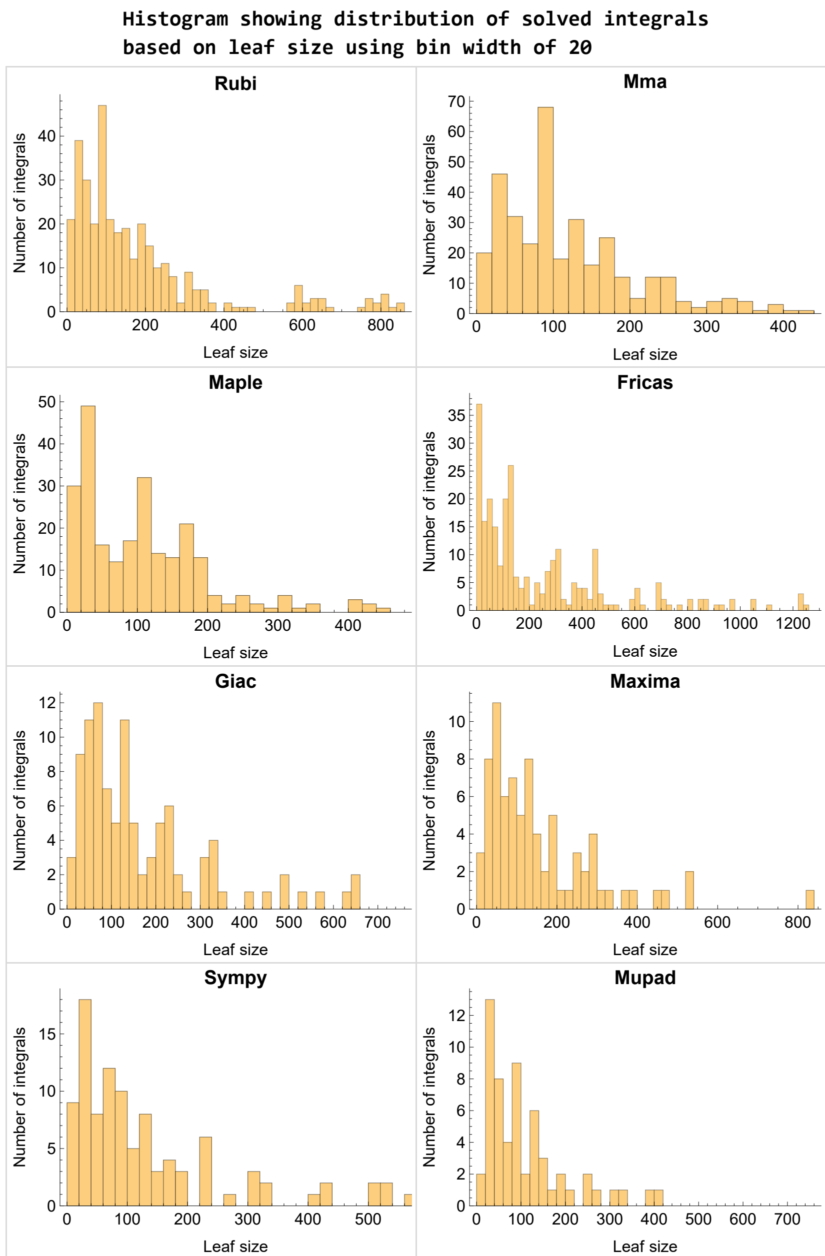


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

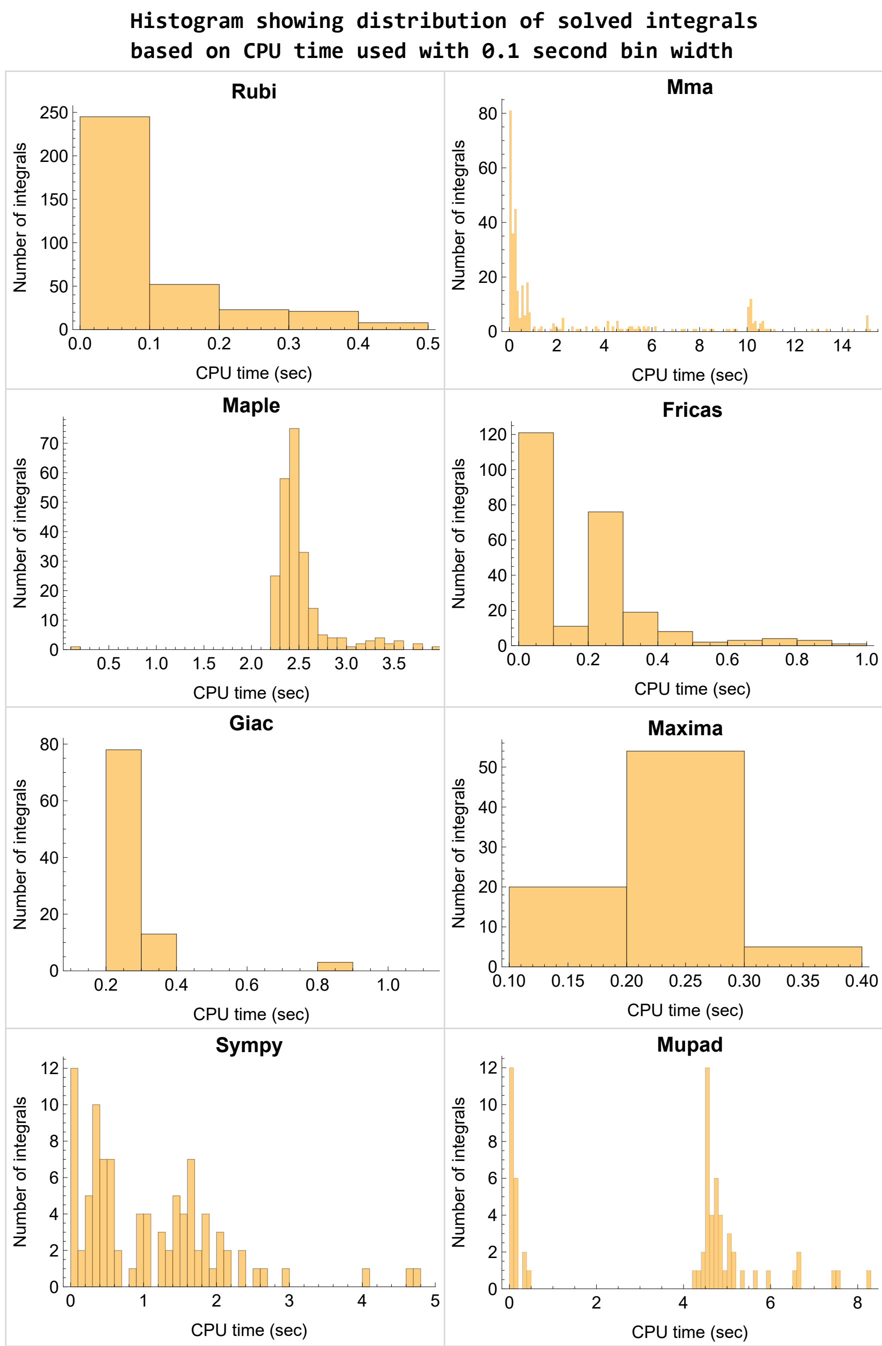


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

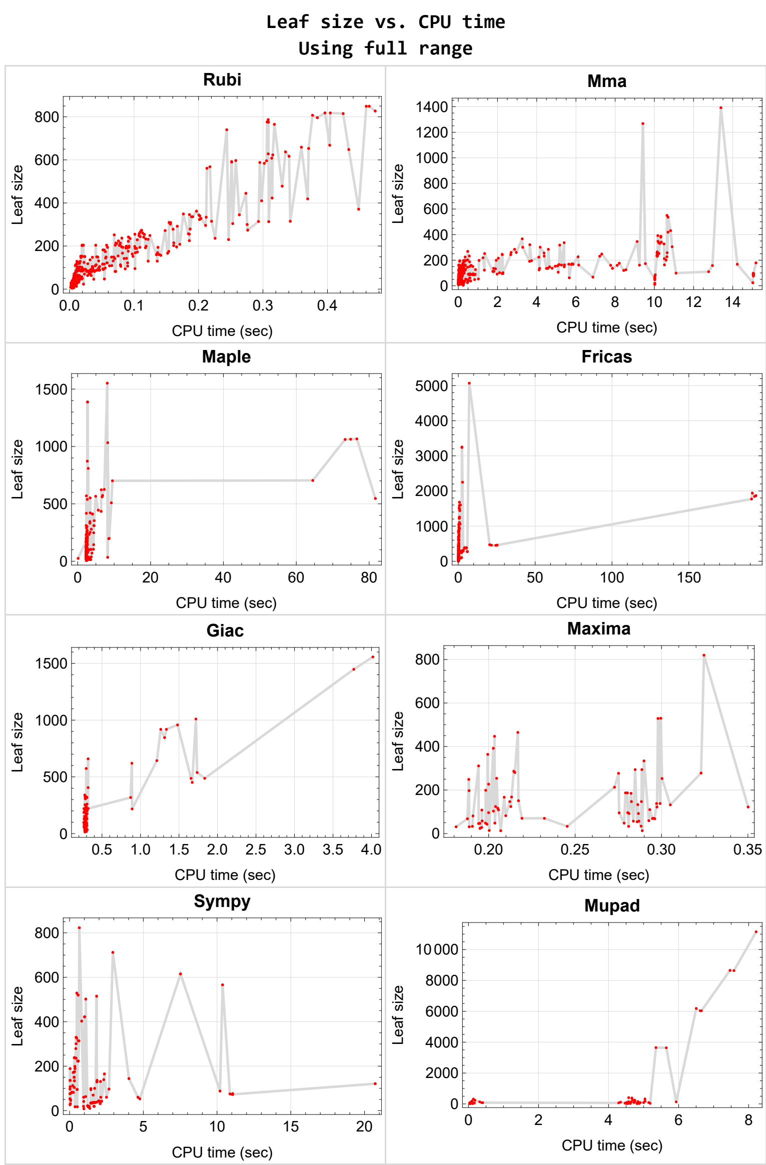


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {59, 69, 110, 112, 114, 115, 119, 120, 121, 124, 126, 127, 128, 130, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 301, 303, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 346, 347, 348}

Maple {148, 156, 157, 158, 159, 305}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
```

```
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

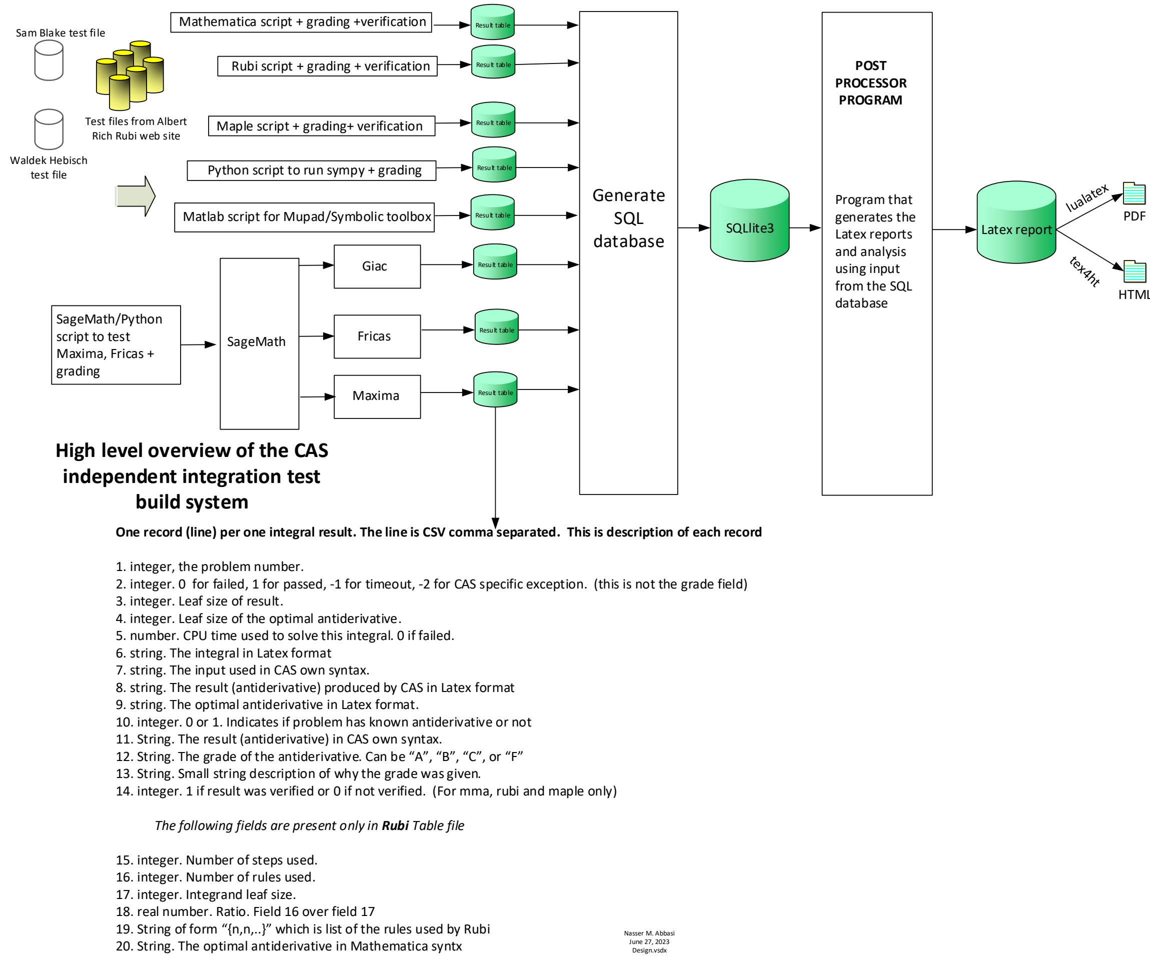
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

B grade { }

C grade { 301 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 137, 148, 149, 163, 164, 165, 168, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 198, 202, 203, 204, 211, 214, 215, 216, 217, 218, 220, 221, 223, 225, 226, 227, 231, 234, 235, 236, 237, 238, 239, 240, 242, 244, 245, 246, 247, 251, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 294, 295, 296, 297, 298, 299, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 326, 342, 343, 344, 345, 349 }

B grade { 51, 224, 243, 293, 341, 346, 347, 348 }

C grade { 88, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 205, 206, 207, 208, 209, 210, 212, 213, 219, 222, 228, 229, 230, 232, 233, 241, 248, 249, 250, 252, 253, 254, 292, 301, 321, 322, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 137, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 277, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 349 }

B grade { 72, 73, 179, 259, 260, 263, 264, 275, 276, 278, 281, 297, 298, 299, 300, 302, 303 }

C grade { 146, 148, 149, 156, 157, 158, 159, 160, 162, 180, 197, 232, 241, 251, 252, 294, 304, 305, 312, 313, 320 }

F normal fail { 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 147, 150, 151, 152, 153, 154, 155, 161, 301, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 30, 31, 32, 39, 43, 44, 45, 46, 47, 48, 49, 53, 54, 55, 56, 57, 58, 62, 63, 64, 65, 66, 67, 71, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 97, 98, 99, 100, 105, 106, 137, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 178, 179, 181, 182, 183, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 295, 299, 300, 303, 349 }

B grade { 12, 13, 18, 19, 26, 27, 28, 29, 33, 34, 35, 36, 37, 38, 40, 41, 42, 50, 51, 52, 59, 60, 61, 68, 69, 70, 72, 73, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 107, 108, 147, 148, 149, 156, 157, 158, 159, 162, 171, 177, 180, 184, 185, 186, 187, 188, 189, 190, 191, 206, 213, 224, 231, 251, 294, 297, 298, 302, 312, 314, 315, 320 }

C grade { 146, 160, 197, 232, 252, 292, 293, 296, 304, 305, 306, 307, 308, 309, 310, 311, 313, 316, 317, 318, 319 }

F normal fail { 109, 110, 111, 113, 116, 117, 118, 122, 123, 124, 125, 129, 130, 131, 135, 136, 138, 301, 341, 342, 343, 344, 345, 346, 347, 348 }

F(-1) timedout fail { 112, 114, 115, 119, 120, 121, 126, 127, 128, 132, 133, 134, 139, 140, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 155, 161, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 98, 99, 100, 108, 137, 163 }

B grade { 26, 33, 34, 41, 42, 72, 73, 97 }

C grade { }

F normal fail { 49, 50, 51, 52, 57, 58, 59, 60, 61, 66, 67, 68, 69, 70, 71, 78, 79, 80, 86, 87, 88, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 62, 63, 64, 65, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 105, 107, 108, 180 }

B grade { 50, 51, 52, 58, 59, 60, 61, 67, 68, 69, 70, 71, 72, 73, 79, 80, 87, 88, 94, 95, 96, 102, 103, 104, 106 }

C grade { }

F normal fail { 49, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 66, 297, 298, 299, 300 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 56, 65, 71, 72, 73, 76, 77, 84, 85, 92, 93, 97, 98, 99, 100, 105, 106, 108, 137, 345, 349 }

C grade { }

F normal fail { }

F(-1) timedout fail { 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 74, 75, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 94, 95, 96, 101, 102, 103, 104, 107, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 39, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 65, 74, 75, 76, 77, 84, 85, 109, 110, 111, 116, 117, 118, 122, 123, 124, 125, 131, 138, 180, 184, 185, 186, 188, 189, 190, 218, 220, 222, 226, 231, 235, 236, 237, 246, 254, 256, 291 }

B grade { 5, 6, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 35, 36, 37, 38, 62, 63, 64, 92, 93, 99, 100, 108, 187, 221, 224, 225, 227, 234 }

C grade { 241, 245, 247, 342, 343, 344, 345 }

F normal fail { 49, 50, 51, 57, 58, 59, 66, 67, 68, 70, 71, 72, 73, 78, 79, 81, 82, 83, 86, 87, 89, 90, 91, 94, 95, 97, 98, 101, 102, 103, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 219, 223, 228, 229, 230, 232, 233, 238, 239, 240, 242, 243, 244, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340 }

F(-1) timedout fail { 25, 26, 32, 33, 34, 40, 41, 42, 52, 60, 61, 69, 80, 88, 96, 104, 341, 346, 347, 348, 349 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	95	96	96	107	98	88
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.04	0.94
time (sec)	N/A	0.052	0.024	2.974	0.200	0.242	0.035	0.262	0.053

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	71	70	70	76	73	65
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.031	0.015	2.292	0.219	0.234	0.032	0.305	4.288

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.020	0.012	2.297	0.204	0.232	0.027	0.275	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.89
time (sec)	N/A	0.011	0.008	0.113	0.195	0.245	0.029	0.287	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	34	99	82	34	31
N.S.	1	1.00	1.00	0.85	0.85	2.48	2.05	0.85	0.78
time (sec)	N/A	0.014	0.023	8.221	0.288	0.260	0.150	0.277	0.061

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	182	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.89	1.78	0.90	0.81
time (sec)	N/A	0.015	0.040	2.273	0.287	0.255	0.214	0.266	4.473

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	82	77	92	300	150	78	82
N.S.	1	1.00	0.89	0.84	1.00	3.26	1.63	0.85	0.89
time (sec)	N/A	0.025	0.044	2.291	0.287	0.261	0.311	0.269	4.578

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	122	124	124	136	131	116
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.058	0.020	2.283	0.204	0.241	0.035	0.275	4.342

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	86	82	82	97	91	75
N.S.	1	1.00	1.00	1.05	1.00	1.00	1.18	1.11	0.91
time (sec)	N/A	0.035	0.015	2.265	0.210	0.234	0.028	0.278	0.046

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	48	53	50	48
N.S.	1	1.00	1.00	0.98	0.96	0.96	1.06	1.00	0.96
time (sec)	N/A	0.020	0.007	2.270	0.198	0.244	0.024	0.289	0.047

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	68	179	172	72	90
N.S.	1	1.00	0.94	1.02	1.08	2.84	2.73	1.14	1.43
time (sec)	N/A	0.033	0.035	2.275	0.296	0.266	0.245	0.289	4.513

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	89	92	96	302	236	95	124
N.S.	1	1.00	1.09	1.12	1.17	3.68	2.88	1.16	1.51
time (sec)	N/A	0.072	0.048	2.277	0.280	0.259	0.404	0.286	4.573

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	106	124	138	449	223	126	130
N.S.	1	1.00	0.91	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.054	0.060	2.287	0.299	0.275	0.594	0.290	4.576

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	161	171	167	167	189	187	152
N.S.	1	1.00	1.05	1.11	1.08	1.08	1.23	1.21	0.99
time (sec)	N/A	0.075	0.019	2.257	0.209	0.260	0.034	0.265	4.841

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	122	124	124	136	131	116
N.S.	1	1.00	1.00	1.00	1.02	1.02	1.11	1.07	0.95
time (sec)	N/A	0.058	0.017	2.284	0.213	0.239	0.034	0.274	4.735

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	71	70	70	76	73	65
N.S.	1	1.00	1.00	1.01	1.00	1.00	1.09	1.04	0.93
time (sec)	N/A	0.032	0.011	2.270	0.295	0.242	0.032	0.276	0.033

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	93	116	122	290	238	130	145
N.S.	1	1.00	0.95	1.18	1.24	2.96	2.43	1.33	1.48
time (sec)	N/A	0.055	0.045	2.304	0.350	0.270	0.319	0.268	4.747

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	107	138	147	444	314	152	181
N.S.	1	1.00	1.00	1.29	1.37	4.15	2.93	1.42	1.69
time (sec)	N/A	0.090	0.047	2.332	0.283	0.250	0.637	0.285	0.102

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	141	167	187	618	422	180	240
N.S.	1	1.00	1.08	1.28	1.44	4.75	3.25	1.38	1.85
time (sec)	N/A	0.122	0.053	2.293	0.280	0.256	1.027	0.291	0.185

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	136	196	187	428	326	198	216
N.S.	1	1.00	0.96	1.38	1.32	3.01	2.30	1.39	1.52
time (sec)	N/A	0.070	0.061	2.326	0.279	0.261	0.482	0.276	4.749

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	92	116	122	292	238	129	146
N.S.	1	1.00	0.94	1.18	1.24	2.98	2.43	1.32	1.49
time (sec)	N/A	0.045	0.045	2.261	0.297	0.260	0.328	0.277	0.083

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	59	64	69	181	172	72	90
N.S.	1	1.00	0.94	1.02	1.10	2.87	2.73	1.14	1.43
time (sec)	N/A	0.028	0.035	2.275	0.295	0.252	0.268	0.270	4.723

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	34	33	98	82	33	32
N.S.	1	1.00	1.03	0.87	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.011	0.019	2.272	0.281	0.262	0.164	0.309	0.063

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	55	54	292	712	54	135
N.S.	1	1.00	0.87	0.79	0.77	4.17	10.17	0.77	1.93
time (sec)	N/A	0.023	0.030	2.335	0.286	0.266	2.933	0.279	0.321

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	95	93	133	711	0	122	3637
N.S.	1	1.00	0.87	0.85	1.22	6.52	0.00	1.12	33.37
time (sec)	N/A	0.066	0.116	2.377	0.286	0.352	0.000	0.279	5.649

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	151	158	277	1585	0	217	6033
N.S.	1	1.00	0.94	0.99	1.73	9.91	0.00	1.36	37.71
time (sec)	N/A	0.135	0.207	2.427	0.275	0.746	0.000	0.297	6.652

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	192	290	294	810	502	306	386
N.S.	1	1.00	1.00	1.51	1.53	4.22	2.61	1.59	2.01
time (sec)	N/A	0.118	0.065	2.406	0.285	0.291	1.097	0.285	4.666

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	142	206	213	612	403	220	261
N.S.	1	1.00	1.00	1.45	1.50	4.31	2.84	1.55	1.84
time (sec)	N/A	0.085	0.060	2.331	0.273	0.259	0.818	0.265	4.683

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	106	139	147	442	314	152	182
N.S.	1	1.00	1.00	1.31	1.39	4.17	2.96	1.43	1.72
time (sec)	N/A	0.079	0.042	2.318	0.288	0.279	0.583	0.273	0.103

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	88	94	95	297	236	94	124
N.S.	1	1.00	1.07	1.15	1.16	3.62	2.88	1.15	1.51
time (sec)	N/A	0.071	0.043	2.307	0.275	0.280	0.440	0.286	4.645

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	57	181	112	57	51
N.S.	1	1.00	1.00	0.90	0.90	2.87	1.78	0.90	0.81
time (sec)	N/A	0.015	0.032	2.284	0.284	0.259	0.214	0.277	4.592

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	109	95	132	699	0	121	3649
N.S.	1	1.00	1.01	0.88	1.22	6.47	0.00	1.12	33.79
time (sec)	N/A	0.062	0.088	2.357	0.305	0.359	0.000	0.279	5.352

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	136	133	294	1681	0	232	6183
N.S.	1	1.00	0.81	0.80	1.76	10.07	0.00	1.39	37.02
time (sec)	N/A	0.152	0.207	2.419	0.289	0.723	0.000	0.296	6.504

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	197	198	529	3239	0	332	8649
N.S.	1	1.00	0.86	0.86	2.30	14.08	0.00	1.44	37.60
time (sec)	N/A	0.247	0.311	2.540	0.298	2.340	0.000	0.281	7.467

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	196	311	334	1044	615	340	409
N.S.	1	1.00	1.00	1.59	1.70	5.33	3.14	1.73	2.09
time (sec)	N/A	0.166	0.078	2.325	0.290	0.262	7.521	0.276	4.575

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	160	231	253	817	515	254	318
N.S.	1	1.00	1.00	1.44	1.58	5.11	3.22	1.59	1.99
time (sec)	N/A	0.139	0.064	2.323	0.300	0.257	1.833	0.278	0.141

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	139	170	185	606	422	178	240
N.S.	1	1.00	1.07	1.31	1.42	4.66	3.25	1.37	1.85
time (sec)	N/A	0.136	0.060	2.322	0.282	0.282	1.013	0.289	4.750

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	106	124	138	449	223	126	130
N.S.	1	1.00	0.91	1.07	1.19	3.87	1.92	1.09	1.12
time (sec)	N/A	0.051	0.058	2.307	0.297	0.257	0.545	0.275	4.733

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	76	92	301	150	78	81
N.S.	1	1.00	0.91	0.83	1.00	3.27	1.63	0.85	0.88
time (sec)	N/A	0.022	0.046	2.293	0.281	0.252	0.313	0.287	4.524

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	139	158	278	1587	0	218	6033
N.S.	1	1.00	0.86	0.98	1.73	9.86	0.00	1.35	37.47
time (sec)	N/A	0.136	0.234	2.443	0.323	0.802	0.000	0.279	6.618

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	197	196	530	3251	0	333	8635
N.S.	1	1.00	0.83	0.83	2.25	13.78	0.00	1.41	36.59
time (sec)	N/A	0.226	0.260	8.528	0.300	2.414	0.000	0.281	7.581

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	233	257	820	5070	0	574	11150
N.S.	1	1.00	0.74	0.82	2.60	16.10	0.00	1.82	35.40
time (sec)	N/A	0.342	0.560	2.534	0.325	7.150	0.000	0.296	8.214

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	23	33	33	31	20	31
N.S.	1	1.00	0.71	0.68	0.97	0.97	0.91	0.59	0.91
time (sec)	N/A	0.007	0.009	2.304	0.191	0.244	0.057	0.280	4.622

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	41	33	48	67	46	54	47
N.S.	1	1.00	0.87	0.70	1.02	1.43	0.98	1.15	1.00
time (sec)	N/A	0.018	0.011	2.348	0.278	0.249	0.082	0.296	0.046

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	180	159	281	398	301	201	0
N.S.	1	1.00	0.78	0.69	1.22	1.72	1.30	0.87	0.00
time (sec)	N/A	0.122	0.252	2.447	0.215	0.319	0.444	0.308	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	122	110	168	264	189	129	0
N.S.	1	1.00	0.82	0.74	1.13	1.77	1.27	0.87	0.00
time (sec)	N/A	0.068	0.157	2.360	0.213	0.290	0.369	0.305	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	74	62	81	158	104	70	0
N.S.	1	1.00	0.85	0.71	0.93	1.82	1.20	0.80	0.00
time (sec)	N/A	0.020	0.085	2.324	0.191	0.264	0.314	0.285	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.76
time (sec)	N/A	0.007	0.009	2.276	0.196	0.289	0.958	0.279	4.492

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	99	112	0	596	0	0	0
N.S.	1	1.00	1.21	1.37	0.00	7.27	0.00	0.00	0.00
time (sec)	N/A	0.039	0.230	2.517	0.000	0.292	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	99	69	0	369	0	217	0
N.S.	1	1.00	1.21	0.84	0.00	4.50	0.00	2.65	0.00
time (sec)	N/A	0.026	0.301	2.380	0.000	0.322	0.000	0.891	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	1268	120	0	698	0	487	0
N.S.	1	1.00	8.51	0.81	0.00	4.68	0.00	3.27	0.00
time (sec)	N/A	0.065	9.417	2.429	0.000	0.355	0.000	1.834	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	227	199	0	1220	0	958	0
N.S.	1	1.00	1.09	0.96	0.00	5.87	0.00	4.61	0.00
time (sec)	N/A	0.171	10.643	8.636	0.000	0.639	0.000	1.481	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	225	208	364	502	529	260	0
N.S.	1	1.00	0.83	0.76	1.34	1.85	1.94	0.96	0.00
time (sec)	N/A	0.148	0.348	2.485	0.199	0.422	0.489	0.306	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	158	142	227	344	330	175	0
N.S.	1	1.00	0.81	0.72	1.16	1.76	1.68	0.89	0.00
time (sec)	N/A	0.081	0.226	2.399	0.200	0.298	0.441	0.307	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	87	116	210	175	103	0
N.S.	1	1.00	0.84	0.74	0.98	1.78	1.48	0.87	0.00
time (sec)	N/A	0.028	0.137	2.351	0.205	0.283	0.340	0.282	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	60	48	43	124	70	49	37
N.S.	1	1.00	0.92	0.74	0.66	1.91	1.08	0.75	0.57
time (sec)	N/A	0.011	0.013	2.293	0.199	0.286	1.597	0.278	4.552

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	126	116	0	721	0	0	0
N.S.	1	1.00	1.12	1.03	0.00	6.38	0.00	0.00	0.00
time (sec)	N/A	0.079	0.288	2.446	0.000	0.375	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	145	147	0	907	0	317	0
N.S.	1	1.00	1.11	1.12	0.00	6.92	0.00	2.42	0.00
time (sec)	N/A	0.068	0.474	2.428	0.000	0.355	0.000	0.304	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	163	94	0	526	0	451	0
N.S.	1	1.00	1.44	0.83	0.00	4.65	0.00	3.99	0.00
time (sec)	N/A	0.041	10.514	2.496	0.000	0.327	0.000	1.672	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	212	179	0	972	0	919	0
N.S.	1	1.00	1.07	0.90	0.00	4.88	0.00	4.62	0.00
time (sec)	N/A	0.088	10.509	2.585	0.000	0.440	0.000	1.336	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	305	270	0	1604	0	1557	0
N.S.	1	1.00	1.02	0.90	0.00	5.35	0.00	5.19	0.00
time (sec)	N/A	0.275	10.908	2.600	0.000	1.374	0.000	4.018	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	269	247	447	608	823	321	0
N.S.	1	1.00	0.77	0.71	1.28	1.74	2.36	0.92	0.00
time (sec)	N/A	0.177	0.481	2.538	0.203	0.520	0.660	0.303	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	190	172	286	420	520	221	0
N.S.	1	1.00	0.79	0.71	1.19	1.74	2.16	0.92	0.00
time (sec)	N/A	0.115	0.312	2.443	0.215	0.363	0.579	0.321	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	123	108	151	260	279	135	0
N.S.	1	1.00	0.83	0.72	1.01	1.74	1.87	0.91	0.00
time (sec)	N/A	0.044	0.189	2.388	0.217	0.320	0.423	0.293	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	71	59	58	146	97	63	37
N.S.	1	1.00	0.85	0.70	0.69	1.74	1.15	0.75	0.44
time (sec)	N/A	0.020	0.023	2.322	0.196	0.263	2.672	0.288	4.525

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	157	136	0	935	0	0	0
N.S.	1	1.00	1.00	0.87	0.00	5.96	0.00	0.00	0.00
time (sec)	N/A	0.142	0.348	2.454	0.000	0.843	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	187	180	0	1236	0	405	0
N.S.	1	1.00	1.07	1.03	0.00	7.06	0.00	2.31	0.00
time (sec)	N/A	0.154	0.575	2.586	0.000	0.659	0.000	0.320	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	252	194	0	1517	0	659	0
N.S.	1	1.00	1.30	1.00	0.00	7.82	0.00	3.40	0.00
time (sec)	N/A	0.137	1.340	2.649	0.000	0.498	0.000	0.319	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	201	144	0	706	0	846	0
N.S.	1	1.00	1.40	1.00	0.00	4.90	0.00	5.88	0.00
time (sec)	N/A	0.056	10.597	2.674	0.000	0.371	0.000	1.313	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	258	234	0	1258	0	1448	0
N.S.	1	1.00	1.04	0.94	0.00	5.05	0.00	5.82	0.00
time (sec)	N/A	0.108	10.655	3.119	0.000	0.786	0.000	3.770	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	33	0	42	0	95	83
N.S.	1	1.00	1.53	1.10	0.00	1.40	0.00	3.17	2.77
time (sec)	N/A	0.010	0.079	2.615	0.000	0.246	0.000	0.277	0.407

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	57	59	67	0	70	59
N.S.	1	1.00	1.81	2.11	2.19	2.48	0.00	2.59	2.19
time (sec)	N/A	0.010	0.090	2.431	0.293	0.254	0.000	0.287	0.169

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	37	56	110	74	0	118	85
N.S.	1	1.00	1.48	2.24	4.40	2.96	0.00	4.72	3.40
time (sec)	N/A	0.009	0.046	2.348	0.293	0.243	0.000	0.280	5.001

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	138	118	199	300	199	150	0
N.S.	1	1.00	0.82	0.70	1.18	1.78	1.18	0.89	0.00
time (sec)	N/A	0.102	0.164	2.418	0.198	0.285	0.358	0.292	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	90	78	109	192	134	90	0
N.S.	1	1.00	0.83	0.72	1.01	1.78	1.24	0.83	0.00
time (sec)	N/A	0.040	0.094	2.351	0.206	0.280	0.329	0.287	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	65	47	47	113	82	49	86
N.S.	1	1.00	1.12	0.81	0.81	1.95	1.41	0.84	1.48
time (sec)	N/A	0.012	0.155	2.292	0.195	0.258	0.255	0.285	5.149

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	0.80
time (sec)	N/A	0.004	0.002	2.279	0.207	0.265	0.522	0.282	0.124

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	67	42	0	241	0	70	0
N.S.	1	1.00	1.37	0.86	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.014	0.103	2.329	0.000	0.298	0.000	0.278	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	118	87	0	463	0	242	0
N.S.	1	1.00	1.17	0.86	0.00	4.58	0.00	2.40	0.00
time (sec)	N/A	0.036	0.267	2.388	0.000	0.364	0.000	0.294	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	160	149	0	864	0	538	0
N.S.	1	1.00	0.98	0.91	0.00	5.30	0.00	3.30	0.00
time (sec)	N/A	0.097	0.656	2.557	0.000	0.487	0.000	1.736	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	257	257	198	192	311	584	0	235	0
N.S.	1	1.00	0.77	0.75	1.21	2.27	0.00	0.91	0.00
time (sec)	N/A	0.184	0.395	2.476	0.194	0.352	0.000	0.296	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	139	137	197	416	0	157	0
N.S.	1	1.00	0.82	0.81	1.17	2.46	0.00	0.93	0.00
time (sec)	N/A	0.137	0.250	2.429	0.189	0.287	0.000	0.291	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	105	95	93	108	276	0	92	0
N.S.	1	1.17	1.06	1.03	1.20	3.07	0.00	1.02	0.00
time (sec)	N/A	0.047	0.164	2.394	0.196	0.267	0.000	0.302	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	58	55	46	167	60	50	53
N.S.	1	1.00	1.07	1.02	0.85	3.09	1.11	0.93	0.98
time (sec)	N/A	0.013	0.081	2.306	0.194	0.267	2.117	0.274	4.818

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	0.88
time (sec)	N/A	0.002	0.001	2.286	0.200	0.251	0.370	0.278	0.042

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	96	92	0	441	0	107	0
N.S.	1	1.00	1.22	1.16	0.00	5.58	0.00	1.35	0.00
time (sec)	N/A	0.029	0.231	2.358	0.000	0.324	0.000	0.293	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	151	150	0	864	0	318	0
N.S.	1	1.00	1.06	1.05	0.00	6.04	0.00	2.22	0.00
time (sec)	N/A	0.088	0.590	2.447	0.000	0.491	0.000	0.872	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	1392	214	0	1482	0	643	0
N.S.	1	1.00	6.19	0.95	0.00	6.59	0.00	2.86	0.00
time (sec)	N/A	0.186	13.395	2.661	0.000	0.878	0.000	1.213	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	202	198	392	684	0	237	0
N.S.	1	1.00	0.79	0.78	1.54	2.68	0.00	0.93	0.00
time (sec)	N/A	0.186	0.476	2.547	0.203	0.398	0.000	0.296	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	143	142	254	486	0	158	0
N.S.	1	1.00	0.83	0.83	1.48	2.83	0.00	0.92	0.00
time (sec)	N/A	0.105	0.270	2.479	0.205	0.298	0.000	0.293	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	91	105	147	318	0	103	0
N.S.	1	1.00	0.87	1.00	1.40	3.03	0.00	0.98	0.00
time (sec)	N/A	0.035	0.172	2.363	0.212	0.274	0.000	0.282	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	68	54	144	40	33
N.S.	1	1.00	0.79	0.72	1.45	1.15	3.06	0.85	0.70
time (sec)	N/A	0.007	0.077	2.323	0.188	0.253	4.024	0.284	4.550

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.002	2.293	0.181	0.290	0.502	0.279	4.517

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	130	124	0	764	0	320	0
N.S.	1	1.00	1.07	1.02	0.00	6.26	0.00	2.62	0.00
time (sec)	N/A	0.090	0.362	2.398	0.000	0.483	0.000	0.283	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	219	208	0	1440	0	620	0
N.S.	1	1.00	1.08	1.03	0.00	7.13	0.00	3.07	0.00
time (sec)	N/A	0.161	1.262	2.628	0.000	0.952	0.000	0.888	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	367	302	0	2250	0	1010	0
N.S.	1	1.00	1.17	0.96	0.00	7.19	0.00	3.23	0.00
time (sec)	N/A	0.309	3.262	2.567	0.000	2.764	0.000	1.720	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	163	150	465	229	0	218	326
N.S.	1	1.00	0.73	0.67	2.08	1.02	0.00	0.97	1.46
time (sec)	N/A	0.076	0.354	2.415	0.217	0.416	0.000	0.299	5.039

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	107	96	249	151	0	138	176
N.S.	1	1.00	0.61	0.55	1.43	0.87	0.00	0.79	1.01
time (sec)	N/A	0.056	0.194	2.382	0.188	0.292	0.000	0.300	5.040

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	52	103	87	566	72	87
N.S.	1	1.00	0.66	0.57	1.13	0.96	6.22	0.79	0.96
time (sec)	N/A	0.019	0.115	2.418	0.203	0.278	10.373	0.288	4.904

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	0.72
time (sec)	N/A	0.004	0.053	2.312	0.189	0.245	0.518	0.285	4.547

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	99	92	0	442	0	107	0
N.S.	1	1.00	1.25	1.16	0.00	5.59	0.00	1.35	0.00
time (sec)	N/A	0.032	0.239	2.389	0.000	0.324	0.000	0.281	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	122	88	0	459	0	225	0
N.S.	1	1.00	1.22	0.88	0.00	4.59	0.00	2.25	0.00
time (sec)	N/A	0.036	0.287	2.395	0.000	0.350	0.000	0.283	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	151	121	0	698	0	487	0
N.S.	1	1.00	1.01	0.81	0.00	4.68	0.00	3.27	0.00
time (sec)	N/A	0.057	0.742	2.527	0.000	0.380	0.000	1.657	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	179	179	0	972	0	919	0
N.S.	1	1.00	0.90	0.90	0.00	4.88	0.00	4.62	0.00
time (sec)	N/A	0.092	15.182	2.482	0.000	0.483	0.000	1.263	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	27	0	18	18
N.S.	1	1.00	1.00	0.95	0.00	1.35	0.00	0.90	0.90
time (sec)	N/A	0.006	0.034	2.306	0.000	0.265	0.000	0.297	4.812

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	0	23	0	51	79
N.S.	1	1.00	1.00	0.96	0.00	0.92	0.00	2.04	3.16
time (sec)	N/A	0.005	0.054	2.347	0.000	0.267	0.000	0.283	0.380

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	41	0	241	0	70	0
N.S.	1	1.00	1.43	0.84	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.016	0.099	2.351	0.000	0.307	0.000	0.276	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	14	13	44	31	25	27
N.S.	1	1.00	1.93	0.93	0.87	2.93	2.07	1.67	1.80
time (sec)	N/A	0.003	0.041	2.381	0.289	0.271	2.076	0.276	0.041

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	648	648	99	0	0	0	136	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.433	11.106	0.000	0.000	0.000	1.863	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	617	617	176	0	0	0	99	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.340	8.213	0.000	0.000	0.000	1.441	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	588	588	62	0	0	0	63	0	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.295	5.662	0.000	0.000	0.000	1.089	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	740	740	162	0	0	0	0	0	0
N.S.	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	6.139	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	584	584	86	0	0	0	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	10.049	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	818	818	252	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.404	10.239	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	849	849	265	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.465	10.145	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	668	668	110	0	0	0	139	0	0
N.S.	1	1.00	0.16	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.404	12.767	0.000	0.000	0.000	2.317	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	637	637	173	0	0	0	131	0	0
N.S.	1	1.00	0.27	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.335	9.542	0.000	0.000	0.000	2.097	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	608	608	68	0	0	0	100	0	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.314	6.866	0.000	0.000	0.000	1.699	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	765	765	231	0	0	0	0	0	0
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.318	7.215	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	775	775	235	0	0	0	0	0	0
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	10.159	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	815	815	252	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.424	10.168	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	659	659	98	0	0	0	165	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.25	0.00	0.00
time (sec)	N/A	0.360	15.045	0.000	0.000	0.000	2.360	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	628	628	88	0	0	0	129	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.21	0.00	0.00
time (sec)	N/A	0.308	15.038	0.000	0.000	0.000	1.856	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	597	597	158	0	0	0	94	0	0
N.S.	1	1.00	0.26	0.00	0.00	0.00	0.16	0.00	0.00
time (sec)	N/A	0.258	12.964	0.000	0.000	0.000	1.446	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	568	568	62	0	0	0	60	0	0
N.S.	1	1.00	0.11	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.218	10.021	0.000	0.000	0.000	0.952	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	5.261	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	787	787	234	0	0	0	0	0	0
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.308	10.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	818	818	255	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.396	10.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	623	623	76	0	0	0	0	0	0
N.S.	1	1.00	0.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.315	15.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	592	592	168	0	0	0	0	0	0
N.S.	1	1.00	0.28	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	14.230	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	561	561	53	0	0	0	60	0	0
N.S.	1	1.00	0.09	0.00	0.00	0.00	0.11	0.00	0.00
time (sec)	N/A	0.213	10.018	0.000	0.000	0.000	2.503	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	776	776	226	0	0	0	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	6.107	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	807	807	236	0	0	0	0	0	0
N.S.	1	1.00	0.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.377	10.143	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	849	849	256	0	0	0	0	0	0
N.S.	1	1.00	0.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.460	10.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	653	653	96	0	0	0	0	0	0
N.S.	1	1.00	0.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.371	15.063	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	596	83	0	0	0	0	0	0
N.S.	1	1.00	0.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.306	15.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	33	42	0	0	27
N.S.	1	1.00	0.55	0.55	0.75	0.95	0.00	0.00	0.61
time (sec)	N/A	0.015	15.021	2.400	0.245	0.263	0.000	0.000	4.878

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	590	590	74	0	0	0	60	0	0
N.S.	1	1.00	0.13	0.00	0.00	0.00	0.10	0.00	0.00
time (sec)	N/A	0.252	10.027	0.000	0.000	0.000	4.637	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	796	796	248	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	7.317	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	827	827	259	0	0	0	0	0	0
N.S.	1	1.00	0.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.474	10.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	252	252	163	0	0	0	0	0	0
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.069	5.409	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	202	202	166	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.020	5.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	153	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.041	4.944	0.000	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	162	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	0.027	0.000	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	204	204	156	0	0	0	0	0	0
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.021	5.065	0.000	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	118	704	0	1232	0	0	0
N.S.	1	1.00	1.04	6.23	0.00	10.90	0.00	0.00	0.00
time (sec)	N/A	0.009	0.031	64.534	0.000	0.689	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	124	0	0	1103	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	10.12	0.00	0.00	0.00
time (sec)	N/A	0.009	0.022	0.000	0.000	0.701	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	96	155	1033	0	285	0	0	0
N.S.	1	1.00	1.61	10.76	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.014	0.242	8.234	0.000	3.208	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	151	1552	0	315	0	0	0
N.S.	1	1.00	1.59	16.34	0.00	3.32	0.00	0.00	0.00
time (sec)	N/A	0.011	0.182	8.109	0.000	3.181	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	169	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	5.641	0.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	167	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	5.771	0.000	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	168	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.024	5.857	0.000	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	153	153	172	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	5.803	0.000	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	148	0	0	0	0	0	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	5.404	0.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	147	147	148	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	4.655	0.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	123	123	136	1066	0	1843	0	0	0
N.S.	1	1.00	1.11	8.67	0.00	14.98	0.00	0.00	0.00
time (sec)	N/A	0.013	4.505	76.625	0.000	192.780	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	123	123	136	547	0	1867	0	0	0
N.S.	1	1.00	1.11	4.45	0.00	15.18	0.00	0.00	0.00
time (sec)	N/A	0.012	4.148	81.705	0.000	193.663	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	119	119	136	1063	0	1771	0	0	0
N.S.	1	1.00	1.14	8.93	0.00	14.88	0.00	0.00	0.00
time (sec)	N/A	0.012	4.501	74.917	0.000	190.797	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	119	119	136	1061	0	1939	0	0	0
N.S.	1	1.00	1.14	8.92	0.00	16.29	0.00	0.00	0.00
time (sec)	N/A	0.014	4.572	73.391	0.000	191.298	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	124	623	0	1059	0	0	0
N.S.	1	1.00	1.77	8.90	0.00	15.13	0.00	0.00	0.00
time (sec)	N/A	0.007	4.122	6.474	0.000	1.169	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	104	104	137	0	0	0	0	0	0
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.012	4.786	0.000	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	125	539	0	269	0	0	0
N.S.	1	1.00	1.69	7.28	0.00	3.64	0.00	0.00	0.00
time (sec)	N/A	0.008	8.550	2.627	0.000	0.542	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	91	57	94	70	315	0	0	0
N.S.	1	1.15	0.72	1.19	0.89	3.99	0.00	0.00	0.00
time (sec)	N/A	0.015	0.102	2.436	0.232	0.287	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	91	54	94	0	303	0	0	0
N.S.	1	1.23	0.73	1.27	0.00	4.09	0.00	0.00	0.00
time (sec)	N/A	0.012	0.072	2.362	0.000	0.281	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	91	57	94	0	314	0	0	0
N.S.	1	1.20	0.75	1.24	0.00	4.13	0.00	0.00	0.00
time (sec)	N/A	0.012	0.091	2.355	0.000	0.297	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	243	412	0	232	0	0	0
N.S.	1	1.00	0.74	1.26	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.204	2.246	4.065	0.000	0.097	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	198	279	0	153	0	0	0
N.S.	1	1.00	0.80	1.12	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.125	1.058	3.732	0.000	0.088	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	114	0	0	0
N.S.	1	1.00	0.42	0.50	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.071	0.795	2.365	0.000	0.082	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	133	181	0	122	0	0	0
N.S.	1	1.00	1.58	2.15	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.017	1.897	2.367	0.000	0.092	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	243	418	0	333	0	0	0
N.S.	1	1.00	1.03	1.76	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.081	2.675	2.373	0.000	0.091	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	285	570	0	689	0	0	0
N.S.	1	1.00	0.92	1.84	0.00	2.23	0.00	0.00	0.00
time (sec)	N/A	0.157	2.871	2.394	0.000	0.106	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	302	566	0	316	0	0	0
N.S.	1	1.00	0.74	1.38	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.298	4.175	4.935	0.000	0.096	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	336	336	246	411	0	233	0	0	0
N.S.	1	1.00	0.73	1.22	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.191	2.236	4.045	0.000	0.097	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	199	310	0	166	0	0	0
N.S.	1	1.00	0.73	1.14	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.112	1.935	4.418	0.000	0.101	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	191	332	0	226	0	0	0
N.S.	1	1.00	0.72	1.24	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.110	3.634	3.344	0.000	0.095	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	232	421	0	294	0	0	0
N.S.	1	1.00	1.01	1.84	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.105	4.380	3.422	0.000	0.089	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	285	552	0	635	0	0	0
N.S.	1	1.00	0.90	1.75	0.00	2.02	0.00	0.00	0.00
time (sec)	N/A	0.220	4.594	3.411	0.000	0.100	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	127	252	0	149	0	0	0
N.S.	1	1.00	0.54	1.07	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.094	0.876	3.396	0.000	0.082	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	22	75	0	41	0	0	0
N.S.	1	1.00	0.58	1.97	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.006	0.118	2.753	0.000	0.075	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	38	0	38	17	17	0
N.S.	1	1.00	0.75	1.90	0.00	1.90	0.85	0.85	0.00
time (sec)	N/A	0.003	0.002	2.357	0.000	0.252	1.252	0.290	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	124	0	0	0
N.S.	1	1.00	0.20	0.20	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.052	0.631	2.366	0.000	0.082	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	60	78	0	67	0	0	0
N.S.	1	1.00	0.66	0.86	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.052	0.579	2.401	0.000	0.076	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	60	53	0	111	0	0	0
N.S.	1	1.00	0.40	0.35	0.00	0.74	0.00	0.00	0.00
time (sec)	N/A	0.042	0.588	2.383	0.000	0.082	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	0	36	34	0	0
N.S.	1	1.00	1.00	1.15	0.00	1.80	1.70	0.00	0.00
time (sec)	N/A	0.005	0.261	2.408	0.000	0.079	1.489	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	0	53	36	0	0
N.S.	1	1.00	1.00	0.86	0.00	2.52	1.71	0.00	0.00
time (sec)	N/A	0.005	0.264	2.404	0.000	0.080	1.539	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	29	0	66	34	0	0
N.S.	1	1.00	1.00	1.45	0.00	3.30	1.70	0.00	0.00
time (sec)	N/A	0.005	0.274	2.414	0.000	0.088	1.543	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	0	41	10	0	0
N.S.	1	1.00	1.00	1.25	0.00	10.25	2.50	0.00	0.00
time (sec)	N/A	0.004	0.254	2.382	0.000	0.083	1.370	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	76	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	3.80	1.80	0.00	0.00
time (sec)	N/A	0.005	0.253	2.410	0.000	0.083	1.657	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	19	0	76	37	0	0
N.S.	1	1.00	1.00	0.90	0.00	3.62	1.76	0.00	0.00
time (sec)	N/A	0.005	0.265	2.414	0.000	0.088	1.755	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	78	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	3.90	1.80	0.00	0.00
time (sec)	N/A	0.005	0.267	2.416	0.000	0.077	1.744	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	0	31	0	0	0
N.S.	1	1.00	0.92	1.08	0.00	2.38	0.00	0.00	0.00
time (sec)	N/A	0.011	0.250	2.384	0.000	0.079	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	27	0	36	0	0	0
N.S.	1	1.00	0.87	0.87	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.014	0.248	2.408	0.000	0.081	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	53	0	0	0
N.S.	1	1.00	0.77	0.89	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.015	0.264	2.396	0.000	0.075	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	31	0	53	0	0	0
N.S.	1	1.00	0.77	0.89	0.00	1.51	0.00	0.00	0.00
time (sec)	N/A	0.014	0.270	2.421	0.000	0.078	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	27	30	0	64	0	0	0
N.S.	1	1.00	0.21	0.23	0.00	0.49	0.00	0.00	0.00
time (sec)	N/A	0.045	0.243	2.378	0.000	0.079	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	27	26	0	64	0	0	0
N.S.	1	1.00	0.20	0.19	0.00	0.47	0.00	0.00	0.00
time (sec)	N/A	0.032	0.255	2.388	0.000	0.081	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	27	20	0	54	0	0	0
N.S.	1	1.00	0.18	0.14	0.00	0.36	0.00	0.00	0.00
time (sec)	N/A	0.035	0.250	2.394	0.000	0.088	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	35	32	0	36	0	0	0
N.S.	1	1.00	0.88	0.80	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.010	0.260	2.485	0.000	0.083	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	423	423	321	626	0	364	0	0	0
N.S.	1	1.00	0.76	1.48	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.314	3.650	7.294	0.000	0.091	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	260	435	0	271	0	0	0
N.S.	1	1.00	0.76	1.26	0.00	0.79	0.00	0.00	0.00
time (sec)	N/A	0.201	2.692	6.428	0.000	0.087	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	216	310	0	189	0	0	0
N.S.	1	1.00	0.83	1.19	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.113	2.044	4.509	0.000	0.094	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	130	0	0	0
N.S.	1	1.00	0.44	0.81	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.083	0.743	2.390	0.000	0.086	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	42	0	0	0
N.S.	1	1.00	0.99	1.15	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.022	0.562	3.013	0.000	0.077	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	112	248	0	187	0	0	0
N.S.	1	1.00	0.41	0.91	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.112	1.821	4.389	0.000	0.093	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	261	445	0	441	0	0	0
N.S.	1	1.00	1.02	1.75	0.00	1.73	0.00	0.00	0.00
time (sec)	N/A	0.101	2.957	5.629	0.000	0.097	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	301	573	0	845	0	0	0
N.S.	1	1.00	0.90	1.72	0.00	2.53	0.00	0.00	0.00
time (sec)	N/A	0.212	3.284	6.832	0.000	0.107	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	445	445	318	701	0	472	0	0	0
N.S.	1	1.00	0.71	1.58	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.274	5.175	9.533	0.000	0.103	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	256	509	0	338	0	0	0
N.S.	1	1.00	0.74	1.47	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.186	4.375	9.223	0.000	0.089	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	196	345	0	224	0	0	0
N.S.	1	1.00	0.76	1.34	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.108	3.733	3.345	0.000	0.102	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	136	188	0	133	0	0	0
N.S.	1	1.00	1.62	2.24	0.00	1.58	0.00	0.00	0.00
time (sec)	N/A	0.012	1.801	2.379	0.000	0.093	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	112	144	0	159	0	0	0
N.S.	1	1.00	0.58	0.74	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.109	1.762	4.378	0.000	0.089	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	224	354	0	407	0	0	0
N.S.	1	1.00	0.93	1.46	0.00	1.68	0.00	0.00	0.00
time (sec)	N/A	0.091	4.107	4.518	0.000	0.094	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	323	323	337	561	0	856	0	0	0
N.S.	1	1.00	1.04	1.74	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.201	5.399	6.708	0.000	0.126	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	86	100	0	42	0	0	0
N.S.	1	1.00	0.99	1.15	0.00	0.48	0.00	0.00	0.00
time (sec)	N/A	0.014	0.028	2.943	0.000	0.078	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	103	0	40	0	0	0
N.S.	1	1.00	1.00	1.18	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.038	0.612	3.574	0.000	0.082	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	89	103	0	40	0	0	0
N.S.	1	1.00	1.02	1.18	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.042	0.634	3.596	0.000	0.087	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	104	0	39	0	0	0
N.S.	1	1.00	1.00	1.18	0.00	0.44	0.00	0.00	0.00
time (sec)	N/A	0.046	0.636	4.310	0.000	0.088	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	9	19	0	0
N.S.	1	1.00	1.00	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.005	0.212	3.595	0.000	0.078	1.670	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	58	14	0	9	0	0	0
N.S.	1	1.00	5.80	1.40	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.005	10.024	2.912	0.000	0.089	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	0	9	19	0	0
N.S.	1	1.00	1.00	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.004	0.197	3.356	0.000	0.080	1.649	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	9	73	0	0
N.S.	1	1.00	1.00	1.00	0.00	0.90	7.30	0.00	0.00
time (sec)	N/A	0.004	10.019	2.467	0.000	0.079	11.024	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	0	9	19	0	0
N.S.	1	1.00	1.50	1.17	0.00	0.75	1.58	0.00	0.00
time (sec)	N/A	0.004	10.024	2.811	0.000	0.084	1.287	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	0	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.005	0.188	2.494	0.000	0.075	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	26	8	0	68	19	0	0
N.S.	1	1.00	3.25	1.00	0.00	8.50	2.38	0.00	0.00
time (sec)	N/A	0.002	0.006	2.467	0.000	0.264	0.946	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	34	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	2.83	0.00	0.00
time (sec)	N/A	0.005	0.191	2.528	0.000	0.080	1.430	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	9	39	0	0
N.S.	1	1.00	1.00	1.10	0.00	0.90	3.90	0.00	0.00
time (sec)	N/A	0.007	0.060	2.623	0.000	0.078	1.882	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	9	34	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.75	2.83	0.00	0.00
time (sec)	N/A	0.005	0.195	2.619	0.000	0.073	1.450	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	11	0	0	0
N.S.	1	1.00	0.37	0.33	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.007	0.200	2.474	0.000	0.075	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	17	15	0	11	0	0	0
N.S.	1	1.00	0.35	0.31	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.007	0.070	2.506	0.000	0.089	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	19	17	0	11	0	0	0
N.S.	1	1.00	0.37	0.33	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.007	0.187	2.589	0.000	0.079	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	0	34	8	0	0
N.S.	1	1.00	1.00	1.00	0.00	4.25	1.00	0.00	0.00
time (sec)	N/A	0.002	0.005	2.482	0.000	0.264	0.971	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	19	15	0	11	0	0	0
N.S.	1	1.00	0.40	0.32	0.00	0.23	0.00	0.00	0.00
time (sec)	N/A	0.006	0.183	2.514	0.000	0.094	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	14	0	9	0	0	0
N.S.	1	1.00	1.90	1.40	0.00	0.90	0.00	0.00	0.00
time (sec)	N/A	0.005	10.023	2.682	0.000	0.075	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	10	0	9	76	0	0
N.S.	1	1.00	1.00	1.00	0.00	0.90	7.60	0.00	0.00
time (sec)	N/A	0.004	0.005	2.504	0.000	0.077	11.057	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	17	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	0.85	1.80	0.00	0.00
time (sec)	N/A	0.005	0.188	3.147	0.000	0.087	1.594	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	0	10	41	0	0
N.S.	1	1.00	1.00	0.94	0.00	0.62	2.56	0.00	0.00
time (sec)	N/A	0.005	0.054	2.707	0.000	0.073	2.055	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	0	17	36	0	0
N.S.	1	1.00	1.00	0.95	0.00	0.85	1.80	0.00	0.00
time (sec)	N/A	0.007	0.185	3.285	0.000	0.077	1.605	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	9	0	0	0
N.S.	1	1.00	1.00	1.16	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.011	0.190	2.711	0.000	0.083	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	34	0	9	0	0	0
N.S.	1	1.00	1.00	1.13	0.00	0.30	0.00	0.00	0.00
time (sec)	N/A	0.010	0.068	2.514	0.000	0.081	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	37	0	9	0	0	0
N.S.	1	1.00	1.00	1.16	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.010	0.201	2.605	0.000	0.072	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	46	30	0	9	75	0	0
N.S.	1	1.00	1.84	1.20	0.00	0.36	3.00	0.00	0.00
time (sec)	N/A	0.007	0.060	2.528	0.000	0.080	10.873	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	34	0	9	0	0	0
N.S.	1	1.00	1.00	1.06	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.009	0.180	2.595	0.000	0.074	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	47	28	0	9	0	0	0
N.S.	1	1.00	3.92	2.33	0.00	0.75	0.00	0.00	0.00
time (sec)	N/A	0.006	10.025	2.748	0.000	0.080	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	24	0	34	0	0	0
N.S.	1	1.00	0.93	0.83	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.002	0.033	2.582	0.000	0.274	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	9	37	0	0
N.S.	1	1.00	1.25	0.91	0.00	0.28	1.16	0.00	0.00
time (sec)	N/A	0.013	0.190	3.289	0.000	0.074	1.684	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	27	0	9	42	0	0
N.S.	1	1.00	1.20	0.90	0.00	0.30	1.40	0.00	0.00
time (sec)	N/A	0.010	0.071	2.891	0.000	0.081	2.173	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	29	0	9	37	0	0
N.S.	1	1.00	1.25	0.91	0.00	0.28	1.16	0.00	0.00
time (sec)	N/A	0.010	0.184	3.231	0.000	0.075	1.639	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	11	0	0	0
N.S.	1	1.00	0.74	0.68	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.007	0.202	2.877	0.000	0.079	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	37	33	0	11	0	0	0
N.S.	1	1.00	0.73	0.65	0.00	0.22	0.00	0.00	0.00
time (sec)	N/A	0.009	0.070	2.565	0.000	0.082	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	39	36	0	11	0	0	0
N.S.	1	1.00	0.74	0.68	0.00	0.21	0.00	0.00	0.00
time (sec)	N/A	0.009	0.189	2.932	0.000	0.077	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	9	0	104	0	0	0
N.S.	1	1.00	0.93	0.32	0.00	3.71	0.00	0.00	0.00
time (sec)	N/A	0.003	0.036	2.494	0.000	0.264	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	53	33	0	17	0	0	0
N.S.	1	1.00	1.08	0.67	0.00	0.35	0.00	0.00	0.00
time (sec)	N/A	0.006	1.024	2.462	0.000	0.087	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	39	34	0	17	0	0	0
N.S.	1	1.00	1.26	1.10	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.013	0.199	2.451	0.000	0.077	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	65	48	30	0	9	73	0	0
N.S.	1	1.55	1.14	0.71	0.00	0.21	1.74	0.00	0.00
time (sec)	N/A	0.012	0.059	2.427	0.000	0.071	11.064	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	23	0	0	0
N.S.	1	1.00	1.00	0.85	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.012	0.196	2.489	0.000	0.083	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	34	0	16	44	0	0
N.S.	1	1.00	1.00	0.94	0.00	0.44	1.22	0.00	0.00
time (sec)	N/A	0.010	0.079	2.471	0.000	0.073	1.981	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	0	23	0	0	0
N.S.	1	1.00	1.00	0.85	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.013	0.211	2.467	0.000	0.071	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	130	0	0	0
N.S.	1	1.00	1.00	1.20	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.032	0.738	2.441	0.000	0.087	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	131	0	0	0
N.S.	1	1.00	1.00	1.87	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.034	0.758	2.476	0.000	0.084	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	128	0	0	0
N.S.	1	1.00	1.00	1.90	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.033	0.549	2.546	0.000	0.085	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	133	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.036	0.540	2.558	0.000	0.094	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	105	0	113	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.036	0.789	2.473	0.000	0.084	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	161	0	116	0	0	0
N.S.	1	1.00	1.00	1.81	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.035	0.770	2.492	0.000	0.085	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	162	0	115	0	0	0
N.S.	1	1.00	1.00	1.82	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.047	0.536	2.558	0.000	0.087	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	107	0	114	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.27	0.00	0.00	0.00
time (sec)	N/A	0.044	0.511	2.555	0.000	0.084	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	86	158	0	130	0	0	0
N.S.	1	1.00	0.44	0.81	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.059	0.028	2.444	0.000	0.087	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	89	104	0	135	0	0	0
N.S.	1	1.00	0.44	0.51	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.079	0.789	2.469	0.000	0.085	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	89	108	0	134	0	0	0
N.S.	1	1.00	0.44	0.53	0.00	0.66	0.00	0.00	0.00
time (sec)	N/A	0.077	0.770	2.472	0.000	0.084	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	92	165	0	135	0	0	0
N.S.	1	1.00	0.43	0.78	0.00	0.64	0.00	0.00	0.00
time (sec)	N/A	0.073	0.586	2.571	0.000	0.085	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	115	0	0	0
N.S.	1	1.00	0.47	0.85	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.102	0.785	2.546	0.000	0.080	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	114	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.089	0.735	2.483	0.000	0.091	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	117	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.092	0.581	2.573	0.000	0.085	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	120	0	0	0
N.S.	1	1.00	0.47	0.84	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.088	0.585	2.563	0.000	0.086	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	104	0	130	0	0	0
N.S.	1	1.00	1.00	1.20	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.032	0.772	2.474	0.000	0.083	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	168	0	131	0	0	0
N.S.	1	1.00	1.00	1.87	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.036	0.798	2.425	0.000	0.091	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	167	0	128	0	0	0
N.S.	1	1.00	1.00	1.90	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.032	0.629	2.480	0.000	0.083	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	91	108	0	133	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.034	0.599	2.413	0.000	0.084	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	160	0	127	0	0	0
N.S.	1	1.00	1.00	1.82	0.00	1.44	0.00	0.00	0.00
time (sec)	N/A	0.046	0.859	2.436	0.000	0.084	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	130	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.46	0.00	0.00	0.00
time (sec)	N/A	0.033	0.817	2.457	0.000	0.089	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	0	129	0	0	0
N.S.	1	1.00	1.00	1.19	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.044	0.709	2.424	0.000	0.088	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	163	0	128	0	0	0
N.S.	1	1.00	1.00	1.81	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.033	0.752	2.433	0.000	0.085	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	86	101	0	114	0	0	0
N.S.	1	1.00	0.42	0.50	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.057	0.025	2.390	0.000	0.090	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	89	161	0	119	0	0	0
N.S.	1	1.00	0.42	0.75	0.00	0.56	0.00	0.00	0.00
time (sec)	N/A	0.079	0.833	2.424	0.000	0.080	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	89	162	0	118	0	0	0
N.S.	1	1.00	0.42	0.76	0.00	0.55	0.00	0.00	0.00
time (sec)	N/A	0.072	0.831	2.417	0.000	0.097	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	222	222	92	111	0	119	0	0	0
N.S.	1	1.00	0.41	0.50	0.00	0.54	0.00	0.00	0.00
time (sec)	N/A	0.091	0.762	2.492	0.000	0.093	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	89	161	0	115	0	0	0
N.S.	1	1.00	0.47	0.85	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.104	0.851	2.428	0.000	0.086	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	90	107	0	114	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.087	0.795	2.447	0.000	0.080	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	92	109	0	117	0	0	0
N.S.	1	1.00	0.47	0.56	0.00	0.60	0.00	0.00	0.00
time (sec)	N/A	0.090	0.730	2.448	0.000	0.088	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	93	166	0	120	0	0	0
N.S.	1	1.00	0.47	0.84	0.00	0.61	0.00	0.00	0.00
time (sec)	N/A	0.101	0.588	2.454	0.000	0.085	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	37	38	0	35	0	0	0
N.S.	1	1.00	0.47	0.49	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.010	0.500	2.442	0.000	0.085	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	38	0	15	20	0	0
N.S.	1	1.00	1.03	0.97	0.00	0.38	0.51	0.00	0.00
time (sec)	N/A	0.014	0.503	3.719	0.000	0.085	1.322	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	53	0	16	0	0	0
N.S.	1	1.00	0.77	0.87	0.00	0.26	0.00	0.00	0.00
time (sec)	N/A	0.013	0.461	2.422	0.000	0.085	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	27	25	0	6	0	0	0
N.S.	1	1.00	4.50	4.17	0.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.007	0.226	2.483	0.000	0.075	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	28	0	73	0	0	0
N.S.	1	1.00	1.04	1.22	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.022	0.467	2.468	0.000	0.084	0.000	0.000	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	37	37	0	124	0	0	0
N.S.	1	1.00	0.20	0.20	0.00	0.68	0.00	0.00	0.00
time (sec)	N/A	0.054	0.008	2.430	0.000	0.081	0.000	0.000	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	35	37	0	72	0	0	0
N.S.	1	1.00	1.84	1.95	0.00	3.79	0.00	0.00	0.00
time (sec)	N/A	0.008	0.331	2.501	0.000	0.089	0.000	0.000	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	95	809	0	318	0	0	0
N.S.	1	1.00	1.00	8.52	0.00	3.35	0.00	0.00	0.00
time (sec)	N/A	0.101	2.238	2.888	0.000	0.137	0.000	0.000	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	95	1388	0	410	0	0	0
N.S.	1	1.00	1.01	14.77	0.00	4.36	0.00	0.00	0.00
time (sec)	N/A	0.086	2.275	2.725	0.000	0.102	0.000	0.000	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	478	478	102	1388	0	415	0	0	0
N.S.	1	1.00	0.21	2.90	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.330	2.276	2.689	0.000	0.107	0.000	0.000	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	102	872	0	287	0	0	0
N.S.	1	1.00	0.47	4.06	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.162	2.105	2.632	0.000	0.128	0.000	0.000	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	62	23	122	0	0	0	0	0	0
N.S.	1	0.37	1.97	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.011	1.330	0.000	0.000	0.000	0.000	0.000	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	117	0	118	0	0	0
N.S.	1	1.00	1.04	2.54	0.00	2.57	0.00	0.00	0.00
time (sec)	N/A	0.038	0.833	2.514	0.000	0.096	0.000	0.000	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	91	197	0	40	0	0	0
N.S.	1	1.00	1.94	4.19	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.054	1.980	2.610	0.000	0.079	0.000	0.000	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	119	187	0	289	0	0	0
N.S.	1	1.00	0.92	1.45	0.00	2.24	0.00	0.00	0.00
time (sec)	N/A	0.011	0.251	2.450	0.000	1.886	0.000	0.000	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	120	120	119	188	0	289	0	0	0
N.S.	1	1.00	0.99	1.57	0.00	2.41	0.00	0.00	0.00
time (sec)	N/A	0.011	0.239	2.472	0.000	1.934	0.000	0.000	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	119	0	0	378	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	2.93	0.00	0.00	0.00
time (sec)	N/A	0.016	0.273	0.000	0.000	5.275	0.000	0.000	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	388	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.015	0.355	0.000	0.000	5.254	0.000	0.000	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	383	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	3.19	0.00	0.00	0.00
time (sec)	N/A	0.015	0.306	0.000	0.000	4.027	0.000	0.000	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	381	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	3.18	0.00	0.00	0.00
time (sec)	N/A	0.012	0.311	0.000	0.000	4.051	0.000	0.000	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	120	119	0	0	451	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.76	0.00	0.00	0.00
time (sec)	N/A	0.015	0.322	0.000	0.000	24.396	0.000	0.000	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	123	0	0	459	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	3.70	0.00	0.00	0.00
time (sec)	N/A	0.022	0.328	0.000	0.000	25.300	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	104	0	0	0
N.S.	1	1.00	0.92	2.26	0.00	1.70	0.00	0.00	0.00
time (sec)	N/A	0.007	0.158	2.517	0.000	2.156	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	138	0	243	0	0	0
N.S.	1	1.00	0.92	2.26	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	0.007	0.155	2.500	0.000	2.069	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	77	77	67	0	0	274	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	3.56	0.00	0.00	0.00
time (sec)	N/A	0.009	0.172	0.000	0.000	5.884	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	69	0	0	273	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	3.46	0.00	0.00	0.00
time (sec)	N/A	0.010	0.186	0.000	0.000	5.867	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	375	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	4.41	0.00	0.00	0.00
time (sec)	N/A	0.013	0.202	0.000	0.000	4.091	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	373	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	4.39	0.00	0.00	0.00
time (sec)	N/A	0.010	0.205	0.000	0.000	4.885	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	101	101	88	0	0	457	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.52	0.00	0.00	0.00
time (sec)	N/A	0.019	0.210	0.000	0.000	21.579	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	90	0	0	469	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	4.55	0.00	0.00	0.00
time (sec)	N/A	0.016	0.194	0.000	0.000	20.416	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	48	117	0	91	0	0	0
N.S.	1	1.00	0.91	2.21	0.00	1.72	0.00	0.00	0.00
time (sec)	N/A	0.007	0.125	2.511	0.000	1.810	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	362	362	346	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.197	9.123	0.000	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	348	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.156	10.186	0.000	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	244	244	161	0	0	0	0	0	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.110	9.242	0.000	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	199	199	160	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	8.106	0.000	0.000	0.000	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	167	167	160	0	0	0	0	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	7.771	0.000	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	152	152	120	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	8.438	0.000	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	233	233	327	0	0	0	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.118	10.493	0.000	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	254	254	331	0	0	0	0	0	0
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	10.279	0.000	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	274	274	419	0	0	0	0	0	0
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.276	10.696	0.000	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	304	304	431	0	0	0	0	0	0
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.253	10.828	0.000	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	340	340	340	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.202	10.347	0.000	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	279	279	341	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.187	10.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	309	309	232	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.154	10.157	0.000	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	278	278	232	0	0	0	0	0	0
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.161	10.144	0.000	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	336	336	392	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.189	10.178	0.000	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	292	292	336	0	0	0	0	0	0
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.167	10.224	0.000	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	314	314	380	0	0	0	0	0	0
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.293	10.331	0.000	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	345	345	387	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.263	10.336	0.000	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	371	371	536	0	0	0	0	0	0
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.448	10.700	0.000	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	419	419	550	0	0	0	0	0	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.369	10.646	0.000	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	0
N.S.	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.031	0.227	0.000	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	296	136	0	0	0	121	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.41	0.00	0.00
time (sec)	N/A	0.211	7.865	0.000	0.000	0.000	20.723	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	88	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.50	0.00	0.00
time (sec)	N/A	0.091	5.097	0.000	0.000	0.000	10.201	0.000	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	53	0	0
N.S.	1	0.91	0.97	0.00	0.00	0.00	0.57	0.00	0.00
time (sec)	N/A	0.028	0.070	0.000	0.000	0.000	4.763	0.000	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	0.93
time (sec)	N/A	0.007	0.002	0.000	0.000	0.000	1.218	0.000	5.188

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.195	0.000	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.017	0.222	0.000	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	0
N.S.	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.019	0.256	0.000	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
N.S.	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.018	0.375	3.913	0.000	0.289	0.000	0.000	5.932

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [329] had the largest ratio of [.4285999999999999981]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	2	2	1.00	17	0.118
6	A	2	2	1.00	17	0.118
7	A	3	3	1.00	17	0.176
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	3	2	1.00	19	0.105
12	A	4	3	1.00	19	0.158
13	A	3	3	1.00	19	0.158
14	A	2	1	1.00	19	0.053
15	A	2	1	1.00	19	0.053
16	A	2	1	1.00	17	0.059
17	A	3	2	1.00	19	0.105
18	A	4	3	1.00	19	0.158
19	A	5	4	1.00	19	0.210
20	A	3	2	1.00	19	0.105
21	A	3	2	1.00	19	0.105
22	A	3	2	1.00	19	0.105
23	A	2	2	1.00	17	0.118
24	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	4	3	1.00	19	0.158
26	A	5	4	1.00	19	0.210
27	A	4	3	1.00	19	0.158
28	A	4	3	1.00	19	0.158
29	A	4	3	1.00	19	0.158
30	A	4	3	1.00	19	0.158
31	A	2	2	1.00	17	0.118
32	A	4	3	1.00	19	0.158
33	A	5	4	1.00	19	0.210
34	A	6	4	1.00	19	0.210
35	A	5	4	1.00	19	0.210
36	A	5	4	1.00	19	0.210
37	A	5	4	1.00	19	0.210
38	A	3	3	1.00	19	0.158
39	A	3	3	1.00	17	0.176
40	A	5	4	1.00	19	0.210
41	A	6	4	1.00	19	0.210
42	A	7	4	1.00	19	0.210
43	A	3	3	1.00	15	0.200
44	A	5	3	1.00	15	0.200
45	A	6	6	1.00	21	0.286
46	A	5	5	1.00	21	0.238
47	A	4	4	1.00	19	0.210
48	A	3	3	1.00	11	0.273
49	A	5	5	1.00	21	0.238
50	A	3	3	1.00	21	0.143
51	A	4	4	1.00	21	0.190
52	A	6	5	1.00	21	0.238
53	A	7	6	1.00	21	0.286
54	A	6	5	1.00	21	0.238
55	A	5	4	1.00	19	0.210
56	A	4	3	1.00	11	0.273
57	A	6	6	1.00	21	0.286
58	A	6	6	1.00	21	0.286
59	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	4	1.00	21	0.190
61	A	7	5	1.00	21	0.238
62	A	8	6	1.00	21	0.286
63	A	7	5	1.00	21	0.238
64	A	6	4	1.00	19	0.210
65	A	5	3	1.00	11	0.273
66	A	7	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	7	7	1.00	21	0.333
69	A	5	3	1.00	21	0.143
70	A	6	4	1.00	21	0.190
71	A	4	4	1.00	19	0.210
72	A	4	4	1.00	17	0.235
73	A	4	4	1.00	21	0.190
74	A	5	5	1.00	21	0.238
75	A	4	4	1.00	21	0.190
76	A	3	3	1.00	19	0.158
77	A	2	2	1.00	11	0.182
78	A	2	2	1.00	21	0.095
79	A	3	3	1.00	21	0.143
80	A	5	5	1.00	21	0.238
81	A	6	5	1.00	21	0.238
82	A	5	5	1.00	21	0.238
83	A	4	4	1.17	21	0.190
84	A	3	3	1.00	19	0.158
85	A	1	1	1.00	11	0.091
86	A	3	3	1.00	21	0.143
87	A	5	5	1.00	21	0.238
88	A	6	5	1.00	21	0.238
89	A	6	6	1.00	21	0.286
90	A	5	5	1.00	21	0.238
91	A	4	4	1.00	21	0.190
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	11	0.182
94	A	5	5	1.00	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	6	5	1.00	21	0.238
96	A	7	5	1.00	21	0.238
97	A	5	3	1.00	21	0.143
98	A	4	3	1.00	21	0.143
99	A	3	3	1.00	19	0.158
100	A	2	2	1.00	11	0.182
101	A	3	3	1.00	21	0.143
102	A	3	3	1.00	21	0.143
103	A	4	4	1.00	21	0.190
104	A	5	4	1.00	21	0.190
105	A	2	2	1.00	26	0.077
106	A	2	2	1.00	19	0.105
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	15	0.133
109	A	8	8	1.00	24	0.333
110	A	7	7	1.00	24	0.292
111	A	6	6	1.00	22	0.273
112	A	6	6	1.00	24	0.250
113	A	6	6	1.00	24	0.250
114	A	8	8	1.00	24	0.333
115	A	9	8	1.00	24	0.333
116	A	9	8	1.00	24	0.333
117	A	8	7	1.00	24	0.292
118	A	7	6	1.00	22	0.273
119	A	7	7	1.00	24	0.292
120	A	7	7	1.00	24	0.292
121	A	9	9	1.00	24	0.375
122	A	8	7	1.00	24	0.292
123	A	7	7	1.00	24	0.292
124	A	6	6	1.00	24	0.250
125	A	5	5	1.00	22	0.227
126	A	1	1	1.00	24	0.042
127	A	7	7	1.00	24	0.292
128	A	8	8	1.00	24	0.333
129	A	7	7	1.00	24	0.292

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	6	6	1.00	24	0.250
131	A	5	5	1.00	22	0.227
132	A	7	7	1.00	24	0.292
133	A	8	8	1.00	24	0.333
134	A	9	8	1.00	24	0.333
135	A	8	8	1.00	24	0.333
136	A	7	7	1.00	24	0.292
137	A	2	2	1.00	24	0.083
138	A	6	6	1.00	22	0.273
139	A	8	8	1.00	24	0.333
140	A	9	8	1.00	24	0.333
141	A	1	1	1.00	26	0.038
142	A	1	1	1.00	24	0.042
143	A	1	1	1.00	23	0.043
144	A	1	1	1.00	24	0.042
145	A	1	1	1.00	22	0.045
146	A	1	1	1.00	19	0.053
147	A	1	1	1.00	19	0.053
148	A	1	1	1.00	24	0.042
149	A	1	1	1.00	22	0.045
150	A	1	1	1.00	27	0.037
151	A	1	1	1.00	28	0.036
152	A	1	1	1.00	29	0.034
153	A	1	1	1.00	30	0.033
154	A	1	1	1.00	26	0.038
155	A	1	1	1.00	26	0.038
156	A	1	1	1.00	23	0.043
157	A	1	1	1.00	23	0.043
158	A	1	1	1.00	23	0.043
159	A	1	1	1.00	23	0.043
160	A	1	1	1.00	17	0.059
161	A	1	1	1.00	21	0.048
162	A	1	1	1.00	21	0.048
163	A	3	3	1.15	29	0.103
164	A	3	3	1.23	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	3	3	1.20	29	0.103
166	A	6	6	1.00	23	0.261
167	A	5	5	1.00	23	0.217
168	A	4	4	1.00	23	0.174
169	A	1	1	1.00	23	0.043
170	A	4	4	1.00	23	0.174
171	A	5	5	1.00	23	0.217
172	A	7	6	1.00	23	0.261
173	A	6	6	1.00	23	0.261
174	A	5	5	1.00	23	0.217
175	A	5	5	1.00	23	0.217
176	A	4	4	1.00	23	0.174
177	A	5	5	1.00	23	0.217
178	A	5	5	1.00	23	0.217
179	A	3	3	1.00	23	0.130
180	A	2	1	1.00	23	0.043
181	A	4	4	1.00	23	0.174
182	A	5	5	1.00	23	0.217
183	A	4	4	1.00	21	0.190
184	A	1	1	1.00	23	0.043
185	A	1	1	1.00	23	0.043
186	A	1	1	1.00	23	0.043
187	A	1	1	1.00	21	0.048
188	A	1	1	1.00	21	0.048
189	A	1	1	1.00	21	0.048
190	A	1	1	1.00	23	0.043
191	A	4	4	1.00	21	0.190
192	A	3	3	1.00	23	0.130
193	A	3	3	1.00	23	0.130
194	A	3	3	1.00	23	0.130
195	A	4	4	1.00	21	0.190
196	A	4	4	1.00	21	0.190
197	A	4	4	1.00	23	0.174
198	A	2	2	1.00	23	0.087
199	A	7	6	1.00	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
200	A	6	6	1.00	23	0.261
201	A	5	5	1.00	23	0.217
202	A	4	4	1.00	23	0.174
203	A	1	1	1.00	23	0.043
204	A	6	6	1.00	23	0.261
205	A	4	4	1.00	23	0.174
206	A	5	5	1.00	23	0.217
207	A	7	6	1.00	23	0.261
208	A	6	6	1.00	23	0.261
209	A	5	5	1.00	23	0.217
210	A	1	1	1.00	23	0.043
211	A	6	6	1.00	23	0.261
212	A	4	4	1.00	23	0.174
213	A	5	5	1.00	23	0.217
214	A	1	1	1.00	23	0.043
215	A	3	2	1.00	24	0.083
216	A	3	2	1.00	24	0.083
217	A	3	2	1.00	25	0.080
218	A	1	1	1.00	23	0.043
219	A	1	1	1.00	23	0.043
220	A	1	1	1.00	23	0.043
221	A	2	2	1.00	23	0.087
222	A	1	1	1.00	21	0.048
223	A	1	1	1.00	23	0.043
224	A	2	2	1.00	23	0.087
225	A	1	1	1.00	23	0.043
226	A	1	1	1.00	23	0.043
227	A	1	1	1.00	23	0.043
228	A	1	1	1.00	21	0.048
229	A	1	1	1.00	21	0.048
230	A	1	1	1.00	21	0.048
231	A	2	2	1.00	21	0.095
232	A	1	1	1.00	19	0.053
233	A	1	1	1.00	21	0.048
234	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
235	A	1	1	1.00	21	0.048
236	A	1	1	1.00	21	0.048
237	A	1	1	1.00	21	0.048
238	A	2	2	1.00	21	0.095
239	A	2	2	1.00	21	0.095
240	A	2	2	1.00	21	0.095
241	A	2	2	1.00	21	0.095
242	A	2	2	1.00	19	0.105
243	A	1	1	1.00	21	0.048
244	A	2	2	1.00	21	0.095
245	A	2	2	1.00	21	0.095
246	A	2	2	1.00	21	0.095
247	A	2	2	1.00	21	0.095
248	A	1	1	1.00	23	0.043
249	A	1	1	1.00	23	0.043
250	A	1	1	1.00	23	0.043
251	A	2	2	1.00	23	0.087
252	A	1	1	1.00	21	0.048
253	A	2	2	1.00	23	0.087
254	A	2	2	1.55	23	0.087
255	A	2	2	1.00	23	0.087
256	A	2	2	1.00	23	0.087
257	A	2	2	1.00	23	0.087
258	A	3	3	1.00	24	0.125
259	A	3	3	1.00	27	0.111
260	A	3	3	1.00	25	0.120
261	A	3	3	1.00	28	0.107
262	A	3	3	1.00	25	0.120
263	A	3	3	1.00	26	0.115
264	A	3	3	1.00	26	0.115
265	A	3	3	1.00	27	0.111
266	A	4	4	1.00	23	0.174
267	A	4	4	1.00	26	0.154
268	A	4	4	1.00	26	0.154
269	A	4	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
270	A	7	6	1.00	24	0.250
271	A	7	6	1.00	25	0.240
272	A	7	6	1.00	27	0.222
273	A	7	6	1.00	28	0.214
274	A	3	3	1.00	24	0.125
275	A	3	3	1.00	27	0.111
276	A	3	3	1.00	25	0.120
277	A	3	3	1.00	28	0.107
278	A	3	3	1.00	25	0.120
279	A	3	3	1.00	26	0.115
280	A	3	3	1.00	26	0.115
281	A	3	3	1.00	27	0.111
282	A	4	4	1.00	23	0.174
283	A	4	4	1.00	26	0.154
284	A	4	4	1.00	26	0.154
285	A	4	4	1.00	29	0.138
286	A	7	6	1.00	24	0.250
287	A	7	6	1.00	25	0.240
288	A	7	6	1.00	27	0.222
289	A	7	6	1.00	28	0.214
290	A	1	1	1.00	23	0.043
291	A	2	2	1.00	23	0.087
292	A	1	1	1.00	21	0.048
293	A	1	1	1.00	23	0.043
294	A	4	4	1.00	28	0.143
295	A	4	4	1.00	23	0.174
296	A	1	1	1.00	23	0.043
297	A	1	1	1.00	59	0.017
298	A	1	1	1.00	59	0.017
299	A	4	4	1.00	59	0.068
300	A	3	3	1.00	59	0.051
301	C	1	1	0.37	21	0.048
302	A	2	2	1.00	26	0.077
303	A	1	1	1.00	41	0.024
304	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
305	A	1	1	1.00	21	0.048
306	A	1	1	1.00	21	0.048
307	A	1	1	1.00	23	0.043
308	A	1	1	1.00	23	0.043
309	A	1	1	1.00	23	0.043
310	A	1	1	1.00	23	0.043
311	A	1	1	1.00	25	0.040
312	A	1	1	1.00	21	0.048
313	A	1	1	1.00	21	0.048
314	A	1	1	1.00	21	0.048
315	A	1	1	1.00	23	0.043
316	A	1	1	1.00	25	0.040
317	A	1	1	1.00	25	0.040
318	A	1	1	1.00	25	0.040
319	A	1	1	1.00	27	0.037
320	A	1	1	1.00	19	0.053
321	A	13	8	1.00	21	0.381
322	A	12	8	1.00	21	0.381
323	A	8	7	1.00	21	0.333
324	A	8	7	1.00	21	0.333
325	A	4	3	1.00	21	0.143
326	A	5	4	1.00	21	0.190
327	A	7	6	1.00	21	0.286
328	A	9	8	1.00	21	0.381
329	A	10	9	1.00	21	0.429
330	A	10	9	1.00	21	0.429
331	A	9	8	1.00	21	0.381
332	A	9	8	1.00	21	0.381
333	A	9	8	1.00	21	0.381
334	A	9	8	1.00	21	0.381
335	A	9	8	1.00	21	0.381
336	A	9	8	1.00	21	0.381
337	A	10	9	1.00	21	0.429
338	A	10	9	1.00	21	0.429
339	A	11	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	11	9	1.00	21	0.429
341	A	3	2	1.00	19	0.105
342	A	5	5	1.00	19	0.263
343	A	4	4	1.00	19	0.210
344	A	3	3	0.91	17	0.176
345	A	2	2	1.00	9	0.222
346	A	2	2	1.00	19	0.105
347	A	2	2	1.00	19	0.105
348	A	2	2	1.00	19	0.105
349	A	1	1	1.00	50	0.020

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx^2)(c+dx^2)^4 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)^4 \, dx = ac^4x + \frac{1}{3}c^3(bc+4ad)x^3 + \frac{2}{5}c^2d(2bc+3ad)x^5 + \frac{2}{7}cd^2(3bc+2ad)x^7 + \frac{1}{9}d^3(4bc+ad)x^9 + \frac{1}{11}bd^4x^{11}$$

```
[Out] a*c^4*x+1/3*c^3*(4*a*d+b*c)*x^3+2/5*c^2*d*(3*a*d+2*b*c)*x^5+2/7*c*d^2*(2*a*
d+3*b*c)*x^7+1/9*d^3*(a*d+4*b*c)*x^9+1/11*b*d^4*x^11
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^2)(c+dx^2)^4 \, dx = \frac{1}{3}c^3x^3(4ad+bc) + \frac{2}{5}c^2dx^5(3ad+2bc) + \frac{1}{9}d^3x^9(ad+4bc) + \frac{2}{7}cd^2x^7(2ad+3bc) + ac^4x + \frac{1}{11}bd^4x^{11}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^4+c^3(bc+4ad)x^2+2c^2d(2bc+3ad)x^4+2cd^2(3bc+2ad)x^6 + d^3(4bc+ad)x^8+bd^4x^{10}\right)\,dx\\ &= ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5+\frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9 + \frac{1}{11}bd^4x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx = ac^4x+\frac{1}{3}c^3(bc+4ad)x^3+\frac{2}{5}c^2d(2bc+3ad)x^5 + \frac{2}{7}cd^2(3bc+2ad)x^7+\frac{1}{9}d^3(4bc+ad)x^9+\frac{1}{11}bd^4x^{11}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^3)/3 + (2*c^2*d*(2*b*c + 3*a*d)*x^5)/5 + (2*
c*d^2*(3*b*c + 2*a*d)*x^7)/7 + (d^3*(4*b*c + a*d)*x^9)/9 + (b*d^4*x^11)/11
```

Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result
norman	$\frac{b\,d^4x^{11}}{11}+\left(\frac{1}{9}a\,d^4+\frac{4}{9}bc\,d^3\right)x^9+\left(\frac{4}{7}ac\,d^3+\frac{6}{7}b\,c^2d^2\right)x^7+\left(\frac{6}{5}a\,c^2d^2+\frac{4}{5}b\,c^3d\right)x^5+\left(\frac{4}{3}a\,c^3d+\frac{1}{3}b\,c^4\right.$
default	$\frac{b\,d^4x^{11}}{11}+\frac{(a\,d^4+4bc\,d^3)x^9}{9}+\frac{(4ac\,d^3+6b\,c^2d^2)x^7}{7}+\frac{(6a\,c^2d^2+4b\,c^3d)x^5}{5}+\frac{(4a\,c^3d+b\,c^4)x^3}{3}+a\,c^4x$
gosper	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+\frac{1}{3}$
risch	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+\frac{1}{3}$
parallelrisch	$\frac{1}{11}b\,d^4x^{11}+\frac{1}{9}x^9a\,d^4+\frac{4}{9}x^9bc\,d^3+\frac{4}{7}x^7ac\,d^3+\frac{6}{7}x^7b\,c^2d^2+\frac{6}{5}x^5a\,c^2d^2+\frac{4}{5}x^5b\,c^3d+\frac{4}{3}x^3a\,c^3d+\frac{1}{3}$

```
[In] int((b*x^2+a)*(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^4*x^11+(1/9*a*d^4+4/9*b*c*d^3)*x^9+(4/7*a*c*d^3+6/7*b*c^2*d^2)*x^7
+(6/5*a*c^2*d^2+4/5*b*c^3*d)*x^5+(4/3*a*c^3*d+1/3*b*c^4)*x^3+a*c^4*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)\,(c+dx^2)^4\,dx = \frac{1}{11}\,bd^4x^{11} + \frac{1}{9}\,(4\,bcd^3 + ad^4)x^9 + \frac{2}{7}\,(3\,bc^2d^2 + 2\,acd^3)x^7 + ac^4x + \frac{2}{5}\,(2\,bc^3d + 3\,ac^2d^2)x^5 + \frac{1}{3}\,(bc^4 + 4\,ac^3d)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int (a+bx^2)\,(c+dx^2)^4\,dx = ac^4x + \frac{bd^4x^{11}}{11} + x^9\left(\frac{ad^4}{9} + \frac{4bcd^3}{9}\right) + x^7\cdot\left(\frac{4acd^3}{7} + \frac{6bc^2d^2}{7}\right) + x^5\cdot\left(\frac{6ac^2d^2}{5} + \frac{4bc^3d}{5}\right) + x^3\cdot\left(\frac{4ac^3d}{3} + \frac{bc^4}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**11/11 + x**9*(a*d**4/9 + 4*b*c*d**3/9) + x**7*(4*a*c*d
**3/7 + 6*b*c**2*d**2/7) + x**5*(6*a*c**2*d**2/5 + 4*b*c**3*d/5) + x**3*(4*
a*c**3*d/3 + b*c**4/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int (a+bx^2)\,(c+dx^2)^4\,dx = \frac{1}{11}\,bd^4x^{11} + \frac{1}{9}\,(4\,bcd^3 + ad^4)x^9 + \frac{2}{7}\,(3\,bc^2d^2 + 2\,acd^3)x^7 + ac^4x + \frac{2}{5}\,(2\,bc^3d + 3\,ac^2d^2)x^5 + \frac{1}{3}\,(bc^4 + 4\,ac^3d)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^4*x^11 + 1/9*(4*b*c*d^3 + a*d^4)*x^9 + 2/7*(3*b*c^2*d^2 + 2*a*c*d^
3)*x^7 + a*c^4*x + 2/5*(2*b*c^3*d + 3*a*c^2*d^2)*x^5 + 1/3*(b*c^4 + 4*a*c^3
*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx=\frac{1}{11}\,bd^4x^{11}+\frac{4}{9}\,bcd^3x^9+\frac{1}{9}\,ad^4x^9+\frac{6}{7}\,bc^2d^2x^7+\frac{4}{7}\,acd^3x^7+\frac{4}{5}\,bc^3dx^5+\frac{6}{5}\,ac^2d^2x^5+\frac{1}{3}\,bc^4x^3+\frac{4}{3}\,ac^3dx^3+ac^4x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] 1/11*b*d^4*x^11 + 4/9*b*c*d^3*x^9 + 1/9*a*d^4*x^9 + 6/7*b*c^2*d^2*x^7 + 4/7
*a*c*d^3*x^7 + 4/5*b*c^3*d*x^5 + 6/5*a*c^2*d^2*x^5 + 1/3*b*c^4*x^3 + 4/3*a*
c^3*d*x^3 + a*c^4*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^4\,dx=x^3\left(\frac{b\,c^4}{3}+\frac{4\,a\,d\,c^3}{3}\right)+x^9\left(\frac{a\,d^4}{9}+\frac{4\,b\,c\,d^3}{9}\right)+\frac{b\,d^4\,x^{11}}{11}+a\,c^4\,x+\frac{2\,c^2\,d\,x^5\,\left(3\,a\,d+2\,b\,c\right)}{5}+\frac{2\,c\,d^2\,x^7\,\left(2\,a\,d+3\,b\,c\right)}{7}$$

```
[In] int((a + b*x^2)*(c + d*x^2)^4,x)
```

```
[Out] x^3*((b*c^4)/3 + (4*a*c^3*d)/3) + x^9*((a*d^4)/9 + (4*b*c*d^3)/9) + (b*d^4*
x^11)/11 + a*c^4*x + (2*c^2*d*x^5*(3*a*d + 2*b*c))/5 + (2*c*d^2*x^7*(2*a*d
+ 3*b*c))/7
```

3.2 $\int (a+bx^2)(c+dx^2)^3 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^2)(c+dx^2)^3 \, dx = ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

```
[Out] a*c^3*x+1/3*c^2*(3*a*d+b*c)*x^3+3/5*c*d*(a*d+b*c)*x^5+1/7*d^2*(a*d+3*b*c)*x
^7+1/9*b*d^3*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^2)(c+dx^2)^3 \, dx = \frac{1}{3}c^2x^3(3ad+bc)+\frac{1}{7}d^2x^7(ad+3bc)+\frac{3}{5}cdx^5(ad+bc)+ac^3x+\frac{1}{9}bd^3x^9$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ac^3+c^2(bc+3ad)x^2+3cd(bc+ad)x^4+d^2(3bc+ad)x^6+bd^3x^8\right) dx \\ &= ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right) \left(c+dx^2\right)^3 \, dx = ac^3x+\frac{1}{3}c^2(bc+3ad)x^3+\frac{3}{5}cd(bc+ad)x^5+\frac{1}{7}d^2(3bc+ad)x^7+\frac{1}{9}bd^3x^9$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^3)/3 + (3*c*d*(b*c + a*d)*x^5)/5 + (d^2*(3*b
*c + a*d)*x^7)/7 + (b*d^3*x^9)/9
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
norman	$\frac{b\,d^3x^9}{9}+\left(\frac{1}{7}a\,d^3+\frac{3}{7}bc\,d^2\right)x^7+\left(\frac{3}{5}ac\,d^2+\frac{3}{5}b\,c^2d\right)x^5+\left(a\,c^2d+\frac{1}{3}c^3b\right)x^3+a\,c^3x$	71
default	$\frac{b\,d^3x^9}{9}+\frac{(a\,d^3+3bc\,d^2)x^7}{7}+\frac{(3ac\,d^2+3b\,c^2d)x^5}{5}+\frac{(3a\,c^2d+c^3b)x^3}{3}+a\,c^3x$	73
gosper	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74
risch	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74
parallelrisch	$\frac{1}{9}b\,d^3x^9+\frac{1}{7}x^7a\,d^3+\frac{3}{7}x^7bc\,d^2+\frac{3}{5}x^5ac\,d^2+\frac{3}{5}x^5b\,c^2d+x^3a\,c^2d+\frac{1}{3}x^3c^3b+a\,c^3x$	74

```
[In] int((b*x^2+a)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^3*x^9+(1/7*a*d^3+3/7*b*c*d^2)*x^7+(3/5*a*c*d^2+3/5*b*c^2*d)*x^5+(a*
c^2*d+1/3*c^3*b)*x^3+a*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right) \left(c+dx^2\right)^3 \, dx = \frac{1}{9}\,bd^3x^9+\frac{1}{7}\left(3\,bcd^2+ad^3\right)x^7 + \frac{3}{5}\left(bc^2d+acd^2\right)x^5+ac^3x+\frac{1}{3}\left(bc^3+3\,ac^2d\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx = ac^3x+\frac{bd^3x^9}{9}+x^7\left(\frac{ad^3}{7}+\frac{3bcd^2}{7}\right)+x^5\cdot\left(\frac{3acd^2}{5}+\frac{3bc^2d}{5}\right)+x^3\left(ac^2d+\frac{bc^3}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**9/9 + x**7*(a*d**3/7 + 3*b*c*d**2/7) + x**5*(3*a*c*d**
2/5 + 3*b*c**2*d/5) + x**3*(a*c**2*d + b*c**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx = \frac{1}{9}\,bd^3x^9+\frac{1}{7}\,\left(3\,bcd^2+ad^3\right)x^7+\frac{3}{5}\,\left(bc^2d+acd^2\right)x^5+ac^3x+\frac{1}{3}\,\left(bc^3+3\,ac^2d\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^3*x^9 + 1/7*(3*b*c*d^2 + a*d^3)*x^7 + 3/5*(b*c^2*d + a*c*d^2)*x^5 +
 a*c^3*x + 1/3*(b*c^3 + 3*a*c^2*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\,dx = \frac{1}{9}\,bd^3x^9+\frac{3}{7}\,bcd^2x^7+\frac{1}{7}\,ad^3x^7+\frac{3}{5}\,bc^2dx^5+\frac{3}{5}\,acd^2x^5+\frac{1}{3}\,bc^3x^3+ac^2dx^3+ac^3x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b*d^3*x^9 + 3/7*b*c*d^2*x^7 + 1/7*a*d^3*x^7 + 3/5*b*c^2*d*x^5 + 3/5*a*c
*d^2*x^5 + 1/3*b*c^3*x^3 + a*c^2*d*x^3 + a*c^3*x
```

Mupad [B] (verification not implemented)

Time = 4.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int (a+bx^2)(c+dx^2)^3\,dx = x^3\left(\frac{b\,c^3}{3}+a\,d\,c^2\right)+x^7\left(\frac{a\,d^3}{7}+\frac{3\,b\,c\,d^2}{7}\right)+\frac{b\,d^3\,x^9}{9}+a\,c^3\,x+\frac{3\,c\,d\,x^5\,(a\,d+b\,c)}{5}$$

```
[In] int((a + b*x^2)*(c + d*x^2)^3,x)
```

```
[Out] x^3*((b*c^3)/3 + a*c^2*d) + x^7*((a*d^3)/7 + (3*b*c*d^2)/7) + (b*d^3*x^9)/9
 + a*c^3*x + (3*c*d*x^5*(a*d + b*c))/5
```

3.3 $\int (a+bx^2)(c+dx^2)^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)(c+dx^2)^2 \, dx = ac^2x + \frac{1}{3}c(bc+2ad)x^3 + \frac{1}{5}d(2bc+ad)x^5 + \frac{1}{7}bd^2x^7$$

```
[Out] a*c^2*x+1/3*c*(2*a*d+b*c)*x^3+1/5*d*(a*d+2*b*c)*x^5+1/7*b*d^2*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {380}

$$\int (a+bx^2)(c+dx^2)^2 \, dx = \frac{1}{5}dx^5(ad+2bc) + \frac{1}{3}cx^3(2ad+bc) + ac^2x + \frac{1}{7}bd^2x^7$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2,x]
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(ac^2 + c(bc+2ad)x^2 + d(2bc+ad)x^4 + bd^2x^6\right) dx \\ &= ac^2x + \frac{1}{3}c(bc+2ad)x^3 + \frac{1}{5}d(2bc+ad)x^5 + \frac{1}{7}bd^2x^7 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2\,dx = ac^2x + \frac{1}{3}c(bc+2ad)x^3 + \frac{1}{5}d(2bc+ad)x^5 + \frac{1}{7}bd^2x^7$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2,x]
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^3)/3 + (d*(2*b*c + a*d)*x^5)/5 + (b*d^2*x^7)/7
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b\,d^2x^7}{7}+\frac{(a\,d^2+2bcd)x^5}{5}+\frac{(2acd+b\,c^2)x^3}{3}+a\,c^2x$	49
norman	$\frac{b\,d^2x^7}{7}+\left(\frac{1}{5}a\,d^2+\frac{2}{5}bcd\right)x^5+\left(\frac{2}{3}acd+\frac{1}{3}b\,c^2\right)x^3+a\,c^2x$	49
gosper	$\frac{1}{7}b\,d^2x^7+\frac{1}{5}x^5a\,d^2+\frac{2}{5}x^5bcd+\frac{2}{3}x^3acd+\frac{1}{3}x^3b\,c^2+a\,c^2x$	51
risch	$\frac{1}{7}b\,d^2x^7+\frac{1}{5}x^5a\,d^2+\frac{2}{5}x^5bcd+\frac{2}{3}x^3acd+\frac{1}{3}x^3b\,c^2+a\,c^2x$	51
parallelrisch	$\frac{1}{7}b\,d^2x^7+\frac{1}{5}x^5a\,d^2+\frac{2}{5}x^5bcd+\frac{2}{3}x^3acd+\frac{1}{3}x^3b\,c^2+a\,c^2x$	51

```
[In] int((b*x^2+a)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
[Out] 1/7*b*d^2*x^7+1/5*(a*d^2+2*b*c*d)*x^5+1/3*(2*a*c*d+b*c^2)*x^3+a*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)^2\,dx = \frac{1}{7}\,bd^2x^7 + \frac{1}{5}\,(2\,bcd+ad^2)x^5 + ac^2x + \frac{1}{3}\,(bc^2+2\,acd)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="fricas")
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx=ac^2x+\frac{bd^2x^7}{7}+x^5\left(\frac{ad^2}{5}+\frac{2bcd}{5}\right)+x^3\cdot\left(\frac{2acd}{3}+\frac{bc^2}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**7/7 + x**5*(a*d**2/5 + 2*b*c*d/5) + x**3*(2*a*c*d/3 +
b*c**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx=\frac{1}{7}\,bd^2x^7+\frac{1}{5}\left(2\,bcd+ad^2\right)x^5+ac^2x+\frac{1}{3}\left(bc^2+2\,acd\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b*d^2*x^7 + 1/5*(2*b*c*d + a*d^2)*x^5 + a*c^2*x + 1/3*(b*c^2 + 2*a*c*d)
*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx=\frac{1}{7}\,bd^2x^7+\frac{2}{5}\,bcdx^5+\frac{1}{5}\,ad^2x^5+\frac{1}{3}\,bc^2x^3+\frac{2}{3}\,acdx^3+ac^2x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b*d^2*x^7 + 2/5*b*c*d*x^5 + 1/5*a*d^2*x^5 + 1/3*b*c^2*x^3 + 2/3*a*c*d*x
^3 + a*c^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\,dx = x^3\left(\frac{b\,c^2}{3}+\frac{2\,a\,d\,c}{3}\right)+x^5\left(\frac{a\,d^2}{5}+\frac{2\,b\,c\,d}{5}\right)+\frac{b\,d^2\,x^7}{7}+a\,c^2\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2)^2,x)
```

```
[Out] x^3*((b*c^2)/3 + (2*a*c*d)/3) + x^5*((a*d^2)/5 + (2*b*c*d)/5) + (b*d^2*x^7)
/7 + a*c^2*x
```

3.4 $\int (a+bx^2)(c+dx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int (a+bx^2)(c+dx^2)\,dx = acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5$$

```
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{380\}$

$$\int (a+bx^2)(c+dx^2)\,dx = \frac{1}{3}x^3(ad+bc)+acx+\frac{1}{5}bdx^5$$

```
[In] Int[(a + b*x^2)*(c + d*x^2),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac+(bc+ad)x^2+bdx^4\right)dx\\ &= acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx=acx+\frac{1}{3}(bc+ad)x^3+\frac{1}{5}bdx^5$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2),x]
[Out] a*c*x + ((b*c + a*d)*x^3)/3 + (b*d*x^5)/5
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$acx+\frac{(ad+bc)x^3}{3}+\frac{bd\,x^5}{5}$	25
norman	$\frac{bd\,x^5}{5}+\left(\frac{ad}{3}+\frac{bc}{3}\right)x^3+acx$	26
gosper	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27
risch	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27
parallelrisch	$\frac{1}{5}bd\,x^5+\frac{1}{3}x^3ad+\frac{1}{3}bc\,x^3+acx$	27

```
[In] int((b*x^2+a)*(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] a*c*x+1/3*(a*d+b*c)*x^3+1/5*b*d*x^5
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx=\frac{1}{5}\,bdx^5+\frac{1}{3}\,(bc+ad)x^3+acx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="fricas")
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx = acx+\frac{bdx^5}{5}+x^3\left(\frac{ad}{3}+\frac{bc}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c),x)
[Out] a*c*x + b*d*x**5/5 + x**3*(a*d/3 + b*c/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx = \frac{1}{5}\,bdx^5+\frac{1}{3}\,(bc+ad)x^3+acx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="maxima")
[Out] 1/5*b*d*x^5 + 1/3*(b*c + a*d)*x^3 + a*c*x
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx = \frac{1}{5}\,bdx^5+\frac{1}{3}\,bcx^3+\frac{1}{3}\,adx^3+acx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c),x, algorithm="giac")
[Out] 1/5*b*d*x^5 + 1/3*b*c*x^3 + 1/3*a*d*x^3 + a*c*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\,dx = \frac{b\,d\,x^5}{5}+\left(\frac{a\,d}{3}+\frac{b\,c}{3}\right)\,x^3+a\,c\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2),x)
[Out] x^3*((a*d)/3 + (b*c)/3) + a*c*x + (b*d*x^5)/5
```

3.5 $\int \frac{a+bx^2}{c+dx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{a+bx^2}{c+dx^2}\,dx=\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

```
[Out] b*x/d-(-a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/d^(3/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {396, 211}

$$\int \frac{a+bx^2}{c+dx^2}\,dx=\frac{bx}{d}-\frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

```
[In] Int[(a + b*x^2)/(c + d*x^2),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^2}\,dx}{d}\\ &= \frac{bx}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{3/2}}$$

```
[In] Integrate[(a + b*x^2)/(c + d*x^2),x]
[Out] (b*x)/d - ((b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^(3/2))
```

Maple [A] (verified)

Time = 8.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{bx}{d}+\frac{(ad-bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d\sqrt{cd}}$	34
risch	$\frac{bx}{d}-\frac{\ln\left(dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a}{2\sqrt{-cd}}-\frac{\ln\left(-dx+\sqrt{-cd}\right)bc}{2d\sqrt{-cd}}$	98

```
[In] int((b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] b*x/d+1/d*(a*d-b*c)/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \left[\frac{2\,bcdx + (bc-ad)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)}{2\,cd^2}, \frac{bcdx - (bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{c}\right)}{cd^2}\right]$$

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/2*(2*b*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(
d*x^2 + c)))/(c*d^2), (b*c*d*x - (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/c
))/(c*d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.15 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.05

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{\sqrt{-\frac{1}{cd^3}}(ad-bc)\log\left(-cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^3}}(ad-bc)\log\left(cd\sqrt{-\frac{1}{cd^3}}+x\right)}{2}$$

```
[In] integrate((b*x**2+a)/(d*x**2+c),x)
```

```
[Out] b*x/d - sqrt(-1/(c*d**3))*(a*d - b*c)*log(-c*d*sqrt(-1/(c*d**3)) + x)/2 + s
qrt(-1/(c*d**3))*(a*d - b*c)*log(c*d*sqrt(-1/(c*d**3)) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{bx}{d} - \frac{(bc-ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \frac{a+bx^2}{c+dx^2}\,dx = \frac{b\,x}{d} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,\left(a\,d-b\,c\right)}{\sqrt{c}\,d^{3/2}}$$

```
[In] int((a + b*x^2)/(c + d*x^2),x)
[Out] (b*x)/d + (atan((d^(1/2)*x)/c^(1/2))*(a*d - b*c))/(c^(1/2)*d^(3/2))
```

3.6 $\int \frac{a+bx^2}{\left(c+dx^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2} \, dx = -\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

```
[Out] -1/2*(-a*d+b*c)*x/c/d/(d*x^2+c)+1/2*(a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(
3/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 211}

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2} \, dx = \frac{(ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}-\frac{x(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[In] Int[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(3/2)*d^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\int \frac{1}{c+dx^2}\,dx}{2cd} \\ &= -\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^2}\,dx=-\frac{(bc-ad)x}{2cd\left(c+dx^2\right)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{3/2}}$$

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x)/(c*d*(c + d*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sq
rt[c]])/(2*c^(3/2)*d^(3/2))
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{(ad-bc)x}{2cd(d\,x^2+c)}+\frac{(ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2cd\sqrt{cd}}$	57
risch	$\frac{(ad-bc)x}{2cd(d\,x^2+c)}-\frac{\ln\left(dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c}-\frac{\ln\left(dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a}{4\sqrt{-cd}\,c}+\frac{\ln\left(-dx+\sqrt{-cd}\right)b}{4\sqrt{-cd}\,d}$	122

```
[In] int((b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(a*d-b*c)/c/d*x/(d*x^2+c)+1/2*(a*d+b*c)/c/d/(c*d)^(1/2)*arctan(d*x/(c*d
)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.89

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx$$
$$= \left[-\frac{(bc^2+acd+(bcd+ad^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2(bc^2d-acd^2)x}{4(c^2d^3x^2+c^3d^2)}, \frac{(bc^2+acd+(bcd+ad^2)x^2)\cdots}{2(c^2\cdots}\right.$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt
(-c*d)*x - c)/(d*x^2 + c)) + 2*(b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^
2), 1/2*((b*c^2 + a*c*d + (b*c*d + a*d^2)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x
/c) - (b*c^2*d - a*c*d^2)*x)/(c^2*d^3*x^2 + c^3*d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = \frac{x(ad-bc)}{2c^2d+2cd^2x^2} - \frac{\sqrt{-\frac{1}{c^3d^3}}(ad+bc)\log\left(-c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^3}}(ad+bc)\log\left(c^2d\sqrt{-\frac{1}{c^3d^3}}+x\right)}{4}$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(2*c**2*d + 2*c*d**2*x**2) - sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(-c**2*d*sqrt(-1/(c**3*d**3)) + x)/4 + sqrt(-1/(c**3*d**3))*(a*d + b*c)
*log(c**2*d*sqrt(-1/(c**3*d**3)) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x}{2\,(cd^2x^2+c^2d)} + \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] -1/2*(b*c - a*d)*x/(c*d^2*x^2 + c^2*d) + 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*
d))/(sqrt(c*d)*c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = \frac{(bc+ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd} - \frac{bcx-adx}{2\,(dx^2+c)cd}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
[Out] 1/2*(b*c + a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d) - 1/2*(b*c*x - a*d*x)
/((d*x^2 + c)*c*d)
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{a+bx^2}{(c+dx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(a\,d+b\,c)}{2\,c^{3/2}\,d^{3/2}} + \frac{x\,(a\,d-b\,c)}{2\,c\,d\,\left(d\,x^2+c\right)}$$

```
[In] int((a + b*x^2)/(c + d*x^2)^2,x)
[Out] (atan((d^(1/2)*x)/c^(1/2))*(a*d + b*c))/(2*c^(3/2)*d^(3/2)) + (x*(a*d - b*c
))/(2*c*d*(c + d*x^2))
```

3.7 $\int \frac{a+bx^2}{(c+dx^2)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{a+bx^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x}{4cd(c+dx^2)^2} + \frac{(bc+3ad)x}{8c^2d(c+dx^2)} + \frac{(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

```
[Out] -1/4*(-a*d+b*c)*x/c/d/(d*x^2+c)^2+1/8*(3*a*d+b*c)*x/c^2/d/(d*x^2+c)+1/8*(3*
a*d+b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {393, 205, 211}

$$\int \frac{a+bx^2}{(c+dx^2)^3} dx = \frac{(3ad+bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}} + \frac{x(3ad+bc)}{8c^2d(c+dx^2)} - \frac{x(bc-ad)}{4cd(c+dx^2)^2}$$

```
[In] Int[(a + b*x^2)/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x)/(c*d*(c + d*x^2)^2) + ((b*c + 3*a*d)*x)/(8*c^2*d*(c +
d*x^2)) + ((b*c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)\int\frac{1}{\left(c+dx^2\right)^2}\,dx}{4cd}\\
&= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\int\frac{1}{c+dx^2}\,dx}{8c^2d}\\
&= -\frac{(bc-ad)x}{4cd\left(c+dx^2\right)^2}+\frac{(bc+3ad)x}{8c^2d\left(c+dx^2\right)}+\frac{(bc+3ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int\frac{a+bx^2}{\left(c+dx^2\right)^3}\,dx=\frac{x\left(bc\left(-c+dx^2\right)+ad\left(5c+3dx^2\right)\right)}{8c^2d\left(c+dx^2\right)^2}+\frac{(bc+3ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{3/2}}$$

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^3,x]
[Out] (x*(b*c*(-c + d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2*d*(c + d*x^2)^2) + ((b*
c + 3*a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(3/2))
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(d\,x^2+c)^2}+\frac{(3ad+bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d\sqrt{cd}}$	77
risch	$\frac{\frac{(3ad+bc)x^3}{8c^2}+\frac{(5ad-bc)x}{8cd}}{(d\,x^2+c)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}+\frac{3\ln\left(-dx+\sqrt{-cd}\right)a}{16\sqrt{-cd}\,c^2}+\frac{\ln\left(-dx+\sqrt{-cd}\right)b}{16\sqrt{-cd}\,dc}$	147

```
[In] int((b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*d+b*c)/c^2*x^3+1/8*(5*a*d-b*c)/c/d*x)/(d*x^2+c)^2+1/8*(3*a*d+b*c)
/c^2/d/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{2\,(bc^2d^2+3\,acd^3)x^3-((bcd^2+3\,ad^3)x^4+bc^3+3\,ac^2d+2\,(bc^2d+3\,acd^2)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)-}{16\,(c^3d^4x^4+2\,c^4d^3x^2+c^5d^2)}\right.$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(b*c^2*d^2 + 3*a*c*d^3)*x^3 - ((b*c*d^2 + 3*a*d^3)*x^4 + b*c^3 + 3
*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d
)*x - c)/(d*x^2 + c)) - 2*(b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d
^3*x^2 + c^5*d^2), 1/8*((b*c^2*d^2 + 3*a*c*d^3)*x^3 + ((b*c*d^2 + 3*a*d^3)*
x^4 + b*c^3 + 3*a*c^2*d + 2*(b*c^2*d + 3*a*c*d^2)*x^2)*sqrt(c*d)*arctan(sqr
t(c*d)*x/c) - (b*c^3*d - 5*a*c^2*d^2)*x)/(c^3*d^4*x^4 + 2*c^4*d^3*x^2 + c^5
*d^2)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(-c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^3}}\cdot(3ad+bc)\log\left(c^3d\sqrt{-\frac{1}{c^5d^3}}+x\right)}{16} + \frac{x^3\cdot(3ad^2+bcd)+x(5acd-bc^2)}{8c^4d+16c^3d^2x^2+8c^2d^3x^4}$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(-c**3*d*sqrt(-1/(c**5*d**3)) + x)/1
6 + sqrt(-1/(c**5*d**3))*(3*a*d + b*c)*log(c**3*d*sqrt(-1/(c**5*d**3)) + x)
/16 + (x**3*(3*a*d**2 + b*c*d) + x*(5*a*c*d - b*c**2))/(8*c**4*d + 16*c**3*
d**2*x**2 + 8*c**2*d**3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{(bcd+3\,ad^2)x^3-(bc^2-5\,acd)x}{8\,(c^2d^3x^4+2\,c^3d^2x^2+c^4d)} + \frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b*c*d + 3*a*d^2)*x^3 - (b*c^2 - 5*a*c*d)*x)/(c^2*d^3*x^4 + 2*c^3*d^2*
x^2 + c^4*d) + 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{(bc+3\,ad)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d} + \frac{bcdx^3+3\,ad^2x^3-bc^2x+5\,acdx}{8\,(dx^2+c)^2c^2d}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b*c + 3*a*d)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d) + 1/8*(b*c*d*x^3
+ 3*a*d^2*x^3 - b*c^2*x + 5*a*c*d*x)/((d*x^2 + c)^2*c^2*d)
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{(c+dx^2)^3}\,dx = \frac{\frac{x^3\,(3\,a\,d+b\,c)}{8\,c^2}+\frac{x\,(5\,a\,d-b\,c)}{8\,c\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(3\,a\,d+b\,c)}{8\,c^{5/2}\,d^{3/2}}$$

```
[In] int((a + b*x^2)/(c + d*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*d + b*c))/(8*c^2) + (x*(5*a*d - b*c))/(8*c*d))/(c^2 + d^2*x^4 +
2*c*d*x^2) + (atan((d^(1/2)*x)/c^(1/2))*(3*a*d + b*c))/(8*c^(5/2)*d^(3/2))
```

3.8 $\int (a+bx^2)^2 (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2c^3x + \frac{1}{3}ac^2(2bc+3ad)x^3 + \frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5 + \frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7 + \frac{1}{9}bd^2(3bc+2ad)x^9 + \frac{1}{11}b^2d^3x^{11}$$

```
[Out] a^2*c^3*x+1/3*a*c^2*(3*a*d+2*b*c)*x^3+1/5*c*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x
^5+1/7*d*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x^7+1/9*b*d^2*(2*a*d+3*b*c)*x^9+1/11
*b^2*d^3*x^11
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {380}

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{1}{7}dx^7\left(a^2d^2+6abcd+3b^2c^2\right) + \frac{1}{5}cx^5\left(3a^2d^2+6abcd+b^2c^2\right) + a^2c^3x + \frac{1}{3}ac^2x^3(3ad+2bc) + \frac{1}{9}bd^2x^9(2ad+3bc) + \frac{1}{11}b^2d^3x^{11}$$

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a^2c^3+ac^2(2bc+3ad)x^2+c\left(b^2c^2+6abcd+3a^2d^2\right)x^4\right.\\
&\qquad \left.+d\left(3b^2c^2+6abcd+a^2d^2\right)x^6+bd^2(3bc+2ad)x^8+b^2d^3x^{10}\right)\,dx\\
&= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5\\
&\quad +\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+\frac{1}{11}b^2d^3x^{11}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int \left(a+bx^2\right)^2\left(c+dx^2\right)^3\,dx &= a^2c^3x+\frac{1}{3}ac^2(2bc+3ad)x^3+\frac{1}{5}c\left(b^2c^2+6abcd+3a^2d^2\right)x^5\\
&\quad +\frac{1}{7}d\left(3b^2c^2+6abcd+a^2d^2\right)x^7+\frac{1}{9}bd^2(3bc+2ad)x^9+\frac{1}{11}b^2d^3x^{11}
\end{aligned}
$$

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^3)/3 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^5)/5 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d^2*(3*b*c
+ 2*a*d)*x^9)/9 + (b^2*d^3*x^11)/11
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^2d^3x^{11}}{11}+\left(\frac{2}{9}ab\,d^3+\frac{1}{3}b^2c\,d^2\right)x^9+\left(\frac{1}{7}a^2d^3+\frac{6}{7}abc\,d^2+\frac{3}{7}b^2c^2d\right)x^7+\left(\frac{3}{5}c\,a^2d^2+\frac{6}{5}ab\,c^2d+\frac{1}{5}b^2c^3\right)$
default	$\frac{b^2d^3x^{11}}{11}+\frac{(2ab\,d^3+3b^2c\,d^2)x^9}{9}+\frac{(a^2d^3+6abc\,d^2+3b^2c^2d)x^7}{7}+\frac{(3c\,a^2d^2+6ab\,c^2d+b^2c^3)x^5}{5}+\frac{(3a^2c^2d+2ab\,c^3)x^3}{3}+a^2$
gosper	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2d$
risch	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2d$
parallelrisch	$\frac{1}{11}b^2d^3x^{11}+\frac{2}{9}x^9ab\,d^3+\frac{1}{3}x^9b^2c\,d^2+\frac{1}{7}x^7a^2d^3+\frac{6}{7}x^7abc\,d^2+\frac{3}{7}x^7b^2c^2d+\frac{3}{5}x^5c\,a^2d^2+\frac{6}{5}x^5ab\,c^2d$

```
[In] int((b*x^2+a)^2*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*d^3*x^11+(2/9*a*b*d^3+1/3*b^2*c*d^2)*x^9+(1/7*a^2*d^3+6/7*a*b*c*d^
2+3/7*b^2*c^2*d)*x^7+(3/5*c*a^2*d^2+6/5*a*b*c^2*d+1/5*b^2*c^3)*x^5+(a^2*c^2
*d+2/3*a*b*c^3)*x^3+a^2*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = \frac{1}{11} b^2 d^3 x^{11} + \frac{1}{9} \left(3 b^2 c d^2 + 2 abd^3\right) x^9 + \frac{1}{7} \left(3 b^2 c^2 d + 6 abcd^2 + a^2 d^3\right) x^7 + a^2 c^3 x + \frac{1}{5} \left(b^2 c^3 + 6 abc^2 d + 3 a^2 c d^2\right) x^5 + \frac{1}{3} \left(2 abc^3 + 3 a^2 c^2 d\right) x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int (a+bx^2)^2 (c+dx^2)^3 \, dx = a^2 c^3 x + \frac{b^2 d^3 x^{11}}{11} + x^9 \cdot \left(\frac{2abd^3}{9} + \frac{b^2 c d^2}{3}\right) + x^7 \left(\frac{a^2 d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2 c^2 d}{7}\right) + x^5 \cdot \left(\frac{3a^2 c d^2}{5} + \frac{6abc^2 d}{5} + \frac{b^2 c^3}{5}\right) + x^3 \left(a^2 c^2 d + \frac{2abc^3}{3}\right)$$

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**11/11 + x**9*(2*a*b*d**3/9 + b**2*c*d**2/3) + x*
*7*(a**2*d**3/7 + 6*a*b*c*d**2/7 + 3*b**2*c**2*d/7) + x**5*(3*a**2*c*d**2/5
 + 6*a*b*c**2*d/5 + b**2*c**3/5) + x**3*(a**2*c**2*d + 2*a*b*c**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int (a+bx^2)^2 (c+dx^2)^3 dx = \frac{1}{11} b^2d^3x^{11} + \frac{1}{9}(3b^2cd^2 + 2abd^3)x^9 + \frac{1}{7}(3b^2c^2d + 6abcd^2 + a^2d^3)x^7 + a^2c^3x + \frac{1}{5}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^5 + \frac{1}{3}(2abc^3 + 3a^2c^2d)x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/9*(3*b^2*c*d^2 + 2*a*b*d^3)*x^9 + 1/7*(3*b^2*c^2*d +
6*a*b*c*d^2 + a^2*d^3)*x^7 + a^2*c^3*x + 1/5*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2
*c*d^2)*x^5 + 1/3*(2*a*b*c^3 + 3*a^2*c^2*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^2 (c+dx^2)^3 dx = \frac{1}{11} b^2d^3x^{11} + \frac{1}{3} b^2cd^2x^9 + \frac{2}{9} abd^3x^9 + \frac{3}{7} b^2c^2dx^7 + \frac{6}{7} abcd^2x^7 + \frac{1}{7} a^2d^3x^7 + \frac{1}{5} b^2c^3x^5 + \frac{6}{5} abc^2dx^5 + \frac{3}{5} a^2cd^2x^5 + \frac{2}{3} abc^3x^3 + a^2c^2dx^3 + a^2c^3x$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/11*b^2*d^3*x^11 + 1/3*b^2*c*d^2*x^9 + 2/9*a*b*d^3*x^9 + 3/7*b^2*c^2*d*x^7
 + 6/7*a*b*c*d^2*x^7 + 1/7*a^2*d^3*x^7 + 1/5*b^2*c^3*x^5 + 6/5*a*b*c^2*d*x^
5 + 3/5*a^2*c*d^2*x^5 + 2/3*a*b*c^3*x^3 + a^2*c^2*d*x^3 + a^2*c^3*x
```

Mupad [B] (verification not implemented)

Time = 4.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int (a+bx^2)^2 (c+dx^2)^3 dx = x^5\left(\frac{3a^2cd^2}{5} + \frac{6abc^2d}{5} + \frac{b^2c^3}{5}\right) + x^7\left(\frac{a^2d^3}{7} + \frac{6abcd^2}{7} + \frac{3b^2c^2d}{7}\right) + a^2c^3x + \frac{b^2d^3x^{11}}{11} + \frac{ac^2x^3(3ad+2bc)}{3} + \frac{bd^2x^9(2ad+3bc)}{9}$$

```
[In] int((a + b*x^2)^2*(c + d*x^2)^3,x)
```

```
[Out] x^5*((b^2*c^3)/5 + (3*a^2*c*d^2)/5 + (6*a*b*c^2*d)/5) + x^7*((a^2*d^3)/7 +
(3*b^2*c^2*d)/7 + (6*a*b*c*d^2)/7) + a^2*c^3*x + (b^2*d^3*x^11)/11 + (a*c^2
*x^3*(3*a*d + 2*b*c))/3 + (b*d^2*x^9*(2*a*d + 3*b*c))/9
```

3.9 $\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5 +\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

```
[Out] a^2*c^2*x+2/3*a*c*(a*d+b*c)*x^3+1/5*(a^2*d^2+4*a*b*c*d+b^2*c^2)*x^5+2/7*b*d
*(a*d+b*c)*x^7+1/9*b^2*d^2*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{380\}$

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{5}x^5\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x +\frac{2}{7}bdx^7(ad+bc)+\frac{2}{3}acx^3(ad+bc)+\frac{1}{9}b^2d^2x^9$$

```
[In] Int[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \int \left(a^2c^2+2ac(bc+ad)x^2+\left(b^2c^2+4abcd+a^2d^2\right)x^4+2bd(bc+ad)x^6+b^2d^2x^8\right)\,dx$$
$$= a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5+\frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2\left(c+dx^2\right)^2\,dx = a^2c^2x+\frac{2}{3}ac(bc+ad)x^3+\frac{1}{5}\left(b^2c^2+4abcd+a^2d^2\right)x^5 + \frac{2}{7}bd(bc+ad)x^7+\frac{1}{9}b^2d^2x^9$$

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^3)/3 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
5)/5 + (2*b*d*(b*c + a*d)*x^7)/7 + (b^2*d^2*x^9)/9
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
norman	$\frac{b^2d^2x^9}{9}+\left(\frac{2}{7}ab\,d^2+\frac{2}{7}b^2cd\right)x^7+\left(\frac{1}{5}a^2d^2+\frac{4}{5}abcd+\frac{1}{5}b^2c^2\right)x^5+\left(\frac{2}{3}a^2cd+\frac{2}{3}b\,c^2a\right)x^3+a^2c^2x$
default	$\frac{b^2d^2x^9}{9}+\frac{(2ab\,d^2+2b^2cd)x^7}{7}+\frac{(a^2d^2+4abcd+b^2c^2)x^5}{5}+\frac{(2a^2cd+2b\,c^2a)x^3}{3}+a^2c^2x$
gosper	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3b\,c^2a+a^2c^2$
risch	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3b\,c^2a+a^2c^2$
parallelrisch	$\frac{1}{9}b^2d^2x^9+\frac{2}{7}x^7ab\,d^2+\frac{2}{7}x^7b^2cd+\frac{1}{5}x^5a^2d^2+\frac{4}{5}x^5abcd+\frac{1}{5}x^5b^2c^2+\frac{2}{3}x^3a^2cd+\frac{2}{3}x^3b\,c^2a+a^2c^2$

```
[In] int((b*x^2+a)^2*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*d^2*x^9+(2/7*a*b*d^2+2/7*b^2*c*d)*x^7+(1/5*a^2*d^2+4/5*a*b*c*d+1/5*
b^2*c^2)*x^5+(2/3*a^2*c*d+2/3*b*c^2*a)*x^3+a^2*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9} b^2 d^2 x^9 + \frac{2}{7} \left(b^2 cd + abd^2\right) x^7 + \frac{1}{5} \left(b^2 c^2 + 4 \, abcd + a^2 d^2\right) x^5 + a^2 c^2 x + \frac{2}{3} \left(abc^2 + a^2 cd\right) x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = a^2 c^2 x + \frac{b^2 d^2 x^9}{9} + x^7 \cdot \left(\frac{2abd^2}{7} + \frac{2b^2 cd}{7}\right) + x^5 \left(\frac{a^2 d^2}{5} + \frac{4abcd}{5} + \frac{b^2 c^2}{5}\right) + x^3 \cdot \left(\frac{2a^2 cd}{3} + \frac{2abc^2}{3}\right)$$

```
[In] integrate((b*x**2+a)**2*(d*x**2+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**9/9 + x**7*(2*a*b*d**2/7 + 2*b**2*c*d/7) + x**5*
(a**2*d**2/5 + 4*a*b*c*d/5 + b**2*c**2/5) + x**3*(2*a**2*c*d/3 + 2*a*b*c**2
/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9} b^2 d^2 x^9 + \frac{2}{7} \left(b^2 cd + abd^2\right) x^7 + \frac{1}{5} \left(b^2 c^2 + 4 \, abcd + a^2 d^2\right) x^5 + a^2 c^2 x + \frac{2}{3} \left(abc^2 + a^2 cd\right) x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*(b^2*c*d + a*b*d^2)*x^7 + 1/5*(b^2*c^2 + 4*a*b*c*d +
a^2*d^2)*x^5 + a^2*c^2*x + 2/3*(a*b*c^2 + a^2*c*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = \frac{1}{9}\,b^2d^2x^9+\frac{2}{7}\,b^2cdx^7+\frac{2}{7}\,abd^2x^7+\frac{1}{5}\,b^2c^2x^5 + \frac{4}{5}\,abcdx^5+\frac{1}{5}\,a^2d^2x^5+\frac{2}{3}\,abc^2x^3+\frac{2}{3}\,a^2cdx^3+a^2c^2x$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*d^2*x^9 + 2/7*b^2*c*d*x^7 + 2/7*a*b*d^2*x^7 + 1/5*b^2*c^2*x^5 + 4/5
*a*b*c*d*x^5 + 1/5*a^2*d^2*x^5 + 2/3*a*b*c^2*x^3 + 2/3*a^2*c*d*x^3 + a^2*c^
2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)^2 \, dx = x^5\left(\frac{a^2\,d^2}{5}+\frac{4\,a\,b\,c\,d}{5}+\frac{b^2\,c^2}{5}\right)+a^2\,c^2\,x+\frac{b^2\,d^2\,x^9}{9} + \frac{2\,a\,c\,x^3\,\left(a\,d+b\,c\right)}{3}+\frac{2\,b\,d\,x^7\,\left(a\,d+b\,c\right)}{7}$$

```
[In] int((a + b*x^2)^2*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^2*d^2)/5 + (b^2*c^2)/5 + (4*a*b*c*d)/5) + a^2*c^2*x + (b^2*d^2*x^9)
/9 + (2*a*c*x^3*(a*d + b*c))/3 + (2*b*d*x^7*(a*d + b*c))/7
```

3.10 $\int (a+bx^2)^2 (c+dx^2) \, dx$

Optimal result

Integrand size = 17, antiderivative size = 50

$$\int (a+bx^2)^2 (c+dx^2) \, dx = a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7$$

```
[Out] a^2*c*x+1/3*a*(a*d+2*b*c)*x^3+1/5*b*(2*a*d+b*c)*x^5+1/7*b^2*d*x^7
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^2)^2 (c+dx^2) \, dx = a^2cx + \frac{1}{5}bx^5(2ad+bc) + \frac{1}{3}ax^3(ad+2bc) + \frac{1}{7}b^2dx^7$$

```
[In] Int[(a + b*x^2)^2*(c + d*x^2),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(a^2c + a(2bc+ad)x^2 + b(bc+2ad)x^4 + b^2dx^6\right) dx \\ &= a^2cx + \frac{1}{3}a(2bc+ad)x^3 + \frac{1}{5}b(bc+2ad)x^5 + \frac{1}{7}b^2dx^7 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx = a^2cx+\frac{1}{3}a(2bc+ad)x^3+\frac{1}{5}b(bc+2ad)x^5+\frac{1}{7}b^2dx^7$$

```
[In] Integrate[(a + b*x^2)^2*(c + d*x^2),x]
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^3)/3 + (b*(b*c + 2*a*d)*x^5)/5 + (b^2*d*x^7)/7
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{b^2 d\,x^7}{7}+\frac{(2abd+b^2c)x^5}{5}+\frac{(a^2d+2abc)x^3}{3}+a^2cx$	49
norman	$\frac{b^2 d\,x^7}{7}+\left(\frac{2}{5}abd+\frac{1}{5}b^2c\right)x^5+\left(\frac{1}{3}a^2d+\frac{2}{3}abc\right)x^3+a^2cx$	49
gosper	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51
risch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51
parallelrisch	$\frac{1}{7}b^2d\,x^7+\frac{2}{5}x^5abd+\frac{1}{5}x^5b^2c+\frac{1}{3}x^3a^2d+\frac{2}{3}abc\,x^3+a^2cx$	51

```
[In] int((b*x^2+a)^2*(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] 1/7*b^2*d*x^7+1/5*(2*a*b*d+b^2*c)*x^5+1/3*(a^2*d+2*a*b*c)*x^3+a^2*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\,dx = \frac{1}{7}\,b^2dx^7+\frac{1}{5}\,\left(b^2c+2\,abd\right)x^5+a^2cx+\frac{1}{3}\,\left(2\,abc+a^2d\right)x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="fricas")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = a^2cx + \frac{b^2dx^7}{7} + x^5\cdot\left(\frac{2abd}{5}+\frac{b^2c}{5}\right) + x^3\left(\frac{a^2d}{3}+\frac{2abc}{3}\right)$$

```
[In] integrate((b*x**2+a)**2*(d*x**2+c),x)
[Out] a**2*c*x + b**2*d*x**7/7 + x**5*(2*a*b*d/5 + b**2*c/5) + x**3*(a**2*d/3 + 2
*a*b*c/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = \frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,\left(b^2c+2\,abd\right)x^5 + a^2cx + \frac{1}{3}\,\left(2\,abc+a^2d\right)x^3$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="maxima")
[Out] 1/7*b^2*d*x^7 + 1/5*(b^2*c + 2*a*b*d)*x^5 + a^2*c*x + 1/3*(2*a*b*c + a^2*d)
*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right)\, dx = \frac{1}{7}\,b^2dx^7 + \frac{1}{5}\,b^2cx^5 + \frac{2}{5}\,abdx^5 + \frac{2}{3}\,abcx^3 + \frac{1}{3}\,a^2dx^3 + a^2cx$$

```
[In] integrate((b*x^2+a)^2*(d*x^2+c),x, algorithm="giac")
[Out] 1/7*b^2*d*x^7 + 1/5*b^2*c*x^5 + 2/5*a*b*d*x^5 + 2/3*a*b*c*x^3 + 1/3*a^2*d*x
^3 + a^2*c*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^2 \left(c+dx^2\right) \, dx = x^3 \left(\frac{d\,a^2}{3}+\frac{2\,b\,c\,a}{3}\right)+x^5 \left(\frac{c\,b^2}{5}+\frac{2\,a\,d\,b}{5}\right)+\frac{b^2\,d\,x^7}{7}+a^2\,c\,x$$

```
[In] int((a + b*x^2)^2*(c + d*x^2),x)
[Out] x^3*((a^2*d)/3 + (2*a*b*c)/3) + x^5*((b^2*c)/5 + (2*a*b*d)/5) + (b^2*d*x^7)
/7 + a^2*c*x
```

3.11 $\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}$$

```
[Out] -b*(-2*a*d+b*c)*x/d^2+1/3*b^2*x^3/d+(-a*d+b*c)^2*arctan(x*d^(1/2)/c^(1/2))/
d^(5/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}-\frac{bx(bc-2ad)}{d^2}+\frac{b^2x^3}{3d}$$

```
[In] Int[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^3)/(3*d) + ((b*c - a*d)^2*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^2}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^2\right)}\right)dx \\
&= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^2}\,dx}{d^2} \\
&= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^3}{3d}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$
\int \frac{\left(a+bx^2\right)^2}{c+dx^2}\,dx=\frac{bx\left(-3bc+6ad+bdx^2\right)}{3d^2}+\frac{(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{5/2}}
$$

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2),x]
```

```
[Out] (b*x*(-3*b*c + 6*a*d + b*d*x^2))/(3*d^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/(Sqrt[c]*d^(5/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$\frac{b\left(\frac{1}{3}bd\,x^3+2adx-bcx\right)}{d^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^2\sqrt{cd}}$
risch	$\frac{b^2x^3}{3d}+\frac{2bax}{d}-\frac{b^2cx}{d^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}}+\frac{\ln\left(dx+\sqrt{-cd}\right)abc}{d\sqrt{-cd}}-\frac{\ln\left(dx+\sqrt{-cd}\right)b^2c^2}{2d^2\sqrt{-cd}}+\frac{\ln\left(-dx+\sqrt{-cd}\right)a^2}{2\sqrt{-cd}}-\frac{\ln(-dx+\sqrt{-}}{d\sqrt{-c}}$

```
[In] int((b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^2*(1/3*b*d*x^3+2*a*d*x-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2/(c*d)^(1/
2)*arctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.84

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx$$
$$= \left[\frac{2\,b^2cd^2x^3 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-cd}\log\left(\frac{dx^2 - 2\,\sqrt{-cd}x - c}{dx^2+c}\right) - 6\,(b^2c^2d - 2\,abcd^2)x}{6\,cd^3}, \frac{b^2cd^2x^3 + 3\,(b^2c^2 -}{}\right.$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b^2*c*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d)*log((d
*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) - 6*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c
*d^3), 1/3*(b^2*c*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)*arc
tan(sqrt(c*d)*x/c) - 3*(b^2*c^2*d - 2*a*b*c*d^2)*x)/(c*d^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(56) = 112.

Time = 0.24 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2x^3}{3d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) - \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(-\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^5}}(ad-bc)^2\log\left(\frac{cd^2\sqrt{-\frac{1}{cd^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2} + x\right)}{2}$$

```
[In] integrate((b*x**2+a)**2/(d*x**2+c),x)
```

```
[Out] b**2*x**3/(3*d) + x*(2*a*b/d - b**2*c/d**2) - sqrt(-1/(c*d**5))*(a*d - b*c)
**2*log(-c*d**2*sqrt(-1/(c*d**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b
**2*c**2) + x)/2 + sqrt(-1/(c*d**5))*(a*d - b*c)**2*log(c*d**2*sqrt(-1/(c*d
**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2dx^3-3\,(b^2c-2\,abd)x}{3\,d^2}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d*x^3 - 3*(b^2*c - 2*a*b*d)*x)/d^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^2} + \frac{b^2d^2x^3-3\,b^2cdx+6\,abd^2x}{3\,d^3}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*d^2) + 1/3
*(b^2*d^2*x^3 - 3*b^2*c*d*x + 6*a*b*d^2*x)/d^3
```

Mupad [B] (verification not implemented)

Time = 4.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^2}{c+dx^2}\,dx = \frac{b^2\,x^3}{3\,d} - x\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2}{\sqrt{c}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{c}\,d^{5/2}}$$

```
[In] int((a + b*x^2)^2/(c + d*x^2),x)
```

```
[Out] (b^2*x^3)/(3*d) - x*((b^2*c)/d^2 - (2*a*b)/d) + (atan((d^(1/2)*x*(a*d - b*c
)^2)/(c^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(c^(1/2)*d^(
5/2))
```

3.12 $\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2(c+dx^2)} - \frac{(bc-ad)(3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

```
[Out] b^2*x/d^2+1/2*(-a*d+b*c)^2*x/c/d^2/(d*x^2+c)-1/2*(-a*d+b*c)*(a*d+3*b*c)*arc
tan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)(ad+3bc)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}} + \frac{x(bc-ad)^2}{2cd^2(c+dx^2)} + \frac{b^2x}{d^2}$$

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((b*c - a*d)*(3*b*c
 + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b^2}{d^2} - \frac{b^2c^2 - a^2d^2 + 2bd(bc-ad)x^2}{d^2\left(c+dx^2\right)^2}\right) dx \\
&= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{(c+dx^2)^2}\, dx}{d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{((bc-ad)(3bc+ad))\int \frac{1}{c+dx^2}\, dx}{2cd^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{(bc-ad)(3bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^2}\, dx = \frac{b^2x}{d^2} + \frac{(bc-ad)^2x}{2cd^2\left(c+dx^2\right)} - \frac{\left(3b^2c^2-2abcd-a^2d^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{5/2}}$$

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(2*c*d^2*(c + d*x^2)) - ((3*b^2*c^2 - 2*a*b
*c*d - a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*d^(5/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

method	result
default	$\frac{b^2 x}{d^2}+\frac{\frac{\left(a^2 d^2-2abcd+b^2 c^2\right)x}{2c\left(d\,x^2+c\right)}+\frac{\left(a^2 d^2+2abcd-3b^2 c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}}{d^2}$
risch	$\frac{b^2 x}{d^2}+\frac{(a^2 d^2-2abcd+b^2c^2)x}{2c\,d^2(d\,x^2+c)}-\frac{\ln(dx+\sqrt{-cd})a^2}{4\sqrt{-cd}\,c}-\frac{\ln(dx+\sqrt{-cd})ab}{2d\sqrt{-cd}}+\frac{3c\ln(dx+\sqrt{-cd})b^2}{4d^2\sqrt{-cd}}+\frac{\ln(-dx+\sqrt{-cd})a^2}{4\sqrt{-cd}\,c}+\frac{\ln(-dx+}{2d\sqrt{}}$

```
[In] int((b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2*x/d^2+1/d^2*(1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c*x/(d*x^2+c)+1/2*(a^2*d^2
+2*a*b*c*d-3*b^2*c^2)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.68

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx$$
$$=\left[\frac{4\,b^2c^2d^2x^3+(3\,b^2c^3-2\,abc^2d-a^2cd^2+(3\,b^2c^2d-2\,abcd^2-a^2d^3)x^2)\sqrt{-cd}\log\left(\frac{dx^2-2\sqrt{-cd}x-c}{dx^2+c}\right)+2\,(3\,b^2c}{4\,(c^2d^4x^2+c^3d^3)}\right.$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*b^2*c^2*d^2*x^3 + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2
*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c
)/(d*x^2 + c)) + 2*(3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^
2 + c^3*d^3), 1/2*(2*b^2*c^2*d^2*x^3 - (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/
c) + (3*b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)/(c^2*d^4*x^2 + c^3*d^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(73) = 146.

Time = 0.40 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} + \frac{x(a^2d^2-2abcd+b^2c^2)}{2c^2d^2+2cd^3x^2} - \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(-\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^5}}\,(ad-bc)\,(ad+3bc)\log\left(\frac{c^2d^2\sqrt{-\frac{1}{c^3d^5}}(ad-bc)(ad+3bc)}{a^2d^2+2abcd-3b^2c^2}+x\right)}{4}$$

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*c**2*d**2 + 2*c*d**3
*x**2) - sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(-c**2*d**2*sqrt
(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*b**2*
c**2) + x)/4 + sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)*log(c**2*d**2
*sqrt(-1/(c**3*d**5))*(a*d - b*c)*(a*d + 3*b*c)/(a**2*d**2 + 2*a*b*c*d - 3*
b**2*c**2) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(cd^3x^2+c^2d^2)} + \frac{b^2x}{d^2} - \frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^2 + c^2*d^2) + b^2*x/d^2 - 1
/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c*d^2
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2x}{d^2} - \frac{(3\,b^2c^2-2\,abcd-a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(dx^2+c)cd^2}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(d*x/sqrt(c*d))/(sq
rt(c*d)*c*d^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^2 + c)*c*d
^2)
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^2}\,dx = \frac{b^2\,x}{d^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,c\,(d^3\,x^2+c\,d^2)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{\sqrt{c}\,(a^2\,d^2+2\,a\,b\,c\,d-3\,b^2\,c^2)}\right)\,(a\,d-b\,c)\,(a\,d+3\,b\,c)}{2\,c^{3/2}\,d^{5/2}}$$

```
[In] int((a + b*x^2)^2/(c + d*x^2)^2,x)
```

```
[Out] (b^2*x)/d^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*c*(c*d^2 + d^3*x^2)) +
 (atan((d^(1/2)*x*(a*d - b*c)*(a*d + 3*b*c))/(c^(1/2)*(a^2*d^2 - 3*b^2*c^2
+ 2*a*b*c*d)))*(a*d - b*c)*(a*d + 3*b*c))/(2*c^(3/2)*d^(5/2))
```

3.13 $\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = -\frac{(bc-ad)x(a+bx^2)}{4cd(c+dx^2)^2} + \frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8(c+dx^2)} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^2+a)/c/d/(d*x^2+c)^2+3/8*(a^2/c^2-b^2/d^2)*x/(d*x^2+
c)+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/d^
(5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 393, 211}

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3} dx = \frac{(3a^2d^2+2abcd+3b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}} + \frac{3x\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)}{8(c+dx^2)} - \frac{x(a+bx^2)(bc-ad)}{4cd(c+dx^2)^2}$$

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^2))/(c*d*(c + d*x^2)^2) + (3*(a^2/c^2 - b^2/d^
2)*x)/(8*(c + d*x^2)) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(8*c^(5/2)*d^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^2)}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{a(bc+3ad)+b(3bc+ad)x^2}{\left(c+dx^2\right)^2}\,dx}{4cd}\\
&= -\frac{(bc-ad)x(a+bx^2)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\int \frac{1}{c+dx^2}\,dx}{8c^2d^2}\\
&= -\frac{(bc-ad)x(a+bx^2)}{4cd\left(c+dx^2\right)^2}+\frac{3\left(\frac{a^2}{c^2}-\frac{b^2}{d^2}\right)x}{8\left(c+dx^2\right)}+\frac{\left(3b^2c^2+2abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)x(ad(5c+3dx^2)+bc(3c+5dx^2))}{8c^2d^2\,(c+dx^2)^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{5/2}}$$

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^3,x]
```

```
[Out] -1/8*((b*c - a*d)*x*(a*d*(5*c + 3*d*x^2) + b*c*(3*c + 5*d*x^2)))/(c^2*d^2*(
c + d*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(8*c^(5/2)*d^(5/2))
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{(d\,x^2+c)^2}+\frac{(3a^2d^2+2abcd+3b^2c^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2d^2\sqrt{cd}}$
risch	$\frac{\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x^3}{8c^2d}+\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x}{8c\,d^2}}{(d\,x^2+c)^2}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2}{16\sqrt{-cd}\,c^2}-\frac{\ln\left(dx+\sqrt{-cd}\right)ab}{8\sqrt{-cd}\,dc}-\frac{3\ln\left(dx+\sqrt{-cd}\right)b^2}{16\sqrt{-cd}\,d^2}+\frac{3\ln(-dx+}{16\sqrt{-}}$

```
[In] int((b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a^2*d^2+2*a*b*c*d-5*b^2*c^2)/c^2/d*x^3+1/8*(5*a^2*d^2-2*a*b*c*d-3*b
^2*c^2)/c/d^2*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/c^2/d^2/(c
*d)^(1/2)*arctan(d*x/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(102) = 204$.

Time = 0.28 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.87

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx$$
$$= \Bigg[-\frac{2\,(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 + (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(}{16\,(c^3d^5x^4 + 2\,c^4d^4}$$
$$-\frac{(5\,b^2c^3d^2 - 2\,abc^2d^3 - 3\,a^2cd^4)x^3 - (3\,b^2c^4 + 2\,abc^3d + 3\,a^2c^2d^2 + (3\,b^2c^2d^2 + 2\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(3\,}{8\,(c^3d^5x^4 + 2\,c^4d^4x^2 + c}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b^2*c^3*d^2 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 + (3*b^2*c^4 +
2*a*b*c^3*d + 3*a^2*c^2*d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4
 + 2*(3*b^2*c^3*d + 2*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-c*d)*log((d*x^2
 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(3*b^2*c^4*d + 2*a*b*c^3*d^2 - 5*a^
2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^3), -1/8*((5*b^2*c^3*d^2
 - 2*a*b*c^2*d^3 - 3*a^2*c*d^4)*x^3 - (3*b^2*c^4 + 2*a*b*c^3*d + 3*a^2*c^2*
d^2 + (3*b^2*c^2*d^2 + 2*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(3*b^2*c^3*d + 2*a*
b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (3*b^2*c^4*
d + 2*a*b*c^3*d^2 - 5*a^2*c^2*d^3)*x)/(c^3*d^5*x^4 + 2*c^4*d^4*x^2 + c^5*d^
3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.59 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{c^5d^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(c^3d^2\sqrt{-\frac{1}{c^5d^5}}+x\right)}{16} + \frac{x^3\cdot(3a^2d^3+2abcd^2-5b^2c^2d)+x(5a^2cd^2-2abc^2d-3b^2c^3)}{8c^4d^2+16c^3d^3x^2+8c^2d^4x^4}$$

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] -sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-c**3*d**
2*sqrt(-1/(c**5*d**5)) + x)/16 + sqrt(-1/(c**5*d**5))*(3*a**2*d**2 + 2*a*b*
```

```
c*d + 3*b**2*c**2)*log(c**3*d**2*sqrt(-1/(c**5*d**5)) + x)/16 + (x**3*(3*a*
*2*d**3 + 2*a*b*c*d**2 - 5*b**2*c**2*d) + x*(5*a**2*c*d**2 - 2*a*b*c**2*d -
 3*b**2*c**3))/(8*c**4*d**2 + 16*c**3*d**3*x**2 + 8*c**2*d**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = -\frac{(5\,b^2c^2d - 2\,abcd^2 - 3\,a^2d^3)x^3 + (3\,b^2c^3 + 2\,abc^2d - 5\,a^2cd^2)x}{8\,(c^2d^4x^4 + 2\,c^3d^3x^2 + c^4d^2)} + \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^3 + (3*b^2*c^3 + 2*a*b*c^2*
d - 5*a^2*c*d^2)*x)/(c^2*d^4*x^4 + 2*c^3*d^3*x^2 + c^4*d^2) + 1/8*(3*b^2*c^
2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{(3\,b^2c^2 + 2\,abcd + 3\,a^2d^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^2} - \frac{5\,b^2c^2dx^3 - 2\,abcd^2x^3 - 3\,a^2d^3x^3 + 3\,b^2c^3x + 2\,abc^2dx - 5\,a^2cd^2x}{8\,(dx^2+c)^2c^2d^2}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^
2*d^2) - 1/8*(5*b^2*c^2*d*x^3 - 2*a*b*c*d^2*x^3 - 3*a^2*d^3*x^3 + 3*b^2*c^3
*x + 2*a*b*c^2*d*x - 5*a^2*c*d^2*x)/((d*x^2 + c)^2*c^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 4.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{d}\,x}{\sqrt{c}}\right)\,(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c^{5/2}\,d^{5/2}} - \frac{\frac{x\,(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2)}{8\,c\,d^2}-\frac{x^3\,(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2)}{8\,c^2\,d}}{c^2+2\,c\,d\,x^2+d^2\,x^4}$$

```
[In] int((a + b*x^2)^2/(c + d*x^2)^3,x)
```

```
[Out] (atan((d^(1/2)*x)/c^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*c^(5/2)*
d^(5/2)) - ((x*(3*b^2*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*c*d^2) - (x^3*(3*a^2
*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*c^2*d))/(c^2 + d^2*x^4 + 2*c*d*x^2)
```

3.14 $\int (a+bx^2)^3 (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = a^3c^3x + a^2c^2(bc+ad)x^3 + \frac{3}{5}ac\left(b^2c^2+3abcd+a^2d^2\right)x^5 + \frac{1}{7}(bc+ad)\left(b^2c^2+8abcd+a^2d^2\right)x^7 + \frac{1}{3}bd\left(b^2c^2+3abcd+a^2d^2\right)x^9 + \frac{3}{11}b^2d^2(bc+ad)x^{11} + \frac{1}{13}b^3d^3x^{13}$$

```
[Out] a^3*c^3*x+a^2*c^2*(a*d+b*c)*x^3+3/5*a*c*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^5+1/7
*(a*d+b*c)*(a^2*d^2+8*a*b*c*d+b^2*c^2)*x^7+1/3*b*d*(a^2*d^2+3*a*b*c*d+b^2*c
^2)*x^9+3/11*b^2*d^2*(a*d+b*c)*x^11+1/13*b^3*d^3*x^13
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^2)^3 (c+dx^2)^3 \, dx = a^3c^3x + \frac{1}{3}bdx^9\left(a^2d^2+3abcd+b^2c^2\right) + \frac{1}{7}x^7(ad+bc)\left(a^2d^2+8abcd+b^2c^2\right) + \frac{3}{5}acx^5\left(a^2d^2+3abcd+b^2c^2\right) + a^2c^2x^3(ad+bc) + \frac{3}{11}b^2d^2x^{11}(ad+bc) + \frac{1}{13}b^3d^3x^{13}$$

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b*c + a*d)*(b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b^2*
c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11 + (b^3*
d^3*x^13)/13
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3c^3 + 3a^2c^2(bc+ad)x^2 + 3ac\left(b^2c^2 + 3abcd + a^2d^2\right)x^4 \right. \\ &\quad + (bc+ad)\left(b^2c^2 + 8abcd + a^2d^2\right)x^6 + 3bd\left(b^2c^2 + 3abcd + a^2d^2\right)x^8 \\ &\quad \left. + 3b^2d^2(bc+ad)x^{10} + b^3d^3x^{12}\right)\,dx \\ &= a^3c^3x + a^2c^2(bc+ad)x^3 + \frac{3}{5}ac\left(b^2c^2 + 3abcd + a^2d^2\right)x^5 \\ &\quad + \frac{1}{7}(bc+ad)\left(b^2c^2 + 8abcd + a^2d^2\right)x^7 \\ &\quad + \frac{1}{3}bd\left(b^2c^2 + 3abcd + a^2d^2\right)x^9 + \frac{3}{11}b^2d^2(bc+ad)x^{11} + \frac{1}{13}b^3d^3x^{13}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3\,dx &= a^3c^3x + a^2c^2(bc+ad)x^3 + \frac{3}{5}ac\left(b^2c^2 + 3abcd + a^2d^2\right)x^5 \\ &\quad + \frac{1}{7}\left(b^3c^3 + 9ab^2c^2d + 9a^2bcd^2 + a^3d^3\right)x^7 \\ &\quad + \frac{1}{3}bd\left(b^2c^2 + 3abcd + a^2d^2\right)x^9 + \frac{3}{11}b^2d^2(bc+ad)x^{11} + \frac{1}{13}b^3d^3x^{13}\end{aligned}$$

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^3,x]
```

```
[Out] a^3*c^3*x + a^2*c^2*(b*c + a*d)*x^3 + (3*a*c*(b^2*c^2 + 3*a*b*c*d + a^2*d^2
)*x^5)/5 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7)/7 + (b
*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^9)/3 + (3*b^2*d^2*(b*c + a*d)*x^11)/11
 + (b^3*d^3*x^13)/13
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11

method	result
norman	$\frac{b^{3} d^{3} x^{13}}{13}+\left(\frac{3}{11} a\, b^{2} d^{3}+\frac{3}{11} b^{3} c\, d^{2}\right) x^{11}+\left(\frac{1}{3} a^{2} b\, d^{3}+a\, b^{2} c\, d^{2}+\frac{1}{3} b^{3} c^{2} d\right) x^{9}+\left(\frac{1}{7} a^{3} d^{3}+\frac{9}{7} a^{2} b c\, d^{2}+\right.$
default	$\frac{b^{3} d^{3} x^{13}}{13}+\frac{\left(3 a\, b^{2} d^{3}+3 b^{3} c\, d^{2}\right) x^{11}}{11}+\frac{\left(3 a^{2} b\, d^{3}+9 a\, b^{2} c\, d^{2}+3 b^{3} c^{2} d\right) x^{9}}{9}+\frac{\left(a^{3} d^{3}+9 a^{2} b c\, d^{2}+9 a\, b^{2} c^{2} d+b^{3} c^{3}\right) x^{7}}{7}+\frac{\left(3 a^{3} c\, d^{2}+\right.}{}$
gosper	$\frac{1}{13} b^{3} d^{3} x^{13}+\frac{3}{11} x^{11} a\, b^{2} d^{3}+\frac{3}{11} x^{11} b^{3} c\, d^{2}+\frac{1}{3} x^{9} a^{2} b\, d^{3}+x^{9} a\, b^{2} c\, d^{2}+\frac{1}{3} x^{9} b^{3} c^{2} d+\frac{1}{7} x^{7} a^{3} d^{3}+\frac{9}{7} x^{7}$
risch	$\frac{1}{13} b^{3} d^{3} x^{13}+\frac{3}{11} x^{11} a\, b^{2} d^{3}+\frac{3}{11} x^{11} b^{3} c\, d^{2}+\frac{1}{3} x^{9} a^{2} b\, d^{3}+x^{9} a\, b^{2} c\, d^{2}+\frac{1}{3} x^{9} b^{3} c^{2} d+\frac{1}{7} x^{7} a^{3} d^{3}+\frac{9}{7} x^{7}$
parallelrisch	$\frac{1}{13} b^{3} d^{3} x^{13}+\frac{3}{11} x^{11} a\, b^{2} d^{3}+\frac{3}{11} x^{11} b^{3} c\, d^{2}+\frac{1}{3} x^{9} a^{2} b\, d^{3}+x^{9} a\, b^{2} c\, d^{2}+\frac{1}{3} x^{9} b^{3} c^{2} d+\frac{1}{7} x^{7} a^{3} d^{3}+\frac{9}{7} x^{7}$

```
[In] int((b*x^2+a)^3*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*d^3*x^13+(3/11*a*b^2*d^3+3/11*b^3*c*d^2)*x^11+(1/3*a^2*b*d^3+a*b^2
*c*d^2+1/3*b^3*c^2*d)*x^9+(1/7*a^3*d^3+9/7*a^2*b*c*d^2+9/7*a*b^2*c^2*d+1/7*
b^3*c^3)*x^7+(3/5*a^3*c*d^2+9/5*a^2*b*c^2*d+3/5*b^2*c^3*a)*x^5+(a^3*c^2*d+a
^2*b*c^3)*x^3+a^3*c^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^3\,dx=\frac{1}{13}\,b^3d^3x^{13}+\frac{3}{11}\,\left(b^3cd^2+ab^2d^3\right)x^{11}+\frac{1}{3}\,\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9+\frac{1}{7}\,\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x+\frac{3}{5}\,\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^5+\left(a^2bc^3+a^3c^2d\right)x^3$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx = a^3c^3x+\frac{b^3d^3x^{13}}{13}+x^{11}\cdot\left(\frac{3ab^2d^3}{11}+\frac{3b^3cd^2}{11}\right)+x^9\left(\frac{a^2bd^3}{3}+ab^2cd^2+\frac{b^3c^2d}{3}\right)+x^7\left(\frac{a^3d^3}{7}+\frac{9a^2bcd^2}{7}+\frac{9ab^2c^2d}{7}+\frac{b^3c^3}{7}\right)+x^5\cdot\left(\frac{3a^3cd^2}{5}+\frac{9a^2bc^2d}{5}+\frac{3ab^2c^3}{5}\right)+x^3\left(a^3c^2d+a^2bc^3\right)$$

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**13/13 + x**11*(3*a*b**2*d**3/11 + 3*b**3*c*d**2/
11) + x**9*(a**2*b*d**3/3 + a*b**2*c*d**2 + b**3*c**2*d/3) + x**7*(a**3*d**
3/7 + 9*a**2*b*c*d**2/7 + 9*a*b**2*c**2*d/7 + b**3*c**3/7) + x**5*(3*a**3*c
*d**2/5 + 9*a**2*b*c**2*d/5 + 3*a*b**2*c**3/5) + x**3*(a**3*c**2*d + a**2*b
*c**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx = \frac{1}{13}b^3d^3x^{13}+\frac{3}{11}\left(b^3cd^2+ab^2d^3\right)x^{11}+\frac{1}{3}\left(b^3c^2d+3\,ab^2cd^2+a^2bd^3\right)x^9+\frac{1}{7}\left(b^3c^3+9\,ab^2c^2d+9\,a^2bcd^2+a^3d^3\right)x^7+a^3c^3x+\frac{3}{5}\left(ab^2c^3+3\,a^2bc^2d+a^3cd^2\right)x^5+\left(a^2bc^3+a^3c^2d\right)x^3$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*(b^3*c*d^2 + a*b^2*d^3)*x^11 + 1/3*(b^3*c^2*d + 3*
a*b^2*c*d^2 + a^2*b*d^3)*x^9 + 1/7*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2
 + a^3*d^3)*x^7 + a^3*c^3*x + 3/5*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x
^5 + (a^2*b*c^3 + a^3*c^2*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.21

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx = \frac{1}{13}\, b^3d^3x^{13} + \frac{3}{11}\, b^3cd^2x^{11} + \frac{3}{11}\, ab^2d^3x^{11} + \frac{1}{3}\, b^3c^2dx^9 + ab^2cd^2x^9 + \frac{1}{3}\, a^2bd^3x^9 + \frac{1}{7}\, b^3c^3x^7 + \frac{9}{7}\, ab^2c^2dx^7 + \frac{9}{7}\, a^2bcd^2x^7 + \frac{1}{7}\, a^3d^3x^7 + \frac{3}{5}\, ab^2c^3x^5 + \frac{9}{5}\, a^2bc^2dx^5 + \frac{3}{5}\, a^3cd^2x^5 + a^2bc^3x^3 + a^3c^2dx^3 + a^3c^3x$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^3*d^3*x^13 + 3/11*b^3*c*d^2*x^11 + 3/11*a*b^2*d^3*x^11 + 1/3*b^3*c^2
*d*x^9 + a*b^2*c*d^2*x^9 + 1/3*a^2*b*d^3*x^9 + 1/7*b^3*c^3*x^7 + 9/7*a*b^2*
c^2*d*x^7 + 9/7*a^2*b*c*d^2*x^7 + 1/7*a^3*d^3*x^7 + 3/5*a*b^2*c^3*x^5 + 9/5
*a^2*b*c^2*d*x^5 + 3/5*a^3*c*d^2*x^5 + a^2*b*c^3*x^3 + a^3*c^2*d*x^3 + a^3*
c^3*x
```

Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^3 \, dx = x^7\left(\frac{a^3\,d^3}{7}+\frac{9\,a^2\,b\,c\,d^2}{7}+\frac{9\,a\,b^2\,c^2\,d}{7}+\frac{b^3\,c^3}{7}\right)+a^3\,c^3\,x + \frac{b^3\,d^3\,x^{13}}{13}+\frac{3\,a\,c\,x^5\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5} + \frac{b\,d\,x^9\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{3} + a^2\,c^2\,x^3\,\left(a\,d+b\,c\right)+\frac{3\,b^2\,d^2\,x^{11}\,\left(a\,d+b\,c\right)}{11}$$

```
[In] int((a + b*x^2)^3*(c + d*x^2)^3,x)
```

```
[Out] x^7*((a^3*d^3)/7 + (b^3*c^3)/7 + (9*a*b^2*c^2*d)/7 + (9*a^2*b*c*d^2)/7) + a
^3*c^3*x + (b^3*d^3*x^13)/13 + (3*a*c*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/
5 + (b*d*x^9*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/3 + a^2*c^2*x^3*(a*d + b*c) +
 (3*b^2*d^2*x^11*(a*d + b*c))/11
```

3.15 $\int (a+bx^2)^3 (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = a^3c^2x + \frac{1}{3}a^2c(3bc+2ad)x^3 + \frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5 + \frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)x^7 + \frac{1}{9}b^2d(2bc+3ad)x^9 + \frac{1}{11}b^3d^2x^{11}$$

```
[Out] a^3*c^2*x+1/3*a^2*c*(2*a*d+3*b*c)*x^3+1/5*a*(a^2*d^2+6*a*b*c*d+3*b^2*c^2)*x
^5+1/7*b*(3*a^2*d^2+6*a*b*c*d+b^2*c^2)*x^7+1/9*b^2*d*(3*a*d+2*b*c)*x^9+1/11
*b^3*d^2*x^11
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {380}

$$\int (a+bx^2)^3 (c+dx^2)^2 \, dx = a^3c^2x + \frac{1}{7}bx^7\left(3a^2d^2+6abcd+b^2c^2\right) + \frac{1}{5}ax^5\left(a^2d^2+6abcd+3b^2c^2\right) + \frac{1}{3}a^2cx^3(2ad+3bc) + \frac{1}{9}b^2dx^9(3ad+2bc) + \frac{1}{11}b^3d^2x^{11}$$

```
[In] Int[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3c^2 + a^2c(3bc+2ad)x^2 + a\left(3b^2c^2+6abcd+a^2d^2\right)x^4 \right.\\ &\left.\qquad + b\left(b^2c^2+6abcd+3a^2d^2\right)x^6 + b^2d(2bc+3ad)x^8 + b^3d^2x^{10}\right)\,dx\\ &= a^3c^2x + \frac{1}{3}a^2c(3bc+2ad)x^3 + \frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5\\ &\quad + \frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)x^7 + \frac{1}{9}b^2d(2bc+3ad)x^9 + \frac{1}{11}b^3d^2x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)^2\,dx = a^3c^2x + \frac{1}{3}a^2c(3bc+2ad)x^3 + \frac{1}{5}a\left(3b^2c^2+6abcd+a^2d^2\right)x^5 + \frac{1}{7}b\left(b^2c^2+6abcd+3a^2d^2\right)x^7 + \frac{1}{9}b^2d(2bc+3ad)x^9 + \frac{1}{11}b^3d^2x^{11}$$

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^3)/3 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^5)/5 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^7)/7 + (b^2*d*(2*b*c
+ 3*a*d)*x^9)/9 + (b^3*d^2*x^11)/11
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

method	result
norman	$\frac{b^3d^2x^{11}}{11} + \left(\frac{1}{3}a\,b^2d^2 + \frac{2}{9}b^3dc\right)x^9 + \left(\frac{3}{7}a^2b\,d^2 + \frac{6}{7}a\,b^2cd + \frac{1}{7}c^2b^3\right)x^7 + \left(\frac{1}{5}a^3d^2 + \frac{6}{5}a^2bcd + \frac{3}{5}a\,b^2c\right.$
default	$\frac{b^3d^2x^{11}}{11} + \frac{\left(3a\,b^2d^2+2b^3dc\right)x^9}{9} + \frac{\left(3a^2b\,d^2+6a\,b^2cd+c^2b^3\right)x^7}{7} + \frac{\left(a^3d^2+6a^2bcd+3a\,b^2c^2\right)x^5}{5} + \frac{\left(2a^3cd+3a^2b\,c^2\right)x^3}{3} + a$
gosper	$\frac{1}{11}b^3d^2x^{11} + \frac{1}{3}x^9a\,b^2d^2 + \frac{2}{9}x^9b^3dc + \frac{3}{7}x^7a^2b\,d^2 + \frac{6}{7}x^7a\,b^2cd + \frac{1}{7}b^3c^2x^7 + \frac{1}{5}x^5a^3d^2 + \frac{6}{5}x^5a^2bcd$
risch	$\frac{1}{11}b^3d^2x^{11} + \frac{1}{3}x^9a\,b^2d^2 + \frac{2}{9}x^9b^3dc + \frac{3}{7}x^7a^2b\,d^2 + \frac{6}{7}x^7a\,b^2cd + \frac{1}{7}b^3c^2x^7 + \frac{1}{5}x^5a^3d^2 + \frac{6}{5}x^5a^2bcd$
parallelrisch	$\frac{1}{11}b^3d^2x^{11} + \frac{1}{3}x^9a\,b^2d^2 + \frac{2}{9}x^9b^3dc + \frac{3}{7}x^7a^2b\,d^2 + \frac{6}{7}x^7a\,b^2cd + \frac{1}{7}b^3c^2x^7 + \frac{1}{5}x^5a^3d^2 + \frac{6}{5}x^5a^2bcd$

```
[In] int((b*x^2+a)^3*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*d^2*x^11+(1/3*a*b^2*d^2+2/9*b^3*d*c)*x^9+(3/7*a^2*b*d^2+6/7*a*b^2*
c*d+1/7*c^2*b^3)*x^7+(1/5*a^3*d^2+6/5*a^2*b*c*d+3/5*a*b^2*c^2)*x^5+(2/3*a^3
*c*d+a^2*b*c^2)*x^3+a^3*c^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx &= \frac{1}{11}\, b^3 d^2 x^{11} + \frac{1}{9}\left(2\, b^3 cd + 3\, ab^2 d^2\right) x^9 \\ &\quad + \frac{1}{7}\left(b^3 c^2 + 6\, ab^2 cd + 3\, a^2 bd^2\right) x^7 + a^3 c^2 x \\ &\quad + \frac{1}{5}\left(3\, ab^2 c^2 + 6\, a^2 bcd + a^3 d^2\right) x^5 + \frac{1}{3}\left(3\, a^2 bc^2 + 2\, a^3 cd\right) x^3\end{aligned}$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx &= a^3c^2x + \frac{b^3d^2x^{11}}{11} + x^9\left(\frac{ab^2d^2}{3} + \frac{2b^3cd}{9}\right) \\ &\quad + x^7 \cdot \left(\frac{3a^2bd^2}{7} + \frac{6ab^2cd}{7} + \frac{b^3c^2}{7}\right) \\ &\quad + x^5\left(\frac{a^3d^2}{5} + \frac{6a^2bcd}{5} + \frac{3ab^2c^2}{5}\right) + x^3 \cdot \left(\frac{2a^3cd}{3} + a^2bc^2\right)\end{aligned}$$

```
[In] integrate((b*x**2+a)**3*(d*x**2+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**11/11 + x**9*(a*b**2*d**2/3 + 2*b**3*c*d/9) + x*
*7*(3*a**2*b*d**2/7 + 6*a*b**2*c*d/7 + b**3*c**2/7) + x**5*(a**3*d**2/5 + 6
*a**2*b*c*d/5 + 3*a*b**2*c**2/5) + x**3*(2*a**3*c*d/3 + a**2*b*c**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx = \frac{1}{11} b^3 d^2 x^{11} + \frac{1}{9} \left(2 b^3 c d + 3 a b^2 d^2\right) x^9 + \frac{1}{7} \left(b^3 c^2 + 6 a b^2 c d + 3 a^2 b d^2\right) x^7 + a^3 c^2 x + \frac{1}{5} \left(3 a b^2 c^2 + 6 a^2 b c d + a^3 d^2\right) x^5 + \frac{1}{3} \left(3 a^2 b c^2 + 2 a^3 c d\right) x^3$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^2*x^11 + 1/9*(2*b^3*c*d + 3*a*b^2*d^2)*x^9 + 1/7*(b^3*c^2 + 6*a*
b^2*c*d + 3*a^2*b*d^2)*x^7 + a^3*c^2*x + 1/5*(3*a*b^2*c^2 + 6*a^2*b*c*d + a
^3*d^2)*x^5 + 1/3*(3*a^2*b*c^2 + 2*a^3*c*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx = \frac{1}{11} b^3 d^2 x^{11} + \frac{2}{9} b^3 c d x^9 + \frac{1}{3} a b^2 d^2 x^9 + \frac{1}{7} b^3 c^2 x^7 + \frac{6}{7} a b^2 c d x^7 + \frac{3}{7} a^2 b d^2 x^7 + \frac{3}{5} a b^2 c^2 x^5 + \frac{6}{5} a^2 b c d x^5 + \frac{1}{5} a^3 d^2 x^5 + a^2 b c^2 x^3 + \frac{2}{3} a^3 c d x^3 + a^3 c^2 x$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^2*x^11 + 2/9*b^3*c*d*x^9 + 1/3*a*b^2*d^2*x^9 + 1/7*b^3*c^2*x^7 +
 6/7*a*b^2*c*d*x^7 + 3/7*a^2*b*d^2*x^7 + 3/5*a*b^2*c^2*x^5 + 6/5*a^2*b*c*d*
x^5 + 1/5*a^3*d^2*x^5 + a^2*b*c^2*x^3 + 2/3*a^3*c*d*x^3 + a^3*c^2*x
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)^2 \, dx = x^5 \left(\frac{a^3 d^2}{5} + \frac{6 a^2 b c d}{5} + \frac{3 a b^2 c^2}{5}\right) + x^7 \left(\frac{3 a^2 b d^2}{7} + \frac{6 a b^2 c d}{7} + \frac{b^3 c^2}{7}\right) + a^3 c^2 x + \frac{b^3 d^2 x^{11}}{11} + \frac{a^2 c x^3 \left(2 a d + 3 b c\right)}{3} + \frac{b^2 d x^9 \left(3 a d + 2 b c\right)}{9}$$

```
[In] int((a + b*x^2)^3*(c + d*x^2)^2,x)
```

```
[Out] x^5*((a^3*d^2)/5 + (3*a*b^2*c^2)/5 + (6*a^2*b*c*d)/5) + x^7*((b^3*c^2)/7 +
(3*a^2*b*d^2)/7 + (6*a*b^2*c*d)/7) + a^3*c^2*x + (b^3*d^2*x^11)/11 + (a^2*c
*x^3*(2*a*d + 3*b*c))/3 + (b^2*d*x^9*(3*a*d + 2*b*c))/9
```

3.16 $\int (a+bx^2)^3 (c+dx^2)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int (a+bx^2)^3 (c+dx^2)\, dx = a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9$$

```
[Out] a^3*c*x+1/3*a^2*(a*d+3*b*c)*x^3+3/5*a*b*(a*d+b*c)*x^5+1/7*b^2*(3*a*d+b*c)*x
^7+1/9*b^3*d*x^9
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {380}

$$\int (a+bx^2)^3 (c+dx^2)\, dx = a^3cx+\frac{1}{3}a^2x^3(ad+3bc)+\frac{1}{7}b^2x^7(3ad+bc)+\frac{3}{5}abx^5(ad+bc)+\frac{1}{9}b^3dx^9$$

```
[In] Int[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(a^3c+a^2(3bc+ad)x^2+3ab(bc+ad)x^4+b^2(bc+3ad)x^6+b^3dx^8\right) dx \\ &= a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right) \, dx = a^3cx+\frac{1}{3}a^2(3bc+ad)x^3+\frac{3}{5}ab(bc+ad)x^5+\frac{1}{7}b^2(bc+3ad)x^7+\frac{1}{9}b^3dx^9$$

```
[In] Integrate[(a + b*x^2)^3*(c + d*x^2),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^3)/3 + (3*a*b*(b*c + a*d)*x^5)/5 + (b^2*(b*c
 + 3*a*d)*x^7)/7 + (b^3*d*x^9)/9
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

method	result	size
norman	$\frac{b^3 d\,x^9}{9}+\left(\frac{3}{7}a\,b^2 d+\frac{1}{7}b^3 c\right)x^7+\left(\frac{3}{5}a^2 bd+\frac{3}{5}a\,b^2 c\right)x^5+\left(\frac{1}{3}a^3 d+a^2 bc\right)x^3+a^3 cx$	71
default	$\frac{b^3 d\,x^9}{9}+\frac{(3a\,b^2 d+b^3 c)x^7}{7}+\frac{(3a^2 bd+3a\,b^2 c)x^5}{5}+\frac{(a^3 d+3a^2 bc)x^3}{3}+a^3 cx$	73
gosper	$\frac{1}{9}b^3 d\,x^9+\frac{3}{7}x^7 a\,b^2 d+\frac{1}{7}x^7 b^3 c+\frac{3}{5}x^5 a^2 bd+\frac{3}{5}a\,b^2 c\,x^5+\frac{1}{3}x^3 a^3 d+x^3 a^2 bc+a^3 cx$	74
risch	$\frac{1}{9}b^3 d\,x^9+\frac{3}{7}x^7 a\,b^2 d+\frac{1}{7}x^7 b^3 c+\frac{3}{5}x^5 a^2 bd+\frac{3}{5}a\,b^2 c\,x^5+\frac{1}{3}x^3 a^3 d+x^3 a^2 bc+a^3 cx$	74
parallelrisch	$\frac{1}{9}b^3 d\,x^9+\frac{3}{7}x^7 a\,b^2 d+\frac{1}{7}x^7 b^3 c+\frac{3}{5}x^5 a^2 bd+\frac{3}{5}a\,b^2 c\,x^5+\frac{1}{3}x^3 a^3 d+x^3 a^2 bc+a^3 cx$	74

```
[In] int((b*x^2+a)^3*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^3*d*x^9+(3/7*a*b^2*d+1/7*b^3*c)*x^7+(3/5*a^2*b*d+3/5*a*b^2*c)*x^5+(1/
3*a^3*d+a^2*b*c)*x^3+a^3*c*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right) \, dx = \frac{1}{9}\,b^3dx^9+\frac{1}{7}\,\left(b^3c+3\,ab^2d\right)x^7 + \frac{3}{5}\,\left(ab^2c+a^2bd\right)x^5+a^3cx+\frac{1}{3}\,\left(3\,a^2bc+a^3d\right)x^3$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = a^3cx + \frac{b^3dx^9}{9} + x^7\cdot\left(\frac{3ab^2d}{7}+\frac{b^3c}{7}\right) + x^5\cdot\left(\frac{3a^2bd}{5}+\frac{3ab^2c}{5}\right) + x^3\left(\frac{a^3d}{3}+a^2bc\right)$$

```
[In] integrate((b*x**2+a)**3*(d*x**2+c),x)
[Out] a**3*c*x + b**3*d*x**9/9 + x**7*(3*a*b**2*d/7 + b**3*c/7) + x**5*(3*a**2*b*
d/5 + 3*a*b**2*c/5) + x**3*(a**3*d/3 + a**2*b*c)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = \frac{1}{9}\,b^3dx^9 + \frac{1}{7}\left(b^3c+3\,ab^2d\right)x^7 + \frac{3}{5}\left(ab^2c+a^2bd\right)x^5 + a^3cx + \frac{1}{3}\left(3\,a^2bc+a^3d\right)x^3$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="maxima")
[Out] 1/9*b^3*d*x^9 + 1/7*(b^3*c + 3*a*b^2*d)*x^7 + 3/5*(a*b^2*c + a^2*b*d)*x^5 +
 a^3*c*x + 1/3*(3*a^2*b*c + a^3*d)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \left(a+bx^2\right)^3 \left(c+dx^2\right)\,dx = \frac{1}{9}\,b^3dx^9 + \frac{1}{7}\,b^3cx^7 + \frac{3}{7}\,ab^2dx^7 + \frac{3}{5}\,ab^2cx^5 + \frac{3}{5}\,a^2bdx^5 + a^2bcx^3 + \frac{1}{3}\,a^3dx^3 + a^3cx$$

```
[In] integrate((b*x^2+a)^3*(d*x^2+c),x, algorithm="giac")
[Out] 1/9*b^3*d*x^9 + 1/7*b^3*c*x^7 + 3/7*a*b^2*d*x^7 + 3/5*a*b^2*c*x^5 + 3/5*a^2
*b*d*x^5 + a^2*b*c*x^3 + 1/3*a^3*d*x^3 + a^3*c*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)^3\left(c+dx^2\right)\,dx=x^7\left(\frac{c\,b^3}{7}+\frac{3\,a\,d\,b^2}{7}\right)+x^3\left(\frac{d\,a^3}{3}+b\,c\,a^2\right)+\frac{b^3\,d\,x^9}{9}+a^3\,c\,x+\frac{3\,a\,b\,x^5\,(a\,d+b\,c)}{5}$$

```
[In] int((a + b*x^2)^3*(c + d*x^2),x)
```

```
[Out] x^7*((b^3*c)/7 + (3*a*b^2*d)/7) + x^3*((a^3*d)/3 + a^2*b*c) + (b^3*d*x^9)/9
 + a^3*c*x + (3*a*b*x^5*(a*d + b*c))/5
```

3.17 $\int \frac{(a+bx^2)^3}{c+dx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{(a+bx^2)^3}{c+dx^2} \, dx = \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3} - \frac{b^2(bc-3ad)x^3}{3d^2} + \frac{b^3x^5}{5d} - \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

```
[Out] b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)*x/d^3-1/3*b^2*(-3*a*d+b*c)*x^3/d^2+1/5*b^3*
x^5/d-(-a*d+b*c)^3*arctan(x*d^(1/2)/c^(1/2))/d^(7/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\int \frac{(a+bx^2)^3}{c+dx^2} \, dx = \frac{bx(3a^2d^2-3abcd+b^2c^2)}{d^3} - \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}} - \frac{b^2x^3(bc-3ad)}{3d^2} + \frac{b^3x^5}{5d}$$

```
[In] Int[(a + b*x^2)^3/(c + d*x^2),x]
```

```
[Out] (b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*x)/d^3 - (b^2*(b*c - 3*a*d)*x^3)/(3*d^
2) + (b^3*x^5)/(5*d) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]
*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{b(b^2c^2-3abcd+3a^2d^2)}{d^3}-\frac{b^2(bc-3ad)x^2}{d^2}+\frac{b^3x^4}{d}+\frac{-b^3c^3+3ab^2c^2d-3a^2bcd^2+a^3d^3}{d^3\,(c+dx^2)}\right)dx\\
&= \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3}-\frac{b^2(bc-3ad)x^3}{3d^2}+\frac{b^3x^5}{5d}-\frac{(bc-ad)^3\int\frac{1}{c+dx^2}\,dx}{d^3}\\
&= \frac{b(b^2c^2-3abcd+3a^2d^2)\,x}{d^3}-\frac{b^2(bc-3ad)x^3}{3d^2}+\frac{b^3x^5}{5d}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \frac{bx(45a^2d^2+15abd(-3c+dx^2)+b^2(15c^2-5cdx^2+3d^2x^4))}{15d^3}-\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}d^{7/2}}$$

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2),x]
[Out] (b*x*(45*a^2*d^2 + 15*a*b*d*(-3*c + d*x^2) + b^2*(15*c^2 - 5*c*d*x^2 + 3*d^
2*x^4)))/(15*d^3) - ((b*c - a*d)^3*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*d^
(7/2))
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
default	$\frac{b\left(\frac{1}{5}b^2d^2x^5+x^3ab\,d^2-\frac{1}{3}x^3b^2cd+3a^2d^2x-3abcdx+b^2c^2x\right)}{d^3}+\frac{\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{d^3\sqrt{cd}}$
risch	$\frac{b^3x^5}{5d}+\frac{b^2x^3a}{d}-\frac{b^3x^3c}{3d^2}+\frac{3b\,a^2x}{d}-\frac{3b^2acx}{d^2}+\frac{b^3c^2x}{d^3}-\frac{\ln\left(dx+\sqrt{-cd}\right)a^3}{2\sqrt{-cd}}+\frac{3\ln\left(dx+\sqrt{-cd}\right)a^2bc}{2d\sqrt{-cd}}-\frac{3\ln\left(dx+\sqrt{-cd}\right)a\,b^2c^2}{2d^2\sqrt{-cd}}$

```
[In] int((b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] b/d^3*(1/5*b^2*d^2*x^5+x^3*a*b*d^2-1/3*x^3*b^2*c*d+3*a^2*d^2*x-3*a*b*c*d*x+
b^2*c^2*x)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^3/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.96

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \left[\frac{6\,b^3cd^3x^5-10\,(b^3c^2d^2-3\,ab^2cd^3)x^3+15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-cd}\log\left(\frac{dx^2-2\,\sqrt{-cd}x-c}{dx^2+c}\right)+}{30\,cd^4}\right.$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b^3*c*d^3*x^5 - 10*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(
-c*d)*x - c)/(d*x^2 + c)) + 30*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3
)*x)/(c*d^4), 1/15*(3*b^3*c*d^3*x^5 - 5*(b^3*c^2*d^2 - 3*a*b^2*c*d^3)*x^3 -
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(c*d)*arctan(sq
rt(c*d)*x/c) + 15*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3)*x)/(c*d^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(92) = 184.

Time = 0.32 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.43

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = \frac{b^3x^5}{5d} + x^3\left(\frac{ab^2}{d} - \frac{b^3c}{3d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) - \frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(-\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2} + \frac{\sqrt{-\frac{1}{cd^7}}\,(ad-bc)^3\log\left(\frac{cd^3\sqrt{-\frac{1}{cd^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3} + x\right)}{2}$$

```
[In] integrate((b*x**2+a)**3/(d*x**2+c),x)
```

```
[Out] b**3*x**5/(5*d) + x**3*(a*b**2/d - b**3*c/(3*d**2)) + x*(3*a**2*b/d - 3*a*b
**2*c/d**2 + b**3*c**2/d**3) - sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(-c*d**3
*sqrt(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c
**2*d - b**3*c**3) + x)/2 + sqrt(-1/(c*d**7))*(a*d - b*c)**3*log(c*d**3*sqr
t(-1/(c*d**7))*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*
d - b**3*c**3) + x)/2
```

Maxima [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = -\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^3d^2x^5 - 5\,(b^3cd - 3\,ab^2d^2)x^3 + 15\,(b^3c^2 - 3\,ab^2cd + 3\,a^2bd^2)x}{15\,d^3}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^2*x^5 - 5*(b^3*c*d - 3*a*b^2*d^2)*x^3 + 15*
(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*x)/d^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{\sqrt{cd}d^3} + \frac{3\,b^3d^4x^5-5\,b^3cd^3x^3+15\,ab^2d^4x^3+15\,b^3c^2d^2x-45\,ab^2cd^3x+45\,a^2bd^4x}{15\,d^5}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/
(sqrt(c*d)*d^3) + 1/15*(3*b^3*d^4*x^5 - 5*b^3*c*d^3*x^3 + 15*a*b^2*d^4*x^3
+ 15*b^3*c^2*d^2*x - 45*a*b^2*c*d^3*x + 45*a^2*b*d^4*x)/d^5
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^3}{c+dx^2}\,dx = x^3\left(\frac{a\,b^2}{d}-\frac{b^3\,c}{3\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\frac{b^3\,x^5}{5\,d} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^3}{\sqrt{c}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{c}\,d^{7/2}}$$

```
[In] int((a + b*x^2)^3/(c + d*x^2),x)
```

```
[Out] x^3*((a*b^2)/d - (b^3*c)/(3*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3
*c)/d^2))/d) + (b^3*x^5)/(5*d) + (atan((d^(1/2)*x*(a*d - b*c)^3)/(c^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(c^(1/2
)*d^(7/2))
```

3.18 $\int \frac{(a+bx^2)^3}{(c+dx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2} dx = -\frac{b^2(2bc-3ad)x}{d^3} + \frac{b^3x^3}{3d^2} - \frac{(bc-ad)^3x}{2cd^3(c+dx^2)} + \frac{(bc-ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

```
[Out] -b^2*(-3*a*d+2*b*c)*x/d^3+1/3*b^3*x^3/d^2-1/2*(-a*d+b*c)^3*x/c/d^3/(d*x^2+c
)+1/2*(-a*d+b*c)^2*(a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/d^(7/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2} dx = \frac{(ad+5bc)(bc-ad)^2\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}} - \frac{b^2x(2bc-3ad)}{d^3} - \frac{x(bc-ad)^3}{2cd^3(c+dx^2)} + \frac{b^3x^3}{3d^2}$$

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x^2}{d^2}+\frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{d^3\left(c+dx^2\right)^2}\right)dx \\
&= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}+\frac{\int \frac{(bc-ad)^2(2bc+ad)+3bd(bc-ad)^2x^2}{(c+dx^2)^2}\,dx}{d^3} \\
&= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\left(c+dx^2\right)}+\frac{\left((bc-ad)^2(5bc+ad)\right)\int \frac{1}{c+dx^2}\,dx}{2cd^3} \\
&= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\left(c+dx^2\right)}+\frac{(bc-ad)^2(5bc+ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2\right)^3}{\left(c+dx^2\right)^2}\,dx = -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^3}{3d^2}-\frac{(bc-ad)^3x}{2cd^3\left(c+dx^2\right)} + \frac{(bc-ad)^2(5bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}d^{7/2}}$$

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^2,x]
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^3)/(3*d^2) - ((b*c - a*d)^3*x)/(2*c
*d^3*(c + d*x^2)) + ((b*c - a*d)^2*(5*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]
])/(2*c^(3/2)*d^(7/2))
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

method	result
default	$\frac{b^{2}\left(\frac{1}{3} b d \,x^{3}+3 a d x -2 b c x \right)}{d^{3}}+\frac{\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) x}{2 c \left(d \,x^{2}+c \right)}+\frac{\left(a^{3} d^{3}+3 a^{2} b c \,d^{2}-9 a \,b^{2} c^{2} d +5 b^{3} c^{3}\right) \arctan \left(\frac{d x}{\sqrt{c d}}\right)}{2 c \sqrt{c d}}}{d^{3}}$
risch	$\frac{b^{3} x^{3}}{3 d^{2}}+\frac{3 b^{2} a x}{d^{2}}-\frac{2 b^{3} c x}{d^{3}}+\frac{\left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) x}{2 c \,d^{3} \left(d \,x^{2}+c \right)}-\frac{\ln \left(d x +\sqrt{-c d}\right) a^{3}}{4 \sqrt{-c d}\, c}-\frac{3 \ln \left(d x +\sqrt{-c d}\right) a^{2} b}{4 d \sqrt{-c d}}+\frac{9 c \ln \left(d x +\sqrt{-c d}\right) a}{4 d^{2} \sqrt{-c d}}$

[In] int((b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] b^2/d^3*(1/3*b*d*x^3+3*a*d*x-2*b*c*x)+1/d^3*(1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a *b^2*c^2*d-b^3*c^3)/c*x/(d*x^2+c)+1/2*(a^3*d^3+3*a^2*b*c*d^2-9*a*b^2*c^2*d+ 5*b^3*c^3)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(93) = 186$.

Time = 0.25 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.15

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = \left[\frac{4\,b^3c^2d^3x^5-4\,(5\,b^3c^3d^2-9\,ab^2c^2d^3)x^3-3\,(5\,b^3c^4-9\,ab^2c^3d+3\,a^2bc^2d^2+a^3cd^3+(5\,b^3c^3d-9\,ab^2c^2d^2+}{12\,(c^2d^5x^2+c^3}\right.$$

[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [1/12*(4*b^3*c^2*d^3*x^5 - 4*(5*b^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 - 3*(5*b ^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b ^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3*d^4)*x^2)*sqrt(-c*d)*log((d*x^2 - 2*sqrt(- c*d)*x - c)/(d*x^2 + c)) - 6*(5*b^3*c^4*d - 9*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d ^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4), 1/6*(2*b^3*c^2*d^3*x^5 - 2*(5*b ^3*c^3*d^2 - 9*a*b^2*c^2*d^3)*x^3 + 3*(5*b^3*c^4 - 9*a*b^2*c^3*d + 3*a^2*b* c^2*d^2 + a^3*c*d^3 + (5*b^3*c^3*d - 9*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + a^3* d^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) - 3*(5*b^3*c^4*d - 9*a*b^2*c^3*d^ 2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)/(c^2*d^5*x^2 + c^3*d^4)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.64 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.93

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = \frac{b^3x^3}{3d^2} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{x(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)}{2c^2d^3 + 2cd^4x^2} - \frac{\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)\log\left(-\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4} + \frac{\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)\log\left(\frac{c^2d^3\sqrt{-\frac{1}{c^3d^7}}(ad-bc)^2(ad+5bc)}{a^3d^3+3a^2bcd^2-9ab^2c^2d+5b^3c^3} + x\right)}{4}$$

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] b**3*x**3/(3*d**2) + x*(3*a*b**2/d**2 - 2*b**3*c/d**3) + x*(a**3*d**3 - 3*a
**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(2*c**2*d**3 + 2*c*d**4*x**2) -
 sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)*log(-c**2*d**3*sqrt(-1/(
c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a
*b**2*c**2*d + 5*b**3*c**3) + x)/4 + sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a
*d + 5*b*c)*log(c**2*d**3*sqrt(-1/(c**3*d**7))*(a*d - b*c)**2*(a*d + 5*b*c)
/(a**3*d**3 + 3*a**2*b*c*d**2 - 9*a*b**2*c**2*d + 5*b**3*c**3) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = -\frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x}{2\,(cd^4x^2 + c^2d^3)} + \frac{b^3dx^3 - 3\,(2\,b^3c - 3\,ab^2d)x}{3\,d^3} + \frac{(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^3}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(c*d^4*x^2 + c^2
*d^3) + 1/3*(b^3*d*x^3 - 3*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 1/2*(5*b^3*c^3 -
9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c
*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = \frac{(5\,b^3c^3 - 9\,ab^2c^2d + 3\,a^2bcd^2 + a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{cd}cd^3} - \frac{b^3c^3x - 3\,ab^2c^2dx + 3\,a^2bcd^2x - a^3d^3x}{2\,(dx^2+c)cd^3} + \frac{b^3d^4x^3 - 6\,b^3cd^3x + 9\,ab^2d^4x}{3\,d^6}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3)*arctan(d*x/sqrt(c
*d))/(sqrt(c*d)*c*d^3) - 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((d*x^2 + c)*c*d^3) + 1/3*(b^3*d^4*x^3 - 6*b^3*c*d^3*x + 9*a*
b^2*d^4*x)/d^6
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^2}\,dx = x\left(\frac{3\,a\,b^2}{d^2} - \frac{2\,b^3\,c}{d^3}\right) + \frac{b^3\,x^3}{3\,d^2} + \frac{x\,(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3)}{2\,c\,(d^4\,x^2 + c\,d^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{d}\,x\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{\sqrt{c}\,(a^3\,d^3+3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(a\,d+5\,b\,c)}{2\,c^{3/2}\,d^{7/2}}$$

```
[In] int((a + b*x^2)^3/(c + d*x^2)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (b^3*x^3)/(3*d^2) + (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*c*(c*d^3 + d^4*x^2)) + (atan((d^(1/
2)*x*(a*d - b*c)^2*(a*d + 5*b*c))/(c^(1/2)*(a^3*d^3 + 5*b^3*c^3 - 9*a*b^2*c
^2*d + 3*a^2*b*c*d^2)))*(a*d - b*c)^2*(a*d + 5*b*c))/(2*c^(3/2)*d^(7/2))
```

3.19 $\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx = \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3(c+dx^2)} - \frac{3(bc-ad)(4b^2c^2+(bc+ad)^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

```
[Out] b^3*x/d^3-1/4*(-a*d+b*c)^3*x/c/d^3/(d*x^2+c)^2+3/8*(-a*d+b*c)^2*(a*d+3*b*c)
*x/c^2/d^3/(d*x^2+c)-3/8*(-a*d+b*c)*(4*b^2*c^2+(a*d+b*c)^2)*arctan(x*d^(1/2
)/c^(1/2))/c^(5/2)/d^(7/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {398, 1171, 393, 211}

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3} dx = -\frac{3(bc-ad)((ad+bc)^2+4b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}} + \frac{3x(bc-ad)^2(ad+3bc)}{8c^2d^3(c+dx^2)} - \frac{x(bc-ad)^3}{4cd^3(c+dx^2)^2} + \frac{b^3x}{d^3}$$

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(b*c - a*d)*(4*b^2*c^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{b^3}{d^3} - \frac{b^3c^3 - a^3d^3 + 3bd(bc-ad)(bc+ad)x^2 + 3b^2d^2(bc-ad)x^4}{d^3\left(c+dx^2\right)^3}\right) dx \\
&= \frac{b^3x}{d^3} - \frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{(c+dx^2)^3}\, dx}{d^3} \\
&= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2} + \frac{\int \frac{-3(bc-ad)(bc+ad)^2-12b^2cd(bc-ad)x^2}{(c+dx^2)^2}\, dx}{4cd^3} \\
&= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3\left(c+dx^2\right)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3\left(c+dx^2\right)} \\
&\quad - \frac{\left(3(bc-ad)\left(4b^2c^2+(bc+ad)^2\right)\right)\int \frac{1}{c+dx^2}\, dx}{8c^2d^3}
\end{aligned}
$$

$$= \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3(c+dx^2)} - \frac{3(bc-ad)(4b^2c^2+(bc+ad)^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{b^3x}{d^3} - \frac{(bc-ad)^3x}{4cd^3(c+dx^2)^2} + \frac{3(bc-ad)^2(3bc+ad)x}{8c^2d^3(c+dx^2)} - \frac{3(5b^3c^3-3ab^2c^2d-a^2bcd^2-a^3d^3)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}d^{7/2}}$$

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^3,x]
```

```
[Out] (b^3*x)/d^3 - ((b*c - a*d)^3*x)/(4*c*d^3*(c + d*x^2)^2) + (3*(b*c - a*d)^2*
(3*b*c + a*d)*x)/(8*c^2*d^3*(c + d*x^2)) - (3*(5*b^3*c^3 - 3*a*b^2*c^2*d -
a^2*b*c*d^2 - a^3*d^3)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*d^(7/2))
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

method	result
default	$\frac{b^3x}{d^3} + \frac{\frac{\frac{3d\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)x^3}{8c^2}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{3\left(a^3d^3+a^2bc\,d^2+3a\,b^2c^2d-5b^3c^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2\sqrt{cd}}}{d^3}$
risch	$\frac{b^3x}{d^3} + \frac{\frac{3d\left(a^3d^3+a^2bc\,d^2-5a\,b^2c^2d+3b^3c^3\right)x^3}{8c^2}+\frac{\left(5a^3d^3-3a^2bc\,d^2-9a\,b^2c^2d+7b^3c^3\right)x}{8c}}{d^3(d\,x^2+c)^2} - \frac{3\ln\left(dx+\sqrt{-cd}\right)a^3}{16\sqrt{-cd}\,c^2} - \frac{3\ln\left(dx+\sqrt{-cd}\right)a^2b}{16d\sqrt{-cd}\,c} -$

```
[In] int((b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^3*x/d^3+1/d^3*((3/8*d*(a^3*d^3+a^2*b*c*d^2-5*a*b^2*c^2*d+3*b^3*c^3)/c^2*x
^3+1/8*(5*a^3*d^3-3*a^2*b*c*d^2-9*a*b^2*c^2*d+7*b^3*c^3)/c*x)/(d*x^2+c)^2+3
/8*(a^3*d^3+a^2*b*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/c^2/(c*d)^(1/2)*arctan(d*x
/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 618, normalized size of antiderivative = 4.75

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{16\,b^3c^3d^3x^5 + 2\,(25\,b^3c^4d^2 - 15\,ab^2c^3d^3 + 3\,a^2bc^2d^4 + 3\,a^3cd^5)x^3 + 3\,(5\,b^3c^5 - 3\,ab^2c^4d - a^2bc^3d^2 - a^3c^2d^3}{}\right.$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b^3*c^3*d^3*x^5 + 2*(25*b^3*c^4*d^2 - 15*a*b^2*c^3*d^3 + 3*a^2*b*
c^2*d^4 + 3*a^3*c*d^5)*x^3 + 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^2*b*c^3*d^2 -
 a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4 - a^3*d^5)*x^
4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d^4)*x^2)*sqrt
(-c*d)*log((d*x^2 - 2*sqrt(-c*d)*x - c)/(d*x^2 + c)) + 2*(15*b^3*c^5*d - 9*
a*b^2*c^4*d^2 - 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^
5*x^2 + c^5*d^4), 1/8*(8*b^3*c^3*d^3*x^5 + (25*b^3*c^4*d^2 - 15*a*b^2*c^3*d
^3 + 3*a^2*b*c^2*d^4 + 3*a^3*c*d^5)*x^3 - 3*(5*b^3*c^5 - 3*a*b^2*c^4*d - a^
2*b*c^3*d^2 - a^3*c^2*d^3 + (5*b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 - a^2*b*c*d^4
- a^3*d^5)*x^4 + 2*(5*b^3*c^4*d - 3*a*b^2*c^3*d^2 - a^2*b*c^2*d^3 - a^3*c*d
^4)*x^2)*sqrt(c*d)*arctan(sqrt(c*d)*x/c) + (15*b^3*c^5*d - 9*a*b^2*c^4*d^2
- 3*a^2*b*c^3*d^3 + 5*a^3*c^2*d^4)*x)/(c^3*d^6*x^4 + 2*c^4*d^5*x^2 + c^5*d^
4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(122) = 244.

Time = 1.03 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.25

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{b^3x}{d^3}$$
$$- \frac{3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\,(a^2d^2+2abcd+5b^2c^2)\log\left(-\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16}$$
$$+ \frac{3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)\,(a^2d^2+2abcd+5b^2c^2)\log\left(\frac{3c^3d^3\sqrt{-\frac{1}{c^5d^7}}(ad-bc)(a^2d^2+2abcd+5b^2c^2)}{3a^3d^3+3a^2bcd^2+9ab^2c^2d-15b^3c^3}+x\right)}{16}$$
$$+ \frac{x^3\cdot(3a^3d^4+3a^2bcd^3-15ab^2c^2d^2+9b^3c^3d)+x(5a^3cd^3-3a^2bc^2d^2-9ab^2c^3d+7b^3c^4)}{8c^4d^3+16c^3d^4x^2+8c^2d^5x^4}$$

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] b**3*x/d**3 - 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*a*b*c*d + 5
*b**2*c**2)*log(-3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 +
2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**2*c**2*d -
 15*b**3*c**3) + x)/16 + 3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a**2*d**2 + 2*
a*b*c*d + 5*b**2*c**2)*log(3*c**3*d**3*sqrt(-1/(c**5*d**7))*(a*d - b*c)*(a*
*2*d**2 + 2*a*b*c*d + 5*b**2*c**2)/(3*a**3*d**3 + 3*a**2*b*c*d**2 + 9*a*b**
2*c**2*d - 15*b**3*c**3) + x)/16 + (x**3*(3*a**3*d**4 + 3*a**2*b*c*d**3 - 1
5*a*b**2*c**2*d**2 + 9*b**3*c**3*d) + x*(5*a**3*c*d**3 - 3*a**2*b*c**2*d**2
 - 9*a*b**2*c**3*d + 7*b**3*c**4))/(8*c**4*d**3 + 16*c**3*d**4*x**2 + 8*c**
2*d**5*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.44

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx &= \frac{b^3x}{d^3} \\ &\quad + \frac{3\,(3\,b^3c^3d-5\,ab^2c^2d^2+a^2bcd^3+a^3d^4)x^3+(7\,b^3c^4-9\,ab^2c^3d-3\,a^2bc^2d^2+5\,a^3cd^3)x}{8\,(c^2d^5x^4+2\,c^3d^4x^2+c^4d^3)} \\ &\quad - \frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3}\end{aligned}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 + 1/8*(3*(3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3 + a^3*d^4)*
x^3 + (7*b^3*c^4 - 9*a*b^2*c^3*d - 3*a^2*b*c^2*d^2 + 5*a^3*c*d^3)*x)/(c^2*d
^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3) - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*
c*d^2 - a^3*d^3)*arctan(d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.38

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx &= \frac{b^3x}{d^3} - \frac{3\,(5\,b^3c^3-3\,ab^2c^2d-a^2bcd^2-a^3d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,\sqrt{cd}c^2d^3} \\ &\quad + \frac{9\,b^3c^3dx^3-15\,ab^2c^2d^2x^3+3\,a^2bcd^3x^3+3\,a^3d^4x^3+7\,b^3c^4x-9\,ab^2c^3dx-3\,a^2bc^2d^2x+5\,a^3cd^3x}{8\,(dx^2+c)^2c^2d^3}\end{aligned}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3/8*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*arctan(
d*x/sqrt(c*d))/(sqrt(c*d)*c^2*d^3) + 1/8*(9*b^3*c^3*d*x^3 - 15*a*b^2*c^2*d^
2*x^3 + 3*a^2*b*c*d^3*x^3 + 3*a^3*d^4*x^3 + 7*b^3*c^4*x - 9*a*b^2*c^3*d*x -
 3*a^2*b*c^2*d^2*x + 5*a^3*c*d^3*x)/((d*x^2 + c)^2*c^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^3}\,dx = \frac{\frac{x\,\left(5\,a^3\,d^3-3\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+7\,b^3\,c^3\right)}{8\,c}+\frac{3\,x^3\,\left(a^3\,d^4+a^2\,b\,c\,d^3-5\,a\,b^2\,c^2\,d^2+3\,b^3\,c^3\,d\right)}{8\,c^2}}{c^2\,d^3+2\,c\,d^4\,x^2+d^5\,x^4}+\frac{b^3\,x}{d^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{d}\,x\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{\sqrt{c}\,\left(a^3\,d^3+a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+5\,b^2\,c^2\right)}{8\,c^{5/2}\,d^{7/2}}$$

```
[In] int((a + b*x^2)^3/(c + d*x^2)^3,x)
```

```
[Out] ((x*(5*a^3*d^3 + 7*b^3*c^3 - 9*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(8*c) + (3*x^3
*(a^3*d^4 + 3*b^3*c^3*d - 5*a*b^2*c^2*d^2 + a^2*b*c*d^3))/(8*c^2))/(c^2*d^3
 + d^5*x^4 + 2*c*d^4*x^2) + (b^3*x)/d^3 + (3*atan((d^(1/2)*x*(a*d - b*c)*(a
^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(c^(1/2)*(a^3*d^3 - 5*b^3*c^3 + 3*a*b^2*c^
2*d + a^2*b*c*d^2)))*(a*d - b*c)*(a^2*d^2 + 5*b^2*c^2 + 2*a*b*c*d))/(8*c^(5
/2)*d^(7/2))
```

3.20 $\int \frac{(c+dx^2)^4}{a+bx^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{(c+dx^2)^4}{a+bx^2} dx = \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4} + \frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3} + \frac{d^3(4bc-ad)x^5}{5b^2} + \frac{d^4x^7}{7b} + \frac{(bc-ad)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}$$

```
[Out] d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/3*d^2*(a^2*d^2-4*a*b*c
*d+6*b^2*c^2)*x^3/b^3+1/5*d^3*(-a*d+4*b*c)*x^5/b^2+1/7*d^4*x^7/b+(-a*d+b*c)
^4*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\int \frac{(c+dx^2)^4}{a+bx^2} dx = \frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4} + \frac{d^2x^3\left(a^2d^2-4abcd+6b^2c^2\right)}{3b^3} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^4}{\sqrt{a}b^{9/2}} + \frac{d^3x^5(4bc-ad)}{5b^2} + \frac{d^4x^7}{7b}$$

```
[In] Int[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^3)/(3*b^3) + (d^3*(4*b*c - a*d)*x^5)/(5*b^2) + (d
```

```
^4*x^7)/(7*b) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(9/2
))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^2}{b^3}\right.\\
&\quad \left.+\frac{d^3(4bc-ad)x^4}{b^2}+\frac{d^4x^6}{b}+\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{b^4\left(a+bx^2\right)}\right)dx\\
&= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3}\\
&\quad +\frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x^7}{7b}+\frac{(bc-ad)^4\int \frac{1}{a+bx^2}\,dx}{b^4}\\
&= \frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{3b^3}\\
&\quad +\frac{d^3(4bc-ad)x^5}{5b^2}+\frac{d^4x^7}{7b}+\frac{(bc-ad)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\begin{aligned}
&\int \frac{\left(c+dx^2\right)^4}{a+bx^2}\,dx\\
&= \frac{dx\left(-105a^3d^3+35a^2bd^2\left(12c+dx^2\right)-7ab^2d\left(90c^2+20cdx^2+3d^2x^4\right)+3b^3\left(140c^3+70c^2dx^2+28cd^2x^4+5\right.\right.}{105b^4}\\
&\quad +\frac{(bc-ad)^4\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{9/2}}
\end{aligned}$$

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2),x]
```

```
[Out] (d*x*(-105*a^3*d^3 + 35*a^2*b*d^2*(12*c + d*x^2) - 7*a*b^2*d*(90*c^2 + 20*c
*d*x^2 + 3*d^2*x^4) + 3*b^3*(140*c^3 + 70*c^2*d*x^2 + 28*c*d^2*x^4 + 5*d^3*
x^6)))/(105*b^4) + ((b*c - a*d)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(
9/2))
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.38

method	result
default	$-\frac{d\left(-\frac{b^{3} d^{3} x^{7}}{7}+\frac{\left(\left(a d-2 b c\right) b^{2} d^{2}-2 b^{3} c\,d^{2}\right) x^{5}}{5}+\frac{\left(2\left(a d-2 b c\right) b^{2} c d-b d\left(a^{2} d^{2}-2 a b c d+2 b^{2} c^{2}\right)\right) x^{3}}{3}+\left(a d-2 b c\right)\left(a^{2} d^{2}-2 a b c d+2 b^{2} c^{2}\right) x\right)}{b^{4}}+\frac{(a^{4}d$
risch	$\frac{d^{4} x^{7}}{7 b}-\frac{d^{4} x^{5} a}{5 b^{2}}+\frac{4 d^{3} x^{5} c}{5 b}-\frac{4 d^{3} a c\,x^{3}}{3 b^{2}}+\frac{2 d^{2} c^{2} x^{3}}{b}+\frac{d^{4} a^{2} x^{3}}{3 b^{3}}-\frac{d^{4} a^{3} x}{b^{4}}+\frac{4 d^{3} a^{2} c x}{b^{3}}-\frac{6 d^{2} a\,c^{2} x}{b^{2}}+\frac{4 d\,c^{3} x}{b}-\frac{\ln\left(b x+\sqrt{-a b}\right.}{2 b^{4}\sqrt{-a}}$

```
[In] int((d*x^2+c)^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/b^4*(-1/7*b^3*d^3*x^7+1/5*((a*d-2*b*c)*b^2*d^2-2*b^3*c*d^2)*x^5+1/3*(2*(
a*d-2*b*c)*b^2*c*d-b*d*(a^2*d^2-2*a*b*c*d+2*b^2*c^2))*x^3+(a*d-2*b*c)*(a^2*
d^2-2*a*b*c*d+2*b^2*c^2)*x)+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 428, normalized size of antiderivative = 3.01

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx$$
$$= \left[\frac{30\,ab^4d^4x^7 + 42\,(4\,ab^4cd^3 - a^2b^3d^4)x^5 + 70\,(6\,ab^4c^2d^2 - 4\,a^2b^3cd^3 + a^3b^2d^4)x^3 - 105\,(b^4c^4 - 4\,ab^3c^3d + 6}{21}\right.$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*a*b^4*d^4*x^7 + 42*(4*a*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 70*(6*a*b
^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3*b^2*d^4)*x^3 - 105*(b^4*c^4 - 4*a*b^3*c^
3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-a*b)*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(4*a*b^4*c^3*d - 6*a^2*b^3*c^2*d^2 +
 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5), 1/105*(15*a*b^4*d^4*x^7 + 21*(4*a
*b^4*c*d^3 - a^2*b^3*d^4)*x^5 + 35*(6*a*b^4*c^2*d^2 - 4*a^2*b^3*c*d^3 + a^3
*b^2*d^4)*x^3 + 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(4*a*b^4*c^3*d - 6*a
^2*b^3*c^2*d^2 + 4*a^3*b^2*c*d^3 - a^4*b*d^4)*x)/(a*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(136) = 272.

Time = 0.48 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = x^5\left(-\frac{ad^4}{5b^2}+\frac{4cd^3}{5b}\right)+x^3\left(\frac{a^2d^4}{3b^3}-\frac{4acd^3}{3b^2}+\frac{2c^2d^2}{b}\right) + x\left(-\frac{a^3d^4}{b^4}+\frac{4a^2cd^3}{b^3}-\frac{6ac^2d^2}{b^2}+\frac{4c^3d}{b}\right) - \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^4\log\left(-\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab^9}}(ad-bc)^4\log\left(\frac{ab^4\sqrt{-\frac{1}{ab^9}}(ad-bc)^4}{a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4}+x\right)}{2} + \frac{d^4x^7}{7b}$$

```
[In] integrate((d*x**2+c)**4/(b*x**2+a),x)
```

```
[Out] x**5*(-a*d**4/(5*b**2) + 4*c*d**3/(5*b)) + x**3*(a**2*d**4/(3*b**3) - 4*a*c
*d**3/(3*b**2) + 2*c**2*d**2/b) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 -
 6*a*c**2*d**2/b**2 + 4*c**3*d/b) - sqrt(-1/(a*b**9))*(a*d - b*c)**4*log(-a
*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**
2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2 + sqrt(-1/(a*b**9))*
(a*d - b*c)**4*log(a*b**4*sqrt(-1/(a*b**9))*(a*d - b*c)**4/(a**4*d**4 - 4*a
**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x)/2
+ d**4*x**7/(7*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3d^4x^7+21\,(4\,b^3cd^3-ab^2d^4)x^5+35\,(6\,b^3c^2d^2-4\,ab^2cd^3+a^2bd^4)x^3+105\,(4\,b^3c^3d-6\,ab^2c^2d^2+4\,a^2b}{105\,b^4}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*d^4*x^7 + 21*(4*b^3*c*d^
3 - a*b^2*d^4)*x^5 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^3 + 1
05*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = \frac{(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6d^4x^7+84\,b^6cd^3x^5-21\,ab^5d^4x^5+210\,b^6c^2d^2x^3-140\,ab^5cd^3x^3+35\,a^2b^4d^4x^3+420\,b^6c^3dx-630\,ab}{105\,b^7}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*d^4*x^7 + 84*b^6*c*d^3*x
^5 - 21*a*b^5*d^4*x^5 + 210*b^6*c^2*d^2*x^3 - 140*a*b^5*c*d^3*x^3 + 35*a^2*
b^4*d^4*x^3 + 420*b^6*c^3*d*x - 630*a*b^5*c^2*d^2*x + 420*a^2*b^4*c*d^3*x -
 105*a^3*b^3*d^4*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx^2)^4}{a+bx^2}\,dx = x\left(\frac{4\,c^3\,d}{b}-\frac{a\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{b}+\frac{6\,c^2\,d^2}{b}\right)}{b}\right)-x^5\left(\frac{a\,d^4}{5\,b^2}-\frac{4\,c\,d^3}{5\,b}\right) + x^3\left(\frac{a\left(\frac{a\,d^4}{b^2}-\frac{4\,c\,d^3}{b}\right)}{3\,b}+\frac{2\,c^2\,d^2}{b}\right)+\frac{d^4\,x^7}{7\,b} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4}{\sqrt{a}\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}\right)\,(a\,d-b\,c)^4}{\sqrt{a}\,b^{9/2}}$$

```
[In] int((c + d*x^2)^4/(a + b*x^2),x)
```

```
[Out] x*((4*c^3*d)/b - (a*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b))/b)
 - x^5*((a*d^4)/(5*b^2) - (4*c*d^3)/(5*b)) + x^3*((a*((a*d^4)/b^2 - (4*c*d^
3)/b))/(3*b) + (2*c^2*d^2)/b) + (d^4*x^7)/(7*b) + (atan((b^(1/2)*x*(a*d - b
*c)^4)/(a^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*
a^3*b*c*d^3)))*(a*d - b*c)^4)/(a^(1/2)*b^(9/2))
```

3.21 $\int \frac{(c+dx^2)^3}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{(c+dx^2)^3}{a+bx^2} \, dx = \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3} + \frac{d^2(3bc-ad)x^3}{3b^2} + \frac{d^3x^5}{5b} + \frac{(bc-ad)^3 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

```
[Out] d*(a^2*d^2-3*a*b*c*d+3*b^2*c^2)*x/b^3+1/3*d^2*(-a*d+3*b*c)*x^3/b^2+1/5*d^3*
x^5/b+(-a*d+b*c)^3*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {398, 211}

$$\int \frac{(c+dx^2)^3}{a+bx^2} \, dx = \frac{dx(a^2d^2-3abcd+3b^2c^2)}{b^3} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3}{\sqrt{a}b^{7/2}} + \frac{d^2x^3(3bc-ad)}{3b^2} + \frac{d^3x^5}{5b}$$

```
[In] Int[(c + d*x^2)^3/(a + b*x^2),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^3)/(3*b^
2) + (d^3*x^5)/(5*b) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]
*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(3b^2c^2-3abcd+a^2d^2)}{b^3}+\frac{d^2(3bc-ad)x^2}{b^2}+\frac{d^3x^4}{b}\right.\\
&\qquad\left.+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^2\right)}\right)dx\\
&= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^2}\,dx}{b^3}\\
&= \frac{d(3b^2c^2-3abcd+a^2d^2)\,x}{b^3}+\frac{d^2(3bc-ad)x^3}{3b^2}+\frac{d^3x^5}{5b}+\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\begin{aligned}
\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx &= \frac{dx(15a^2d^2-5abd(9c+dx^2)+3b^2(15c^2+5cdx^2+d^2x^4))}{15b^3}\\
&\quad+\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}
\end{aligned}$$

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2),x]
[Out] (d*x*(15*a^2*d^2 - 5*a*b*d*(9*c + d*x^2) + 3*b^2*(15*c^2 + 5*c*d*x^2 + d^2*
x^4)))/(15*b^3) + ((b*c - a*d)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7
/2))
```

Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
default	$\frac{d\left(\frac{1}{5}b^{2}d^{2}x^{5}-\frac{1}{3}x^{3}ab\,d^{2}+x^{3}b^{2}cd+a^{2}d^{2}x-3abcdx+3b^{2}c^{2}x\right)}{b^{3}}+\frac{\left(-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^{3}\sqrt{ab}}$
risch	$\frac{d^{3}x^{5}}{5b}-\frac{d^{3}x^{3}a}{3b^{2}}+\frac{d^{2}x^{3}c}{b}+\frac{d^{3}a^{2}x}{b^{3}}-\frac{3d^{2}acx}{b^{2}}+\frac{3d\,c^{2}x}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^{3}d^{3}}{2b^{3}\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)a^{2}c\,d^{2}}{2b^{2}\sqrt{-ab}}-\frac{3\ln\left(bx-\sqrt{-ab}\right)a\,c}{2b\sqrt{-ab}}$

```
[In] int((d*x^2+c)^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] d/b^3*(1/5*b^2*d^2*x^5-1/3*x^3*a*b*d^2+x^3*b^2*c*d+a^2*d^2*x-3*a*b*c*d*x+3*
b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^3/(a*b)^(1/2)*a
rctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{\left(c+dx^2\right)^3}{a+bx^2}\,dx$$
$$=\left[\frac{6\,ab^3d^3x^5+10\,(3\,ab^3cd^2-a^2b^2d^3)x^3+15\,(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+}{30\,ab^4}\right.$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*d^3*x^5 + 10*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 + 15*(b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 30*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3
)*x)/(a*b^4), 1/15*(3*a*b^3*d^3*x^5 + 5*(3*a*b^3*c*d^2 - a^2*b^2*d^3)*x^3 +
 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) + 15*(3*a*b^3*c^2*d - 3*a^2*b^2*c*d^2 + a^3*b*d^3)*x)/(a*b^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(92) = 184.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x^3\left(-\frac{ad^3}{3b^2}+\frac{cd^2}{b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(-\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}(ad-bc)^3\log\left(\frac{ab^3\sqrt{-\frac{1}{ab^7}}(ad-bc)^3}{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}+x\right)}{2}+\frac{d^3x^5}{5b}$$

```
[In] integrate((d*x**2+c)**3/(b*x**2+a),x)
```

```
[Out] x**3*(-a*d**3/(3*b**2) + c*d**2/b) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b**2 +
3*c**2*d/b) + sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(-a*b**3*sqrt(-1/(a*b**7)
)*(a*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3
) + x)/2 - sqrt(-1/(a*b**7))*(a*d - b*c)**3*log(a*b**3*sqrt(-1/(a*b**7))*(a
*d - b*c)**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) +
x)/2 + d**3*x**5/(5*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2d^3x^5+5\,(3\,b^2cd^2-abd^3)x^3+15\,(3\,b^2c^2d-3\,abcd^2+a^2d^3)x}{15\,b^3}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^2*d^3*x^5 + 5*(3*b^2*c*d^2 - a*b*d^3)*x^3 + 15*(
3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4d^3x^5+15\,b^4cd^2x^3-5\,ab^3d^3x^3+45\,b^4c^2dx-45\,ab^3cd^2x+15\,a^2b^2d^3x}{15\,b^5}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^3) + 1/15*(3*b^4*d^3*x^5 + 15*b^4*c*d^2*x^3 - 5*a*b^3*d^3*x^3 +
 45*b^4*c^2*d*x - 45*a*b^3*c*d^2*x + 15*a^2*b^2*d^3*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx^2)^3}{a+bx^2}\,dx = x\left(\frac{3\,c^2\,d}{b}+\frac{a\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^3\left(\frac{a\,d^3}{3\,b^2}-\frac{c\,d^2}{b}\right)+\frac{d^3\,x^5}{5\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3}{\sqrt{a}\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{\sqrt{a}\,b^{7/2}}$$

```
[In] int((c + d*x^2)^3/(a + b*x^2),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^3*((a*d^3)/(3*b^2)
- (c*d^2)/b) + (d^3*x^5)/(5*b) - (atan((b^(1/2)*x*(a*d - b*c)^3)/(a^(1/2)*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b*c)^3)/(a^(1/2
)*b^(7/2))
```

3.22 $\int \frac{(c+dx^2)^2}{a+bx^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{(c+dx^2)^2}{a+bx^2} \, dx = \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

```
[Out] d*(-a*d+2*b*c)*x/b^2+1/3*d^2*x^3/b+(-a*d+b*c)^2*arctan(x*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {398, 211}

$$\int \frac{(c+dx^2)^2}{a+bx^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2}{\sqrt{a}b^{5/2}} + \frac{dx(2bc-ad)}{b^2} + \frac{d^2x^3}{3b}$$

```
[In] Int[(c + d*x^2)^2/(a + b*x^2),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^3)/(3*b) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d(2bc-ad)}{b^2} + \frac{d^2x^2}{b} + \frac{b^2c^2-2abcd+a^2d^2}{b^2(a+bx^2)} \right) dx \\
&= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \int \frac{1}{a+bx^2}\,dx}{b^2} \\
&= \frac{d(2bc-ad)x}{b^2} + \frac{d^2x^3}{3b} + \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{dx(6bc-3ad+bdx^2)}{3b^2} + \frac{(bc-ad)^2 \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}$$

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2),x]
[Out] (d*x*(6*b*c - 3*a*d + b*d*x^2))/(3*b^2) + ((b*c - a*d)^2*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(Sqrt[a]*b^(5/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

method	result
default	$-\frac{d\left(-\frac{1}{3}bd\,x^3+adx-2bcx\right)}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^2\sqrt{ab}}$
risch	$\frac{d^2x^3}{3b}-\frac{d^2ax}{b^2}+\frac{2dcx}{b}-\frac{\ln\left(bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}+\frac{\ln\left(bx+\sqrt{-ab}\right)acd}{b\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)c^2}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)a^2d^2}{2b^2\sqrt{-ab}}-$

```
[In] int((d*x^2+c)^2/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -d/b^2*(-1/3*b*d*x^3+a*d*x-2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx$$
$$= \left[\frac{2\,ab^2d^2x^3 - 3\,(b^2c^2 - 2\,abcd + a^2d^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right) + 6\,(2\,ab^2cd - a^2bd^2)x}{6\,ab^3}, \frac{ab^2d^2x^3 + 3\,(b^2c^2\right.$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a*b^2*d^2*x^3 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-a*b)*log((b
*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a
*b^3), 1/3*(a*b^2*d^2*x^3 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 3*(2*a*b^2*c*d - a^2*b*d^2)*x)/(a*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(56) = 112.

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.73

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = x\left(-\frac{ad^2}{b^2}+\frac{2cd}{b}\right) - \frac{\sqrt{-\frac{1}{ab^5}}(ad-bc)^2\log\left(-\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{1}{ab^5}}(ad-bc)^2\log\left(\frac{ab^2\sqrt{-\frac{1}{ab^5}}(ad-bc)^2}{a^2d^2-2abcd+b^2c^2}+x\right)}{2} + \frac{d^2x^3}{3b}$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) - sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(-a*b**2*s
qrt(-1/(a*b**5))*(a*d - b*c)**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2
+ sqrt(-1/(a*b**5))*(a*d - b*c)**2*log(a*b**2*sqrt(-1/(a*b**5))*(a*d - b*c)
**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)/2 + d**2*x**3/(3*b)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bd^2x^3+3\,(2\,bcd-ad^2)x}{3\,b^2}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b*d^2*x^3 + 3*(2*b*c*d - a*d^2)*x)/b^2
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2d^2x^3+6\,b^2cdx-3\,abd^2x}{3\,b^3}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3
*(b^2*d^2*x^3 + 6*b^2*c*d*x - 3*a*b*d^2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 4.72 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^2}{a+bx^2}\,dx = \frac{d^2\,x^3}{3\,b} - x\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2}{\sqrt{a}\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{\sqrt{a}\,b^{5/2}}$$

```
[In] int((c + d*x^2)^2/(a + b*x^2),x)
```

```
[Out] (d^2*x^3)/(3*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (atan((b^(1/2)*x*(a*d - b*c
)^2)/(a^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2)/(a^(1/2)*b^(
5/2))
```

3.23 $\int \frac{c+dx^2}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{dx}{b}+\frac{(bc-ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

```
[Out] d*x/b+(-a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {396, 211}

$$\int \frac{c+dx^2}{a+bx^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)}{\sqrt{a}b^{3/2}}+\frac{dx}{b}$$

```
[In] Int[(c + d*x^2)/(a + b*x^2),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{dx}{b} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^2),x]
[Out] (d*x)/b - ((-(b*c) + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2))
```

Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{dx}{b}+\frac{(-ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	34
risch	$\frac{dx}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}+\frac{\ln\left(-bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}-\frac{\ln\left(-bx-\sqrt{-ab}\right)c}{2\sqrt{-ab}}$	106

```
[In] int((d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] d*x/b+(-a*d+b*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.51

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \left[\frac{2\,abdx+\sqrt{-ab}(bc-ad)\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab^2}, \frac{abdx+\sqrt{ab}(bc-ad)\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab^2}\right]$$

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x + sqrt(-a*b)*(b*c - a*d)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*d*x + sqrt(a*b)*(b*c - a*d)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(34) = 68.

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^3}}(ad-bc)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2} + \frac{dx}{b}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*d - b*c)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*d - b*c)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + d*x/b
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} + \frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{dx}{b} + \frac{(bc-ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a),x, algorithm="giac")
[Out] d*x/b + (b*c - a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2}{a+bx^2}\,dx = \frac{d\,x}{b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d-b\,c)}{\sqrt{a}\,b^{3/2}}$$

```
[In] int((c + d*x^2)/(a + b*x^2),x)
[Out] (d*x)/b - (atan((b^(1/2)*x)/a^(1/2))*(a*d - b*c))/(a^(1/2)*b^(3/2))
```

3.24 $\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 70

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/(-a*d+b*c)/a^(1/2)-arctan(x*d^(1/2)/c^(1/
2))*d^(1/2)/(-a*d+b*c)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {400, 211}

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b*c - a*d)) - (Sqrt[d]*ArcT
an[(Sqrt[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 400

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{1}{a+bx^2}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^2}\,dx}{bc-ad}\\ &= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)} - \frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{d}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{bc-ad}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)),x]
[Out] ((Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] - (Sqrt[d]*ArcTan[(Sqrt[d]*x
)/Sqrt[c]])/Sqrt[c])/(b*c - a*d)
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)\sqrt{ab}} + \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)\sqrt{cd}}$	55
risch	$\frac{\sqrt{-cd}\,\ln\left(dx+\sqrt{-cd}\right)}{2c(ad-bc)} - \frac{\sqrt{-cd}\,\ln\left(dx-\sqrt{-cd}\right)}{2c(ad-bc)} + \frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a(ad-bc)} - \frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a(ad-bc)}$	136

```
[In] int(1/(b*x^2+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] -b/(a*d-b*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)/(c*d)^(1/2)*ar
ctan(d*x/(c*d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)},\right.$$

$$-\frac{2\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)+\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{2\,(bc-ad)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)-\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{2\,(bc-ad)}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), -1/2*(2*s
qrt(d/c)*arctan(x*sqrt(d/c)) + sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a
)/(b*x^2 + a)))/(b*c - a*d), 1/2*(2*sqrt(b/a)*arctan(x*sqrt(b/a)) - sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)))/(b*c - a*d), (sqrt(b/a
)*arctan(x*sqrt(b/a)) - sqrt(d/c)*arctan(x*sqrt(d/c)))/(b*c - a*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(60) = 120$.

Time = 2.93 (sec) , antiderivative size = 712, normalized size of antiderivative = 10.17

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx$$
$$= \frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$-\frac{\sqrt{-\frac{b}{a}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{b}{a}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{b}{a}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{b}{a}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$+\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{-\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}-\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$
$$-\frac{\sqrt{-\frac{d}{c}}\log\left(x+\frac{\frac{a^4cd^3\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^3bc^2d^2\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}-\frac{a^2b^2c^3d\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{a^2d^2\sqrt{-\frac{d}{c}}}{ad-bc}+\frac{ab^3c^4\left(-\frac{d}{c}\right)^{\frac{3}{2}}}{(ad-bc)^3}+\frac{b^2c^2\sqrt{-\frac{d}{c}}}{ad-bc}}{bd}\right)}{2(ad-bc)}$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c),x)
```

```
[Out] sqrt(-b/a)*log(x + (-a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2
*d**2*(-b/a)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d -
b*c)**3 - a**2*d**2*sqrt(-b/a)/(a*d - b*c) - a*b**3*c**4*(-b/a)**(3/2)/(a*d
 - b*c)**3 - b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqr
t(-b/a)*log(x + (a**4*c*d**3*(-b/a)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**
2*(-b/a)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-b/a)**(3/2)/(a*d - b*c)
**3 + a**2*d**2*sqrt(-b/a)/(a*d - b*c) + a*b**3*c**4*(-b/a)**(3/2)/(a*d - b
*c)**3 + b**2*c**2*sqrt(-b/a)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) + sqrt(-d
/c)*log(x + (-a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 + a**3*b*c**2*d**2*(
-d/c)**(3/2)/(a*d - b*c)**3 + a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3
 - a**2*d**2*sqrt(-d/c)/(a*d - b*c) - a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)
**3 - b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c)) - sqrt(-d/c)
*log(x + (a**4*c*d**3*(-d/c)**(3/2)/(a*d - b*c)**3 - a**3*b*c**2*d**2*(-d/c
)**(3/2)/(a*d - b*c)**3 - a**2*b**2*c**3*d*(-d/c)**(3/2)/(a*d - b*c)**3 + a
**2*d**2*sqrt(-d/c)/(a*d - b*c) + a*b**3*c**4*(-d/c)**(3/2)/(a*d - b*c)**3
+ b**2*c**2*sqrt(-d/c)/(a*d - b*c))/(b*d))/(2*(a*d - b*c))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}(bc-ad)} - \frac{d\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(bc-ad)\sqrt{cd}}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*(b*c - a*d)) - d*arctan(d*x/sqrt(c*d))/(
(b*c - a*d)*sqrt(c*d))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.93

$$\int \frac{1}{(a+bx^2)(c+dx^2)}\,dx = \frac{\ln\left(b\,x-\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,a^2\,d-2\,a\,b\,c} - \frac{\ln\left(d\,x+\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,\left(b\,c^2-a\,c\,d\right)} - \frac{\ln\left(b\,x+\sqrt{-a\,b}\right)\,\sqrt{-a\,b}}{2\,\left(a^2\,d-a\,b\,c\right)} + \frac{\ln\left(d\,x-\sqrt{-c\,d}\right)\,\sqrt{-c\,d}}{2\,b\,c^2-2\,a\,c\,d}$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)),x)
```

```
[Out] (log(b*x - (-a*b)^(1/2))*(-a*b)^(1/2))/(2*a^2*d - 2*a*b*c) - (log(d*x + (-c
*d)^(1/2))*(-c*d)^(1/2))/(2*(b*c^2 - a*c*d)) - (log(b*x + (-a*b)^(1/2))*(-a
*b)^(1/2))/(2*(a^2*d - a*b*c)) + (log(d*x - (-c*d)^(1/2))*(-c*d)^(1/2))/(2*
b*c^2 - 2*a*c*d)
```

3.25 $\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = -\frac{dx}{2c(bc-ad)(c+dx^2)}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c
)^2/a^(1/2)-1/2*(-a*d+3*b*c)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(3/2)/(-a*
d+b*c)^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {425, 536, 211}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}-\frac{dx}{2c(c+dx^2)(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*(c + d*x^2)) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(Sqrt[a]*(b*c - a*d)^2) - (Sqrt[d]*(3*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(2*c^(3/2)*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{dx}{2c(bc-ad)\,(c+dx^2)}+\frac{\int\frac{2bc-ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2c(bc-ad)}\\ &= -\frac{dx}{2c(bc-ad)\,(c+dx^2)}+\frac{b^2\int\frac{1}{a+bx^2}\,dx}{(bc-ad)^2}-\frac{(d(3bc-ad))\int\frac{1}{c+dx^2}\,dx}{2c(bc-ad)^2}\\ &= -\frac{dx}{2c(bc-ad)\,(c+dx^2)}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^2}-\frac{\sqrt{d}(3bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87

$$\int\frac{1}{(a+bx^2)\,(c+dx^2)^2}\,dx=\frac{\frac{d(-bc+ad)x}{c(c+dx^2)}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\sqrt{d}(-3bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{2(bc-ad)^2}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^2),x]
```

```
[Out] ((d*(-(b*c) + a*d)*x)/(c*(c + d*x^2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/Sqrt[a] + (Sqrt[d]*(-3*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2)
)/(2*(b*c - a*d)^2)
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\left(a d-b c\right)^{2} \sqrt{a b}}+\frac{d \left(\frac{\left(a d-b c\right) x}{2 c \left(d \,x^{2}+c \right)}+\frac{\left(a d-3 b c \right) \arctan\left(\frac{d x}{\sqrt{c d}}\right)}{2 c \sqrt{c d}}\right)}{\left(a d-b c \right)^{2}}$	93
risch	Expression too large to display	1039

[In] int(1/(b*x^2+a)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] b^2/(a*d-b*c)^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^2*(1/2*(a*d -b*c)/c*x/(d*x^2+c)+1/2*(a*d-3*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 711, normalized size of antiderivative = 6.52

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx = \left[\frac{2\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)-}{4\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}\right.$$

$$-\frac{\left(3\,bc^2-acd+\left(3\,bcd-ad^2\right)x^2\right)\sqrt{\frac{d}{c}}\arctan\left(x\sqrt{\frac{d}{c}}\right)-\left(bcdx^2+bc^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+\left(bcd-\right.}{2\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x^2\right)}$$

[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [1/4*(2*(b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)*log((d*x ^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2 *a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), -1 /2*((3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d*x^2 + b*c^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3 *d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^2), 1/4*(4*(b*c*d*x^2 + b*c^2)*sqrt(b/a)* arctan(x*sqrt(b/a)) - (3*b*c^2 - a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(-d/c)* log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b*c*d - a*d^2)*x)/(b^2 *c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*

```
x^2), 1/2*(2*(b*c*d*x^2 + b*c^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - (3*b*c^2 -
 a*c*d + (3*b*c*d - a*d^2)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (b*c*d - a*
d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time $=0.29$ (sec) , antiderivative size $=133$, normalized size of antiderivative $=1.22$

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^2}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{dx}{2\,(bc^3-ac^2d+(bc^2d-acd^2)x^2)} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^2) - 1/2*(3*b*c*d - a*d^2)*ar
ctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqrt(c*d))
```

Giac [A] (verification not implemented)

none

Time $=0.28$ (sec) , antiderivative size $=122$, normalized size of antiderivative $=1.12$

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^2}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{ab}} - \frac{(3\,bcd-ad^2)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^2c^3-2\,abc^2d+a^2cd^2)\sqrt{cd}} - \frac{dx}{2\,(bc^2-acd)(dx^2+c)}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*arctan(b*x/sqrt(a*b))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(a*b)) - 1/2
*(3*b*c*d - a*d^2)*arctan(d*x/sqrt(c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^
2)*sqrt(c*d)) - 1/2*d*x/((b*c^2 - a*c*d)*(d*x^2 + c))
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 3637, normalized size of antiderivative = 33.37

$$\int \frac{1}{(a+bx^2)(c+dx^2)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^2),x)
```

```
[Out] (d*x)/(2*c*(c + d*x^2)*(a*d - b*c)) - (atan((((-c^3*d)^(1/2)*(a*d - 3*b*c)*
((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7
+ 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 -
a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3
*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a
*d - 3*b*c))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a
^2*c^3*d^2 - 2*a*b*c^4*d)) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5
 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)) + (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4
*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*
a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^
7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4
*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d
)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*
(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*1i)/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)))/(((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5 - a^3*c^2*d^3 + 3*a
^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*
d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)) - (((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*
c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 +
 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) - (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7
*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*
a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^
3*d)*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/
(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a
*b*c^4*d)) - ((-c^3*d)^(1/2)*(a*d - 3*b*c)*((x*(a^2*b^3*d^5 + 13*b^5*c^2*d^
3 - 6*a*b^4*c*d^4))/(2*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)) + (((4*b^7*c^
```

```
6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^
4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 -
3*a*b^2*c^4*d) + (x*(-c^3*d)^(1/2)*(a*d - 3*b*c)*(16*b^7*c^7*d^2 - 48*a*b^6
*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 1
6*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(b^2*c^5 + a^2
*c^3*d^2 - 2*a*b*c^4*d)))*(-c^3*d)^(1/2)*(a*d - 3*b*c))/(4*(b^2*c^5 + a^2*c
^3*d^2 - 2*a*b*c^4*d))))/(4*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d))))*(-c^3*
d)^(1/2)*(a*d - 3*b*c)*1i)/(2*(b^2*c^5 + a^2*c^3*d^2 - 2*a*b*c^4*d)) - (ata
n((((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 +
 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (x*(-a*b^3)^(1/2)*(16*b
^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 4
8*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*
c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 +
a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^
4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*b^2*c^2 - 2
*a^2*b*c*d) - ((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*
b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(
2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^
(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*
c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d
^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*
(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) + (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)))*1i)/(a^3*d^2 + a*
b^2*c^2 - 2*a^2*b*c*d))/(((-a*b^3)^(1/2)*((((4*b^7*c^6*d^2 - 18*a*b^6*c^5*d
^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a^3*b^4*c^3*d^5 + 12*a^4*b^3
*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d)) - (
x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*d^3 + 32*a^2*b^5*c^5*d^4 +
32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5*b^2*c^2*d^7))/(8*(b^2*c^4
+ a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)
^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)) - (x*(a^2*b^3*d^5 + 13*b^5*
c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d))))/(a^3*
d^2 + a*b^2*c^2 - 2*a^2*b*c*d) - ((a*b^4*d^4)/2 - (3*b^5*c*d^3)/2)/(b^3*c^5
 - a^3*c^2*d^3 + 3*a^2*b*c^3*d^2 - 3*a*b^2*c^4*d) + ((-a*b^3)^(1/2)*((((4*b
^7*c^6*d^2 - 18*a*b^6*c^5*d^3 - 2*a^5*b^2*c*d^7 + 32*a^2*b^5*c^4*d^4 - 28*a
^3*b^4*c^3*d^5 + 12*a^4*b^3*c^2*d^6)/(2*(b^3*c^5 - a^3*c^2*d^3 + 3*a^2*b*c^
3*d^2 - 3*a*b^2*c^4*d)) + (x*(-a*b^3)^(1/2)*(16*b^7*c^7*d^2 - 48*a*b^6*c^6*
d^3 + 32*a^2*b^5*c^5*d^4 + 32*a^3*b^4*c^4*d^5 - 48*a^4*b^3*c^3*d^6 + 16*a^5
*b^2*c^2*d^7))/(8*(b^2*c^4 + a^2*c^2*d^2 - 2*a*b*c^3*d)*(a^3*d^2 + a*b^2*c^
2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2))/(2*(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d))
+ (x*(a^2*b^3*d^5 + 13*b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(b^2*c^4 + a^2*c^2*
d^2 - 2*a*b*c^3*d))))/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)))*(-a*b^3)^(1/2)*
1i)/(a^3*d^2 + a*b^2*c^2 - 2*a^2*b*c*d)
```

3.26 $\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^2)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2(c+dx^2)} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}(15b^2c^2-10abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2-1/8*d*(-3*a*d+7*b*c)*x/c^2/(-a*d+b*c)^2/(
d*x^2+c)+b^(5/2)*arctan(x*b^(1/2)/a^(1/2))/(-a*d+b*c)^3/a^(1/2)-1/8*(3*a^2*
d^2-10*a*b*c*d+15*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))*d^(1/2)/c^(5/2)/(-a*d+
b*c)^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {425, 541, 536, 211}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = -\frac{\sqrt{d}(3a^2d^2-10abcd+15b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{dx(7bc-3ad)}{8c^2(c+dx^2)(bc-ad)^2} - \frac{dx}{4c(c+dx^2)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(c + d*x^2)^2) - (d*(7*b*c - 3*a*d)*x)/(8*c^2*(b*
c - a*d)^2*(c + d*x^2)) + (b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(b
*c - a*d)^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[
d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-3bdx^2}{(a+bx^2)(c+dx^2)^2}\,dx}{4c(bc-ad)}\\ &= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-7abcd+3a^2d^2-bd(7bc-3ad)x^2}{(a+bx^2)(c+dx^2)}\,dx}{8c^2(bc-ad)^2}\end{aligned}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^3\int \frac{1}{a+bx^2}\,dx}{(bc-ad)^3} - \frac{\left(d\left(15b^2c^2-10abcd+3a^2d^2\right)\right)\int \frac{1}{c+dx^2}\,dx}{8c^2(bc-ad)^3}$$

$$= -\frac{dx}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{d(7bc-3ad)x}{8c^2(bc-ad)^2\left(c+dx^2\right)} + \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(bc-ad)^3} - \frac{\sqrt{d}\left(15b^2c^2-10abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^3}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx = \frac{1}{8}\left(\frac{dx\left(ad\left(5c+3dx^2\right)-bc\left(9c+7dx^2\right)\right)}{c^2(bc-ad)^2\left(c+dx^2\right)^2} - \frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}(-bc+ad)^3} - \frac{\sqrt{d}\left(15b^2c^2-10abcd+3a^2d^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^3}\right)$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^3),x]
```

```
[Out] ((d*x*(a*d*(5*c + 3*d*x^2) - b*c*(9*c + 7*d*x^2)))/(c^2*(b*c - a*d)^2*(c +
d*x^2)^2) - (8*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*(-(b*c) + a*d)
^3) - (Sqrt[d]*(15*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(5/2)*(b*c - a*d)^3))/8
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(ad-bc)^3\sqrt{ab}} + \frac{d\left(\frac{\frac{d\left(3a^2d^2-10abcd+7b^2c^2\right)x^3}{8c^2}+\frac{\left(5a^2d^2-14abcd+9b^2c^2\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2-10abcd+15b^2c^2\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8c^2\sqrt{cd}}\right)}{(ad-bc)^3}$	158
risch	Expression too large to display	2285

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))+d/(a*d-b*c)^3*((1/8*d*
(3*a^2*d^2-10*a*b*c*d+7*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-14*a*b*c*d+9*b^2*c^
```

```
2)/c*x)/(d*x^2+c)^2+1/8*(3*a^2*d^2-10*a*b*c*d+15*b^2*c^2)/c^2/(c*d)^(1/2)*a
rctan(d*x/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(138) = 276.

Time = 0.75 (sec) , antiderivative size = 1585, normalized size of antiderivative = 9.91

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="fricas")
[Out] [-1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 + 8*(b^2*c^2*d^2*x
^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*
d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*
a^2*c*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c))
+ 2*(9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*
d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*
b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4
*d^3 - a^3*c^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*
x^3 + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + 4*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 +
b^2*c^4)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (9*b^
2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2
*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4
 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^
3*c^3*d^4)*x^2), -1/16*(2*(7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 -
16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/
a)) + (15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b
*c*d^3 + 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x
^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(9*b^2*c
^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*
c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 -
a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c
^3*d^4)*x^2), -1/8*((7*b^2*c^2*d^2 - 10*a*b*c*d^3 + 3*a^2*d^4)*x^3 - 8*(b^2
*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (
15*b^2*c^4 - 10*a*b*c^3*d + 3*a^2*c^2*d^2 + (15*b^2*c^2*d^2 - 10*a*b*c*d^3
+ 3*a^2*d^4)*x^4 + 2*(15*b^2*c^3*d - 10*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqr
t(d/c)*arctan(x*sqrt(d/c)) + (9*b^2*c^3*d - 14*a*b*c^2*d^2 + 5*a^2*c*d^3)*x
)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2 - a^3*c^4*d^3 + (b^3*c^5*d^2 -
 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^4 + 2*(b^3*c^6*d - 3*a*
b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(138) = 276$.

Time = 0.28 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^3}\,dx = \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} - \frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}} - \frac{(7\,bcd^2-3\,ad^3)x^3+(9\,bc^2d-5\,acd^2)x}{8\,(b^2c^6-2\,abc^5d+a^2c^4d^2+(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4)x^4+2\,(b^2c^5d-2\,abc^4d^2+a^2c^3d^3)x^2)}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*((7*b*c*d^2 - 3*a*d^3)*x^3 + (9*b*c^2*d - 5*a*c*d^2)*x)/(b^2*c^6
 - 2*a*b*c^5*d + a^2*c^4*d^2 + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*
x^4 + 2*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^3}\,dx = \frac{b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{ab}} - \frac{(15\,b^2c^2d-10\,abcd^2+3\,a^2d^3)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{cd}} - \frac{7\,bcd^2x^3-3\,ad^3x^3+9\,bc^2dx-5\,acd^2x}{8\,(b^2c^4-2\,abc^3d+a^2c^2d^2)(dx^2+c)^2}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*arctan(b*x/sqrt(a*b))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*sqrt(a*b)) - 1/8*(15*b^2*c^2*d - 10*a*b*c*d^2 + 3*a^2*d^3)*arctan(d*x/s
qrt(c*d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(c
*d)) - 1/8*(7*b*c*d^2*x^3 - 3*a*d^3*x^3 + 9*b*c^2*d*x - 5*a*c*d^2*x)/((b^2*
c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(d*x^2 + c)^2)
```

Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 6033, normalized size of antiderivative = 37.71

$$\int \frac{1}{(a+bx^2)(c+dx^2)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^3),x)
```

```
[Out] ((x^3*(3*a*d^3 - 7*b*c*d^2))/(8*c^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*a*d^2 - 9*b*c*d))/(8*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(c^2 + d^2*x^4 +
 2*c*d*x^2) - (atan((((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 -
 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8
+ a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-
a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^
4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 30
40*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^
10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^
3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*
d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1
280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7
*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*
(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*
d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(
a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) + ((-a*b^5)^(1/2)*(
(x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6
+ 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^
2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 176
0*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^
6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d
^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 1
5*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d
)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b
^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6
*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^
3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d
^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*
```

b^2*c^2*d - 3*a^3*b*c*d^2)))*1i)/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 - 51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)) - ((-a*b^5)^(1/2)*((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + ((-a*b^5)^(1/2)*((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-a*b^5)^(1/2)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(64*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))/(2*(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2))))*(-a*b^5)^(1/2)*1i)/(a^4*d^3 - a*b^3*c^3 + 3*a^2*b^2*c^2*d - 3*a^3*b*c*d^2) - (atan((((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3

+ 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)) + (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))/((9*a^3*b^5*d^6 - 105*b^8*c^3*d^3 + 115*a*b^7*c^2*d^4 - 51*a^2*b^6*c*d^5)/(32*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) - (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)) - (((x*(9*a^4*b^3*d^7 + 289*b^7*c^4*d^3 - 300*a*b^6*c^3*d^4 - 60*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(b^4*c^8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d)) + (((256*b^10*c^10*d^2 - 1760*a*b^9*c^9*d^3 + 5280*a^2*b^8*c^8*d^4 - 9056*a^3*b^7*c^7*d^5 + 9760*a^4*b^6*c^6*d^6 - 6816*a^5*b^5*c^5*d^7 + 3040*a^6*b^4*c^4*d^8 - 800*a^7*b^3*c^3*d^9 + 96*a^8*b^2*c^2*d^10)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + (x*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*(256*b^9*c^11*d^2 - 1280*a*b^8*c^10*d^3 + 2304*a^2*b^7*c^9*d^4 - 1280*a^3*b^6*c^8*d^5 - 1280*a^4*b^5*c^7*d^6 + 2304*a^5*b^4*c^6*d^7 - 1280*a^6*b^3*c^5*d^8 + 256*a^7*b^2*c^4*d

```
^9))/(512*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)*(b^4*c^
8 + a^4*c^4*d^4 - 4*a^3*b*c^5*d^3 + 6*a^2*b^2*c^6*d^2 - 4*a*b^3*c^7*d))*(-
c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*
d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d)))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2
*c^2 - 10*a*b*c*d))/(16*(b^3*c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*
c^7*d))))*(-c^5*d)^(1/2)*(3*a^2*d^2 + 15*b^2*c^2 - 10*a*b*c*d)*1i)/(8*(b^3*
c^8 - a^3*c^5*d^3 + 3*a^2*b*c^6*d^2 - 3*a*b^2*c^7*d))
```

3.27 $\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc-2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc-ad)^5x}{2ab^5\,(a+bx^2)} + \frac{(bc-ad)^4(bc+9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

```
[Out] d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/3*d^3*(3*
a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^3/b^4+1/5*d^4*(-2*a*d+5*b*c)*x^5/b^3+1/7*d
^5*x^7/b^2+1/2*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)+1/2*(-a*d+b*c)^4*(9*a*d+b*c)*
arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(11/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^4(9ad+bc)}{2a^{3/2}b^{11/2}} + \frac{d^3x^3(3a^2d^2-10abcd+10b^2c^2)}{3b^4} + \frac{d^2x(-4a^3d^3+15a^2bcd^2-20ab^2c^2d+10b^3c^3)}{b^5} + \frac{x(bc-ad)^5}{2ab^5\,(a+bx^2)} + \frac{d^4x^5(5bc-2ad)}{5b^3} + \frac{d^5x^7}{7b^2}$$

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^2}{b^4}\right.\\
&\qquad \left.+\frac{d^4(5bc-2ad)x^4}{b^3}+\frac{d^5x^6}{b^2}+\frac{(bc-ad)^4(bc+4ad)+5bd(bc-ad)^4x^2}{b^5\left(a+bx^2\right)^2}\right)\,dx\\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4}\\
&\quad +\frac{d^4(5bc-2ad)x^5}{5b^3}+\frac{d^5x^7}{7b^2}+\frac{\int \frac{(bc-ad)^4(bc+4ad)+5bd(bc-ad)^4x^2}{(a+bx^2)^2}\,dx}{b^5}\\
&= \frac{d^2(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3)\,x}{b^5}+\frac{d^3(10b^2c^2-10abcd+3a^2d^2)\,x^3}{3b^4}\\
&\quad +\frac{d^4(5bc-2ad)x^5}{5b^3}+\frac{d^5x^7}{7b^2}+\frac{(bc-ad)^5x}{2ab^5\left(a+bx^2\right)}+\frac{((bc-ad)^4(bc+9ad))\int\frac{1}{a+bx^2}\,dx}{2ab^5}
\end{aligned}
$$

$$= \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc - 2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc - ad)^5x}{2ab^5\,(a + bx^2)} + \frac{(bc - ad)^4(bc + 9ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00

$$\int \frac{(c + dx^2)^5}{(a + bx^2)^2}\,dx = \frac{d^2(10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)\,x}{b^5} + \frac{d^3(10b^2c^2 - 10abcd + 3a^2d^2)\,x^3}{3b^4} + \frac{d^4(5bc - 2ad)x^5}{5b^3} + \frac{d^5x^7}{7b^2} + \frac{(bc - ad)^5x}{2ab^5\,(a + bx^2)} + \frac{(bc - ad)^4(bc + 9ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{11/2}}$$

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^3)/(3*b^4) + (d^4*(5*b*c - 2*a*d
)*x^5)/(5*b^3) + (d^5*x^7)/(7*b^2) + ((b*c - a*d)^5*x)/(2*a*b^5*(a + b*x^2)
) + ((b*c - a*d)^4*(b*c + 9*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^
(11/2))
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.51

method	result
default	$-\frac{d^2\left(-\frac{1}{7}b^3d^3x^7+\frac{2}{5}a\,b^2d^3x^5-b^3c\,d^2x^5-a^2b\,d^3x^3+\frac{10}{3}a\,b^2c\,d^2x^3-\frac{10}{3}b^3c^2d\,x^3+4a^3d^3x-15a^2bc\,d^2x+20a\,b^2c^2dx-10b^3c^3x\right)}{b^5}+$
risch	$\frac{d^5x^7}{7b^2}-\frac{2d^5a\,x^5}{5b^3}+\frac{d^4c\,x^5}{b^2}+\frac{d^5a^2x^3}{b^4}-\frac{10d^4ac\,x^3}{3b^3}+\frac{10d^3c^2x^3}{3b^2}-\frac{4d^5a^3x}{b^5}+\frac{15d^4a^2cx}{b^4}-\frac{20d^3a\,c^2x}{b^3}+\frac{10d^2c^3x}{b^2}-$

```
[In] int((d*x^2+c)^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/b^5*(-1/7*b^3*d^3*x^7+2/5*a*b^2*d^3*x^5-b^3*c*d^2*x^5-a^2*b*d^3*x^3+10
/3*a*b^2*c*d^2*x^3-10/3*b^3*c^2*d*x^3+4*a^3*d^3*x-15*a^2*b*c*d^2*x+20*a*b^2
*c^2*d*x-10*b^3*c^3*x)+1/b^5*(-1/2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^
3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/a*x/(b*x^2+a)+1/2*(9*a^5*d^5-35
*a^4*b*c*d^4+50*a^3*b^2*c^2*d^3-30*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+b^5*c^5)/a
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(174) = 348$.

Time = 0.29 (sec) , antiderivative size = 810, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{60\,a^2b^5d^5x^9 + 12\,(35\,a^2b^5cd^4 - 9\,a^3b^4d^5)x^7 + 28\,(50\,a^2b^5c^2d^3 - 35\,a^3b^4cd^4 + 9\,a^4b^3d^5)x^5 + 140\,(30\,a^2b^5c^3d}{}\right.$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*a^2*b^5*d^5*x^9 + 12*(35*a^2*b^5*c*d^4 - 9*a^3*b^4*d^5)*x^7 + 28
*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*a^4*b^3*d^5)*x^5 + 140*(30*a^2*
b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3*c*d^4 - 9*a^5*b^2*d^5)*x^3 -
105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3*c^3*d^2 + 50*a^4*b^2*c^2*d^3
- 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a*b^5*c^4*d - 30*a^2*b^4*c^3*d^
2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9*a^5*b*d^5)*x^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 210*(a*b^6*c^5 - 5*a^2*b^5*c^
4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a^6*b*
d^5)*x)/(a^2*b^7*x^2 + a^3*b^6), 1/210*(30*a^2*b^5*d^5*x^9 + 6*(35*a^2*b^5*
c*d^4 - 9*a^3*b^4*d^5)*x^7 + 14*(50*a^2*b^5*c^2*d^3 - 35*a^3*b^4*c*d^4 + 9*
a^4*b^3*d^5)*x^5 + 70*(30*a^2*b^5*c^3*d^2 - 50*a^3*b^4*c^2*d^3 + 35*a^4*b^3
*c*d^4 - 9*a^5*b^2*d^5)*x^3 + 105*(a*b^5*c^5 + 5*a^2*b^4*c^4*d - 30*a^3*b^3
*c^3*d^2 + 50*a^4*b^2*c^2*d^3 - 35*a^5*b*c*d^4 + 9*a^6*d^5 + (b^6*c^5 + 5*a
*b^5*c^4*d - 30*a^2*b^4*c^3*d^2 + 50*a^3*b^3*c^2*d^3 - 35*a^4*b^2*c*d^4 + 9
*a^5*b*d^5)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 105*(a*b^6*c^5 - 5*a^2*b
^5*c^4*d + 30*a^3*b^4*c^3*d^2 - 50*a^4*b^3*c^2*d^3 + 35*a^5*b^2*c*d^4 - 9*a
^6*b*d^5)*x)/(a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(185) = 370$.

Time = 1.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.61

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = x^5\left(-\frac{2ad^5}{5b^3}+\frac{cd^4}{b^2}\right)+x^3\left(\frac{a^2d^5}{b^4}-\frac{10acd^4}{3b^3}+\frac{10c^2d^3}{3b^2}\right) + x\left(-\frac{4a^3d^5}{b^5}+\frac{15a^2cd^4}{b^4}-\frac{20ac^2d^3}{b^3}+\frac{10c^3d^2}{b^2}\right) + \frac{x(-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5)}{2a^2b^5+2ab^6x^2} - \frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(-\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)\log\left(\frac{a^2b^5\sqrt{-\frac{1}{a^3b^{11}}}(ad-bc)^4\cdot(9ad+bc)}{9a^5d^5-35a^4bcd^4+50a^3b^2c^2d^3-30a^2b^3c^3d^2+5ab^4c^4d+b^5c^5}+x\right)}{4} + \frac{d^5x^7}{7b^2}$$

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*d**5/(5*b**3) + c*d**4/b**2) + x**3*(a**2*d**5/b**4 - 10*a*c*d**
4/(3*b**3) + 10*c**2*d**3/(3*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*d**4
/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a**4*b
*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d
 + b**5*c**5)/(2*a**2*b**5 + 2*a*b**6*x**2) - sqrt(-1/(a**3*b**11))*(a*d -
b*c)**4*(9*a*d + b*c)*log(-a**2*b**5*sqrt(-1/(a**3*b**11))*(a*d - b*c)**4*(
9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c**2*d**3 - 30*
a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + sqrt(-1/(a**3*b
**11))*(a*d - b*c)**4*(9*a*d + b*c)*log(a**2*b**5*sqrt(-1/(a**3*b**11))*(a*
d - b*c)**4*(9*a*d + b*c)/(9*a**5*d**5 - 35*a**4*b*c*d**4 + 50*a**3*b**2*c*
*2*d**3 - 30*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + b**5*c**5) + x)/4 + d*
*5*x**7/(7*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx = \frac{(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5)x}{2\,(ab^6x^2 + a^2b^5)}$$
$$+ \frac{15\,b^3d^5x^7 + 21\,(5\,b^3cd^4 - 2\,ab^2d^5)x^5 + 35\,(10\,b^3c^2d^3 - 10\,ab^2cd^4 + 3\,a^2bd^5)x^3 + 105\,(10\,b^3c^3d^2 - 20\,ab^2c^2}{105\,b^5}$$
$$+ \frac{(b^5c^5 + 5\,ab^4c^4d - 30\,a^2b^3c^3d^2 + 50\,a^3b^2c^2d^3 - 35\,a^4bcd^4 + 9\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^5}$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^2 + a^2*b^5) + 1/105*(15*b^3*d^5*x^7 + 21
*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^5 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3*
a^2*b*d^5)*x^3 + 105*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 -
4*a^3*d^5)*x)/b^5 + 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*
a^3*b^2*c^2*d^3 - 35*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$= \frac{(b^5c^5 + 5\,ab^4c^4d - 30\,a^2b^3c^3d^2 + 50\,a^3b^2c^2d^3 - 35\,a^4bcd^4 + 9\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^5}$$
$$+ \frac{b^5c^5x - 5\,ab^4c^4dx + 10\,a^2b^3c^3d^2x - 10\,a^3b^2c^2d^3x + 5\,a^4bcd^4x - a^5d^5x}{2\,(bx^2+a)ab^5}$$
$$+ \frac{15\,b^{12}d^5x^7 + 105\,b^{12}cd^4x^5 - 42\,ab^{11}d^5x^5 + 350\,b^{12}c^2d^3x^3 - 350\,ab^{11}cd^4x^3 + 105\,a^2b^{10}d^5x^3 + 1050\,b^{12}c^3d^2x}{105\,b^{14}}$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*c^5 + 5*a*b^4*c^4*d - 30*a^2*b^3*c^3*d^2 + 50*a^3*b^2*c^2*d^3 - 35
*a^4*b*c*d^4 + 9*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^5) + 1/2*(b^
5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5
*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^2 + a)*a*b^5) + 1/105*(15*b^12*d^5*x^7 +
105*b^12*c*d^4*x^5 - 42*a*b^11*d^5*x^5 + 350*b^12*c^2*d^3*x^3 - 350*a*b^11*
c*d^4*x^3 + 105*a^2*b^10*d^5*x^3 + 1050*b^12*c^3*d^2*x - 2100*a*b^11*c^2*d^
3*x + 1575*a^2*b^10*c*d^4*x - 420*a^3*b^9*d^5*x)/b^14
```

Mupad [B] (verification not implemented)

Time = 4.67 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^2}\,dx$$
$$= x\left(\frac{10\,c^3\,d^2}{b^2}-\frac{2\,a\,\left(\frac{2\,a\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b}-\frac{a^2\,d^5}{b^4}+\frac{10\,c^2\,d^3}{b^2}\right)}{b}+\frac{a^2\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{b^2}\right)$$
$$-x^5\left(\frac{2\,a\,d^5}{5\,b^3}-\frac{c\,d^4}{b^2}\right)+x^3\left(\frac{2\,a\,\left(\frac{2\,a\,d^5}{b^3}-\frac{5\,c\,d^4}{b^2}\right)}{3\,b}-\frac{a^2\,d^5}{3\,b^4}+\frac{10\,c^2\,d^3}{3\,b^2}\right)+\frac{d^5\,x^7}{7\,b^2}$$
$$-\frac{x\,(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5)}{2\,a\,(b^6\,x^2+a\,b^5)}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^4\,(9\,a\,d+b\,c)}{\sqrt{a}\,(9\,a^5\,d^5-35\,a^4\,b\,c\,d^4+50\,a^3\,b^2\,c^2\,d^3-30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+b^5\,c^5)}\right)\,(a\,d-b\,c)^4\,(9\,a\,d+b\,c)}{2\,a^{3/2}\,b^{11/2}}$$

```
[In] int((c + d*x^2)^5/(a + b*x^2)^2,x)
```

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b - (a^2*
d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2
) - x^5*((2*a*d^5)/(5*b^3) - (c*d^4)/b^2) + x^3*((2*a*((2*a*d^5)/b^3 - (5*c
*d^4)/b^2))/(3*b) - (a^2*d^5)/(3*b^4) + (10*c^2*d^3)/(3*b^2)) + (d^5*x^7)/(
7*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(2*a*(a*b^5 + b^6*x^2)) + (atan((b^(1/2)*x*
(a*d - b*c)^4*(9*a*d + b*c))/(a^(1/2)*(9*a^5*d^5 + b^5*c^5 - 30*a^2*b^3*c^3
*d^2 + 50*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 35*a^4*b*c*d^4)))*(a*d - b*c)^4
*(9*a*d + b*c))/(2*a^(3/2)*b^(11/2))
```

3.28 $\int \frac{(c+dx^2)^4}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 142

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2} dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4} + \frac{2d^3(2bc-ad)x^3}{3b^3} + \frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4\,(a+bx^2)} + \frac{(bc-ad)^3(bc+7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

```
[Out] d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/3*d^3*(-a*d+2*b*c)*x^3/b^3+1/5*
d^4*x^5/b^2+1/2*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a)+1/2*(-a*d+b*c)^3*(7*a*d+b*c)
*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^3(7ad+bc)}{2a^{3/2}b^{9/2}} + \frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} + \frac{x(bc-ad)^4}{2ab^4\,(a+bx^2)} + \frac{2d^3x^3(2bc-ad)}{3b^3} + \frac{d^4x^5}{5b^2}$$

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4}+\frac{2d^3(2bc-ad)x^2}{b^3}+\frac{d^4x^4}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{b^4\left(a+bx^2\right)^2}\right)dx \\
&= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^2}{\left(a+bx^2\right)^2}\,dx}{b^4} \\
&= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4\left(a+bx^2\right)}+\frac{\left((bc-ad)^3(bc+7ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^4} \\
&= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2} + \frac{(bc-ad)^4x}{2ab^4\left(a+bx^2\right)}+\frac{(bc-ad)^3(bc+7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{d^2(6b^2c^2-8abcd+3a^2d^2)\,x}{b^4}+\frac{2d^3(2bc-ad)x^3}{3b^3}+\frac{d^4x^5}{5b^2}+\frac{(bc-ad)^4x}{2ab^4\,(a+bx^2)}+\frac{(bc-ad)^3(bc+7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{9/2}}$$

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^3)
/(3*b^3) + (d^4*x^5)/(5*b^2) + ((b*c - a*d)^4*x)/(2*a*b^4*(a + b*x^2)) + ((
b*c - a*d)^3*(b*c + 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(9/2))
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.45

method	result
default	$\frac{d^2\left(\frac{1}{5}b^2d^2x^5-\frac{2}{3}x^3ab\,d^2+\frac{4}{3}x^3b^2cd+3a^2d^2x-8abcdx+6b^2c^2x\right)}{b^4}-\frac{-\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(7a^4d^4-20a^3bc\,d^3+1\right.}{}}{b^4}$
risch	$\frac{d^4x^5}{5b^2}-\frac{2d^4x^3a}{3b^3}+\frac{4d^3x^3c}{3b^2}+\frac{3d^4a^2x}{b^4}-\frac{8d^3acx}{b^3}+\frac{6d^2c^2x}{b^2}+\frac{\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)x}{2a\,b^4\left(b\,x^2+a\right)}-\frac{7a^3\ln\left(bx-\sqrt{\right.}}{4b^4\sqrt{-}}$

```
[In] int((d*x^2+c)^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/b^4*(1/5*b^2*d^2*x^5-2/3*x^3*a*b*d^2+4/3*x^3*b^2*c*d+3*a^2*d^2*x-8*a*b*
c*d*x+6*b^2*c^2*x)-1/b^4*(-1/2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/a*x/(b*x^2+a)+1/2*(7*a^4*d^4-20*a^3*b*c*d^3+18*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d-b^4*c^4)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(126) = 252$.

Time = 0.26 (sec) , antiderivative size = 612, normalized size of antiderivative = 4.31

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{12\,a^2b^4d^4x^7+4\,(20\,a^2b^4cd^3-7\,a^3b^3d^4)x^5+20\,(18\,a^2b^4c^2d^2-20\,a^3b^3cd^3+7\,a^4b^2d^4)x^3+15\,(ab^4c^4+4\,a^2b}{}\right.$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a^2*b^4*d^4*x^7 + 4*(20*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*x^5 + 20*(
18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^2*d^4)*x^3 + 15*(a*b^4*c^4
+ 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*b*c*d^3 - 7*a^5*d^4 + (b^5*
c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^3*b^2*c*d^3 - 7*a^4*b*d^4)*
x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*b^5*c
^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^4)
*x)/(a^2*b^6*x^2 + a^3*b^5), 1/30*(6*a^2*b^4*d^4*x^7 + 2*(20*a^2*b^4*c*d^3
- 7*a^3*b^3*d^4)*x^5 + 10*(18*a^2*b^4*c^2*d^2 - 20*a^3*b^3*c*d^3 + 7*a^4*b^
2*d^4)*x^3 + 15*(a*b^4*c^4 + 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 20*a^4*
b*c*d^3 - 7*a^5*d^4 + (b^5*c^4 + 4*a*b^4*c^3*d - 18*a^2*b^3*c^2*d^2 + 20*a^
3*b^2*c*d^3 - 7*a^4*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(a*b^5
*c^4 - 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 20*a^4*b^2*c*d^3 + 7*a^5*b*d^
4)*x)/(a^2*b^6*x^2 + a^3*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(133) = 266$.

Time = 0.82 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.84

$$\begin{aligned}
&\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx \\
&= x^3\left(-\frac{2ad^4}{3b^3}+\frac{4cd^3}{3b^2}\right)+x\left(\frac{3a^2d^4}{b^4}-\frac{8acd^3}{b^3}+\frac{6c^2d^2}{b^2}\right) \\
&\quad+\frac{x(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{2a^2b^4+2ab^5x^2} \\
&\quad+\frac{\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)\log\left(-\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4} \\
&\quad-\frac{\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)\log\left(\frac{a^2b^4\sqrt{-\frac{1}{a^3b^9}}(ad-bc)^3\cdot(7ad+bc)}{7a^4d^4-20a^3bcd^3+18a^2b^2c^2d^2-4ab^3c^3d-b^4c^4}+x\right)}{4}+\frac{d^4x^5}{5b^2}
\end{aligned}$$

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*d**4/(3*b**3) + 4*c*d**3/(3*b**2)) + x*(3*a**2*d**4/b**4 - 8*a*c
*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b*
*2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(2*a**2*b**4 + 2*a*b**5*x**2) +
 sqrt(-1/(a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(-a**2*b**4*sqrt(-1/(
a**3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 +
18*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 - sqrt(-1/(a**
3*b**9))*(a*d - b*c)**3*(7*a*d + b*c)*log(a**2*b**4*sqrt(-1/(a**3*b**9))*(a
*d - b*c)**3*(7*a*d + b*c)/(7*a**4*d**4 - 20*a**3*b*c*d**3 + 18*a**2*b**2*c
**2*d**2 - 4*a*b**3*c**3*d - b**4*c**4) + x)/4 + d**4*x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4)x}{2\,(ab^5x^2 + a^2b^4)} + \frac{3\,b^2d^4x^5 + 10\,(2\,b^2cd^3 - abd^4)x^3 + 15\,(6\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x}{15\,b^4} + \frac{(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^4}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*x/(a*b^5*x^2 + a^2*b^4) + 1/15*(3*b^2*d^4*x^5 + 10*(2*b^2*c*d^3 - a*b*d^4)
*x^3 + 15*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 1/2*(b^4*c^4 +
 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*d^4)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.55

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx = \frac{(b^4c^4 + 4\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 20\,a^3bcd^3 - 7\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^4} + \frac{b^4c^4x - 4\,ab^3c^3dx + 6\,a^2b^2c^2d^2x - 4\,a^3bcd^3x + a^4d^4x}{2\,(bx^2+a)ab^4} + \frac{3\,b^8d^4x^5 + 20\,b^8cd^3x^3 - 10\,ab^7d^4x^3 + 90\,b^8c^2d^2x - 120\,ab^7cd^3x + 45\,a^2b^6d^4x}{15\,b^{10}}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^4*c^4 + 4*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 7*a^4*
d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/2*(b^4*c^4*x - 4*a*b^3*c^3
*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^2 + a)*a*b^
4) + 1/15*(3*b^8*d^4*x^5 + 20*b^8*c*d^3*x^3 - 10*a*b^7*d^4*x^3 + 90*b^8*c^2
*d^2*x - 120*a*b^7*c*d^3*x + 45*a^2*b^6*d^4*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 4.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^2}\,dx$$
$$= x\left(\frac{2\,a\,\left(\frac{2\,a\,d^4}{b^3}-\frac{4\,c\,d^3}{b^2}\right)}{b}-\frac{a^2\,d^4}{b^4}+\frac{6\,c^2\,d^2}{b^2}\right)-x^3\left(\frac{2\,a\,d^4}{3\,b^3}-\frac{4\,c\,d^3}{3\,b^2}\right)$$
$$+\frac{d^4\,x^5}{5\,b^2}+\frac{x\,(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4)}{2\,a\,\left(b^5\,x^2+a\,b^4\right)}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3\,(7\,a\,d+b\,c)}{\sqrt{a}\,(-7\,a^4\,d^4+20\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+b^4\,c^4)}\right)\,(a\,d-b\,c)^3\,(7\,a\,d+b\,c)}{2\,a^{3/2}\,b^{9/2}}$$

```
[In] int((c + d*x^2)^4/(a + b*x^2)^2,x)
```

```
[Out] x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^
2) - x^3*((2*a*d^4)/(3*b^3) - (4*c*d^3)/(3*b^2)) + (d^4*x^5)/(5*b^2) + (x*(
a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(2*
a*(a*b^4 + b^5*x^2)) + (atan((b^(1/2)*x*(a*d - b*c)^3*(7*a*d + b*c))/(a^(1/
2)*(b^4*c^4 - 7*a^4*d^4 - 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d + 20*a^3*b*c*d
^3)))*(a*d - b*c)^3*(7*a*d + b*c))/(2*a^(3/2)*b^(9/2))
```

3.29 $\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{d^2(3bc-2ad)x}{b^3} + \frac{d^3x^3}{3b^2} + \frac{(bc-ad)^3x}{2ab^3(a+bx^2)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[Out] d^2*(-2*a*d+3*b*c)*x/b^3+1/3*d^3*x^3/b^2+1/2*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)
+1/2*(-a*d+b*c)^2*(5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5ad+bc)(bc-ad)^2}{2a^{3/2}b^{7/2}} + \frac{d^2x(3bc-2ad)}{b^3} + \frac{x(bc-ad)^3}{2ab^3(a+bx^2)} + \frac{d^3x^3}{3b^2}$$

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^2}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{b^3\left(a+bx^2\right)^2}\right)dx \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^2}{(a+bx^2)^2}\,dx}{b^3} \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{\left((bc-ad)^2(bc+5ad)\right)\int \frac{1}{a+bx^2}\,dx}{2ab^3} \\
&= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)}+\frac{(bc-ad)^2(bc+5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx = \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^3}{3b^2}+\frac{(bc-ad)^3x}{2ab^3\left(a+bx^2\right)} + \frac{(bc-ad)^2(bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^3)/(3*b^2) + ((b*c - a*d)^3*x)/(2*a*b^
3*(a + b*x^2)) + ((b*c - a*d)^2*(b*c + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(2*a^(3/2)*b^(7/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

method	result
default	$-\frac{d^{2}\left(-\frac{1}{3}bd\,x^{3}+2adx-3bcx\right)}{b^{3}}+\frac{-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)x}{2a\left(b\,x^{2}+a\right)}+\frac{\left(5a^{3}d^{3}-9a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d+b^{3}c^{3}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^{3}}$
risch	$\frac{d^{3}x^{3}}{3b^{2}}-\frac{2d^{3}ax}{b^{3}}+\frac{3d^{2}cx}{b^{2}}-\frac{\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)x}{2a\,b^{3}\left(b\,x^{2}+a\right)}-\frac{5a^{2}\ln\left(bx+\sqrt{-ab}\right)d^{3}}{4b^{3}\sqrt{-ab}}+\frac{9a\ln\left(bx+\sqrt{-ab}\right)c\,d^{2}}{4b^{2}\sqrt{-ab}}-\frac{3\ln\left(bx+\sqrt{-\right.}{4b\sqrt{-a}}$

[In] int((d*x^2+c)^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] -d^2/b^3*(-1/3*b*d*x^3+2*a*d*x-3*b*c*x)+1/b^3*(-1/2*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a*x/(b*x^2+a)+1/2*(5*a^3*d^3-9*a^2*b*c*d^2+3*a*b^2*c^2*d+b^3*c^3)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(92) = 184$.

Time = 0.28 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.17

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{4\,a^2b^3d^3x^5+4\left(9\,a^2b^3cd^2-5\,a^3b^2d^3\right)x^3-3\left(ab^3c^3+3\,a^2b^2c^2d-9\,a^3bcd^2+5\,a^4d^3+\left(b^4c^3+3\,ab^3c^2d-9\,a\right.\right.}{12\left(a^2b^5x^2+a^3\right.}\right.$$

[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/12*(4*a^2*b^3*d^3*x^5 + 4*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 - 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/6*(2*a^2*b^3*d^3*x^5 + 2*(9*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^3 + 3*(a*b^3*c^3 + 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 5*a^4*d^3 + (b^4*c^3 + 3*a*b^3*c^2*d - 9*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 9*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x)/(a^2*b^5*x^2 + a^3*b^4)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.58 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.96

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{x(-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3)}{2a^2b^3+2ab^4x^2} - \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(-\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)\log\left(\frac{a^2b^3\sqrt{-\frac{1}{a^3b^7}}(ad-bc)^2\cdot(5ad+bc)}{5a^3d^3-9a^2bcd^2+3ab^2c^2d+b^3c^3}+x\right)}{4} + \frac{d^3x^3}{3b^2}$$

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + x*(-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*
b**2*c**2*d + b**3*c**3)/(2*a**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7
))*(a*d - b*c)**2*(5*a*d + b*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7))*(a*d -
b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a**2*b*c*d**2 + 3*a*b**2*c**2*d + b*
*3*c**3) + x)/4 + sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)*log(a**
2*b**3*sqrt(-1/(a**3*b**7))*(a*d - b*c)**2*(5*a*d + b*c)/(5*a**3*d**3 - 9*a
**2*b*c*d**2 + 3*a*b**2*c**2*d + b**3*c**3) + x)/4 + d**3*x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)x}{2\,(ab^4x^2+a^2b^3)}+\frac{bd^3x^3+3\,(3\,bcd^2-2\,ad^3)x}{3\,b^3} + \frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^2 + a^2*
b^3) + 1/3*(b*d^3*x^3 + 3*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/2*(b^3*c^3 + 3*a
*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{(b^3c^3+3\,ab^2c^2d-9\,a^2bcd^2+5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3} + \frac{b^3c^3x-3\,ab^2c^2dx+3\,a^2bcd^2x-a^3d^3x}{2\,(bx^2+a)ab^3} + \frac{b^4d^3x^3+9\,b^4cd^2x-6\,ab^3d^3x}{3\,b^6}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a*b^3) + 1/2*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x
 - a^3*d^3*x)/((b*x^2 + a)*a*b^3) + 1/3*(b^4*d^3*x^3 + 9*b^4*c*d^2*x - 6*a*
b^3*d^3*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^2}\,dx = \frac{d^3\,x^3}{3\,b^2} - x\left(\frac{2\,a\,d^3}{b^3} - \frac{3\,c\,d^2}{b^2}\right) - \frac{x\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}{2\,a\,(b^4\,x^2+a\,b^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{\sqrt{a}\,(5\,a^3\,d^3-9\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3)}\right)\,(a\,d-b\,c)^2\,(5\,a\,d+b\,c)}{2\,a^{3/2}\,b^{7/2}}$$

```
[In] int((c + d*x^2)^3/(a + b*x^2)^2,x)
```

```
[Out] (d^3*x^3)/(3*b^2) - x*((2*a*d^3)/b^3 - (3*c*d^2)/b^2) - (x*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/
2)*x*(a*d - b*c)^2*(5*a*d + b*c))/(a^(1/2)*(5*a^3*d^3 + b^3*c^3 + 3*a*b^2*c
^2*d - 9*a^2*b*c*d^2)))*(a*d - b*c)^2*(5*a*d + b*c))/(2*a^(3/2)*b^(7/2))
```

3.30 $\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{2ab^2(a+bx^2)} + \frac{(bc-ad)(bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}$$

```
[Out] d^2*x/b^2+1/2*(-a*d+b*c)^2*x/a/b^2/(b*x^2+a)+1/2*(-a*d+b*c)*(3*a*d+b*c)*arc
tan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {398, 393, 211}

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(3ad+bc)}{2a^{3/2}b^{5/2}} + \frac{x(bc-ad)^2}{2ab^2(a+bx^2)} + \frac{d^2x}{b^2}$$

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b*c - a*d)*(b*c +
 3*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{b^2\left(a+bx^2\right)^2}\right)dx\\
&= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{b^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{((bc-ad)(bc+3ad))\int \frac{1}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{(bc-ad)(bc+3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$
\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^2}\,dx=\frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{2ab^2\left(a+bx^2\right)}+\frac{\left(b^2c^2+2abcd-3a^2d^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}
$$

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(2*a*b^2*(a + b*x^2)) + ((b^2*c^2 + 2*a*b*c
*d - 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(5/2))
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

method	result
default	$\frac{d^2x}{b^2}-\frac{-\frac{(a^2d^2-2abcd+b^2c^2)x}{2a(bx^2+a)}+\frac{(3a^2d^2-2abcd-b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^2}$
risch	$\frac{d^2x}{b^2}+\frac{(a^2d^2-2abcd+b^2c^2)x}{2a\,b^2(bx^2+a)}-\frac{3a\ln(bx-\sqrt{-ab})d^2}{4b^2\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})cd}{2b\sqrt{-ab}}+\frac{\ln(bx-\sqrt{-ab})c^2}{4\sqrt{-ab}\,a}+\frac{3a\ln(-bx-\sqrt{-ab})d^2}{4b^2\sqrt{-ab}}-\frac{\ln(-$

[In] int((d*x^2+c)^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] d^2*x/b^2-1/b^2*(-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/a*x/(b*x^2+a)+1/2*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.62

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{4a^2b^2d^2x^3+(ab^2c^2+2a^2bcd-3a^3d^2+(b^3c^2+2ab^2cd-3a^2bd^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+2(ab}{4(a^2b^4x^2+a^3b^3)}\right.$$

[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="fricas")

[Out] [1/4*(4*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3), 1/2*(2*a^2*b^2*d^2*x^3 + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2 + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (a*b^3*c^2 - 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x)/(a^2*b^4*x^2 + a^3*b^3)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(73) = 146.

Time = 0.44 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.88

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{x(a^2d^2-2abcd+b^2c^2)}{2a^2b^2+2ab^3x^2} + \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(-\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} - \frac{\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)\log\left(\frac{a^2b^2\sqrt{-\frac{1}{a^3b^5}}(ad-bc)(3ad+bc)}{3a^2d^2-2abcd-b^2c^2}+x\right)}{4} + \frac{d^2x}{b^2}$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(2*a**2*b**2 + 2*a*b**3*x**2) + sqrt(
-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5
))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x)/4 -
 sqrt(-1/(a**3*b**5))*(a*d - b*c)*(3*a*d + b*c)*log(a**2*b**2*sqrt(-1/(a**3
*b**5))*(a*d - b*c)*(3*a*d + b*c)/(3*a**2*d**2 - 2*a*b*c*d - b**2*c**2) + x
)/4 + d**2*x/b**2
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(a*b^3*x^2 + a^2*b^2) + d^2*x/b^2 + 1
/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2
)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2x}{b^2} + \frac{(b^2c^2+2\,abcd-3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^2} + \frac{b^2c^2x-2\,abcdx+a^2d^2x}{2\,(bx^2+a)ab^2}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a*b^2) + 1/2*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^2 + a)*a*b
^2)
```

Mupad [B] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^2}\,dx = \frac{d^2\,x}{b^2} + \frac{x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{2\,a\,\left(b^3\,x^2+a\,b^2\right)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)\,(3\,a\,d+b\,c)}{\sqrt{a}\,(-3\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)\,\left(3\,a\,d+b\,c\right)}{2\,a^{3/2}\,b^{5/2}}$$

```
[In] int((c + d*x^2)^2/(a + b*x^2)^2,x)
```

```
[Out] (d^2*x)/b^2 + (x*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(2*a*(a*b^2 + b^3*x^2)) +
 (atan((b^(1/2)*x*(a*d - b*c)*(3*a*d + b*c))/(a^(1/2)*(b^2*c^2 - 3*a^2*d^2
+ 2*a*b*c*d)))*(a*d - b*c)*(3*a*d + b*c))/(2*a^(3/2)*b^(5/2))
```

3.31 $\int \frac{c+dx^2}{\left(a+bx^2\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2} \, dx = \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] 1/2*(-a*d+b*c)*x/a/b/(b*x^2+a)+1/2*(a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3
/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {393, 211}

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2} \, dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+bc)}{2a^{3/2}b^{3/2}}+\frac{x(bc-ad)}{2ab\left(a+bx^2\right)}$$

```
[In] Int[(c + d*x^2)/(a + b*x^2)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*a^(3/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\int\frac{1}{a+bx^2}\,dx}{2ab}\\ &= \frac{(bc-ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int\frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx=-\frac{(-bc+ad)x}{2ab\left(a+bx^2\right)}+\frac{(bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^2,x]
[Out] -1/2*((-(b*c) + a*d)*x)/(a*b*(a + b*x^2)) + ((b*c + a*d)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(3/2)*b^(3/2))
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}+\frac{(ad+bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2ab\sqrt{ab}}$	57
risch	$-\frac{(ad-bc)x}{2ab(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}-\frac{\ln\left(bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)c}{4\sqrt{-ab}\,a}$	122

```
[In] int((d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
[Out] -1/2*(a*d-b*c)/a/b*x/(b*x^2+a)+1/2*(a*d+b*c)/a/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx$$
$$= \left[-\frac{(abc+a^2d+(b^2c+abd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\left(ab^2c-a^2bd\right)x}{4\left(a^2b^3x^2+a^3b^2\right)}, \frac{(abc+a^2d+(b^2c+abd)x^2)\cdot}{2\left(a^2\right.}\right.$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) - 2*(a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^
2), 1/2*((a*b*c + a^2*d + (b^2*c + a*b*d)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x
/a) + (a*b^2*c - a^2*b*d)*x)/(a^2*b^3*x^2 + a^3*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(54) = 108.

Time = 0.21 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.78

$$\int \frac{c+dx^2}{(a+bx^2)^2}\,dx = \frac{x(-ad+bc)}{2a^2b+2ab^2x^2} - \frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b^3}}\,(ad+bc)\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{4}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-a*d + b*c)/(2*a**2*b + 2*a*b**2*x**2) - sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/4 + sqrt(-1/(a**3*b**3))*(a*d + b*c
)*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx = \frac{(bc-ad)x}{2\left(ab^2x^2+a^2b\right)} + \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c - a*d)*x/(a*b^2*x^2 + a^2*b) + 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx = \frac{(bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} + \frac{bcx-adx}{2\left(bx^2+a\right)ab}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(b*c*x - a*d*x)
/((b*x^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(a\,d+b\,c\right)}{2\,a^{3/2}\,b^{3/2}} - \frac{x\,\left(a\,d-b\,c\right)}{2\,a\,b\,\left(b\,x^2+a\right)}$$

```
[In] int((c + d*x^2)/(a + b*x^2)^2,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(a*d + b*c))/(2*a^(3/2)*b^(3/2)) - (x*(a*d - b*c
))/(2*a*b*(a + b*x^2))
```

3.32 $\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 108

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{bx}{2a(bc-ad)(a+bx^2)} + \frac{\sqrt{b}(bc-3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}$$

```
[Out] 1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)+1/2*(-3*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))*b
^(1/2)/a^(3/2)/(-a*d+b*c)^2+d^(3/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^2/
c^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {425, 536, 211}

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-3ad)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2} + \frac{bx}{2a(a+bx^2)(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*(a + b*x^2)) + (Sqrt[b]*(b*c - 3*a*d)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*a^(3/2)*(b*c - a*d)^2) + (d^(3/2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} - \frac{\int \frac{-bc+2ad-bdx^2}{(a+bx^2)(c+dx^2)}\,dx}{2a(bc-ad)} \\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} + \frac{d^2\int \frac{1}{c+dx^2}\,dx}{(bc-ad)^2} + \frac{(b(bc-3ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^2} \\
&= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)} + \frac{\sqrt{b}(bc-3ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^2} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$
\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx = -\frac{bx}{2a(-bc+ad)\left(a+bx^2\right)} - \frac{\sqrt{b}(-bc+3ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(-bc+ad)^2} + \frac{d^{3/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^2}
$$

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)),x]
[Out] -1/2*(b*x)/(a*(-(b*c) + a*d)*(a + b*x^2)) - (Sqrt[b]*(-(b*c) + 3*a*d)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*(-(b*c) + a*d)^2) + (d^(3/2)*ArcTan[(Sqr
t[d]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^2)
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b\left(\frac{(ad-bc)x}{2a\left(b\,x^{2}+a\right)}+\frac{(3ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^{2}}+\frac{d^{2}\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^{2}\sqrt{cd}}$	95
risch	Expression too large to display	1035

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(a*d-b*c)^2*b*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(3*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2
))
```

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 699, normalized size of antiderivative = 6.47

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx$$
$$= \left[-\frac{(abc-3\,a^2d+(b^2c-3\,abd)x^2)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2\,(abdx^2+a^2d)\sqrt{-\frac{d}{c}}\log\left(\frac{dx^2+2\,cx\sqrt{-\frac{d}{c}}-c}{dx^2+c}\right)}{4\,(a^2b^2c^2-2\,a^3bcd+a^4d^2+(ab^3c^2-2\,a^2b^2cd+a^3bd^2)x^2)}\right.$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*
a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2*(a*b*d*x^2 + a^2*d)*sqrt(-d/c)*log((d*
x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) - 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^
2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1
/4*(4*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) - (a*b*c - 3*a^2*d
+ (b^2*c - 3*a*b*d)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x
^2 + a)) + 2*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b
^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c -
3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) + (a*b*d*x^2 + a^2*d)*sqrt(-d/c
)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (b^2*c - a*b*d)*x)/(a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*
x^2), 1/2*((a*b*c - 3*a^2*d + (b^2*c - 3*a*b*d)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + 2*(a*b*d*x^2 + a^2*d)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (b^2*c - a*
b*d)*x)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d +
 a^3*b*d^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{bx}{2\,(a^2bc-a^3d+(ab^2c-a^2bd)x^2)} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="maxima")
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*b*x/(a^2*b*c - a^3*d + (a*b^2*c - a^2*b*d)*x^2) + 1/2*(b^2*c - 3*a*b*d)*ar
ctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(a*b))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \frac{d^2\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{cd}} + \frac{(b^2c-3\,abd)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^2c^2-2\,a^2bcd+a^3d^2)\sqrt{ab}} + \frac{bx}{2\,(abc-a^2d)(bx^2+a)}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c),x, algorithm="giac")
[Out] d^2*arctan(d*x/sqrt(c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d)) + 1/2
*(b^2*c - 3*a*b*d)*arctan(b*x/sqrt(a*b))/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^
2)*sqrt(a*b)) + 1/2*b*x/((a*b*c - a^2*d)*(b*x^2 + a))
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 3649, normalized size of antiderivative = 33.79

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)}\,dx = \text{Too large to display}$$

[In] int(1/((a + b*x^2)^2*(c + d*x^2)),x)

[Out] (atan((((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*1i)/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))/(((3*a*b^3*d^5)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - ((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) - (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) - (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)) + ((-a^3*b)^(1/2)*(3*a*d - b*c)*((x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(2*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)) + (((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2) + (x*(-a^3*b)^(1/2)*(3*a*d - b*c)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d)))*(-a^3*b)^(1/2)*(3*a*d - b*c))/(4*(a^5*d^2 + a^3*b^2*c^2 - 2*a^4*b*c*d))))/(4*(a^5*d^2 +

```
a^3*b^2*c^2 - 2*a^4*b*c*d))))*(-a^3*b)^(1/2)*(3*a*d - b*c)*1i)/(2*(a^5*d^2
 + a^3*b^2*c^2 - 2*a^4*b*c*d)) - (atan((((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 -
 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d
^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^
4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2
*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5
))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^
2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2
*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*
b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d) - ((-c*d^3)^(1/2)*((((4*a^
6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^
3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c
^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d
^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*
b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2
 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) +
 (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)))*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d))/(((3*a*b^3*d^5
)/2 - (b^4*c*d^4)/2)/(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d
^2) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*b^3*c*d^
6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(2*(a^5*d
^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) - (x*(-c*d^3)^(1/2)*(1
6*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*c^4*d^3
+ 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c^2 - 2*a
^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*(b^2*c^3
 + a^2*c*d^2 - 2*a*b*c^2*d)) - (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6*a*b^4*c
*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c*d^2 - 2
*a*b*c^2*d) + ((-c*d^3)^(1/2)*((((4*a^6*b^2*d^7 - 2*a*b^7*c^5*d^2 - 18*a^5*
b^3*c*d^6 + 12*a^2*b^6*c^4*d^3 - 28*a^3*b^5*c^3*d^4 + 32*a^4*b^4*c^2*d^5)/(
2*(a^5*d^3 - a^2*b^3*c^3 + 3*a^3*b^2*c^2*d - 3*a^4*b*c*d^2)) + (x*(-c*d^3)^
(1/2)*(16*a^7*b^2*d^7 - 48*a^6*b^3*c*d^6 + 16*a^2*b^7*c^5*d^2 - 48*a^3*b^6*
c^4*d^3 + 32*a^4*b^5*c^3*d^4 + 32*a^5*b^4*c^2*d^5))/(8*(a^4*d^2 + a^2*b^2*c
^2 - 2*a^3*b*c*d)*(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2))/(2*
(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d)) + (x*(13*a^2*b^3*d^5 + b^5*c^2*d^3 - 6
*a*b^4*c*d^4))/(4*(a^4*d^2 + a^2*b^2*c^2 - 2*a^3*b*c*d))))/(b^2*c^3 + a^2*c
*d^2 - 2*a*b*c^2*d)))*(-c*d^3)^(1/2)*1i)/(b^2*c^3 + a^2*c*d^2 - 2*a*b*c^2*d
) - (b*x)/(2*a*(a + b*x^2)*(a*d - b*c))
```

3.33 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx = \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

```
[Out] 1/2*d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)+1/2*b*x/a/(-a*d+b*c)/(b*x^2+a)
/(d*x^2+c)+1/2*b^(3/2)*(-5*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d
+b*c)^3+1/2*d^(3/2)*(-a*d+5*b*c)*arctan(x*d^(1/2)/c^(1/2))/c^(3/2)/(-a*d+b*
c)^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {425, 541, 536, 211}

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2} dx = \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-5ad)}{2a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3} + \frac{bx}{2a(a+bx^2)(c+dx^2)(bc-ad)} + \frac{dx(ad+bc)}{2ac(c+dx^2)(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*(c + d*x^2)) + (b*x)/(2*a*(b*c - a*d
)*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(b*c - 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt
```

```
[a]])/(2*a^(3/2)*(b*c - a*d)^3) + (d^(3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)
/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}-\frac{\int \frac{-bc+2ad-3bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{2a(bc-ad)}\\
&= \frac{d(bc+ad)x}{2ac(bc-ad)^2\left(c+dx^2\right)}+\frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)}\\
&\quad-\frac{\int \frac{-2\left(b^2c^2-4abcd+a^2d^2\right)-2bd(bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{4ac(bc-ad)^2}
\end{aligned}
$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{(b^2(bc-5ad))\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^3} + \frac{(d^2(5bc-ad))\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{2ac(bc-ad)^2(c+dx^2)} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)} + \frac{b^{3/2}(bc-5ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^3} + \frac{d^{3/2}(5bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^3}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^2}\,dx = \frac{1}{2}\left(\frac{b^{3/2}(-bc+5ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(-bc+ad)^3} + \frac{(bc-ad)x\left(\frac{b^2}{a^2+abx^2}+\frac{d^2}{c^2+cdx^2}\right)+\frac{d^{3/2}(5bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}}}{(bc-ad)^3}\right)$$

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^2),x]
```

```
[Out] ((b^(3/2)*(-(b*c) + 5*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(-(b*c) +
a*d)^3) + ((b*c - a*d)*x*(b^2/(a^2 + a*b*x^2) + d^2/(c^2 + c*d*x^2)) + (d^(
3/2)*(5*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(3/2))/(b*c - a*d)^3)/2
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{b^2\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(5ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^2\left(\frac{(ad-bc)x}{2c\left(d\,x^2+c\right)}+\frac{(ad-5bc)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2c\sqrt{cd}}\right)}{(ad-bc)^3}$	133
risch	Expression too large to display	2124

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(5*a*d-b*c)/a/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^3*(1/2*(a*d-b*c)/c*x/(d*x^2+c)+1/2*(
a*d-5*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(143) = 286$.

Time = 0.72 (sec) , antiderivative size = 1681, normalized size of antiderivative = 10.07

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2
*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sq
rt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (5*a^2*b*c^2*d -
 a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c^2*d + 4*a^2*b*c*d
^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c
)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*
a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^
3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d
 + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d^3)*x^3 +
2*(5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2*c
^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (a*b^2
*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 + (b^3*c^3 - 4*a*b^2
*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^
3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*
a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2
*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/4*(2*(b^3*c^2*d - a^2*b*d
^3)*x^3 + 2*(a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*x^4 +
(b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)
) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5*a*b^2
*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-
d/c) - c)/(d*x^2 + c)) + 2*(b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*
x)/(a^2*b^3*c^5 - 3*a^3*b^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*
c^4*d - 3*a^2*b^3*c^3*d^2 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c
^5 - 2*a^2*b^3*c^4*d + 2*a^4*b*c^2*d^3 - a^5*c*d^4)*x^2), 1/2*((b^3*c^2*d -
 a^2*b*d^3)*x^3 + (a*b^2*c^3 - 5*a^2*b*c^2*d + (b^3*c^2*d - 5*a*b^2*c*d^2)*
x^4 + (b^3*c^3 - 4*a*b^2*c^2*d - 5*a^2*b*c*d^2)*x^2)*sqrt(b/a)*arctan(x*sqr
t(b/a)) + (5*a^2*b*c^2*d - a^3*c*d^2 + (5*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (5
*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c))
+ (b^3*c^3 - a*b^2*c^2*d + a^2*b*c*d^2 - a^3*d^3)*x)/(a^2*b^3*c^5 - 3*a^3*b
^2*c^4*d + 3*a^4*b*c^3*d^2 - a^5*c^2*d^3 + (a*b^4*c^4*d - 3*a^2*b^3*c^3*d^2
 + 3*a^3*b^2*c^2*d^3 - a^4*b*c*d^4)*x^4 + (a*b^4*c^5 - 2*a^2*b^3*c^4*d + 2*
a^4*b*c^2*d^3 - a^5*c*d^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(143) = 286$.

Time $= 0.29$ (sec) , antiderivative size $= 294$, normalized size of antiderivative $= 1.76$

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{\left(b^3c-5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{ab}} + \frac{\left(5\,bcd^2-ad^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\sqrt{cd}} + \frac{\left(b^2cd+abd^2\right)x^3+\left(b^2c^2+a^2d^2\right)x}{2\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2+\left(ab^3c^3d-2\,a^2b^2c^2d^2+a^3bcd^3\right)x^4+\left(ab^3c^4-a^2b^2c^3d-a^3bc^2d^2+a^4cd^3\right)x\right.}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*((b^2*c*d + a*b*d^2)*x^3 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 -
 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d
^3)*x^4 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^2)
```

Giac [A] (verification not implemented)

none

Time $= 0.30$ (sec) , antiderivative size $= 232$, normalized size of antiderivative $= 1.39$

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx = \frac{\left(b^3c-5\,ab^2d\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\left(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3\right)\sqrt{ab}} + \frac{\left(5\,bcd^2-ad^3\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\left(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3\right)\sqrt{cd}} + \frac{b^2cdx^3+abd^2x^3+b^2c^2x+a^2d^2x}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)\left(bdx^4+bcx^2+adx^2+ac\right)}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 5*a*b^2*d)*arctan(b*x/sqrt(a*b))/((a*b^3*c^3 - 3*a^2*b^2*c^2*d
 + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(a*b)) + 1/2*(5*b*c*d^2 - a*d^3)*arctan(d*x
/sqrt(c*d))/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(c
*d)) + 1/2*(b^2*c*d*x^3 + a*b*d^2*x^3 + b^2*c^2*x + a^2*d^2*x)/((a*b^2*c^3
- 2*a^2*b*c^2*d + a^3*c*d^2)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c))
```

Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 6183, normalized size of antiderivative = 37.02

$$\int \frac{1}{(a+bx^2)^2\,(c+dx^2)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^2),x)
```

```
[Out] ((x*(a^2*d^2 + b^2*c^2))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^3
*(a*d + b*c))/(2*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^2*(a*d + b*
c) + b*d*x^4) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 -
 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*
a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2
 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7
*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 -
 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b
*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) -
(x*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3
+ 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b
^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^
3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3
*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3
)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*
a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d
 - 3*a^5*b*c*d^2)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 1
0*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^
3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 +
 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c
^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 2
0*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c
^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x
*(5*a*d - b*c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 +
144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4
*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*
c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b
^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^
```

```
(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*
d - b*c)*(-a^3*b^3)^(1/2)*1i)/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d -
 3*a^5*b*c*d^2)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^6*c^2*d
^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d
- 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^
4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*
d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d
- 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c
*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220
*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^
2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*
a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(5*a*d - b*
c)*(-a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*
c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 8
0*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*
b^2*c^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4
*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a
^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a
^3*b^3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)
) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*c*d^6 +
 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a
^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2*c*d^10
 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 220*a^5*
b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*c^2*d^9
)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b
^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(5*a*d - b*c)*(-
a^3*b^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^7*c^7*d
^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 - 80*a^8
*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c
^2*d - 3*a^5*b*c*d^2)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d - 4*a^5*
b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(5*a*d - b*c)*(-a^3*b^3)^(1/2))/(4*(a^6*d^
3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2)))*(5*a*d - b*c)*(-a^3*b^
3)^(1/2))/(4*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2*d - 3*a^5*b*c*d^2))))*(
5*a*d - b*c)*(-a^3*b^3)^(1/2)*1i)/(2*(a^6*d^3 - a^3*b^3*c^3 + 3*a^4*b^2*c^2
*d - 3*a^5*b*c*d^2)) + (atan(((((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^
3*d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d
^4 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*
c^9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*
a^4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^
3*d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d -
6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*
d^4) - (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c
^8*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 14
4*a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 -
 a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4
```

```
- 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-
c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d
)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*
c^4*d^2 - 3*a*b^2*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*
d^4 - 10*a^3*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4
 - 4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^
9*d^2 + 2*a^9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^
4*b^7*c^6*d^5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*
d^8 - 20*a^8*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*
a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^
4) + (x*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8
*d^3 + 144*a^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*
a^7*b^4*c^4*d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a
^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 -
4*a^3*b^3*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^
3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d))
)*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^
4*d^2 - 3*a*b^2*c^5*d)))/(((5*a^3*b^4*d^7)/4 + (5*b^7*c^3*d^4)/4 - (21*a*b^
6*c^2*d^5)/4 - (21*a^2*b^5*c*d^6)/4)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5
*c^7*d - 6*a^7*b*c^3*d^5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6
*b^2*c^4*d^4) - (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3
*b^4*c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3
*c^5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) - (((2*a*b^10*c^9*d^2 + 2*a^
9*b^2*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^
5 + 220*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8
*b^3*c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^
5 + 15*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) - (x*(a*d
 - 5*b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a
^4*b^7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*
d^7 - 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 +
 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^
5*d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)
)/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b
*c)*(-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2
*c^5*d)) + (((x*(a^4*b^3*d^7 + b^7*c^4*d^3 - 10*a*b^6*c^3*d^4 - 10*a^3*b^4*
c*d^6 + 50*a^2*b^5*c^2*d^5))/(2*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*
d - 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)) + (((2*a*b^10*c^9*d^2 + 2*a^9*b^2
*c*d^10 - 20*a^2*b^9*c^8*d^3 + 80*a^3*b^8*c^7*d^4 - 172*a^4*b^7*c^6*d^5 + 2
20*a^5*b^6*c^5*d^6 - 172*a^6*b^5*c^4*d^7 + 80*a^7*b^4*c^3*d^8 - 20*a^8*b^3*
c^2*d^9)/(a^2*b^6*c^8 + a^8*c^2*d^6 - 6*a^3*b^5*c^7*d - 6*a^7*b*c^3*d^5 + 1
5*a^4*b^4*c^6*d^2 - 20*a^5*b^3*c^5*d^3 + 15*a^6*b^2*c^4*d^4) + (x*(a*d - 5*
b*c)*(-c^3*d^3)^(1/2)*(16*a^2*b^9*c^9*d^2 - 80*a^3*b^8*c^8*d^3 + 144*a^4*b^
7*c^7*d^4 - 80*a^5*b^6*c^6*d^5 - 80*a^6*b^5*c^5*d^6 + 144*a^7*b^4*c^4*d^7 -
 80*a^8*b^3*c^3*d^8 + 16*a^9*b^2*c^2*d^9))/(8*(b^3*c^6 - a^3*c^3*d^3 + 3*a^
2*b*c^4*d^2 - 3*a*b^2*c^5*d)*(a^2*b^4*c^6 + a^6*c^2*d^4 - 4*a^3*b^3*c^5*d -
```

```
 4*a^5*b*c^3*d^3 + 6*a^4*b^2*c^4*d^2)))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2))/(4*
(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*(a*d - 5*b*c)*(
-c^3*d^3)^(1/2))/(4*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*
d))))*(a*d - 5*b*c)*(-c^3*d^3)^(1/2)*1i)/(2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*
b*c^4*d^2 - 3*a*b^2*c^5*d))
```

3.34 $\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 230

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{d(2bc+ad)x}{4ac(bc-ad)^2(c+dx^2)^2} + \frac{bx}{2a(bc-ad)(a+bx^2)(c+dx^2)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3(c+dx^2)} + \frac{b^{5/2}(bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4} + \frac{d^{3/2}(35b^2c^2-14abcd+3a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}$$

```
[Out] 1/4*d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)^2+1/2*b*x/a/(-a*d+b*c)/(b*x^
2+a)/(d*x^2+c)^2+1/8*d*(-a*d+4*b*c)*(3*a*d+b*c)*x/a/c^2/(-a*d+b*c)^3/(d*x^2
+c)+1/2*b^(5/2)*(-7*a*d+b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/(-a*d+b*c)^4
+1/8*d^(3/2)*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^
(5/2)/(-a*d+b*c)^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used

$= \{425, 541, 536, 211\}$

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-7ad)}{2a^{3/2}(bc-ad)^4} + \frac{d^{3/2}(3a^2d^2-14abcd+35b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4} + \frac{dx(4bc-ad)(3ad+bc)}{8ac^2(c+dx^2)(bc-ad)^3} + \frac{bx}{2a(a+bx^2)(c+dx^2)^2(bc-ad)} + \frac{dx(ad+2bc)}{4ac(c+dx^2)^2(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^2)^2) + (b*x)/(2*a*(b*c -
 a*d)*(a + b*x^2)*(c + d*x^2)^2) + (d*(4*b*c - a*d)*(b*c + 3*a*d)*x)/(8*a*c
^2*(b*c - a*d)^3*(c + d*x^2)) + (b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(3/2)*(b*c - a*d)^4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*
a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-bc+2ad-5bdx^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{2a(bc-ad)}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2d^2\right)-6bd(2bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{8ac(bc-ad)^2}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)} - \frac{\int \frac{-2\left(4b^3c^3-24ab^2c^2d+11a^2bcd^2-3a^3d^3\right)-2bd(4bc-ad)(bc+3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{16ac^2(bc-ad)^3}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)} + \frac{\left(b^3(bc-7ad)\right)\int \frac{1}{a+bx^2}\,dx}{2a(bc-ad)^4} + \frac{\left(d^2\left(35b^2c^2-14abcd+3a^2d^2\right)\right)\int \frac{1}{c+dx^2}\,dx}{8c^2(bc-ad)^4}$$

$$= \frac{d(2bc+ad)x}{4ac(bc-ad)^2\left(c+dx^2\right)^2} + \frac{bx}{2a(bc-ad)\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{d(4bc-ad)(bc+3ad)x}{8ac^2(bc-ad)^3\left(c+dx^2\right)} + \frac{b^{5/2}(bc-7ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}(bc-ad)^4} + \frac{d^{3/2}\left(35b^2c^2-14abcd+3a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^4}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(a+bx^2\right)^2 \left(c+dx^2\right)^3}\,dx = \frac{1}{8}\left(-\frac{4b^3x}{a(-bc+ad)^3\left(a+bx^2\right)}+\frac{2d^2x}{c(bc-ad)^2\left(c+dx^2\right)^2}+\frac{d^2(11bc-3ad)x}{c^2(bc-ad)^3\left(c+dx^2\right)}+\frac{4b^{5/2}(bc-7ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}(bc-ad)^4}+\frac{d^{3/2}\left(35b^2c^2-14abcd+3a^2d^2\right)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{5/2}(bc-ad)^4}\right)$$

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)^3),x]
```

```
[Out] ((-4*b^3*x)/(a*(-(b*c) + a*d)^3*(a + b*x^2)) + (2*d^2*x)/(c*(b*c - a*d)^2*(
c + d*x^2)^2) + (d^2*(11*b*c - 3*a*d)*x)/(c^2*(b*c - a*d)^3*(c + d*x^2)) +
(4*b^(5/2)*(b*c - 7*a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*(b*c - a*d)^
4) + (d^(3/2)*(35*b^2*c^2 - 14*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt
[c]])/(c^(5/2)*(b*c - a*d)^4))/8
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.86

method	result
default	$-\frac{b^3\left(\frac{(ad-bc)x}{2a\left(b\,x^2+a\right)}+\frac{(7ad-bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d^2\left(\frac{\frac{d\left(3a^2d^2-14abcd+11b^2c^2\right)x^3}{8c^2}+\frac{\left(5a^2d^2-18abcd+13b^2c^2\right)x}{8c}}{\left(d\,x^2+c\right)^2}+\frac{\left(3a^2d^2-14abcd+35b^2c^2\right)}{8c^2\sqrt{cd}}\right.}{(ad-bc)^4}$
risch	Expression too large to display

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^4*(1/2*(a*d-b*c)/a*x/(b*x^2+a)+1/2*(7*a*d-b*c)/a/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))+d^2/(a*d-b*c)^4*((1/8*d*(3*a^2*d^2-14*a*b*c*d+11*
b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-18*a*b*c*d+13*b^2*c^2)/c*x)/(d*x^2+c)^2+1/8
*(3*a^2*d^2-14*a*b*c*d+35*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 785 vs. 2(204) = 408.

Time = 2.34 (sec) , antiderivative size = 3239, normalized size of antiderivative = 14.08

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3 - 9*a^3*b*c*d^4
 + 3*a^4*d^5)*x^3 - 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*c^3*d^2 - 7*a*b^3
*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^3)*x^4 +
(b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 -
 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2
+ 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 +
 (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (
35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 -
 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a
^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^
4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*
c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b
^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^
5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c
^4*d^4 + 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^
2*b^2*c*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2
*c^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + (35*a^2*b^2*c^4*d - 14*a^3*b*c^
3*d^2 + 3*a^4*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)
*x^6 + (70*a*b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x
^4 + (35*a*b^3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) - 2*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4
*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b
^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + (4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/16*(2*(4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^
2*c*d^4 + 3*a^3*b*d^5)*x^5 + 2*(8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c
^2*d^3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 8*(a*b^3*c^5 - 7*a^2*b^2*c^4*d +
```

```
(b^4*c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a
^2*b^2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4
*c^2*d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*
b^3*c^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^
3*c^4*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(-d
/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(4*b^4*c^5 - 4*a*b^
3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*
c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 +
 (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3 + 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5
 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*
d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 -
 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4
+ 2*a^6*c^3*d^5)*x^2), 1/8*((4*b^4*c^3*d^2 + 7*a*b^3*c^2*d^3 - 14*a^2*b^2*c
*d^4 + 3*a^3*b*d^5)*x^5 + (8*b^4*c^4*d + 5*a*b^3*c^3*d^2 - 7*a^2*b^2*c^2*d^
3 - 9*a^3*b*c*d^4 + 3*a^4*d^5)*x^3 + 4*(a*b^3*c^5 - 7*a^2*b^2*c^4*d + (b^4*
c^3*d^2 - 7*a*b^3*c^2*d^3)*x^6 + (2*b^4*c^4*d - 13*a*b^3*c^3*d^2 - 7*a^2*b^
2*c^2*d^3)*x^4 + (b^4*c^5 - 5*a*b^3*c^4*d - 14*a^2*b^2*c^3*d^2)*x^2)*sqrt(b
/a)*arctan(x*sqrt(b/a)) + (35*a^2*b^2*c^4*d - 14*a^3*b*c^3*d^2 + 3*a^4*c^2*
d^3 + (35*a*b^3*c^2*d^3 - 14*a^2*b^2*c*d^4 + 3*a^3*b*d^5)*x^6 + (70*a*b^3*c
^3*d^2 + 7*a^2*b^2*c^2*d^3 - 8*a^3*b*c*d^4 + 3*a^4*d^5)*x^4 + (35*a*b^3*c^4
*d + 56*a^2*b^2*c^3*d^2 - 25*a^3*b*c^2*d^3 + 6*a^4*c*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (4*b^4*c^5 - 4*a*b^3*c^4*d + 13*a^2*b^2*c^3*d^2 - 18*a^
3*b*c^2*d^3 + 5*a^4*c*d^4)*x)/(a^2*b^4*c^8 - 4*a^3*b^3*c^7*d + 6*a^4*b^2*c^
6*d^2 - 4*a^5*b*c^5*d^3 + a^6*c^4*d^4 + (a*b^5*c^6*d^2 - 4*a^2*b^4*c^5*d^3
+ 6*a^3*b^3*c^4*d^4 - 4*a^4*b^2*c^3*d^5 + a^5*b*c^2*d^6)*x^6 + (2*a*b^5*c^7
*d - 7*a^2*b^4*c^6*d^2 + 8*a^3*b^3*c^5*d^3 - 2*a^4*b^2*c^4*d^4 - 2*a^5*b*c^
3*d^5 + a^6*c^2*d^6)*x^4 + (a*b^5*c^8 - 2*a^2*b^4*c^7*d - 2*a^3*b^3*c^6*d^2
 + 8*a^4*b^2*c^5*d^3 - 7*a^5*b*c^4*d^4 + 2*a^6*c^3*d^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3} dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(204) = 408$.

Time = 0.30 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{(b^4c - 7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4 - 4\,a^2b^3c^3d + 6\,a^3b^2c^2d^2 - 4\,a^4bcd^3 + a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2 - 14\,abcd^3 + 3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6 - 4\,ab^3c^5d + 6\,a^2b^2c^4d^2 - 4\,a^3bc^3d^3 + a^4c^2d^4)\sqrt{cd}} + \frac{(4\,b^3c^2d^2 + 11\,ab^2cd^3 - 3\,a^2bd^4)x^5 + (8\,b^3c^3d + 13\,ab^2c^2}{8\,(a^2b^3c^7 - 3\,a^3b^2c^6d + 3\,a^4bc^5d^2 - a^5c^4d^3 + (ab^4c^5d^2 - 3\,a^2b^3c^4d^3 + 3\,a^3b^2c^3d^4 - a^4bc^2d^5)x^6 + (2\,ab^4c^6}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^3*c^3*d
 + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35*b^2*c^
2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 - 4*a*b^3
*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d)) + 1/
8*((4*b^3*c^2*d^2 + 11*a*b^2*c*d^3 - 3*a^2*b*d^4)*x^5 + (8*b^3*c^3*d + 13*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3 - 3*a^3*d^4)*x^3 + (4*b^3*c^4 + 13*a^2*b*c^2*d
^2 - 5*a^3*c*d^3)*x)/(a^2*b^3*c^7 - 3*a^3*b^2*c^6*d + 3*a^4*b*c^5*d^2 - a^5
*c^4*d^3 + (a*b^4*c^5*d^2 - 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^3*d^4 - a^4*b*c
^2*d^5)*x^6 + (2*a*b^4*c^6*d - 5*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^4*d^3 + a^4*
b*c^3*d^4 - a^5*c^2*d^5)*x^4 + (a*b^4*c^7 - a^2*b^3*c^6*d - 3*a^3*b^2*c^5*d
^2 + 5*a^4*b*c^4*d^3 - 2*a^5*c^3*d^4)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2)^3}\,dx = \frac{b^3x}{2\,(ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3)(bx^2 + a)} + \frac{(b^4c - 7\,ab^3d)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,(ab^4c^4 - 4\,a^2b^3c^3d + 6\,a^3b^2c^2d^2 - 4\,a^4bcd^3 + a^5d^4)\sqrt{ab}} + \frac{(35\,b^2c^2d^2 - 14\,abcd^3 + 3\,a^2d^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\,(b^4c^6 - 4\,ab^3c^5d + 6\,a^2b^2c^4d^2 - 4\,a^3bc^3d^3 + a^4c^2d^4)\sqrt{cd}} + \frac{11\,bcd^3x^3 - 3\,ad^4x^3 + 13\,bc^2d^2x - 5\,acd^3x}{8\,(b^3c^5 - 3\,ab^2c^4d + 3\,a^2bc^3d^2 - a^3c^2d^3)(dx^2 + c)^2}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*(b*x^2 +
 a)) + 1/2*(b^4*c - 7*a*b^3*d)*arctan(b*x/sqrt(a*b))/((a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + a^5*d^4)*sqrt(a*b)) + 1/8*(35
*b^2*c^2*d^2 - 14*a*b*c*d^3 + 3*a^2*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^6 -
4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(c*d
)) + 1/8*(11*b*c*d^3*x^3 - 3*a*d^4*x^3 + 13*b*c^2*d^2*x - 5*a*c*d^3*x)/((b^
3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(d*x^2 + c)^2)
```

Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 8649, normalized size of antiderivative = 37.60

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^3),x)
```

```
[Out] (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^
4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7)
)/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*
a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - (((2*a*b^13*c
^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*
c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7
*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 +
(35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^
4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*
b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d
^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^
11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^
7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 +
256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a
^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9
*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2
*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^
4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d))
)*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 +
 a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) + (((x
*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2
009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*
b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8
*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + (((2*a*b^13*c^13*d^2 - 2
8*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*
a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^
```

3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a
^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^
3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9
*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(
256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 71
68*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^
8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^
2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*
d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*
b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))
*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4
*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3
)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(b^4*c^9 + a^4*c^5*d^
4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)))/(((63*a^5*b^5*d^
9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)
/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 -
 a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 -
 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b
^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 2
24*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^
3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c
^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b
^2*c^6*d^4)) - (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*
c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^
8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^
10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13
)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 3
6*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^
4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(-c^5*d^3)^(1/2)*
(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^1
0*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c
^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9
 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5
*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 +
 a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*
a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^
2*c^2 - 14*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b
^2*c^7*d^2 - 4*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14
*a*b*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2
 - 4*a*b^3*c^8*d)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d
^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^
4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b
*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) +
 (((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 -
(987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 11

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97*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b
^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*
c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11
*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84
*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 3
5*b^2*c^2 - 14*a*b*c*d)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5
120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*
a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b
^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(512*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b
*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)*(a^2*b^6*c^10 + a^8*c^4*d^6 -
 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^
3 + 15*a^6*b^2*c^6*d^4)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b
*c*d))/(16*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4
*a*b^3*c^8*d)))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d))/(16
*(b^4*c^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8
*d))))*(-c^5*d^3)^(1/2)*(3*a^2*d^2 + 35*b^2*c^2 - 14*a*b*c*d)*1i)/(8*(b^4*c
^9 + a^4*c^5*d^4 - 4*a^3*b*c^6*d^3 + 6*a^2*b^2*c^7*d^2 - 4*a*b^3*c^8*d)) -
((x^5*(4*b^3*c^2*d^2 - 3*a^2*b*d^4 + 11*a*b^2*c*d^3))/(8*a*c^2*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*b^3*c^3 - 5*a^3*d^3 + 13*a
^2*b*c*d^2))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(
8*b^3*c^3 - 3*a^3*d^3 + 13*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(8*a*c^2*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c^2 + x^2*(b*c^2 + 2*a*c*d) + x^4*(
a*d^2 + 2*b*c*d) + b*d^2*x^6) + (atan(((((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3
 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^
6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b
^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a
^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28
*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2
+ 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a
^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3
*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^
3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3
 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*
b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1
792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 358
4*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*
b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^
4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2
*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^
8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c
^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(
-a^3*b^5)^(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b
^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6
+ 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d -
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 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6
*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*
c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*
b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6
*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)
/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12
*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6
*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^
7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^
10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*
c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^
9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^
4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10
+ a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20
*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4
*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^
(1/2)*1i)/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 -
 4*a^6*b*c*d^3)))/(((63*a^5*b^5*d^9)/64 + (35*b^10*c^5*d^4)/16 - (651*a*b^9
*c^4*d^5)/64 - (267*a^4*b^6*c*d^8)/32 - (1275*a^2*b^8*c^3*d^6)/32 + (451*a^
3*b^7*c^2*d^7)/16)/(a^2*b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10
*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^
4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (((x*(
9*a^6*b^3*d^9 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 200
9*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^
6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d
^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) - ((7*a*d - b*c)*(-a^3*b^5)^
(1/2)*((2*a*b^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/
2 - (987*a^4*b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7
+ 1197*a^7*b^7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^
10*b^4*c^4*d^11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*
b^9*c^13 - a^11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*
c^11*d^2 - 84*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5
+ 84*a^8*b^3*c^7*d^6 - 36*a^9*b^2*c^6*d^7) - (x*(7*a*d - b*c)*(-a^3*b^5)^(1
/2)*(256*a^2*b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4
 - 7168*a^5*b^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 71
68*a^8*b^5*c^7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^
11*b^2*c^4*d^11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2
*c^2*d^2 - 4*a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6
*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d
^4))))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)))*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 -
4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3)) + (((x*(9*a^6*b^3*d^9
 + 16*b^9*c^6*d^3 - 224*a*b^8*c^5*d^4 - 84*a^5*b^4*c*d^8 + 2009*a^2*b^7*c^4
*d^5 - 980*a^3*b^6*c^3*d^6 + 406*a^4*b^5*c^2*d^7))/(32*(a^2*b^6*c^10 + a^8*
c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b
```

```
^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4)) + ((7*a*d - b*c)*(-a^3*b^5)^(1/2)*((2*a*b
^13*c^13*d^2 - 28*a^2*b^12*c^12*d^3 + (315*a^3*b^11*c^11*d^4)/2 - (987*a^4*
b^10*c^10*d^5)/2 + 978*a^5*b^9*c^9*d^6 - 1302*a^6*b^8*c^8*d^7 + 1197*a^7*b^
7*c^7*d^8 - 765*a^8*b^6*c^6*d^9 + 336*a^9*b^5*c^5*d^10 - 98*a^10*b^4*c^4*d^
11 + (35*a^11*b^3*c^3*d^12)/2 - (3*a^12*b^2*c^2*d^13)/2)/(a^2*b^9*c^13 - a^
11*c^4*d^9 - 9*a^3*b^8*c^12*d + 9*a^10*b*c^5*d^8 + 36*a^4*b^7*c^11*d^2 - 84
*a^5*b^6*c^10*d^3 + 126*a^6*b^5*c^9*d^4 - 126*a^7*b^4*c^8*d^5 + 84*a^8*b^3*
c^7*d^6 - 36*a^9*b^2*c^6*d^7) + (x*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*(256*a^2*
b^11*c^13*d^2 - 1792*a^3*b^10*c^12*d^3 + 5120*a^4*b^9*c^11*d^4 - 7168*a^5*b
^8*c^10*d^5 + 3584*a^6*b^7*c^9*d^6 + 3584*a^7*b^6*c^8*d^7 - 7168*a^8*b^5*c^
7*d^8 + 5120*a^9*b^4*c^6*d^9 - 1792*a^10*b^3*c^5*d^10 + 256*a^11*b^2*c^4*d^
11))/(128*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*
a^6*b*c*d^3)*(a^2*b^6*c^10 + a^8*c^4*d^6 - 6*a^3*b^5*c^9*d - 6*a^7*b*c^5*d^
5 + 15*a^4*b^4*c^8*d^2 - 20*a^5*b^3*c^7*d^3 + 15*a^6*b^2*c^6*d^4))))/(4*(a^
7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))
)*(7*a*d - b*c)*(-a^3*b^5)^(1/2))/(4*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3
*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3))))*(7*a*d - b*c)*(-a^3*b^5)^(1/2)*1
i)/(2*(a^7*d^4 + a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3))
```

3.35 $\int \frac{\left(c+dx^2\right)^5}{\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\begin{aligned}\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx &= \frac{d^3(10b^2c^2-15abcd+6a^2d^2)\,x}{b^5}+\frac{d^4(5bc-3ad)x^3}{3b^4}\\ &\quad+\frac{d^5x^5}{5b^3}+\frac{(bc-ad)^5x}{4ab^5\left(a+bx^2\right)^2}+\frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5\left(a+bx^2\right)}\\ &\quad+\frac{(bc-ad)^3\left(3b^2c^2+14abcd+63a^2d^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}\end{aligned}$$

```
[Out] d^3*(6*a^2*d^2-15*a*b*c*d+10*b^2*c^2)*x/b^5+1/3*d^4*(-3*a*d+5*b*c)*x^3/b^4+
1/5*d^5*x^5/b^3+1/4*(-a*d+b*c)^5*x/a/b^5/(b*x^2+a)^2+1/8*(-a*d+b*c)^4*(17*a
*d+3*b*c)*x/a^2/b^5/(b*x^2+a)+1/8*(-a*d+b*c)^3*(63*a^2*d^2+14*a*b*c*d+3*b^2
*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(11/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {398, 1171, 393, 211}

$$\begin{aligned}\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx &= \frac{x(17ad+3bc)(bc-ad)^4}{8a^2b^5\left(a+bx^2\right)}+\frac{d^3x(6a^2d^2-15abcd+10b^2c^2)}{b^5}\\ &\quad+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(63a^2d^2+14abcd+3b^2c^2\right)(bc-ad)^3}{8a^{5/2}b^{11/2}}\\ &\quad+\frac{x(bc-ad)^5}{4ab^5\left(a+bx^2\right)^2}+\frac{d^4x^3(5bc-3ad)}{3b^4}+\frac{d^5x^5}{5b^3}\end{aligned}$$

```
[In] Int[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{d^3(10b^2c^2-15abcd+6a^2d^2)}{b^5}+\frac{d^4(5bc-3ad)x^2}{b^4}+\frac{d^5x^4}{b^3}\right.\\ &\quad \left.+\frac{(bc-ad)^3\left(b^2c^2+3abcd+6a^2d^2\right)+5bd(bc-ad)^3(bc+3ad)x^2+10b^2d^2(bc-ad)^3x^4}{b^5\left(a+bx^2\right)^3}\right)\,dx\end{aligned}$$

$$= \frac{d^3(10b^2c^2 - 15abcd + 6a^2d^2)\,x}{b^5} + \frac{d^4(5bc - 3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{\int \frac{(bc-ad)^3(b^2c^2+3abcd+6a^2d^2)+5bd(bc-ad)^3(bc+3ad)x^2+10b^2d^2(bc-ad)^3x^4}{(a+bx^2)^3}\,dx}{b^5}$$

$$= \frac{d^3(10b^2c^2 - 15abcd + 6a^2d^2)\,x}{b^5} + \frac{d^4(5bc - 3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} - \frac{\int \frac{-(bc-ad)^3(3b^2c^2+14abcd+23a^2d^2)-40abd^2(bc-ad)^3x^2}{(a+bx^2)^2}\,dx}{4ab^5}$$

$$= \frac{d^3(10b^2c^2 - 15abcd + 6a^2d^2)\,x}{b^5} + \frac{d^4(5bc - 3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5(a+bx^2)} + \frac{\left((bc-ad)^3(3b^2c^2+14abcd+63a^2d^2)\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^5}$$

$$= \frac{d^3(10b^2c^2 - 15abcd + 6a^2d^2)\,x}{b^5} + \frac{d^4(5bc - 3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5(a+bx^2)} + \frac{(bc-ad)^3(3b^2c^2+14abcd+63a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx = \frac{d^3(10b^2c^2 - 15abcd + 6a^2d^2)\,x}{b^5} + \frac{d^4(5bc - 3ad)x^3}{3b^4} + \frac{d^5x^5}{5b^3} + \frac{(bc-ad)^5x}{4ab^5(a+bx^2)^2} + \frac{(bc-ad)^4(3bc+17ad)x}{8a^2b^5(a+bx^2)} + \frac{(bc-ad)^3(3b^2c^2+14abcd+63a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{11/2}}$$

```
[In] Integrate[(c + d*x^2)^5/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(10*b^2*c^2 - 15*a*b*c*d + 6*a^2*d^2)*x)/b^5 + (d^4*(5*b*c - 3*a*d)*x^
3)/(3*b^4) + (d^5*x^5)/(5*b^3) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^2)^2)
+ ((b*c - a*d)^4*(3*b*c + 17*a*d)*x)/(8*a^2*b^5*(a + b*x^2)) + ((b*c - a*d)
^3*(3*b^2*c^2 + 14*a*b*c*d + 63*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(5/2)*b^(11/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.59

method	result
default	$\frac{d^{3}\left(\frac{1}{5}b^{2}d^{2}x^{5}-x^{3}ab\,d^{2}+\frac{5}{3}x^{3}b^{2}cd+6a^{2}d^{2}x-15abcdx+10b^{2}c^{2}x\right)}{b^{5}}-\frac{-\frac{b\left(17a^{5}d^{5}-65a^{4}bc\,d^{4}+90a^{3}b^{2}c^{2}d^{3}-50a^{2}b^{3}c^{3}d^{2}+5a\,b^{4}c^{4}d+3b^{5}c^{5}\right)x^{3}}{8a^{2}}-}{\left(b\,x^{2}+a\right)^{2}}$
risch	$\frac{d^{5}x^{5}}{5b^{3}}-\frac{d^{5}x^{3}a}{b^{4}}+\frac{5d^{4}x^{3}c}{3b^{3}}+\frac{6d^{5}a^{2}x}{b^{5}}-\frac{15d^{4}acx}{b^{4}}+\frac{10d^{3}c^{2}x}{b^{3}}+\frac{\frac{b\left(17a^{5}d^{5}-65a^{4}bc\,d^{4}+90a^{3}b^{2}c^{2}d^{3}-50a^{2}b^{3}c^{3}d^{2}+5a\,b^{4}c^{4}d+3b^{5}c^{5}\right)x^{3}}{8a^{2}}}{b^{5}\left(b\,x^{2}+\right.}$

```
[In] int((d*x^2+c)^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3/b^5*(1/5*b^2*d^2*x^5-x^3*a*b*d^2+5/3*x^3*b^2*c*d+6*a^2*d^2*x-15*a*b*c*d
*x+10*b^2*c^2*x)-1/b^5*((-1/8*b*(17*a^5*d^5-65*a^4*b*c*d^4+90*a^3*b^2*c^2*d
^3-50*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d+3*b^5*c^5)/a^2*x^3-5/8*(3*a^5*d^5-11*a^
4*b*c*d^4+14*a^3*b^2*c^2*d^3-6*a^2*b^3*c^3*d^2-a*b^4*c^4*d+b^5*c^5)/a*x)/(b
*x^2+a)^2+1/8*(63*a^5*d^5-175*a^4*b*c*d^4+150*a^3*b^2*c^2*d^3-30*a^2*b^3*c^
3*d^2-5*a*b^4*c^4*d-3*b^5*c^5)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(178) = 356$.

Time = 0.26 (sec) , antiderivative size = 1044, normalized size of antiderivative = 5.33

$$\int \frac{\left(c+dx^{2}\right)^{5}}{\left(a+bx^{2}\right)^{3}}\,dx$$

$$= \left[\frac{48\,a^{3}b^{5}d^{5}x^{9}+16\,(25\,a^{3}b^{5}cd^{4}-9\,a^{4}b^{4}d^{5})x^{7}+16\,(150\,a^{3}b^{5}c^{2}d^{3}-175\,a^{4}b^{4}cd^{4}+63\,a^{5}b^{3}d^{5})x^{5}+10\,(9\,ab^{7}c^{5}\right.}{}$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a^3*b^5*d^5*x^9 + 16*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 16
*(150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 10*(9*a*b
^7*c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875
*a^5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d
 + 30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5
+ (3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 1
75*a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d +
 30*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^
5)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(5*a^
2*b^6*c^5 - 5*a^3*b^5*c^4*d - 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 17
5*a^6*b^2*c*d^4 + 63*a^7*b*d^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6),
```

```
1/120*(24*a^3*b^5*d^5*x^9 + 8*(25*a^3*b^5*c*d^4 - 9*a^4*b^4*d^5)*x^7 + 8*(
150*a^3*b^5*c^2*d^3 - 175*a^4*b^4*c*d^4 + 63*a^5*b^3*d^5)*x^5 + 5*(9*a*b^7*
c^5 + 15*a^2*b^6*c^4*d - 150*a^3*b^5*c^3*d^2 + 750*a^4*b^4*c^2*d^3 - 875*a^
5*b^3*c*d^4 + 315*a^6*b^2*d^5)*x^3 + 15*(3*a^2*b^5*c^5 + 5*a^3*b^4*c^4*d +
30*a^4*b^3*c^3*d^2 - 150*a^5*b^2*c^2*d^3 + 175*a^6*b*c*d^4 - 63*a^7*d^5 + (
3*b^7*c^5 + 5*a*b^6*c^4*d + 30*a^2*b^5*c^3*d^2 - 150*a^3*b^4*c^2*d^3 + 175*
a^4*b^3*c*d^4 - 63*a^5*b^2*d^5)*x^4 + 2*(3*a*b^6*c^5 + 5*a^2*b^5*c^4*d + 30
*a^3*b^4*c^3*d^2 - 150*a^4*b^3*c^2*d^3 + 175*a^5*b^2*c*d^4 - 63*a^6*b*d^5)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(5*a^2*b^6*c^5 - 5*a^3*b^5*c^4*d
- 30*a^4*b^4*c^3*d^2 + 150*a^5*b^3*c^2*d^3 - 175*a^6*b^2*c*d^4 + 63*a^7*b*d
^5)*x)/(a^3*b^8*x^4 + 2*a^4*b^7*x^2 + a^5*b^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(187) = 374$.

Time = 7.52 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.14

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx = x^3\left(-\frac{ad^5}{b^4}+\frac{5cd^4}{3b^3}\right)+x\left(\frac{6a^2d^5}{b^5}-\frac{15acd^4}{b^4}+\frac{10c^2d^3}{b^3}\right)$$
$$+\frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(-\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}\right.}{16}$$
$$-\frac{\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)\log\left(\frac{a^3b^5\sqrt{-\frac{1}{a^5b^{11}}}(ad-bc)^3\cdot(63a^2d^2+14abcd+3b^2c^2)}{63a^5d^5-175a^4bcd^4+150a^3b^2c^2d^3-30a^2b^3c^3d^2-5ab^4c^4d-3b^5c^5}+\right.}{16}$$
$$+\frac{x^3\cdot(17a^5bd^5-65a^4b^2cd^4+90a^3b^3c^2d^3-50a^2b^4c^3d^2+5ab^5c^4d+3b^6c^5)+x(15a^6d^5-55a^5bcd^4+70a^4}{8a^4b^5+16a^3b^6x^2+8a^2b^7x^4}$$
$$+\frac{d^5x^5}{5b^3}$$

```
[In] integrate((d*x**2+c)**5/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*d**5/b**4 + 5*c*d**4/(3*b**3)) + x*(6*a**2*d**5/b**5 - 15*a*c*d**4
/b**4 + 10*c**2*d**3/b**3) + sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2*
d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 - sqrt(-1/(a**5*b**11))*(a*d - b*c)**3*(63*a**2
*d**2 + 14*a*b*c*d + 3*b**2*c**2)*log(a**3*b**5*sqrt(-1/(a**5*b**11))*(a*d
- b*c)**3*(63*a**2*d**2 + 14*a*b*c*d + 3*b**2*c**2)/(63*a**5*d**5 - 175*a**
4*b*c*d**4 + 150*a**3*b**2*c**2*d**3 - 30*a**2*b**3*c**3*d**2 - 5*a*b**4*c*
*4*d - 3*b**5*c**5) + x)/16 + (x**3*(17*a**5*b*d**5 - 65*a**4*b**2*c*d**4 +
 90*a**3*b**3*c**2*d**3 - 50*a**2*b**4*c**3*d**2 + 5*a*b**5*c**4*d + 3*b**6
*c**5) + x*(15*a**6*d**5 - 55*a**5*b*c*d**4 + 70*a**4*b**2*c**2*d**3 - 30*a
```

```
**3*b**3*c**3*d**2 - 5*a**2*b**4*c**4*d + 5*a*b**5*c**5))/(8*a**4*b**5 + 16
*a**3*b**6*x**2 + 8*a**2*b**7*x**4) + d**5*x**5/(5*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$
\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx
= \frac{(3\,b^6c^5+5\,ab^5c^4d-50\,a^2b^4c^3d^2+90\,a^3b^3c^2d^3-65\,a^4b^2cd^4+17\,a^5bd^5)x^3+5\,(ab^5c^5-a^2b^4c^4d-6\,a^3b^3c^3d^2}{8\,(a^2b^7x^4+2\,a^3b^6x^2+a^4b^5)}
+\frac{3\,b^2d^5x^5+5\,(5\,b^2cd^4-3\,abd^5)x^3+15\,(10\,b^2c^2d^3-15\,abcd^4+6\,a^2d^5)x}{15\,b^5}
+\frac{(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^5}
$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^6*c^5 + 5*a*b^5*c^4*d - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 -
 65*a^4*b^2*c*d^4 + 17*a^5*b*d^5)*x^3 + 5*(a*b^5*c^5 - a^2*b^4*c^4*d - 6*a^
3*b^3*c^3*d^2 + 14*a^4*b^2*c^2*d^3 - 11*a^5*b*c*d^4 + 3*a^6*d^5)*x)/(a^2*b^
7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5) + 1/15*(3*b^2*d^5*x^5 + 5*(5*b^2*c*d^4 - 3
*a*b*d^5)*x^3 + 15*(10*b^2*c^2*d^3 - 15*a*b*c*d^4 + 6*a^2*d^5)*x)/b^5 + 1/8
*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 + 17
5*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.73

$$
\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx
= \frac{(3\,b^5c^5+5\,ab^4c^4d+30\,a^2b^3c^3d^2-150\,a^3b^2c^2d^3+175\,a^4bcd^4-63\,a^5d^5)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^5}
+\frac{3\,b^6c^5x^3+5\,ab^5c^4dx^3-50\,a^2b^4c^3d^2x^3+90\,a^3b^3c^2d^3x^3-65\,a^4b^2cd^4x^3+17\,a^5bd^5x^3+5\,ab^5c^5x-5\,a^2b^4c^4}{8\,(bx^2+a)^2a^2b^5}
+\frac{3\,b^{12}d^5x^5+25\,b^{12}cd^4x^3-15\,ab^{11}d^5x^3+150\,b^{12}c^2d^3x-225\,ab^{11}cd^4x+90\,a^2b^{10}d^5x}{15\,b^{15}}
$$

```
[In] integrate((d*x^2+c)^5/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^5*c^5 + 5*a*b^4*c^4*d + 30*a^2*b^3*c^3*d^2 - 150*a^3*b^2*c^2*d^3 +
 175*a^4*b*c*d^4 - 63*a^5*d^5)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^5) +
1/8*(3*b^6*c^5*x^3 + 5*a*b^5*c^4*d*x^3 - 50*a^2*b^4*c^3*d^2*x^3 + 90*a^3*b^
3*c^2*d^3*x^3 - 65*a^4*b^2*c*d^4*x^3 + 17*a^5*b*d^5*x^3 + 5*a*b^5*c^5*x - 5
*a^2*b^4*c^4*d*x - 30*a^3*b^3*c^3*d^2*x + 70*a^4*b^2*c^2*d^3*x - 55*a^5*b*c
*d^4*x + 15*a^6*d^5*x)/((b*x^2 + a)^2*a^2*b^5) + 1/15*(3*b^12*d^5*x^5 + 25*
b^12*c*d^4*x^3 - 15*a*b^11*d^5*x^3 + 150*b^12*c^2*d^3*x - 225*a*b^11*c*d^4*
x + 90*a^2*b^10*d^5*x)/b^15
```

Mupad [B] (verification not implemented)

Time = 4.57 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx^2)^5}{(a+bx^2)^3}\,dx$$
$$= \frac{\frac{5\,x\,\left(3\,a^5\,d^5-11\,a^4\,b\,c\,d^4+14\,a^3\,b^2\,c^2\,d^3-6\,a^2\,b^3\,c^3\,d^2-a\,b^4\,c^4\,d+b^5\,c^5\right)}{8\,a}+\frac{x^3\,\left(17\,a^5\,b\,d^5-65\,a^4\,b^2\,c\,d^4+90\,a^3\,b^3\,c^2\,d^3-50\,a^2\,b^4\,c^3\,d^2+5\,a\,b^5\,c^4\,d+\right.}{8\,a^2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}$$
$$-x^3\left(\frac{a\,d^5}{b^4}-\frac{5\,c\,d^4}{3\,b^3}\right)+x\left(\frac{3\,a\left(\frac{3\,a\,d^5}{b^4}-\frac{5\,c\,d^4}{b^3}\right)}{b}-\frac{3\,a^2\,d^5}{b^5}+\frac{10\,c^2\,d^3}{b^3}\right)+\frac{d^5\,x^5}{5\,b^3}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,(a\,d-b\,c)^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(-63\,a^5\,d^5+175\,a^4\,b\,c\,d^4-150\,a^3\,b^2\,c^2\,d^3+30\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d+3\,b^5\,c^5\right)}\right)\,(a\,d-b\,c)^3\,\left(63\,a^2\,d^2+14\,a\,b\,c\,d+3\,b\right.}{8\,a^{5/2}\,b^{11/2}}$$

```
[In] int((c + d*x^2)^5/(a + b*x^2)^3,x)
```

```
[Out] ((5*x*(3*a^5*d^5 + b^5*c^5 - 6*a^2*b^3*c^3*d^2 + 14*a^3*b^2*c^2*d^3 - a*b^4
*c^4*d - 11*a^4*b*c*d^4))/(8*a) + (x^3*(3*b^6*c^5 + 17*a^5*b*d^5 - 65*a^4*b
^2*c*d^4 - 50*a^2*b^4*c^3*d^2 + 90*a^3*b^3*c^2*d^3 + 5*a*b^5*c^4*d))/(8*a^2
))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) - x^3*((a*d^5)/b^4 - (5*c*d^4)/(3*b^3)
) + x*((3*a*((3*a*d^5)/b^4 - (5*c*d^4)/b^3))/b - (3*a^2*d^5)/b^5 + (10*c^2*
d^3)/b^3) + (d^5*x^5)/(5*b^3) + (atan((b^(1/2)*x*(a*d - b*c)^3*(63*a^2*d^2
+ 3*b^2*c^2 + 14*a*b*c*d))/(a^(1/2)*(3*b^5*c^5 - 63*a^5*d^5 + 30*a^2*b^3*c^
3*d^2 - 150*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d + 175*a^4*b*c*d^4)))*(a*d - b*c
)^3*(63*a^2*d^2 + 3*b^2*c^2 + 14*a*b*c*d))/(8*a^(5/2)*b^(11/2))
```

3.36 $\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx = \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4(a+bx^2)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4(a+bx^2)} + \frac{(bc-ad)^2(3b^2c^2+10abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}$$

```
[Out] d^3*(-3*a*d+4*b*c)*x/b^4+1/3*d^4*x^3/b^3+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^2+a)
^2+1/8*(-a*d+b*c)^3*(13*a*d+3*b*c)*x/a^2/b^4/(b*x^2+a)+1/8*(-a*d+b*c)^2*(35
*a^2*d^2+10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(9/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {398, 1171, 393, 211}

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3} dx = \frac{x(bc-ad)^3(13ad+3bc)}{8a^2b^4(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)^2(35a^2d^2+10abcd+3b^2c^2)}{8a^{5/2}b^{9/2}} + \frac{d^3x(4bc-3ad)}{b^4} + \frac{x(bc-ad)^4}{4ab^4(a+bx^2)^2} + \frac{d^4x^3}{3b^3}$$

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^3(4bc-3ad)}{b^4}+\frac{d^4x^2}{b^3}\right.\\
&\quad\left.+\frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{b^4\left(a+bx^2\right)^3}\right)dx\\
&= \frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{\int\frac{b^4c^4-4a^3bcd^3+3a^4d^4+4bd(bc-ad)^2(bc+2ad)x^2+6b^2d^2(bc-ad)^2x^4}{\left(a+bx^2\right)^3}\,dx}{b^4}\\
&= \frac{d^3(4bc-3ad)x}{b^4}+\frac{d^4x^3}{3b^3}+\frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2}-\frac{\int\frac{-(bc-ad)^2\left(3b^2c^2+10abcd+11a^2d^2\right)-24abd^2(bc-ad)^2x^2}{\left(a+bx^2\right)^2}\,dx}{4ab^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} \\
&\quad + \frac{\left((bc-ad)^2\left(3b^2c^2+10abcd+35a^2d^2\right)\right)\int\frac{1}{a+bx^2}\,dx}{8a^2b^4} \\
&= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} \\
&\quad + \frac{(bc-ad)^2\left(3b^2c^2+10abcd+35a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int\frac{\left(c+dx^2\right)^4}{\left(a+bx^2\right)^3}\,dx &= \frac{d^3(4bc-3ad)x}{b^4} + \frac{d^4x^3}{3b^3} + \frac{(bc-ad)^4x}{4ab^4\left(a+bx^2\right)^2} + \frac{(bc-ad)^3(3bc+13ad)x}{8a^2b^4\left(a+bx^2\right)} \\
&\quad + \frac{(bc-ad)^2\left(3b^2c^2+10abcd+35a^2d^2\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{9/2}}
\end{aligned}
$$

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^3,x]
```

```
[Out] (d^3*(4*b*c - 3*a*d)*x)/b^4 + (d^4*x^3)/(3*b^3) + ((b*c - a*d)^4*x)/(4*a*b^
4*(a + b*x^2)^2) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*x)/(8*a^2*b^4*(a + b*x^2
)) + ((b*c - a*d)^2*(3*b^2*c^2 + 10*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(9/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.44

method	result
default	$-\frac{d^3\left(-\frac{1}{3}bd\,x^3+3adx-4bcx\right)}{b^4}+\frac{\frac{-\frac{b\left(13a^4d^4-36a^3bc\,d^3+30a^2b^2c^2d^2-4a\,b^3c^3d-3b^4c^4\right)x^3}{8a^2}-\frac{\left(11a^4d^4-28a^3bc\,d^3+18a^2b^2c^2d^2+4a\,b^3c^3d-5b^4c\right)}{8a}}{\left(b\,x^2+a\right)^2}}{b^4}$
risch	$\frac{d^4x^3}{3b^3}-\frac{3d^4ax}{b^4}+\frac{4d^3cx}{b^3}+\frac{-\frac{b\left(13a^4d^4-36a^3bc\,d^3+30a^2b^2c^2d^2-4a\,b^3c^3d-3b^4c^4\right)x^3}{8a^2}-\frac{\left(11a^4d^4-28a^3bc\,d^3+18a^2b^2c^2d^2+4a\,b^3c^3d-5b^4c^4\right)}{8a}}{b^4\left(b\,x^2+a\right)^2}$

```
[In] int((d*x^2+c)^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3/b^4*(-1/3*b*d*x^3+3*a*d*x-4*b*c*x)+1/b^4*((-1/8*b*(13*a^4*d^4-36*a^3*b
*c*d^3+30*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d-3*b^4*c^4)/a^2*x^3-1/8*(11*a^4*d^4-
28*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d-5*b^4*c^4)/a*x)/(b*x^2+a)^2
```

```
+1/8*(35*a^4*d^4-60*a^3*b*c*d^3+18*a^2*b^2*c^2*d^2+4*a*b^3*c^3*d+3*b^4*c^4)
/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(144) = 288$.

Time = 0.26 (sec) , antiderivative size = 817, normalized size of antiderivative = 5.11

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,a^3b^4d^4x^7 + 16\,(12\,a^3b^4cd^3 - 7\,a^4b^3d^4)x^5 + 2\,(9\,ab^6c^4 + 12\,a^2b^5c^3d - 90\,a^3b^4c^2d^2 + 300\,a^4b^3cd^3 - 175\,a\right.$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^3*b^4*d^4*x^7 + 16*(12*a^3*b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + 2*(
9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 1
75*a^5*b^2*d^4)*x^3 - 3*(3*a^2*b^4*c^4 + 4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d
^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*
c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b
^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*a^2*b^5*c^4 - 4*
a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d^4)*x)/(a
^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5), 1/24*(8*a^3*b^4*d^4*x^7 + 8*(12*a^3*
b^4*c*d^3 - 7*a^4*b^3*d^4)*x^5 + (9*a*b^6*c^4 + 12*a^2*b^5*c^3*d - 90*a^3*b
^4*c^2*d^2 + 300*a^4*b^3*c*d^3 - 175*a^5*b^2*d^4)*x^3 + 3*(3*a^2*b^4*c^4 +
4*a^3*b^3*c^3*d + 18*a^4*b^2*c^2*d^2 - 60*a^5*b*c*d^3 + 35*a^6*d^4 + (3*b^6
*c^4 + 4*a*b^5*c^3*d + 18*a^2*b^4*c^2*d^2 - 60*a^3*b^3*c*d^3 + 35*a^4*b^2*d
^4)*x^4 + 2*(3*a*b^5*c^4 + 4*a^2*b^4*c^3*d + 18*a^3*b^3*c^2*d^2 - 60*a^4*b^
2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*a^2*b^5
*c^4 - 4*a^3*b^4*c^3*d - 18*a^4*b^3*c^2*d^2 + 60*a^5*b^2*c*d^3 - 35*a^6*b*d
^4)*x)/(a^3*b^7*x^4 + 2*a^4*b^6*x^2 + a^5*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(150) = 300$.

Time = 1.83 (sec) , antiderivative size = 515, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = x\left(-\frac{3ad^4}{b^4}+\frac{4cd^3}{b^3}\right) - \frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(-\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)\log\left(\frac{a^3b^4\sqrt{-\frac{1}{a^5b^9}}(ad-bc)^2\cdot(35a^2d^2+10abcd+3b^2c^2)}{35a^4d^4-60a^3bcd^3+18a^2b^2c^2d^2+4ab^3c^3d+3b^4c^4}+x\right)}{16} + \frac{x^3(-13a^4bd^4+36a^3b^2cd^3-30a^2b^3c^2d^2+4ab^4c^3d+3b^5c^4)+x(-11a^5d^4+28a^4bcd^3-18a^3b^2c^2d^2-4a^2b}{8a^4b^4+16a^3b^5x^2+8a^2b^6x^4} + \frac{d^4x^3}{3b^3}$$

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*d**4/b**4 + 4*c*d**3/b**3) - sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(3
5*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**4*sqrt(-1/(a**5*b**9))
*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 6
0*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) +
 x)/16 + sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2 + 10*a*b*c*d + 3
*b**2*c**2)*log(a**3*b**4*sqrt(-1/(a**5*b**9))*(a*d - b*c)**2*(35*a**2*d**2
 + 10*a*b*c*d + 3*b**2*c**2)/(35*a**4*d**4 - 60*a**3*b*c*d**3 + 18*a**2*b**
2*c**2*d**2 + 4*a*b**3*c**3*d + 3*b**4*c**4) + x)/16 + (x**3*(-13*a**4*b*d*
*4 + 36*a**3*b**2*c*d**3 - 30*a**2*b**3*c**2*d**2 + 4*a*b**4*c**3*d + 3*b**
5*c**4) + x*(-11*a**5*d**4 + 28*a**4*b*c*d**3 - 18*a**3*b**2*c**2*d**2 - 4*
a**2*b**3*c**3*d + 5*a*b**4*c**4))/(8*a**4*b**4 + 16*a**3*b**5*x**2 + 8*a**
2*b**6*x**4) + d**4*x**3/(3*b**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \frac{(3\,b^5c^4+4\,ab^4c^3d-30\,a^2b^3c^2d^2+36\,a^3b^2cd^3-13\,a^4bd^4)x^3+(5\,ab^4c^4-4\,a^2b^3c^3d-18\,a^3b^2c^2d^2+28\,a^4bcd}{8\,(a^2b^6x^4+2\,a^3b^5x^2+a^4b^4)} + \frac{bd^4x^3+3\,(4\,bcd^3-3\,ad^4)x}{3\,b^4} + \frac{(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^5*c^4 + 4*a*b^4*c^3*d - 30*a^2*b^3*c^2*d^2 + 36*a^3*b^2*c*d^3 - 1
3*a^4*b*d^4)*x^3 + (5*a*b^4*c^4 - 4*a^2*b^3*c^3*d - 18*a^3*b^2*c^2*d^2 + 28
*a^4*b*c*d^3 - 11*a^5*d^4)*x)/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4) + 1/3
*(b*d^4*x^3 + 3*(4*b*c*d^3 - 3*a*d^4)*x)/b^4 + 1/8*(3*b^4*c^4 + 4*a*b^3*c^3
*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a^4*d^4)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \frac{(3\,b^4c^4+4\,ab^3c^3d+18\,a^2b^2c^2d^2-60\,a^3bcd^3+35\,a^4d^4)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^4} + \frac{3\,b^5c^4x^3+4\,ab^4c^3dx^3-30\,a^2b^3c^2d^2x^3+36\,a^3b^2cd^3x^3-13\,a^4bd^4x^3+5\,ab^4c^4x-4\,a^2b^3c^3dx-18\,a^3b^2c^2d}{8\,(bx^2+a)^2a^2b^4} + \frac{b^6d^4x^3+12\,b^6cd^3x-9\,ab^5d^4x}{3\,b^9}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^4*c^4 + 4*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 60*a^3*b*c*d^3 + 35*a
^4*d^4)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^4) + 1/8*(3*b^5*c^4*x^3 + 4*
a*b^4*c^3*d*x^3 - 30*a^2*b^3*c^2*d^2*x^3 + 36*a^3*b^2*c*d^3*x^3 - 13*a^4*b*
d^4*x^3 + 5*a*b^4*c^4*x - 4*a^2*b^3*c^3*d*x - 18*a^3*b^2*c^2*d^2*x + 28*a^4
*b*c*d^3*x - 11*a^5*d^4*x)/((b*x^2 + a)^2*a^2*b^4) + 1/3*(b^6*d^4*x^3 + 12*
b^6*c*d^3*x - 9*a*b^5*d^4*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.99

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^3}\,dx = \frac{d^4\,x^3}{3\,b^3} - x\left(\frac{3\,a\,d^4}{b^4}-\frac{4\,c\,d^3}{b^3}\right) - \frac{\frac{x\,\left(11\,a^4\,d^4-28\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d-5\,b^4\,c^4\right)}{8\,a}-\frac{x^3\,\left(-13\,a^4\,b\,d^4+36\,a^3\,b^2\,c\,d^3-30\,a^2\,b^3\,c^2\,d^2+4\,a\,b^4\,c^3\,d+3\,b^5\,c^4\right)}{8\,a^2}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{\sqrt{a}\,\left(35\,a^4\,d^4-60\,a^3\,b\,c\,d^3+18\,a^2\,b^2\,c^2\,d^2+4\,a\,b^3\,c^3\,d+3\,b^4\,c^4\right)}\right)\,{\left(a\,d-b\,c\right)}^2\,\left(35\,a^2\,d^2+10\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{9/2}}$$

```
[In] int((c + d*x^2)^4/(a + b*x^2)^3,x)
```

```
[Out] (d^4*x^3)/(3*b^3) - x*((3*a*d^4)/b^4 - (4*c*d^3)/b^3) - ((x*(11*a^4*d^4 - 5
*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 28*a^3*b*c*d^3))/(8*a) - (x
^3*(3*b^5*c^4 - 13*a^4*b*d^4 + 36*a^3*b^2*c*d^3 - 30*a^2*b^3*c^2*d^2 + 4*a*
b^4*c^3*d))/(8*a^2))/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (atan((b^(1/2)*x*(
a*d - b*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(a^(1/2)*(35*a^4*d^4 +
3*b^4*c^4 + 18*a^2*b^2*c^2*d^2 + 4*a*b^3*c^3*d - 60*a^3*b*c*d^3)))*(a*d - b
*c)^2*(35*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/(8*a^(5/2)*b^(9/2))
```

3.37 $\int \frac{(c+dx^2)^3}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3} \, dx = \frac{d^3x}{b^3} + \frac{(bc-ad)^3x}{4ab^3(a+bx^2)^2} + \frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3(a+bx^2)} + \frac{3(bc-ad)(4a^2d^2+(bc+ad)^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

```
[Out] d^3*x/b^3+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^2+a)^2+3/8*(-a*d+b*c)^2*(3*a*d+b*c)
*x/a^2/b^3/(b*x^2+a)+3/8*(-a*d+b*c)*(4*a^2*d^2+(a*d+b*c)^2)*arctan(x*b^(1/2
)/a^(1/2))/a^(5/2)/b^(7/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {398, 1171, 393, 211}

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3} \, dx = \frac{3x(bc-ad)^2(3ad+bc)}{8a^2b^3(a+bx^2)} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bc-ad)(4a^2d^2+(ad+bc)^2)}{8a^{5/2}b^{7/2}} + \frac{x(bc-ad)^3}{4ab^3(a+bx^2)^2} + \frac{d^3x}{b^3}$$

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b*c - a*d)*(4*a^2*d^2 + (b*c
 + a*d)^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 398

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{d^3}{b^3}+\frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{b^3\left(a+bx^2\right)^3}\right)dx \\
&= \frac{d^3x}{b^3}+\frac{\int \frac{b^3c^3-a^3d^3+3bd(bc-ad)(bc+ad)x^2+3b^2d^2(bc-ad)x^4}{\left(a+bx^2\right)^3}\,dx}{b^3} \\
&= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}-\frac{\int \frac{-3(bc-ad)(bc+ad)^2-12abd^2(bc-ad)x^2}{\left(a+bx^2\right)^2}\,dx}{4ab^3} \\
&= \frac{d^3x}{b^3}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2}+\frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)} \\
&\quad +\frac{\left(3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\right)\int \frac{1}{a+bx^2}\,dx}{8a^2b^3}
\end{aligned}
$$

$$
= \frac{d^3x}{b^3} + \frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2} + \frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)} + \frac{3(bc-ad)\left(4a^2d^2+(bc+ad)^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.07

$$
\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^3}\,dx = \frac{d^3x}{b^3} + \frac{(bc-ad)^3x}{4ab^3\left(a+bx^2\right)^2} + \frac{3(bc-ad)^2(bc+3ad)x}{8a^2b^3\left(a+bx^2\right)} + \frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}
$$

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^3,x]
```

```
[Out] (d^3*x)/b^3 + ((b*c - a*d)^3*x)/(4*a*b^3*(a + b*x^2)^2) + (3*(b*c - a*d)^2*
(b*c + 3*a*d)*x)/(8*a^2*b^3*(a + b*x^2)) + (3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(7/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

method	result
default	$\frac{d^3x}{b^3} - \frac{\frac{-\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}-\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{3\left(5a^3d^3-3a^2bc\,d^2-a\,b^2c^2d-b^3c^3\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$
risch	$\frac{d^3x}{b^3} + \frac{\frac{3b\left(3a^3d^3-5a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)x^3}{8a^2}+\frac{\left(7a^3d^3-9a^2bc\,d^2-3a\,b^2c^2d+5b^3c^3\right)x}{8a}}{b^3\left(b\,x^2+a\right)^2} - \frac{15a\ln\left(bx-\sqrt{-ab}\right)d^3}{16b^3\sqrt{-ab}} + \frac{9\ln\left(bx-\sqrt{-ab}\right)c\,d^2}{16b^2\sqrt{-ab}}$

```
[In] int((d*x^2+c)^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^3*x/b^3-1/b^3*((-3/8*b*(3*a^3*d^3-5*a^2*b*c*d^2+a*b^2*c^2*d+b^3*c^3)/a^2*
x^3-1/8*(7*a^3*d^3-9*a^2*b*c*d^2-3*a*b^2*c^2*d+5*b^3*c^3)/a*x)/(b*x^2+a)^2+
3/8*(5*a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(116) = 232.

Time = 0.28 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.66

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,a^3b^3d^3x^5 + 2\,(3\,ab^5c^3 + 3\,a^2b^4c^2d - 15\,a^3b^3cd^2 + 25\,a^4b^2d^3)x^3 + 3\,(a^2b^3c^3 + a^3b^2c^2d + 3\,a^4bcd^2 - 5\,a^5d^3}{}\right.$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*d^3*x^5 + 2*(3*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 15*a^3*b^3*c
*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2
 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt
(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^4*c^3 - 3
*a^3*b^3*c^2*d - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^
5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*d^3*x^5 + (3*a*b^5*c^3 + 3*a^2*b^4*c^2*d -
 15*a^3*b^3*c*d^2 + 25*a^4*b^2*d^3)*x^3 + 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d +
3*a^4*b*c*d^2 - 5*a^5*d^3 + (b^5*c^3 + a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - 5*a^
3*b^2*d^3)*x^4 + 2*(a*b^4*c^3 + a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - 5*a^4*b*d
^3)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^4*c^3 - 3*a^3*b^3*c^2*d
 - 9*a^4*b^2*c*d^2 + 15*a^5*b*d^3)*x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^
4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. 2(122) = 244.

Time = 1.01 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.25

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx$$
$$= \frac{3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(-\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}$$
$$-\frac{3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)\left(5a^2d^2+2abcd+b^2c^2\right)\log\left(\frac{3a^3b^3\sqrt{-\frac{1}{a^5b^7}}(ad-bc)(5a^2d^2+2abcd+b^2c^2)}{15a^3d^3-9a^2bcd^2-3ab^2c^2d-3b^3c^3}+x\right)}{16}$$
$$+\frac{x^3\cdot(9a^3bd^3-15a^2b^2cd^2+3ab^3c^2d+3b^4c^3)+x(7a^4d^3-9a^3bcd^2-3a^2b^2c^2d+5ab^3c^3)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}$$
$$+\frac{d^3x}{b^3}$$

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**3,x)
```

```
[Out] 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b**2*c**2)*lo
g(-3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d +
b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*b**3*c**3)
 + x)/16 - 3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*a*b*c*d + b*
*2*c**2)*log(3*a**3*b**3*sqrt(-1/(a**5*b**7))*(a*d - b*c)*(5*a**2*d**2 + 2*
a*b*c*d + b**2*c**2)/(15*a**3*d**3 - 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 3*
b**3*c**3) + x)/16 + (x**3*(9*a**3*b*d**3 - 15*a**2*b**2*c*d**2 + 3*a*b**3*
c**2*d + 3*b**4*c**3) + x*(7*a**4*d**3 - 9*a**3*b*c*d**2 - 3*a**2*b**2*c**2
*d + 5*a*b**3*c**3))/(8*a**4*b**3 + 16*a**3*b**4*x**2 + 8*a**2*b**5*x**4) +
 d**3*x/b**3
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{d^3x}{b^3} + \frac{3\,(b^4c^3+ab^3c^2d-5\,a^2b^2cd^2+3\,a^3bd^3)x^3+(5\,ab^3c^3-3\,a^2b^2c^2d-9\,a^3bcd^2+7\,a^4d^3)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 + 1/8*(3*(b^4*c^3 + a*b^3*c^2*d - 5*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*
x^3 + (5*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 9*a^3*b*c*d^2 + 7*a^4*d^3)*x)/(a^2*b
^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*
d^2 - 5*a^3*d^3)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{d^3x}{b^3} + \frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4c^3x^3+3\,ab^3c^2dx^3-15\,a^2b^2cd^2x^3+9\,a^3bd^3x^3+5\,ab^3c^3x-3\,a^2b^2c^2dx-9\,a^3bcd^2x+7\,a^4d^3x}{8\,(bx^2+a)^2a^2b^3}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3/8*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c^3*x^3 + 3*a*b^3*c^2*d*x^3
 - 15*a^2*b^2*c*d^2*x^3 + 9*a^3*b*d^3*x^3 + 5*a*b^3*c^3*x - 3*a^2*b^2*c^2*d
*x - 9*a^3*b*c*d^2*x + 7*a^4*d^3*x)/((b*x^2 + a)^2*a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.85

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^3}\,dx = \frac{\frac{x\,\left(7\,a^3\,d^3-9\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+5\,b^3\,c^3\right)}{8\,a}+\frac{3\,x^3\,\left(3\,a^3\,b\,d^3-5\,a^2\,b^2\,c\,d^2+a\,b^3\,c^2\,d+b^4\,c^3\right)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{d^3\,x}{b^3} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{\sqrt{a}\,\left(-5\,a^3\,d^3+3\,a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}\right)\,\left(a\,d-b\,c\right)\,\left(5\,a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,a^{5/2}\,b^{7/2}}$$

```
[In] int((c + d*x^2)^3/(a + b*x^2)^3,x)
```

```
[Out] ((x*(7*a^3*d^3 + 5*b^3*c^3 - 3*a*b^2*c^2*d - 9*a^2*b*c*d^2))/(8*a) + (3*x^3
*(b^4*c^3 + 3*a^3*b*d^3 - 5*a^2*b^2*c*d^2 + a*b^3*c^2*d))/(8*a^2))/(a^2*b^3
 + b^5*x^4 + 2*a*b^4*x^2) + (d^3*x)/b^3 + (3*atan((b^(1/2)*x*(a*d - b*c)*(5
*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(a^(1/2)*(b^3*c^3 - 5*a^3*d^3 + a*b^2*c^2*
d + 3*a^2*b*c*d^2)))*(a*d - b*c)*(5*a^2*d^2 + b^2*c^2 + 2*a*b*c*d))/(8*a^(5
/2)*b^(7/2))
```

3.38 $\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

```
[Out] 3/8*(c^2/a^2-d^2/b^2)*x/(b*x^2+a)+1/4*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)^
2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(
5/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {424, 393, 211}

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{3x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{8(a+bx^2)}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3a^2d^2+2abcd+3b^2c^2)}{8a^{5/2}b^{5/2}}+\frac{x(c+dx^2)(bc-ad)}{4ab(a+bx^2)^2}$$

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] (3*(c^2/a^2 - d^2/b^2)*x)/(8*(a + b*x^2)) + ((b*c - a*d)*x*(c + d*x^2))/(4*
a*b*(a + b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(5/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{\int \frac{c(3bc+ad)+d(bc+3ad)x^2}{(a+bx^2)^2}\,dx}{4ab}\\
&= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\,(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\int\frac{1}{a+bx^2}\,dx}{8a^2b^2}\\
&= \frac{3\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{8\,(a+bx^2)}+\frac{(bc-ad)x(c+dx^2)}{4ab\,(a+bx^2)^2}+\frac{(3b^2c^2+2abcd+3a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(bc-ad)x(3a^2d+3b^2cx^2+5ab(c+dx^2))}{8a^2b^2\,(a+bx^2)^2} + \frac{(3b^2c^2+2abcd+3a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}}$$

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x*(3*a^2*d + 3*b^2*c*x^2 + 5*a*b*(c + d*x^2)))/(8*a^2*b^2*(a +
 b*x^2)^2) + ((3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(5/2)*b^(5/2))
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.07

method	result
default	$\frac{-\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x^3}{8a^2b}-\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}+\frac{\left(3a^2d^2+2abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b^2\sqrt{ab}}$
risch	$\frac{-\frac{\left(5a^2d^2-2abcd-3b^2c^2\right)x^3}{8a^2b}-\frac{\left(3a^2d^2+2abcd-5b^2c^2\right)x}{8a\,b^2}}{\left(b\,x^2+a\right)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)d^2}{16\sqrt{-ab}\,b^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)cd}{8\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c^2}{16\sqrt{-ab}\,a^2}+\frac{3\ln(-bx}{16\sqrt{}}$

```
[In] int((d*x^2+c)^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*(5*a^2*d^2-2*a*b*c*d-3*b^2*c^2)/a^2/b*x^3-1/8*(3*a^2*d^2+2*a*b*c*d-5*
b^2*c^2)/a/b^2*x)/(b*x^2+a)^2+1/8*(3*a^2*d^2+2*a*b*c*d+3*b^2*c^2)/a^2/b^2/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(102) = 204$.

Time = 0.26 (sec) , antiderivative size = 449, normalized size of antiderivative = 3.87

$$\int \frac{\left(c+dx^2\right)^2}{\left(a+bx^2\right)^3}\,dx = \left[\frac{2\left(3\,ab^4c^2+2\,a^2b^3cd-5\,a^3b^2d^2\right)x^3-\left(3\,a^2b^2c^2+2\,a^3bcd+3\,a^4d^2+\left(3\,b^4c^2+2\,ab^3cd+3\,a^2b^2d^2\right)x^4+2\,(\cdots\right.}{16\left(a^3b^5x^4+2\,a^4b^4\cdots\right.}\right.$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^4*c^2 + 2*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 - (3*a^2*b^2*c^2
 + 2*a^3*b*c*d + 3*a^4*d^2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4
+ 2*(3*a*b^3*c^2 + 2*a^2*b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(-a*b)*log((b*x^2
- 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^3*c^2 - 2*a^3*b^2*c*d - 3*a
^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3), 1/8*((3*a*b^4*c^2 + 2
*a^2*b^3*c*d - 5*a^3*b^2*d^2)*x^3 + (3*a^2*b^2*c^2 + 2*a^3*b*c*d + 3*a^4*d^
2 + (3*b^4*c^2 + 2*a*b^3*c*d + 3*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 + 2*a^2*
b^2*c*d + 3*a^3*b*d^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (5*a^2*b^3*c^
2 - 2*a^3*b^2*c*d - 3*a^4*b*d^2)*x)/(a^3*b^5*x^4 + 2*a^4*b^4*x^2 + a^5*b^3)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(110) = 220.

Time = 0.55 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^2d^2+2abcd+3b^2c^2)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{16} + \frac{x^3(-5a^2bd^2+2ab^2cd+3b^3c^2)+x(-3a^3d^2-2a^2bcd+5ab^2c^2)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*c*d + 3*b**2*c**2)*log(-a**3*b**
2*sqrt(-1/(a**5*b**5)) + x)/16 + sqrt(-1/(a**5*b**5))*(3*a**2*d**2 + 2*a*b*
c*d + 3*b**2*c**2)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/16 + (x**3*(-5*a
**2*b*d**2 + 2*a*b**2*c*d + 3*b**3*c**2) + x*(-3*a**3*d**2 - 2*a**2*b*c*d +
 5*a*b**2*c**2))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b**4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^3c^2+2\,ab^2cd-5\,a^2bd^2)x^3+(5\,ab^2c^2-2\,a^2bcd-3\,a^3d^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^3*c^2 + 2*a*b^2*c*d - 5*a^2*b*d^2)*x^3 + (5*a*b^2*c^2 - 2*a^2*b*c
*d - 3*a^3*d^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(3*b^2*c^2
 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^2c^2+2\,abcd+3\,a^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^2} + \frac{3\,b^3c^2x^3+2\,ab^2cdx^3-5\,a^2bd^2x^3+5\,ab^2c^2x-2\,a^2bcdx-3\,a^3d^2x}{8\,(bx^2+a)^2a^2b^2}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^2*c^2 + 2*a*b*c*d + 3*a^2*d^2)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
2*b^2) + 1/8*(3*b^3*c^2*x^3 + 2*a*b^2*c*d*x^3 - 5*a^2*b*d^2*x^3 + 5*a*b^2*c
^2*x - 2*a^2*b*c*d*x - 3*a^3*d^2*x)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 4.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^{5/2}\,b^{5/2}} - \frac{\frac{x\,\left(3\,a^2\,d^2+2\,a\,b\,c\,d-5\,b^2\,c^2\right)}{8\,a\,b^2}-\frac{x^3\,\left(-5\,a^2\,d^2+2\,a\,b\,c\,d+3\,b^2\,c^2\right)}{8\,a^2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

```
[In] int((c + d*x^2)^2/(a + b*x^2)^3,x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^2*d^2 + 3*b^2*c^2 + 2*a*b*c*d))/(8*a^(5/2)*
b^(5/2)) - ((x*(3*a^2*d^2 - 5*b^2*c^2 + 2*a*b*c*d))/(8*a*b^2) - (x^3*(3*b^2
*c^2 - 5*a^2*d^2 + 2*a*b*c*d))/(8*a^2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

3.39 $\int \frac{c+dx^2}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(bc-ad)x}{4ab\,(a+bx^2)^2} + \frac{(3bc+ad)x}{8a^2b\,(a+bx^2)} + \frac{(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[Out] 1/4*(-a*d+b*c)*x/a/b/(b*x^2+a)^2+1/8*(a*d+3*b*c)*x/a^2/b/(b*x^2+a)+1/8*(a*d
+3*b*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {393, 205, 211}

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(ad+3bc)}{8a^{5/2}b^{3/2}} + \frac{x(ad+3bc)}{8a^2b\,(a+bx^2)} + \frac{x(bc-ad)}{4ab\,(a+bx^2)^2}$$

```
[In] Int[(c + d*x^2)/(a + b*x^2)^3,x]
```

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^2)^2) + ((3*b*c + a*d)*x)/(8*a^2*b*(a + b*x
^2)) + ((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)\int\frac{1}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bc+ad)\int\frac{1}{a+bx^2}\,dx}{8a^2b}\\
&= \frac{(bc-ad)x}{4ab\left(a+bx^2\right)^2}+\frac{(3bc+ad)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bc+ad)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int\frac{c+dx^2}{\left(a+bx^2\right)^3}\,dx=\frac{x\left(-a^2d+3b^2cx^2+ab\left(5c+dx^2\right)\right)}{8a^2b\left(a+bx^2\right)^2}+\frac{(3bc+ad)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^3,x]
[Out] (x*(-(a^2*d) + 3*b^2*c*x^2 + a*b*(5*c + d*x^2)))/(8*a^2*b*(a + b*x^2)^2) +
((3*b*c + a*d)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(b\,x^2+a)^2}+\frac{(ad+3bc)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	76
risch	$\frac{\frac{(ad+3bc)x^3}{8a^2}-\frac{(ad-5bc)x}{8ab}}{(b\,x^2+a)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}+\frac{\ln\left(-bx+\sqrt{-ab}\right)d}{16\sqrt{-ab}\,ba}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)c}{16\sqrt{-ab}\,a^2}$	146

```
[In] int((d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(a*d+3*b*c)/a^2*x^3-1/8*(a*d-5*b*c)/a/b*x)/(b*x^2+a)^2+1/8*(a*d+3*b*c)
/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.27

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^3}\,dx = \left[\frac{2\,(3\,ab^3c+a^2b^2d)x^3-((3\,b^3c+ab^2d)x^4+3\,a^2bc+a^3d+2\,(3\,ab^2c+a^2bd)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)+}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right.$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a*b^3*c + a^2*b^2*d)*x^3 - ((3*b^3*c + a*b^2*d)*x^4 + 3*a^2*b*c
 + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) + 2*(5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b
^3*x^2 + a^5*b^2), 1/8*((3*a*b^3*c + a^2*b^2*d)*x^3 + ((3*b^3*c + a*b^2*d)*
x^4 + 3*a^2*b*c + a^3*d + 2*(3*a*b^2*c + a^2*b*d)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^2*c - a^3*b*d)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5
*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^5b^3}}(ad+3bc)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^5b^3}}(ad+3bc)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16} + \frac{x^3(abd+3b^2c)+x(-a^2d+5abc)}{8a^4b+16a^3b^2x^2+8a^2b^3x^4}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(a*d + 3*b*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (x**3*(a*b*d + 3*b**2*c) + x*(-a**2*d + 5*a*b*c))/(8*a**4*b + 16*a**3
*b**2*x**2 + 8*a**2*b**3*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(3\,b^2c+abd)x^3+(5\,abc-a^2d)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} + \frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^2*c + a*b*d)*x^3 + (5*a*b*c - a^2*d)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*
x^2 + a^4*b) + 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{(3\,bc+ad)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{3\,b^2cx^3+abdx^3+5\,abcx-a^2dx}{8\,(bx^2+a)^2a^2b}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*c + a*d)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(3*b^2*c*x^
3 + a*b*d*x^3 + 5*a*b*c*x - a^2*d*x)/((b*x^2 + a)^2*a^2*b)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{c+dx^2}{(a+bx^2)^3}\,dx = \frac{\frac{x^3\,(a\,d+3\,b\,c)}{8\,a^2}-\frac{x\,(a\,d-5\,b\,c)}{8\,a\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(a\,d+3\,b\,c)}{8\,a^{5/2}\,b^{3/2}}$$

```
[In] int((c + d*x^2)/(a + b*x^2)^3,x)
```

```
[Out] ((x^3*(a*d + 3*b*c))/(8*a^2) - (x*(a*d - 5*b*c))/(8*a*b))/(a^2 + b^2*x^4 +
2*a*b*x^2) + (atan((b^(1/2)*x)/a^(1/2))*(a*d + 3*b*c))/(8*a^(5/2)*b^(3/2))
```

3.40 $\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2(a+bx^2)} + \frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}$$

```
[Out] 1/4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^2/(b
*x^2+a)+1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)/a^(1/2))*b^(
1/2)/a^(5/2)/(-a*d+b*c)^3-d^(5/2)*arctan(x*d^(1/2)/c^(1/2))/(-a*d+b*c)^3/c^
(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {425, 541, 536, 211}

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = \frac{bx(3bc-7ad)}{8a^2(a+bx^2)(bc-ad)^2} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(15a^2d^2-10abcd+3b^2c^2)}{8a^{5/2}(bc-ad)^3} - \frac{d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3} + \frac{bx}{4a(a+bx^2)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (b*(3*b*c - 7*a*d)*x)/(8*a^2*(b*c -
 a*d)^2*(a + b*x^2)) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^3) - (d^(5/2)*ArcTan[(Sqrt[d
]*x)/Sqrt[c]])/(Sqrt[c]*(b*c - a*d)^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} - \frac{\int \frac{-3bc+4ad-3bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)} + \frac{\int \frac{3b^2c^2-7abcd+8a^2d^2+bd(3bc-7ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{8a^2(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\,(a+bx^2)} - \frac{d^3 \int \frac{1}{c+dx^2}\,dx}{(bc-ad)^3} + \frac{(b(3b^2c^2-10abcd+15a^2d^2)) \int \frac{1}{a+bx^2}\,dx}{8a^2(bc-ad)^3}$$

$$= \frac{bx}{4a(bc-ad)\,(a+bx^2)^2} + \frac{b(3bc-7ad)x}{8a^2(bc-ad)^2\,(a+bx^2)} + \frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^3} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}(bc-ad)^3}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+bx^2)^3\,(c+dx^2)}\,dx = -\frac{\frac{b(-bc+ad)x(-5abc+9a^2d-3b^2cx^2+7abdx^2)}{a^2(a+bx^2)^2} + \frac{\sqrt{b}(3b^2c^2-10abcd+15a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{8d^{5/2}\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{\sqrt{c}}}{8(-bc+ad)^3}$$

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)),x]
```

```
[Out] -1/8*((b*(-(b*c) + a*d)*x*(-5*a*b*c + 9*a^2*d - 3*b^2*c*x^2 + 7*a*b*d*x^2))
/(a^2*(a + b*x^2)^2) + (Sqrt[b]*(3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2) - (8*d^(5/2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/S
qrt[c])/(-(b*c) + a*d)^3
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{b\left(\frac{\frac{b(7a^2d^2-10abcd+3b^2c^2)x^3}{8a^2}+\frac{(9a^2d^2-14abcd+5b^2c^2)x}{8a}}{(bx^2+a)^2}+\frac{(15a^2d^2-10abcd+3b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^3}+\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(ad-bc)^3\sqrt{cd}}$	158
risch	Expression too large to display	2285

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -b/(a*d-b*c)^3*((1/8*b*(7*a^2*d^2-10*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(9*a^2*
d^2-14*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(15*a^2*d^2-10*a*b*c*d+3*b^2
*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^3/(c*d)^(1/2)*
arctan(d*x/(c*d)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(139) = 278$.

Time = 0.80 (sec) , antiderivative size = 1587, normalized size of antiderivative = 9.86

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)} \, dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 - (3*a^2*b^2*c^2 -
10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d^2)*x^4
 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(-b/a)*log((b*x
^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*
x^2 + a^4*d^2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) +
 2*(5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2
*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b
^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*
c*d^2 - a^6*b*d^3)*x^2), 1/16*(2*(3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)
*x^3 - 16*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/c)*arctan(x*
sqrt(d/c)) - (3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a
*b^3*c*d + 15*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b
*d^2)*x^2)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(
5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2
*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c
*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^
2 - a^6*b*d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 +
(3*a^2*b^2*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15
*a^2*b^2*d^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sq
rt(b/a)*arctan(x*sqrt(b/a)) - 4*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^
2)*sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^3*c^
2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6
*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5
*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*
d^3)*x^2), 1/8*((3*b^4*c^2 - 10*a*b^3*c*d + 7*a^2*b^2*d^2)*x^3 + (3*a^2*b^2
*c^2 - 10*a^3*b*c*d + 15*a^4*d^2 + (3*b^4*c^2 - 10*a*b^3*c*d + 15*a^2*b^2*d
^2)*x^4 + 2*(3*a*b^3*c^2 - 10*a^2*b^2*c*d + 15*a^3*b*d^2)*x^2)*sqrt(b/a)*ar
ctan(x*sqrt(b/a)) - 8*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(d/
c)*arctan(x*sqrt(d/c)) + (5*a*b^3*c^2 - 14*a^2*b^2*c*d + 9*a^3*b*d^2)*x)/(a
^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*d^3 + (a^2*b^5*c^3 - 3*a
^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^4 + 2*(a^3*b^4*c^3 - 3*a^4*
b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c),x)
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\begin{aligned}\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx &= -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}+\frac{(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}\\&\quad+\frac{(3\,b^3c-7\,ab^2d)x^3+(5\,ab^2c-9\,a^2bd)x}{8\,(a^4b^2c^2-2\,a^5bcd+a^6d^2+(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2)x^4+2\,(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2)x^2)}\end{aligned}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="maxima")
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*((3*b^3*c - 7*a*b^2*d)*x^3 + (5*a*b^2*c - 9*a^2*b*d)*x)/(a^4*b^
2*c^2 - 2*a^5*b*c*d + a^6*d^2 + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)
*x^4 + 2*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \frac{1}{(a+bx^2)^3(c+dx^2)}\,dx &= -\frac{d^3\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\sqrt{cd}}\\&\quad+\frac{(3\,b^3c^2-10\,ab^2cd+15\,a^2bd^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^3c^3-3\,a^3b^2c^2d+3\,a^4bcd^2-a^5d^3)\sqrt{ab}}\\&\quad+\frac{3\,b^3cx^3-7\,ab^2dx^3+5\,ab^2cx-9\,a^2bdx}{8\,(a^2b^2c^2-2\,a^3bcd+a^4d^2)(bx^2+a)^2}\end{aligned}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c),x, algorithm="giac")
```

```
[Out] -d^3*arctan(d*x/sqrt(c*d))/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*sqrt(c*d)) + 1/8*(3*b^3*c^2 - 10*a*b^2*c*d + 15*a^2*b*d^2)*arctan(b*x/
sqrt(a*b))/((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*sqrt(
a*b)) + 1/8*(3*b^3*c*x^3 - 7*a*b^2*d*x^3 + 5*a*b^2*c*x - 9*a^2*b*d*x)/((a^2
*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 6.62 (sec) , antiderivative size = 6033, normalized size of antiderivative = 37.47

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)} dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)),x)
```

```
[Out] ((x^3*(3*b^3*c - 7*a*b^2*d))/(8*a^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(
5*b^2*c - 9*a*b*d))/(8*a*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2 + b^2*x^4 +
 2*a*b*x^2) + (atan(((((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d
^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 -
 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((
256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*
c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^
6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^
6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*
c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10
*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*
d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/
(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*
b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c
^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(
b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3
*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c
^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 -
 4*a^7*b*c*d^3)) + ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9
 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*
a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^
4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d
^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5
)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 128
0*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*
b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2
*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*
c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a
```

```
*b^2*c^3*d)))*(-c*d^5)^(1/2)*1i)/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2
- 3*a*b^2*c^3*d)))/((105*a^3*b^3*d^8 - 9*b^6*c^3*d^5 + 51*a*b^5*c^2*d^6 - 1
15*a^2*b^4*c*d^7)/(32*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^
4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (((
x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 +
 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6
*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - ((-c*d^5)^(1/2)*((256*a^10*b^2*d^10 - 1760
*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c
^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7
 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15
*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)
) - (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 - 1280*a^10*b^3*c*d^8 + 256*a^4*b^9
*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d
^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3
 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*
d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b
*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2
*b*c^2*d^2 - 3*a*b^2*c^3*d)) + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a
*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4
*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + ((-c*d^5
)^(1/2)*((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 80
0*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*
b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 +
a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 1
5*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-c*d^5)^(1/2)*(256*a^11*b^2*d^9 -
 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^
6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*
c^2*d^7))/(64*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)*(a^8*
d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3))))
/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d)))*(-c*d^5)^(1/2
))/(2*(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d))))*(-c*d^5)^(
1/2)*1i)/(b^3*c^4 - a^3*c*d^3 + 3*a^2*b*c^2*d^2 - 3*a*b^2*c^3*d) + (atan(((
((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6
 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a
^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - (((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^
9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816
*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b
^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*
d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-a^5*
b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280*a^1
0*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5
*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))
/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^4 +
a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-a^5*
b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3
```

```
+ 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2
 - 10*a*b*c*d)*1i)/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c
*d^2)) + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3
*b^4*c*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c
^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + (((256*a^10*b^2*d^10 - 1760*a^
9*b^3*c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*
d^4 - 6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 +
5280*a^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^
6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) +
 (x*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9
- 1280*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a
^6*b^7*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4
*c^2*d^7))/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(
a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3
)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^
5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 +
 3*b^2*c^2 - 10*a*b*c*d)*1i)/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d -
 3*a^7*b*c*d^2)))/((105*a^3*b^3*d^8 - 9*b^6*c^3*d^5 + 51*a*b^5*c^2*d^6 - 11
5*a^2*b^4*c*d^7)/(32*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4
*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (((x
*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c*d^6 +
190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*
b^2*c^2*d^2 - 4*a^7*b*c*d^3)) - (((256*a^10*b^2*d^10 - 1760*a^9*b^3*c*d^9 +
 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 - 6816*a^
5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a^8*b^4*
c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2
 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) - (x*(-a^5*b)^
(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280*a^10*b
^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7*c^5*d^
4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d^7))/(5
12*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^4 + a^4
*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-a^5*b)^
(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3
*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 -
10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))
 + (((x*(289*a^4*b^3*d^7 + 9*b^7*c^4*d^3 - 60*a*b^6*c^3*d^4 - 300*a^3*b^4*c
*d^6 + 190*a^2*b^5*c^2*d^5))/(32*(a^8*d^4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d +
 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)) + (((256*a^10*b^2*d^10 - 1760*a^9*b^3*
c*d^9 + 96*a^2*b^10*c^8*d^2 - 800*a^3*b^9*c^7*d^3 + 3040*a^4*b^8*c^6*d^4 -
6816*a^5*b^7*c^5*d^5 + 9760*a^6*b^6*c^4*d^6 - 9056*a^7*b^5*c^3*d^7 + 5280*a
^8*b^4*c^2*d^8)/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*
c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)) + (x*(-
a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*(256*a^11*b^2*d^9 - 1280
*a^10*b^3*c*d^8 + 256*a^4*b^9*c^7*d^2 - 1280*a^5*b^8*c^6*d^3 + 2304*a^6*b^7
*c^5*d^4 - 1280*a^7*b^6*c^4*d^5 - 1280*a^8*b^5*c^3*d^6 + 2304*a^9*b^4*c^2*d
```

```
^7))/(512*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)*(a^8*d^
4 + a^4*b^4*c^4 - 4*a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3)))*(-
a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*
c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2)))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2
*c^2 - 10*a*b*c*d))/(16*(a^8*d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*
c*d^2))))*(-a^5*b)^(1/2)*(15*a^2*d^2 + 3*b^2*c^2 - 10*a*b*c*d)*1i)/(8*(a^8*
d^3 - a^5*b^3*c^3 + 3*a^6*b^2*c^2*d - 3*a^7*b*c*d^2))
```

3.41 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = {} & \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3(c+dx^2)} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)} \\
& + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2(a+bx^2)(c+dx^2)} \\
& + \frac{b^{3/2}(3b^2c^2-14abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4} \\
& - \frac{d^{5/2}(7bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}
\end{aligned}
$$

```
[Out] 1/8*d*(-4*a*d+b*c)*(a*d+3*b*c)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)+1/4*b*x/a/(-a
*d+b*c)/(b*x^2+a)^2/(d*x^2+c)+3/8*b*(-3*a*d+b*c)*x/a^2/(-a*d+b*c)^2/(b*x^2+
a)/(d*x^2+c)+1/8*b^(3/2)*(35*a^2*d^2-14*a*b*c*d+3*b^2*c^2)*arctan(x*b^(1/2)
/a^(1/2))/a^(5/2)/(-a*d+b*c)^4-1/2*d^(5/2)*(-a*d+7*b*c)*arctan(x*d^(1/2)/c^
(1/2))/c^(3/2)/(-a*d+b*c)^4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used

$= \{425, 541, 536, 211\}$

$$\int \frac{1}{(a+bx^2)^3 (c+dx^2)^2}\,dx = \frac{dx(bc-4ad)(ad+3bc)}{8a^2c(c+dx^2)(bc-ad)^3} + \frac{3bx(bc-3ad)}{8a^2(a+bx^2)(c+dx^2)(bc-ad)^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^2d^2-14abcd+3b^2c^2)}{8a^{5/2}(bc-ad)^4} - \frac{d^{5/2}(7bc-ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4} + \frac{bx}{4a(a+bx^2)^2(c+dx^2)(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] (d*(b*c - 4*a*d)*(3*b*c + a*d)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^2)) + (b*
x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)) + (3*b*(b*c - 3*a*d)*x)/(8*a
^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)) + (b^(3/2)*(3*b^2*c^2 - 14*a*b*c*
d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*(b*c - a*d)^4) - (d
^(5/2)*(7*b*c - a*d)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(2*c^(3/2)*(b*c - a*d)^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} - \frac{\int \frac{-3bc+4ad-5bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^2}\,dx}{4a(bc-ad)} \\
&= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} \\
&\quad + \frac{\int \frac{3b^2c^2-5abcd+8a^2d^2+9bd(bc-3ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{8a^2(bc-ad)^2} \\
&= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} \\
&\quad + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} \\
&\quad + \frac{\int \frac{2\left(3b^3c^3-11ab^2c^2d+24a^2bcd^2-4a^3d^3\right)+2bd(bc-4ad)(3bc+ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{16a^2c(bc-ad)^3} \\
&= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} \\
&\quad + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} - \frac{\left(d^3(7bc-ad)\right)\int \frac{1}{c+dx^2}\,dx}{2c(bc-ad)^4} \\
&\quad + \frac{\left(b^2\left(3b^2c^2-14abcd+35a^2d^2\right)\right)\int \frac{1}{a+bx^2}\,dx}{8a^2(bc-ad)^4} \\
&= \frac{d(bc-4ad)(3bc+ad)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)} \\
&\quad + \frac{3b(bc-3ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)} \\
&\quad + \frac{b^{3/2}\left(3b^2c^2-14abcd+35a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^4} - \frac{d^{5/2}(7bc-ad)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{2c^{3/2}(bc-ad)^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \frac{1}{8}\left(\frac{2b^2x}{a(bc-ad)^2(a+bx^2)^2} + \frac{b^2(-3bc+11ad)x}{a^2(-bc+ad)^3(a+bx^2)} - \frac{4d^3x}{c(bc-ad)^3(c+dx^2)} + \frac{b^{3/2}(3b^2c^2-14abcd+35a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(bc-ad)^4} + \frac{4d^{5/2}(-7bc+ad)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{c^{3/2}(bc-ad)^4}\right)$$

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^2),x]
```

```
[Out] ((2*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^2)^2) + (b^2*(-3*b*c + 11*a*d)*x)/(a^2
*(-(b*c) + a*d)^3*(a + b*x^2)) - (4*d^3*x)/(c*(b*c - a*d)^3*(c + d*x^2)) +
(b^(3/2)*(3*b^2*c^2 - 14*a*b*c*d + 35*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(5/2)*(b*c - a*d)^4) + (4*d^(5/2)*(-7*b*c + a*d)*ArcTan[(Sqrt[d]*x)/Sqr
t[c]])/(c^(3/2)*(b*c - a*d)^4))/8
```

Maple [A] (verified)

Time = 8.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.83

method	result
default	$\frac{b^2\left(\frac{\frac{b\left(11a^2d^2-14abcd+3b^2c^2\right)x^3}{8a^2}+\frac{\left(13a^2d^2-18abcd+5b^2c^2\right)x}{8a}}{\left(bx^2+a\right)^2}+\frac{\left(35a^2d^2-14abcd+3b^2c^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^4}+\frac{d^3\left(\frac{(ad-bc)x}{2c\left(dx^2+c\right)}+\frac{(ad-7bc)\,\mathrm{ar}}{2c}\right.}{(ad-bc)^4}$
risch	Expression too large to display

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/(a*d-b*c)^4*((1/8*b*(11*a^2*d^2-14*a*b*c*d+3*b^2*c^2)/a^2*x^3+1/8*(13*a
^2*d^2-18*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+1/8*(35*a^2*d^2-14*a*b*c*d+3*
b^2*c^2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^4*(1/2*(a*d
-b*c)/c*x/(d*x^2+c)+1/2*(a*d-7*b*c)/c/(c*d)^(1/2)*arctan(d*x/(c*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. 2(210) = 420.

Time = 2.41 (sec) , antiderivative size = 3251, normalized size of antiderivative = 13.78

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d^3 + 4*a^3*b^2*d^4)
*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 +
 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35*a^4*b*c^2*d^2 +
(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 + (3*b^5*c^4 - 8*a*
b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6*a*b^4*c^4 - 25*a
^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 +
 (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*
sqrt(-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/16*(2*(3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a
^2*b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + 2*(3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^
3*c^2*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 - 8*(7*a^4*b*c^2*d^2 - a^5*c
*d^3 + (7*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^
2*c*d^3 - 2*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)
*x^2)*sqrt(d/c)*arctan(x*sqrt(d/c)) + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 3
5*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
 (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (
6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 2*(5*a*b^4*c^4
 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a
^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^
2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*
c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c
^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^
3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7
*b*c^2*d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*
b^3*c*d^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2
*d^2 + 5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3
```

```
*d + 35*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)
*x^6 + (3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x
^4 + (6*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)
*x^2)*sqrt(b/a)*arctan(x*sqrt(b/a)) - 2*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a
^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2
*a^4*b*d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(
-d/c)*log((d*x^2 + 2*c*x*sqrt(-d/c) - c)/(d*x^2 + c)) + (5*a*b^4*c^4 - 18*a
^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*
c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 +
 (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2 + 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4
 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2
+ 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c
^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*
d^4 + a^8*c*d^5)*x^2), 1/8*((3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 7*a^2*b^3*c*d
^3 + 4*a^3*b^2*d^4)*x^5 + (3*b^5*c^4 - 9*a*b^4*c^3*d - 7*a^2*b^3*c^2*d^2 +
5*a^3*b^2*c*d^3 + 8*a^4*b*d^4)*x^3 + (3*a^2*b^3*c^4 - 14*a^3*b^2*c^3*d + 35
*a^4*b*c^2*d^2 + (3*b^5*c^3*d - 14*a*b^4*c^2*d^2 + 35*a^2*b^3*c*d^3)*x^6 +
(3*b^5*c^4 - 8*a*b^4*c^3*d + 7*a^2*b^3*c^2*d^2 + 70*a^3*b^2*c*d^3)*x^4 + (6
*a*b^4*c^4 - 25*a^2*b^3*c^3*d + 56*a^3*b^2*c^2*d^2 + 35*a^4*b*c*d^3)*x^2)*s
qrt(b/a)*arctan(x*sqrt(b/a)) - 4*(7*a^4*b*c^2*d^2 - a^5*c*d^3 + (7*a^2*b^3*
c*d^3 - a^3*b^2*d^4)*x^6 + (7*a^2*b^3*c^2*d^2 + 13*a^3*b^2*c*d^3 - 2*a^4*b*
d^4)*x^4 + (14*a^3*b^2*c^2*d^2 + 5*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(d/c)*ar
ctan(x*sqrt(d/c)) + (5*a*b^4*c^4 - 18*a^2*b^3*c^3*d + 13*a^3*b^2*c^2*d^2 -
4*a^4*b*c*d^3 + 4*a^5*d^4)*x)/(a^4*b^4*c^6 - 4*a^5*b^3*c^5*d + 6*a^6*b^2*c^
4*d^2 - 4*a^7*b*c^3*d^3 + a^8*c^2*d^4 + (a^2*b^6*c^5*d - 4*a^3*b^5*c^4*d^2
+ 6*a^4*b^4*c^3*d^3 - 4*a^5*b^3*c^2*d^4 + a^6*b^2*c*d^5)*x^6 + (a^2*b^6*c^6
 - 2*a^3*b^5*c^5*d - 2*a^4*b^4*c^4*d^2 + 8*a^5*b^3*c^3*d^3 - 7*a^6*b^2*c^2*
d^4 + 2*a^7*b*c*d^5)*x^4 + (2*a^3*b^5*c^6 - 7*a^4*b^4*c^5*d + 8*a^5*b^3*c^4
*d^2 - 2*a^6*b^2*c^3*d^3 - 2*a^7*b*c^2*d^4 + a^8*c*d^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3\,(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**2,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(210) = 420.

Time = 0.30 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \frac{(3\,b^4c^2 - 14\,ab^3cd + 35\,a^2b^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4)\sqrt{ab}} - \frac{(7\,bcd^3 - ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5 - 4\,ab^3c^4d + 6\,a^2b^2c^3d^2 - 4\,a^3bc^2d^3 + a^4cd^4)\sqrt{cd}} + \frac{(3\,b^4c^2d - 11\,ab^3cd^2 - 4\,a^2b^2d^3)x^5 + (3\,b^4c^3 - 6\,ab^3c^2d -}{8\,(a^4b^3c^5 - 3\,a^5b^2c^4d + 3\,a^6bc^3d^2 - a^7c^2d^3 + (a^2b^5c^4d - 3\,a^3b^4c^3d^2 + 3\,a^4b^3c^2d^3 - a^5b^2cd^4)x^6 + (a^2b^5c^5 -}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b))/((a^2
*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4)*s
qrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5 - 4*a*b
^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*d)) + 1/
8*((3*b^4*c^2*d - 11*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*x^5 + (3*b^4*c^3 - 6*a*b^
3*c^2*d - 13*a^2*b^2*c*d^2 - 8*a^3*b*d^3)*x^3 + (5*a*b^3*c^3 - 13*a^2*b^2*c
^2*d - 4*a^4*d^3)*x)/(a^4*b^3*c^5 - 3*a^5*b^2*c^4*d + 3*a^6*b*c^3*d^2 - a^7
*c^2*d^3 + (a^2*b^5*c^4*d - 3*a^3*b^4*c^3*d^2 + 3*a^4*b^3*c^2*d^3 - a^5*b^2
*c*d^4)*x^6 + (a^2*b^5*c^5 - a^3*b^4*c^4*d - 3*a^4*b^3*c^3*d^2 + 5*a^5*b^2*
c^2*d^3 - 2*a^6*b*c*d^4)*x^4 + (2*a^3*b^4*c^5 - 5*a^4*b^3*c^4*d + 3*a^5*b^2
*c^3*d^2 + a^6*b*c^2*d^3 - a^7*c*d^4)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = -\frac{d^3x}{2\,(b^3c^4 - 3\,ab^2c^3d + 3\,a^2bc^2d^2 - a^3cd^3)(dx^2+c)} + \frac{(3\,b^4c^2 - 14\,ab^3cd + 35\,a^2b^2d^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,(a^2b^4c^4 - 4\,a^3b^3c^3d + 6\,a^4b^2c^2d^2 - 4\,a^5bcd^3 + a^6d^4)\sqrt{ab}} - \frac{(7\,bcd^3 - ad^4)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{2\,(b^4c^5 - 4\,ab^3c^4d + 6\,a^2b^2c^3d^2 - 4\,a^3bc^2d^3 + a^4cd^4)\sqrt{cd}} + \frac{3\,b^4cx^3 - 11\,ab^3dx^3 + 5\,ab^3cx - 13\,a^2b^2dx}{8\,(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3)(bx^2+a)^2}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*x/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*(d*x^2
+ c)) + 1/8*(3*b^4*c^2 - 14*a*b^3*c*d + 35*a^2*b^2*d^2)*arctan(b*x/sqrt(a*b
))/((a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^
6*d^4)*sqrt(a*b)) - 1/2*(7*b*c*d^3 - a*d^4)*arctan(d*x/sqrt(c*d))/((b^4*c^5
 - 4*a*b^3*c^4*d + 6*a^2*b^2*c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4)*sqrt(c*
d)) + 1/8*(3*b^4*c*x^3 - 11*a*b^3*d*x^3 + 5*a*b^3*c*x - 13*a^2*b^2*d*x)/((a
^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(b*x^2 + a)^2)
```

Mupad [B] (verification not implemented)

Time = 7.58 (sec) , antiderivative size = 8635, normalized size of antiderivative = 36.59

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^2),x)
```

```
[Out] ((x^5*(4*a^2*b^2*d^3 - 3*b^4*c^2*d + 11*a*b^3*c*d^2))/(8*a^2*c*(a^3*d^3 - b
^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (x*(4*a^3*d^3 - 5*b^3*c^3 + 13*a
*b^2*c^2*d))/(8*a*c*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*x^3*(
8*a^3*d^3 - 3*b^3*c^3 + 6*a*b^2*c^2*d + 13*a^2*b*c*d^2))/(8*a^2*c*(a*d - b*
c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a^2*c + x^2*(a^2*d + 2*a*b*c) + x^4*(
b^2*c + 2*a*b*d) + b^2*d*x^6) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3
 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6
*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b
^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a
^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3
*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b
^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*
d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c
^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8
 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*
b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(-a^5*b^3)^(1
/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5
*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7
*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4
*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5
*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^
8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 -
 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 +
3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a
^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2
- 14*a*b*c*d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*
c^2*d^2 - 4*a^8*b*c*d^3)) + (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8
*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 +
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2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) + (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*1i)/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))/(((35*a^5*b^4*d^10)/16 + (63*b^9*c^5*d^5)/64 - (267*a*b^8*c^4*d^6)/32 - (651*a^4*b^5*c*d^9)/64 + (451*a^2*b^7*c^3*d^7)/16 - (1275*a^3*b^6*c^2*d^8)/32)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) + (((x*(16*a^6*b^3*d^9 +

9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) + (((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(512*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(16*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3))))*(-a^5*b^3)^(1/2)*(35*a^2*d^2 + 3*b^2*c^2 - 14*a*b*c*d)*1i)/(8*(a^9*d^4 + a^5*b^4*c^4 - 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3)) - (atan(((((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(128*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*1i)/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)) + (((x*(16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d -

6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4
*d^4)) + ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^13*b^2*c*d^13 - (3*a^2*b^13*
c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^1
0*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d
^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^
11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10
*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b
^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^
7) + (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^
10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^
7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^
9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^11))/(128*(b^4*c^7 + a^4*c^
3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4*b^6*c^8 +
a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20
*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3
*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)))*(a*d - 7*b*c)*(-c^3*d^5)^
(1/2)*1i)/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 -
4*a*b^3*c^6*d)))/(((35*a^5*b^4*d^10)/16 + (63*b^9*c^5*d^5)/64 - (267*a*b^8
*c^4*d^6)/32 - (651*a^4*b^5*c*d^9)/64 + (451*a^2*b^7*c^3*d^7)/16 - (1275*a^
3*b^6*c^2*d^8)/32)/(a^4*b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12
*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4
- 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (((x*(
16*a^6*b^3*d^9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406
*a^2*b^7*c^4*d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^
6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d
^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) - ((a*d - 7*b*c)*(-c^3*d^5)^
(1/2)*((2*a^13*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3
)/2 - 98*a^4*b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1
197*a^7*b^8*c^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^1
0*b^5*c^4*d^10)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*
b^9*c^11 - a^13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*
c^9*d^2 - 84*a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 +
84*a^10*b^3*c^5*d^6 - 36*a^11*b^2*c^4*d^7) - (x*(a*d - 7*b*c)*(-c^3*d^5)^(1
/2)*(256*a^4*b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4
- 7168*a^7*b^8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168
*a^10*b^5*c^5*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^
13*b^2*c^2*d^11))/(128*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2
*c^5*d^2 - 4*a*b^3*c^6*d)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6
*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d
^4))))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*
a*b^3*c^6*d)))*(a*d - 7*b*c)*(-c^3*d^5)^(1/2))/(4*(b^4*c^7 + a^4*c^3*d^4 -
4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d)) + (((x*(16*a^6*b^3*d^
9 + 9*b^9*c^6*d^3 - 84*a*b^8*c^5*d^4 - 224*a^5*b^4*c*d^8 + 406*a^2*b^7*c^4*
d^5 - 980*a^3*b^6*c^3*d^6 + 2009*a^4*b^5*c^2*d^7))/(32*(a^4*b^6*c^8 + a^10*
c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b

```
^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4)) + ((a*d - 7*b*c)*(-c^3*d^5)^(1/2)*((2*a^1
3*b^2*c*d^13 - (3*a^2*b^13*c^12*d^2)/2 + (35*a^3*b^12*c^11*d^3)/2 - 98*a^4*
b^11*c^10*d^4 + 336*a^5*b^10*c^9*d^5 - 765*a^6*b^9*c^8*d^6 + 1197*a^7*b^8*c
^7*d^7 - 1302*a^8*b^7*c^6*d^8 + 978*a^9*b^6*c^5*d^9 - (987*a^10*b^5*c^4*d^1
0)/2 + (315*a^11*b^4*c^3*d^11)/2 - 28*a^12*b^3*c^2*d^12)/(a^4*b^9*c^11 - a^
13*c^2*d^9 - 9*a^5*b^8*c^10*d + 9*a^12*b*c^3*d^8 + 36*a^6*b^7*c^9*d^2 - 84*
a^7*b^6*c^8*d^3 + 126*a^8*b^5*c^7*d^4 - 126*a^9*b^4*c^6*d^5 + 84*a^10*b^3*c
^5*d^6 - 36*a^11*b^2*c^4*d^7) + (x*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*(256*a^4*
b^11*c^11*d^2 - 1792*a^5*b^10*c^10*d^3 + 5120*a^6*b^9*c^9*d^4 - 7168*a^7*b^
8*c^8*d^5 + 3584*a^8*b^7*c^7*d^6 + 3584*a^9*b^6*c^6*d^7 - 7168*a^10*b^5*c^5
*d^8 + 5120*a^11*b^4*c^4*d^9 - 1792*a^12*b^3*c^3*d^10 + 256*a^13*b^2*c^2*d^
11))/(128*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*
a*b^3*c^6*d)*(a^4*b^6*c^8 + a^10*c^2*d^6 - 6*a^5*b^5*c^7*d - 6*a^9*b*c^3*d^
5 + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4))))/(4*(b^
4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))
)*(a*d - 7*b*c)*(-c^3*d^5)^(1/2))/(4*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d
^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^3*c^6*d))))*(a*d - 7*b*c)*(-c^3*d^5)^(1/2)*1
i)/(2*(b^4*c^7 + a^4*c^3*d^4 - 4*a^3*b*c^4*d^3 + 6*a^2*b^2*c^5*d^2 - 4*a*b^
3*c^6*d))
```

3.42 $\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 315

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = \frac{d(3b^2c^2-13abcd-2a^2d^2)x}{8a^2c(bc-ad)^3(c+dx^2)^2} + \frac{bx}{4a(bc-ad)(a+bx^2)^2(c+dx^2)^2} + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2(a+bx^2)(c+dx^2)^2} + \frac{3d(bc+ad)(b^2c^2-6abcd+a^2d^2)x}{8a^2c^2(bc-ad)^4(c+dx^2)} + \frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5} - \frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}$$

```
[Out] 1/8*d*(-2*a^2*d^2-13*a*b*c*d+3*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(d*x^2+c)^2+1/
4*b*x/a/(-a*d+b*c)/(b*x^2+a)^2/(d*x^2+c)^2+1/8*b*(-11*a*d+3*b*c)*x/a^2/(-a*
d+b*c)^2/(b*x^2+a)/(d*x^2+c)^2+3/8*d*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*
x/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)+3/8*b^(5/2)*(21*a^2*d^2-6*a*b*c*d+b^2*c^2)
*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/(-a*d+b*c)^5-3/8*d^(5/2)*(a^2*d^2-6*a*b*
c*d+21*b^2*c^2)*arctan(x*d^(1/2)/c^(1/2))/c^(5/2)/(-a*d+b*c)^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {425, 541, 536, 211}

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = -\frac{3d^{5/2}(a^2d^2-6abcd+21b^2c^2)\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5} + \frac{3dx(ad+bc)(a^2d^2-6abcd+b^2c^2)}{8a^2c^2(c+dx^2)(bc-ad)^4} + \frac{dx(-2a^2d^2-13abcd+3b^2c^2)}{8a^2c(c+dx^2)^2(bc-ad)^3} + \frac{bx(3bc-11ad)}{8a^2(a+bx^2)(c+dx^2)^2(bc-ad)^2} + \frac{3b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(21a^2d^2-6abcd+b^2c^2)}{8a^{5/2}(bc-ad)^5} + \frac{bx}{4a(a+bx^2)^2(c+dx^2)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 13*a*b*c*d - 2*a^2*d^2)*x)/(8*a^2*c*(b*c - a*d)^3*(c + d*x^
2)^2) + (b*x)/(4*a*(b*c - a*d)*(a + b*x^2)^2*(c + d*x^2)^2) + (b*(3*b*c - 1
1*a*d)*x)/(8*a^2*(b*c - a*d)^2*(a + b*x^2)*(c + d*x^2)^2) + (3*d*(b*c + a*d
)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x)/(8*a^2*c^2*(b*c - a*d)^4*(c + d*x^2))
+ (3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(8*a^(5/2)*(b*c - a*d)^5) - (3*d^(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)
*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/(8*c^(5/2)*(b*c - a*d)^5)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} - \frac{\int \frac{-3bc+4ad-7bdx^2}{\left(a+bx^2\right)^2\left(c+dx^2\right)^3}\,dx}{4a(bc-ad)} \\
&= \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)^2} \\
&\quad + \frac{\int \frac{3b^2c^2-3abcd+8a^2d^2+5bd(3bc-11ad)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^3}\,dx}{8a^2(bc-ad)^2} \\
&= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} \\
&\quad + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)^2} \\
&\quad + \frac{\int \frac{12\left(b^3c^3-3ab^2c^2d+8a^2bcd^2-2a^3d^3\right)+12bd\left(3b^2c^2-13abcd-2a^2d^2\right)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)^2}\,dx}{32a^2c(bc-ad)^3} \\
&= \frac{d\left(3b^2c^2-13abcd-2a^2d^2\right)x}{8a^2c(bc-ad)^3\left(c+dx^2\right)^2} + \frac{bx}{4a(bc-ad)\left(a+bx^2\right)^2\left(c+dx^2\right)^2} \\
&\quad + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\left(a+bx^2\right)\left(c+dx^2\right)^2} + \frac{3d(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x}{8a^2c^2(bc-ad)^4\left(c+dx^2\right)} \\
&\quad + \frac{\int \frac{24\left(b^4c^4-5ab^3c^3d+16a^2b^2c^2d^2-5a^3bcd^3+a^4d^4\right)+24bd(bc+ad)\left(b^2c^2-6abcd+a^2d^2\right)x^2}{\left(a+bx^2\right)\left(c+dx^2\right)}\,dx}{64a^2c^2(bc-ad)^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d(3b^2c^2 - 13abcd - 2a^2d^2)\,x}{8a^2c(bc-ad)^3\,(c+dx^2)^2} + \frac{bx}{4a(bc-ad)\,(a+bx^2)^2\,(c+dx^2)^2} \\
&\quad + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\,(a+bx^2)\,(c+dx^2)^2} + \frac{3d(bc+ad)\,(b^2c^2-6abcd+a^2d^2)\,x}{8a^2c^2(bc-ad)^4\,(c+dx^2)} \\
&\quad - \frac{(3d^3(21b^2c^2-6abcd+a^2d^2))\int \frac{1}{c+dx^2}\,dx}{8c^2(bc-ad)^5} \\
&\quad + \frac{(3b^3(b^2c^2-6abcd+21a^2d^2))\int \frac{1}{a+bx^2}\,dx}{8a^2(bc-ad)^5}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{d(3b^2c^2 - 13abcd - 2a^2d^2)\,x}{8a^2c(bc-ad)^3\,(c+dx^2)^2} + \frac{bx}{4a(bc-ad)\,(a+bx^2)^2\,(c+dx^2)^2} \\
&\quad + \frac{b(3bc-11ad)x}{8a^2(bc-ad)^2\,(a+bx^2)\,(c+dx^2)^2} + \frac{3d(bc+ad)\,(b^2c^2-6abcd+a^2d^2)\,x}{8a^2c^2(bc-ad)^4\,(c+dx^2)} \\
&\quad + \frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}(bc-ad)^5} \\
&\quad - \frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)}{8c^{5/2}(bc-ad)^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.74

$$
\int \frac{1}{(a+bx^2)^3\,(c+dx^2)^3}\,dx = \frac{1}{8}\left(-\frac{3b^{5/2}(b^2c^2-6abcd+21a^2d^2)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}(-bc+ad)^5} + \frac{(bc-ad)x\left(\frac{3b^4c}{a^2(a+bx^2)} + \frac{3ad^4}{c^2(c+dx^2)} + \frac{b^3(2bc-17ad-15bdx^2)}{a(a+bx^2)^2} - \frac{d^3(17bc-2ad+15bdx^2)}{c(c+dx^2)^2}\right) - \frac{3d^{5/2}(21b^2c^2-6abcd+a^2d^2)\arctan}{c^{5/2}}}{(bc-ad)^5}
$$

```
[In] Integrate[1/((a + b*x^2)^3*(c + d*x^2)^3),x]
```

```
[Out] ((-3*b^(5/2)*(b^2*c^2 - 6*a*b*c*d + 21*a^2*d^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(a^(5/2)*(-(b*c) + a*d)^5) + ((b*c - a*d)*x*((3*b^4*c)/(a^2*(a + b*x^2))
+ (3*a*d^4)/(c^2*(c + d*x^2)) + (b^3*(2*b*c - 17*a*d - 15*b*d*x^2))/(a*(a +
 b*x^2)^2) - (d^3*(17*b*c - 2*a*d + 15*b*d*x^2))/(c*(c + d*x^2)^2)) - (3*d^
(5/2)*(21*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(Sqrt[d]*x)/Sqrt[c]])/c^(5/
2))/(b*c - a*d)^5)/8
```

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.82

method	result
default	$-\frac{b^3\left(\frac{\frac{3b(5a^2d^2-6abcd+b^2c^2)x^3}{8a^2}+\frac{(17a^2d^2-22abcd+5b^2c^2)x}{8a}}{(bx^2+a)^2}+\frac{3(21a^2d^2-6abcd+b^2c^2)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(ad-bc)^5}+\frac{d^3\left(\frac{\frac{3d(a^2d^2-6abcd+5b^2c^2)x^3}{8c^2}}{(dx^2}\right.}{}$
risch	Expression too large to display

```
[In] int(1/(b*x^2+a)^3/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b^3/(a*d-b*c)^5*((3/8*b*(5*a^2*d^2-6*a*b*c*d+b^2*c^2)/a^2*x^3+1/8*(17*a^2*
d^2-22*a*b*c*d+5*b^2*c^2)/a*x)/(b*x^2+a)^2+3/8*(21*a^2*d^2-6*a*b*c*d+b^2*c^
2)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))+d^3/(a*d-b*c)^5*((3/8*d*(a^2*d^
2-6*a*b*c*d+5*b^2*c^2)/c^2*x^3+1/8*(5*a^2*d^2-22*a*b*c*d+17*b^2*c^2)/c*x)/(
d*x^2+c)^2+3/8*(a^2*d^2-6*a*b*c*d+21*b^2*c^2)/c^2/(c*d)^(1/2)*arctan(d*x/(c
*d)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(287) = 574.

Time = 7.15 (sec) , antiderivative size = 5070, normalized size of antiderivative = 16.10

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**3/(d*x**2+c)**3,x)
```

```
[Out] Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(287) = 574$.

Time = 0.32 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.60

$$\int \frac{1}{(a+bx^2)^3(c+dx^2)^3}\,dx = \frac{3\left(b^5c^2-6\,ab^4cd+21\,a^2b^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^5c^5-5\,a^3b^4c^4d+10\,a^4b^3c^3d^2-10\,a^5b^2c^2d^3+5\,a^6bcd^4-a^7d^5\right)\sqrt{ab}} - \frac{3\left(21\,b^2c^2d^3-6\,abcd^4+a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^5c^7-5\,ab^4c^6d+10\,a^2b^3c^5d^2-10\,a^3b^2c^4d^3+5\,a^4bc^3d^4-a^5c^2d^5\right)\sqrt{cd}} + \frac{3\left(b^5c^3d^2-5\,ab^4c^2d^3-5\,a^2b^3cd^4+a^3b^2d^5\right)x^7+(6}{8\left(a^4b^4c^8-4\,a^5b^3c^7d+6\,a^6b^2c^6d^2-4\,a^7bc^5d^3+a^8c^4d^4+\left(a^2b^6c^6d^2-4\,a^3b^5c^5d^3+6\,a^4b^4c^4d^4-4\,a^5b^3c^3\right.\right.}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*(b^5*c
^3*d^2 - 5*a*b^4*c^2*d^3 - 5*a^2*b^3*c*d^4 + a^3*b^2*d^5)*x^7 + (6*b^5*c^4*
d - 25*a*b^4*c^3*d^2 - 34*a^2*b^3*c^2*d^3 - 25*a^3*b^2*c*d^4 + 6*a^4*b*d^5)
*x^5 + (3*b^5*c^5 - 5*a*b^4*c^4*d - 34*a^2*b^3*c^3*d^2 - 34*a^3*b^2*c^2*d^3
 - 5*a^4*b*c*d^4 + 3*a^5*d^5)*x^3 + (5*a*b^4*c^5 - 17*a^2*b^3*c^4*d - 17*a^
4*b*c^2*d^3 + 5*a^5*c*d^4)*x)/(a^4*b^4*c^8 - 4*a^5*b^3*c^7*d + 6*a^6*b^2*c^
6*d^2 - 4*a^7*b*c^5*d^3 + a^8*c^4*d^4 + (a^2*b^6*c^6*d^2 - 4*a^3*b^5*c^5*d^
3 + 6*a^4*b^4*c^4*d^4 - 4*a^5*b^3*c^3*d^5 + a^6*b^2*c^2*d^6)*x^8 + 2*(a^2*b
^6*c^7*d - 3*a^3*b^5*c^6*d^2 + 2*a^4*b^4*c^5*d^3 + 2*a^5*b^3*c^4*d^4 - 3*a^
6*b^2*c^3*d^5 + a^7*b*c^2*d^6)*x^6 + (a^2*b^6*c^8 - 9*a^4*b^4*c^6*d^2 + 16*
a^5*b^3*c^5*d^3 - 9*a^6*b^2*c^4*d^4 + a^8*c^2*d^6)*x^4 + 2*(a^3*b^5*c^8 - 3
*a^4*b^4*c^7*d + 2*a^5*b^3*c^6*d^2 + 2*a^6*b^2*c^5*d^3 - 3*a^7*b*c^4*d^4 +
a^8*c^3*d^5)*x^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.82

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx$$
$$= \frac{3\left(b^5c^2-6\,ab^4cd+21\,a^2b^3d^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\left(a^2b^5c^5-5\,a^3b^4c^4d+10\,a^4b^3c^3d^2-10\,a^5b^2c^2d^3+5\,a^6bcd^4-a^7d^5\right)\sqrt{ab}}$$
$$-\frac{3\left(21\,b^2c^2d^3-6\,abcd^4+a^2d^5\right)\arctan\left(\frac{dx}{\sqrt{cd}}\right)}{8\left(b^5c^7-5\,ab^4c^6d+10\,a^2b^3c^5d^2-10\,a^3b^2c^4d^3+5\,a^4bc^3d^4-a^5c^2d^5\right)\sqrt{cd}}$$
$$+\frac{3\,b^5c^3d^2x^7-15\,ab^4c^2d^3x^7-15\,a^2b^3cd^4x^7+3\,a^3b^2d^5x^7+6\,b^5c^4dx^5-25\,ab^4c^3d^2x^5-34\,a^2b^3c^2d^3x^5-25\,a}{8\left(a^2b^4c^6-4\,a^3b^3\right.}$$

```
[In] integrate(1/(b*x^2+a)^3/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b^5*c^2 - 6*a*b^4*c*d + 21*a^2*b^3*d^2)*arctan(b*x/sqrt(a*b))/((a^2*b^
5*c^5 - 5*a^3*b^4*c^4*d + 10*a^4*b^3*c^3*d^2 - 10*a^5*b^2*c^2*d^3 + 5*a^6*b
*c*d^4 - a^7*d^5)*sqrt(a*b)) - 3/8*(21*b^2*c^2*d^3 - 6*a*b*c*d^4 + a^2*d^5)
*arctan(d*x/sqrt(c*d))/((b^5*c^7 - 5*a*b^4*c^6*d + 10*a^2*b^3*c^5*d^2 - 10*
a^3*b^2*c^4*d^3 + 5*a^4*b*c^3*d^4 - a^5*c^2*d^5)*sqrt(c*d)) + 1/8*(3*b^5*c^
3*d^2*x^7 - 15*a*b^4*c^2*d^3*x^7 - 15*a^2*b^3*c*d^4*x^7 + 3*a^3*b^2*d^5*x^7
 + 6*b^5*c^4*d*x^5 - 25*a*b^4*c^3*d^2*x^5 - 34*a^2*b^3*c^2*d^3*x^5 - 25*a^3
*b^2*c*d^4*x^5 + 6*a^4*b*d^5*x^5 + 3*b^5*c^5*x^3 - 5*a*b^4*c^4*d*x^3 - 34*a
^2*b^3*c^3*d^2*x^3 - 34*a^3*b^2*c^2*d^3*x^3 - 5*a^4*b*c*d^4*x^3 + 3*a^5*d^5
*x^3 + 5*a*b^4*c^5*x - 17*a^2*b^3*c^4*d*x - 17*a^4*b*c^2*d^3*x + 5*a^5*c*d^
4*x)/((a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3
+ a^6*c^2*d^4)*(b*d*x^4 + b*c*x^2 + a*d*x^2 + a*c)^2)
```

Mupad [B] (verification not implemented)

Time = 8.21 (sec) , antiderivative size = 11150, normalized size of antiderivative = 35.40

$$\int \frac{1}{\left(a+bx^2\right)^3\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] int(1/((a + b*x^2)^3*(c + d*x^2)^3),x)
```

```
[Out] ((x*(5*a^4*d^4 + 5*b^4*c^4 - 17*a*b^3*c^3*d - 17*a^3*b*c*d^3))/(8*a*c*(a^4*
d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^3*
(34*a^2*b^3*c^3*d^2 - 3*b^5*c^5 - 3*a^5*d^5 + 34*a^3*b^2*c^2*d^3 + 5*a*b^4*
c^4*d + 5*a^4*b*c*d^4))/(8*a^2*c^2*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 -
 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (x^5*(25*a*b^4*c^3*d^2 - 6*b^5*c^4*d - 6
*a^4*b*d^5 + 25*a^3*b^2*c*d^4 + 34*a^2*b^3*c^2*d^3))/(8*a^2*c^2*(a^4*d^4 +
b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (3*b*d*x^7*
(a^3*b*d^4 + b^4*c^3*d - 5*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3))/(8*a^2*c^2*(a^
```

4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/(x^4
*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^2*(2*a*b*c^2 + 2*a^2*c*d) + x^6*(2*a*b
*d^2 + 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^8) - (atan(((((x*(9*a^8*b^3*d^11 +
9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*
d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 +
702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d -
8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c
^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((3*a^2*b^16*c^16
*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^
13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (1
6425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10
)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 - 765*a^13*b
^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^1
6*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12
*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^
8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^
9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10)
 - (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c
^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*
c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 10752*a^10*b^7
*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 8960*a^13*b^4
*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512*(a^10*d^5
 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3
- 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b
*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 -
56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 +
b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^
7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5*b^5)^(1/2)*(21*
a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*
c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) + (((x*(9
*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 +
702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^
5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*
a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^
3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (3*((
(3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4
)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*
c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a
^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12
)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*
d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*
b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*
d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 -
792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^
14*b^2*c^6*d^10) + (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)

```
*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 -
 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7
- 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10
 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13)
)/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*
a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c
^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^
8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)^(1/2
)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b
^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*(-a^5
*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d)*3i)/(16*(a^10*d^5 - a^5*b^5*
c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c
*d^4)))/(((567*a^7*b^5*d^12)/256 + (567*b^12*c^7*d^5)/256 - (6399*a*b^11*c^
6*d^6)/256 - (6399*a^6*b^6*c*d^11)/256 + (27891*a^2*b^10*c^5*d^7)/256 - (49
707*a^3*b^9*c^4*d^8)/256 - (49707*a^4*b^8*c^3*d^9)/256 + (27891*a^5*b^7*c^2
*d^10)/256)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b
*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*
d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 +
 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*(
(x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^
10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 22
68*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8
 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c
^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) -
(3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^1
4*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*
b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16
425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6
*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3
*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12
*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*
c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*
d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 +
66*a^14*b^2*c^6*d^10) - (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b
*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*
d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10
*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7
*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*
d^13))/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2
+ 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*
b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 +
70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-a^5*b^5)
^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^5*c^5 + 5*
a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)))*
(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a^5*b^
```

5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b
*c*d^4)) - (3*((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 1
08*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b
^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12
 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2
 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b
^2*c^6*d^6)) + (3*(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (3
33*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/
2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^
9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (47
43*a^12*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2
 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^1
6*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2
 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924
*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13
*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*x*(-a^5*b^5)^(1/2)*(21*a^2*d^2 +
b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960
*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 107
52*a^9*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 192
00*a^12*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 25
6*a^15*b^2*c^4*d^13))/(512*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a
^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)*(a^4*b^8*c^12 + a^12*c
^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7
*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^
6)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^10*d^5 - a
^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^2*d^3 - 5*
a^9*b*c*d^4)))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(16*(a^
10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 + 10*a^8*b^2*c^
2*d^3 - 5*a^9*b*c*d^4))))*(-a^5*b^5)^(1/2)*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*
d)*3i)/(8*(a^10*d^5 - a^5*b^5*c^5 + 5*a^6*b^4*c^4*d - 10*a^7*b^3*c^3*d^2 +
10*a^8*b^2*c^2*d^3 - 5*a^9*b*c*d^4)) - (atan(((((x*(9*a^8*b^3*d^11 + 9*b^11
*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 -
2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^
6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^1
1*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4
 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((3*a^2*b^16*c^16*d^2)/
2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^1
3*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d^7)/2 + (16425*a
^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^8*c^8*d^10)/2 -
(10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5
*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*
c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*
b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12
*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7
+ 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) - (3*

x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^
2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200*a^7*b^10*c^12*d
^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d
^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d
^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512*(b^5*c^10 - a^5
*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*
b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d
^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9
*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^
2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*
c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*d^5)^(1/2)*(a^2*d^2 +
 21*b^2*c^2 - 6*a*b*c*d)*3i)/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4
+ 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)) + (((x*(9*a^8*b
^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 702*a^
2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5*b^6*
c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^
7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70
*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) + (3*(((3*a^2
*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)/2 -
765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c^11*d
^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^10*b^
8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)/2 -
765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d^15)/
2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c
^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 +
495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a
^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2
*c^6*d^10) + (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*(256*
a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 - 19200
*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 - 1075
2*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10 + 896
0*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))/(512
*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^
2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^11*d
- 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*
c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)*(a^2
*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d
^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*d^5)^
(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*3i)/(16*(b^5*c^10 - a^5*c^5*d^5 +
5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d))
)/(((567*a^7*b^5*d^12)/256 + (567*b^12*c^7*d^5)/256 - (6399*a*b^11*c^6*d^6)
/256 - (6399*a^6*b^6*c*d^11)/256 + (27891*a^2*b^10*c^5*d^7)/256 - (49707*a^
3*b^9*c^4*d^8)/256 - (49707*a^4*b^8*c^3*d^9)/256 + (27891*a^5*b^7*c^2*d^10)
/256)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d
^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 -

792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a
^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*((x*(9*
a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7*b^4*c*d^10 + 7
02*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4*d^7 - 2268*a^5
*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a
^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3
 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)) - (3*(((
3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4*b^14*c^14*d^4)
/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (10371*a^7*b^11*c
^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*d^9 + (16425*a^
10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^12*b^6*c^6*d^12)
/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45*a^15*b^3*c^3*d
^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*d^12 - 12*a^5*b
^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220*a^7*b^9*c^13*d
^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*b^6*c^10*d^6 -
792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c^7*d^9 + 66*a^1
4*b^2*c^6*d^10) - (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*
(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b^11*c^13*d^4 -
19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9*b^8*c^10*d^7 -
 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^12*b^5*c^7*d^10
+ 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15*b^2*c^4*d^13))
/(512*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a
^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8 - 8*a^5*b^7*c^
11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8
*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(-c^5*d^5)^(1/2)
*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*
c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)))*(-c^5*
d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5*d^5
+ 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d
)) - (3*((x*(9*a^8*b^3*d^11 + 9*b^11*c^8*d^3 - 108*a*b^10*c^7*d^4 - 108*a^7
*b^4*c*d^10 + 702*a^2*b^9*c^6*d^5 - 2268*a^3*b^8*c^5*d^6 + 7938*a^4*b^7*c^4
*d^7 - 2268*a^5*b^6*c^3*d^8 + 702*a^6*b^5*c^2*d^9))/(32*(a^4*b^8*c^12 + a^1
2*c^4*d^8 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*
a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6
d^6)) + (3(((3*a^2*b^16*c^16*d^2)/2 - (45*a^3*b^15*c^15*d^3)/2 + (333*a^4
*b^14*c^14*d^4)/2 - 765*a^5*b^13*c^13*d^5 + (4743*a^6*b^12*c^12*d^6)/2 - (1
0371*a^7*b^11*c^11*d^7)/2 + (16425*a^8*b^10*c^10*d^8)/2 - 9558*a^9*b^9*c^9*
d^9 + (16425*a^10*b^8*c^8*d^10)/2 - (10371*a^11*b^7*c^7*d^11)/2 + (4743*a^1
2*b^6*c^6*d^12)/2 - 765*a^13*b^5*c^5*d^13 + (333*a^14*b^4*c^4*d^14)/2 - (45
*a^15*b^3*c^3*d^15)/2 + (3*a^16*b^2*c^2*d^16)/2)/(a^4*b^12*c^16 + a^16*c^4*
d^12 - 12*a^5*b^11*c^15*d - 12*a^15*b*c^5*d^11 + 66*a^6*b^10*c^14*d^2 - 220
*a^7*b^9*c^13*d^3 + 495*a^8*b^8*c^12*d^4 - 792*a^9*b^7*c^11*d^5 + 924*a^10*
b^6*c^10*d^6 - 792*a^11*b^5*c^9*d^7 + 495*a^12*b^4*c^8*d^8 - 220*a^13*b^3*c
^7*d^9 + 66*a^14*b^2*c^6*d^10) + (3*x*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^
2 - 6*a*b*c*d)*(256*a^4*b^13*c^15*d^2 - 2304*a^5*b^12*c^14*d^3 + 8960*a^6*b

```
^11*c^13*d^4 - 19200*a^7*b^10*c^12*d^5 + 23040*a^8*b^9*c^11*d^6 - 10752*a^9
*b^8*c^10*d^7 - 10752*a^10*b^7*c^9*d^8 + 23040*a^11*b^6*c^8*d^9 - 19200*a^1
2*b^5*c^7*d^10 + 8960*a^13*b^4*c^6*d^11 - 2304*a^14*b^3*c^5*d^12 + 256*a^15
*b^2*c^4*d^13))/(512*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3
*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*c^9*d)*(a^4*b^8*c^12 + a^12*c^4*d^8
 - 8*a^5*b^7*c^11*d - 8*a^11*b*c^5*d^7 + 28*a^6*b^6*c^10*d^2 - 56*a^7*b^5*c
^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^10*b^2*c^6*d^6)))*(
-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10 - a^5*c^5
*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3 - 5*a*b^4*
c^9*d)))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d))/(16*(b^5*c^10
 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3*b^2*c^7*d^3
- 5*a*b^4*c^9*d))))*(-c^5*d^5)^(1/2)*(a^2*d^2 + 21*b^2*c^2 - 6*a*b*c*d)*3i)
/(8*(b^5*c^10 - a^5*c^5*d^5 + 5*a^4*b*c^6*d^4 + 10*a^2*b^3*c^8*d^2 - 10*a^3
*b^2*c^7*d^3 - 5*a*b^4*c^9*d))
```

3.43 $\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4} dx = -\frac{x\left(1-x^2\right)^2}{3\left(1+x^2\right)^3} - \frac{2x}{3\left(1+x^2\right)}$$

```
[Out] -1/3*x*(-x^2+1)^2/(x^2+1)^3-2/3*x/(x^2+1)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {424, 21, 391}

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4} dx = -\frac{x\left(1-x^2\right)^2}{3\left(x^2+1\right)^3} - \frac{2x}{3\left(x^2+1\right)}$$

```
[In] Int[(-1 + x^2)^3/(1 + x^2)^4,x]
```

```
[Out] -1/3*(x*(1 - x^2)^2)/(1 + x^2)^3 - (2*x)/(3*(1 + x^2))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
```

```
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{x(1-x^2)^2}{3(1+x^2)^3}+\frac{1}{6}\int \frac{(-1+x^2)(4+4x^2)}{(1+x^2)^3}\,dx\\ &= -\frac{x(1-x^2)^2}{3(1+x^2)^3}+\frac{2}{3}\int \frac{-1+x^2}{(1+x^2)^2}\,dx\\ &= -\frac{x(1-x^2)^2}{3(1+x^2)^3}-\frac{2x}{3(1+x^2)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{x(3+2x^2+3x^4)}{3(1+x^2)^3}$$

```
[In] Integrate[(-1 + x^2)^3/(1 + x^2)^4,x]
[Out] -1/3*(x*(3 + 2*x^2 + 3*x^4))/(1 + x^2)^3
```

Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{x(3x^4+2x^2+3)}{3(x^2+1)^3}$	23
default	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
norman	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
risch	$\frac{-x^5-\frac{2}{3}x^3-x}{(x^2+1)^3}$	23
parallelrisch	$\frac{-3x^5-2x^3-3x}{3(x^2+1)^3}$	24
meijerg	$-\frac{x(15x^4+40x^2+33)}{48(x^2+1)^3}-\frac{x(231x^4+280x^2+105)}{336(x^2+1)^3}+\frac{x(-15x^4+40x^2+15)}{80(x^2+1)^3}-\frac{x(-3x^4-8x^2+3)}{16(x^2+1)^3}$	90

```
[In] int((x^2-1)^3/(x^2+1)^4,x,method=_RETURNVERBOSE)
[Out] -1/3*x*(3*x^4+2*x^2+3)/(x^2+1)^3
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = -\frac{3\,x^5+2\,x^3+3\,x}{3\,(x^6+3\,x^4+3\,x^2+1)}$$

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="fricas")
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(-1+x^2)^3}{(1+x^2)^4}\,dx = \frac{-3x^5-2x^3-3x}{3x^6+9x^4+9x^2+3}$$

```
[In] integrate((x**2-1)**3/(x**2+1)**4,x)
[Out] (-3*x**5 - 2*x**3 - 3*x)/(3*x**6 + 9*x**4 + 9*x**2 + 3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4}\,dx = -\frac{3\,x^5+2\,x^3+3\,x}{3\,\left(x^6+3\,x^4+3\,x^2+1\right)}$$

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="maxima")
[Out] -1/3*(3*x^5 + 2*x^3 + 3*x)/(x^6 + 3*x^4 + 3*x^2 + 1)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4}\,dx = -\frac{3\,\left(x+\frac{1}{x}\right)^2-4}{3\,\left(x+\frac{1}{x}\right)^3}$$

```
[In] integrate((x^2-1)^3/(x^2+1)^4,x, algorithm="giac")
[Out] -1/3*(3*(x + 1/x)^2 - 4)/(x + 1/x)^3
```

Mupad [B] (verification not implemented)

Time = 4.62 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\left(-1+x^2\right)^3}{\left(1+x^2\right)^4}\,dx = \frac{4\,x}{3\,\left(x^2+1\right)^2}-\frac{x}{x^2+1}-\frac{4\,x}{3\,\left(x^2+1\right)^3}$$

```
[In] int((x^2 - 1)^3/(x^2 + 1)^4,x)
[Out] (4*x)/(3*(x^2 + 1)^2) - x/(x^2 + 1) - (4*x)/(3*(x^2 + 1)^3)
```

3.44 $\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx=\frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{3x(1-x^2)}{8(1+x^2)^2}+\frac{3\arctan(x)}{8}$$

```
[Out] 1/4*x*(-x^2+1)^3/(x^2+1)^4+3/8*x*(-x^2+1)/(x^2+1)^2+3/8*arctan(x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {424, 21, 209}

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx=\frac{3\arctan(x)}{8}+\frac{x(1-x^2)^3}{4(x^2+1)^4}+\frac{3x(1-x^2)}{8(x^2+1)^2}$$

```
[In] Int[(-1 + x^2)^4/(1 + x^2)^5,x]
```

```
[Out] (x*(1 - x^2)^3)/(4*(1 + x^2)^4) + (3*x*(1 - x^2))/(8*(1 + x^2)^2) + (3*ArcT
an[x])/8
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{1}{8}\int\frac{(-1+x^2)^2(6+6x^2)}{(1+x^2)^4}\,dx\\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{3}{4}\int\frac{(-1+x^2)^2}{(1+x^2)^3}\,dx\\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{3x(1-x^2)}{8(1+x^2)^2}+\frac{3}{16}\int\frac{2+2x^2}{(1+x^2)^2}\,dx\\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{3x(1-x^2)}{8(1+x^2)^2}+\frac{3}{8}\int\frac{1}{1+x^2}\,dx\\
&= \frac{x(1-x^2)^3}{4(1+x^2)^4}+\frac{3x(1-x^2)}{8(1+x^2)^2}+\frac{3}{8}\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int\frac{(-1+x^2)^4}{(1+x^2)^5}\,dx=\frac{5x-3x^3+3x^5-5x^7+3(1+x^2)^4\arctan(x)}{8(1+x^2)^4}$$

```
[In] Integrate[(-1 + x^2)^4/(1 + x^2)^5,x]
[Out] (5*x - 3*x^3 + 3*x^5 - 5*x^7 + 3*(1 + x^2)^4*ArcTan[x])/(8*(1 + x^2)^4)
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result
default	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
risch	$\frac{-\frac{5}{8}x^7+\frac{3}{8}x^5-\frac{3}{8}x^3+\frac{5}{8}x}{(x^2+1)^4}+\frac{3\arctan(x)}{8}$
parallelrisch	$-\frac{-36i\ln(x+i)x^2+9i\ln(x-i)+9i\ln(x-i)x^8+54i\ln(x-i)x^4+30x^7+36i\ln(x-i)x^2+36i\ln(x-i)x^6-18x^5-9i\ln(x+i)-36i\ln(}{48(x^2+1)^4}$
meijerg	$\frac{x(105x^6+385x^4+511x^2+279)}{384(x^2+1)^4}+\frac{3\arctan(x)}{8}-\frac{x(837x^6+1533x^4+1155x^2+315)}{1152(x^2+1)^4}+\frac{x(-105x^6+511x^4+385x^2+105)}{672(x^2+1)^4}-$

```
[In] int((x^2-1)^4/(x^2+1)^5,x,method=_RETURNVERBOSE)
[Out] (-5/8*x^7+3/8*x^5-3/8*x^3+5/8*x)/(x^2+1)^4+3/8*arctan(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = -\frac{5\,x^7-3\,x^5+3\,x^3-3\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)\arctan(x)-5\,x}{8\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}$$

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="fricas")
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 3*(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)*arctan(x)
 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{-5x^7+3x^5-3x^3+5x}{8x^8+32x^6+48x^4+32x^2+8}+\frac{3\,\mathrm{atan}\,(x)}{8}$$

```
[In] integrate((x**2-1)**4/(x**2+1)**5,x)
[Out] (-5*x**7 + 3*x**5 - 3*x**3 + 5*x)/(8*x**8 + 32*x**6 + 48*x**4 + 32*x**2 + 8
) + 3*atan(x)/8
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = -\frac{5\,x^7-3\,x^5+3\,x^3-5\,x}{8\,(x^8+4\,x^6+6\,x^4+4\,x^2+1)}+\frac{3}{8}\,\arctan(x)$$

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^7 - 3*x^5 + 3*x^3 - 5*x)/(x^8 + 4*x^6 + 6*x^4 + 4*x^2 + 1) + 3/8*
arctan(x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{3}{32}\,\pi\mathrm{sgn}(x)-\frac{5\,\left(x-\frac{1}{x}\right)^3+12\,x-\frac{12}{x}}{8\,\left(\left(x-\frac{1}{x}\right)^2+4\right)^2}+\frac{3}{16}\,\arctan\left(\frac{x^2-1}{2\,x}\right)$$

```
[In] integrate((x^2-1)^4/(x^2+1)^5,x, algorithm="giac")
```

```
[Out] 3/32*pi*sgn(x) - 1/8*(5*(x - 1/x)^3 + 12*x - 12/x)/((x - 1/x)^2 + 4)^2 + 3/
16*arctan(1/2*(x^2 - 1)/x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(-1+x^2)^4}{(1+x^2)^5}\,dx = \frac{3\,\mathrm{atan}(x)}{8}+\frac{-\frac{5\,x^7}{8}+\frac{3\,x^5}{8}-\frac{3\,x^3}{8}+\frac{5\,x}{8}}{x^8+4\,x^6+6\,x^4+4\,x^2+1}$$

```
[In] int((x^2 - 1)^4/(x^2 + 1)^5,x)
```

```
[Out] (3*atan(x))/8 + ((5*x)/8 - (3*x^3)/8 + (3*x^5)/8 - (5*x^7)/8)/(4*x^2 + 6*x^
4 + 4*x^6 + x^8 + 1)
```

3.45 $\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 231

$$\begin{aligned}\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx &= \frac{(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3}\\
&+\frac{d(72b^2c^2-52abcd+15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3}\\
&+\frac{d(12bc-5ad)x(a+bx^2)^{3/2}\left(c+dx^2\right)}{48b^2}\\
&+\frac{dx(a+bx^2)^{3/2}\left(c+dx^2\right)^2}{8b}\\
&+\frac{a(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

```
[Out] 1/192*d*(15*a^2*d^2-52*a*b*c*d+72*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/48*d*(-5
*a*d+12*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)/b^2+1/8*d*x*(b*x^2+a)^(3/2)*(d*x^2
+c)^2/b+1/128*a*(-5*a^3*d^3+24*a^2*b*c*d^2-48*a*b^2*c^2*d+64*b^3*c^3)*arcta
nh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/128*(-5*a^3*d^3+24*a^2*b*c*d^2-48*a
*b^2*c^2*d+64*b^3*c^3)*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {427, 542, 396, 201, 223, 212}

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx=\frac{dx\left(a+bx^2\right)^{3/2}\left(15a^2d^2-52abcd+72b^2c^2\right)}{192b^3}+\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{128b^{7/2}}+\frac{x\sqrt{a+bx^2}\left(-5a^3d^3+24a^2bcd^2-48ab^2c^2d+64b^3c^3\right)}{128b^3}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)(12bc-5ad)}{48b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}{8b}$$

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] ((64*b^3*c^3 - 48*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a + b*x^
2])/(128*b^3) + (d*(72*b^2*c^2 - 52*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(3/
2))/(192*b^3) + (d*(12*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*(c + d*x^2))/(48*b^
2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)^2)/(8*b) + (a*(64*b^3*c^3 - 48*a*b^
2*c^2*d + 24*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(128*b^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

integral

$$= \frac{dx(a+bx^2)^{3/2}(c+dx^2)^2}{8b} + \frac{\int \sqrt{a+bx^2}(c+dx^2)\left(c(8bc-ad)+d(12bc-5ad)x^2\right)dx}{8b}$$

$$= \frac{d(12bc-5ad)x(a+bx^2)^{3/2}(c+dx^2)}{48b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)^2}{8b} + \frac{\int \sqrt{a+bx^2}\left(c(48b^2c^2-18abcd+5a^2d^2)+d(72b^2c^2-52abcd+15a^2d^2)x^2\right)dx}{48b^2}$$

$$= \frac{d(72b^2c^2-52abcd+15a^2d^2)x(a+bx^2)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}(c+dx^2)}{48b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)^2}{8b} + \frac{(64b^3c^3-48ab^2c^2d+24a^2bcd^2-5a^3d^3)\int \sqrt{a+bx^2}\,dx}{64b^3}$$

$$= \frac{(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2 - 52abcd + 15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}\,(c+dx^2)}{48b^2} + \frac{dx(a+bx^2)^{3/2}\,(c+dx^2)^2}{8b} + \frac{(a(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b^3}$$

$$= \frac{(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2 - 52abcd + 15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}\,(c+dx^2)}{48b^2} + \frac{dx(a+bx^2)^{3/2}\,(c+dx^2)^2}{8b} + \frac{(a(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b^3}$$

$$= \frac{(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3)\,x\sqrt{a+bx^2}}{128b^3} + \frac{d(72b^2c^2 - 52abcd + 15a^2d^2)\,x(a+bx^2)^{3/2}}{192b^3} + \frac{d(12bc-5ad)x(a+bx^2)^{3/2}\,(c+dx^2)}{48b^2} + \frac{dx(a+bx^2)^{3/2}\,(c+dx^2)^2}{8b} + \frac{a(64b^3c^3 - 48ab^2c^2d + 24a^2bcd^2 - 5a^3d^3)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.78

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$

$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(15a^3d^3 - 2a^2bd^2(36c+5dx^2) + 8ab^2d(18c^2+6cdx^2+d^2x^4) + 48b^3(4c^3+6c^2dx^2+4cd^2x^4 +}{384b^{7/2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^3*d^3 - 2*a^2*b*d^2*(36*c + 5*d*x^2) + 8*a
*b^2*d*(18*c^2 + 6*c*d*x^2 + d^2*x^4) + 48*b^3*(4*c^3 + 6*c^2*d*x^2 + 4*c*d
^2*x^4 + d^3*x^6)) + 3*a*(-64*b^3*c^3 + 48*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 5
*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(384*b^(7/2))
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{5\left(a\left(a^{3}d^{3}-\frac{24}{5}a^{2}bc\,d^{2}+\frac{48}{5}a\,b^{2}c^{2}d-\frac{64}{5}b^{3}c^{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\left(\frac{64\left(\frac{1}{2}d^{2}x^{4}+cd\,x^{2}+c^{2}\right)\left(\frac{d\,x^{2}}{2}+c\right)b^{\frac{7}{2}}}{5}+\left(\left(\frac{8}{15}d\right.\right.\right.\right.}{128b^{\frac{7}{2}}}$
risch	$\frac{x\left(48b^{3}d^{3}x^{6}+8a\,b^{2}d^{3}x^{4}+192b^{3}c\,d^{2}x^{4}-10x^{2}a^{2}b\,d^{3}+48x^{2}a\,b^{2}c\,d^{2}+288x^{2}b^{3}c^{2}d+15a^{3}d^{3}-72a^{2}bc\,d^{2}+144a\,b^{2}c^{2}d+192b^{3}c^{3}\right)\sqrt{}}{384b^{3}}$
default	$c^{3}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d^{3}\left(\frac{x^{5}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{8b}-\frac{5a\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\right.}{}\right)}{2b}\right)}{8b}\right.$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -5/128*(a*(a^3*d^3-24/5*a^2*b*c*d^2+48/5*a*b^2*c^2*d-64/5*b^3*c^3)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))-x*(b*x^2+a)^(1/2)*(64/5*(1/2*d^2*x^4+c*d*x^2+c^2
)*(1/2*d*x^2+c)*b^(7/2)+((8/15*d^2*x^4+16/5*c*d*x^2+48/5*c^2)*b^(5/2)+d*a*(
(-2/3*d*x^2-24/5*c)*b^(3/2)+a*d*b^(1/2)))*d*a))/b^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.72

$$\int\sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$

$$=\left[-\frac{3\left(64\,ab^{3}c^{3}-48\,a^{2}b^{2}c^{2}d+24\,a^{3}bcd^{2}-5\,a^{4}d^{3}\right)\sqrt{b}\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\left(48\,b^{4}d^{3}x^{7}+\right.}{}\right.$$

$$-\frac{3\left(64\,ab^{3}c^{3}-48\,a^{2}b^{2}c^{2}d+24\,a^{3}bcd^{2}-5\,a^{4}d^{3}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-\left(48\,b^{4}d^{3}x^{7}+8\left(24\,b^{4}cd^{2}+ab^{3}d^{3}\right)\right.}{38}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/768*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*s
qrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*d^3*x^7
+ 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d + 24*a*b^3*c*d^2 - 5*
a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a^2*b^2*c*d^2 + 5*a^
3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/384*(3*(64*a*b^3*c^3 - 48*a^2*b^2*c^2*
d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (48*b^4*d^3*x^7 + 8*(24*b^4*c*d^2 + a*b^3*d^3)*x^5 + 2*(144*b^4*c^2*d +
24*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^3 + 3*(64*b^4*c^3 + 48*a*b^3*c^2*d - 24*a
^2*b^2*c*d^2 + 5*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.30

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^3x^7}{8}+\frac{x^5\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+\frac{x^3\cdot\left(3acd^2-\frac{5a\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+3bc^2d\right)}{4b}+\frac{x\left(3ac^2d-\frac{3a\left(3acd^2-\frac{5a\left(\frac{ad^3}{8}+3bcd^2\right)}{6b}+3bc^2d\right)}{4b}+bc^3\right)}{2b}\right. \\ \sqrt{a}\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(d**3*x**7/8 + x**5*(a*d**3/8 + 3*b*c*d**2)/(6*
b) + x**3*(3*a*c*d**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b) + 3*b*c**2*d)/(4*
b) + x*(3*a*c**2*d - 3*a*(3*a*c*d**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b) +
3*b*c**2*d)/(4*b) + b*c**3)/(2*b)) + (a*c**3 - a*(3*a*c**2*d - 3*a*(3*a*c*d
**2 - 5*a*(a*d**3/8 + 3*b*c*d**2)/(6*b) + 3*b*c**2*d)/(4*b) + b*c**3)/(2*b)
)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x
*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c**3*x + c**2*d*x**3 + 3
*c*d**2*x**5/5 + d**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.22

$$\int \sqrt{a+bx^2}(c+dx^2)^3\,dx = \frac{(bx^2+a)^{\frac{3}{2}}d^3x^5}{8\,b} + \frac{(bx^2+a)^{\frac{3}{2}}cd^2x^3}{2\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}ad^3x^3}{48\,b^2} + \frac{1}{2}\sqrt{bx^2+a}c^3x + \frac{3\,(bx^2+a)^{\frac{3}{2}}c^2dx}{4\,b} - \frac{3\sqrt{bx^2+a}ac^2dx}{8\,b} - \frac{3\,(bx^2+a)^{\frac{3}{2}}acd^2x}{8\,b^2} + \frac{3\sqrt{bx^2+a}a^2cd^2x}{16\,b^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2d^3x}{64\,b^3} - \frac{5\sqrt{bx^2+a}a^3d^3x}{128\,b^3} + \frac{ac^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{3\,a^2c^2d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{3\,a^3cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} - \frac{5\,a^4d^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(3/2)*d^3*x^5/b + 1/2*(b*x^2 + a)^(3/2)*c*d^2*x^3/b - 5/48*
(b*x^2 + a)^(3/2)*a*d^3*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*c^3*x + 3/4*(b*x^2 +
a)^(3/2)*c^2*d*x/b - 3/8*sqrt(b*x^2 + a)*a*c^2*d*x/b - 3/8*(b*x^2 + a)^(3/2
)*a*c*d^2*x/b^2 + 3/16*sqrt(b*x^2 + a)*a^2*c*d^2*x/b^2 + 5/64*(b*x^2 + a)^(
3/2)*a^2*d^3*x/b^3 - 5/128*sqrt(b*x^2 + a)*a^3*d^3*x/b^3 + 1/2*a*c^3*arcsin
h(b*x/sqrt(a*b))/sqrt(b) - 3/8*a^2*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3
/16*a^3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/128*a^4*d^3*arcsinh(b*x/sq
rt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}(c+dx^2)^3\,dx = \frac{1}{384}\left(2\left(4\left(6\,d^3x^2 + \frac{24\,b^6cd^2+ab^5d^3}{b^6}\right)x^2 + \frac{144\,b^6c^2d+24\,ab^5cd^2-5\,a^2b^4d^3}{b^6}\right)x^2 + \frac{3\,(64\,b^6c^3+48\,ab^5c^2\cdots\right. - \frac{(64\,ab^3c^3-48\,a^2b^2c^2d+24\,a^3bcd^2-5\,a^4d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*d^3*x^2 + (24*b^6*c*d^2 + a*b^5*d^3)/b^6)*x^2 + (144*b^6*c^2
*d + 24*a*b^5*c*d^2 - 5*a^2*b^4*d^3)/b^6)*x^2 + 3*(64*b^6*c^3 + 48*a*b^5*c^
2*d - 24*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(64*
a*b^3*c^3 - 48*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 5*a^4*d^3)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^3\,dx=\int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)^3\,dx$$

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^3, x)
```

3.46 $\int \sqrt{a+bx^2}(c+dx^2)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\frac{\left(8b^2c^2-4abcd+a^2d^2\right)x\sqrt{a+bx^2}}{16b^2}+\frac{d(8bc-3ad)x\left(a+bx^2\right)^{3/2}}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}+\frac{a\left(8b^2c^2-4abcd+a^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

```
[Out] 1/24*d*(-3*a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b^2+1/6*d*x*(b*x^2+a)^(3/2)*(d*x^2+
c)/b+1/16*a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2)
)/b^(5/2)+1/16*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {427, 396, 201, 223, 212}

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^{5/2}}+\frac{x\sqrt{a+bx^2}\left(a^2d^2-4abcd+8b^2c^2\right)}{16b^2}+\frac{dx\left(a+bx^2\right)^{3/2}(8bc-3ad)}{24b^2}+\frac{dx\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}{6b}$$

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] ((8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(16*b^2) + (d*(8*b*c
- 3*a*d)*x*(a + b*x^2)^(3/2))/(24*b^2) + (d*x*(a + b*x^2)^(3/2)*(c + d*x^2)
)/(6*b) + (a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(16*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{\int \sqrt{a+bx^2}(c(6bc-ad)+d(8bc-3ad)x^2)\,dx}{6b}$$

$$= \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{(8b^2c^2-4abcd+a^2d^2)\int\sqrt{a+bx^2}\,dx}{8b^2}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{(a(8b^2c^2-4abcd+a^2d^2))\int\frac{1}{\sqrt{a+bx^2}}\,dx}{16b^2}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{(a(8b^2c^2-4abcd+a^2d^2))\,\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16b^2}$$

$$= \frac{(8b^2c^2-4abcd+a^2d^2)\,x\sqrt{a+bx^2}}{16b^2} + \frac{d(8bc-3ad)x(a+bx^2)^{3/2}}{24b^2} + \frac{dx(a+bx^2)^{3/2}(c+dx^2)}{6b} + \frac{a(8b^2c^2-4abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int\sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx$$

$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-3a^2d^2+2abd(6c+dx^2)+8b^2(3c^2+3cdx^2+d^2x^4))-3a(8b^2c^2-4abcd+a^2d^2)\log\left(-\sqrt{\right.}}{48b^{5/2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-3*a^2*d^2 + 2*a*b*d*(6*c + d*x^2) + 8*b^2*(3*c
^2 + 3*c*d*x^2 + d^2*x^4)) - 3*a*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(5/2))
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{a\left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right)-x \sqrt{b\,x^{2}+a}\,\left(\left(-\frac{8}{3} d^{2} x^{4}-8 c d\,x^{2}-8 c^{2}\right) b^{\frac{5}{2}}+d\left(\left(-\frac{2 d\,x^{2}}{3}-4 c\right) b^{\frac{3}{2}}+a d \sqrt{b}\right) a\right)}{16 b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-8 b^{2} d^{2} x^{4}-2 x^{2} a b\,d^{2}-24 x^{2} b^{2} c d+3 a^{2} d^{2}-12 a b c d-24 b^{2} c^{2}\right) \sqrt{b\,x^{2}+a}}{48 b^{2}}+\frac{a\left(a^{2} d^{2}-4 a b c d+8 b^{2} c^{2}\right) \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16 b^{\frac{5}{2}}}$
default	$c^{2}\left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)+d^{2}\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6 b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4 b}-\frac{a\left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)}{2 b}\right.$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(a*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*
(b*x^2+a)^(1/2)*((-8/3*d^2*x^4-8*c*d*x^2-8*c^2)*b^(5/2)+d*((-2/3*d*x^2-4*c)
*b^(3/2)+a*d*b^(1/2))*a))/b^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.77

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx$$
$$=\left[\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3\right.}{96\,b^3}$$
$$-\frac{3\,(8\,ab^2c^2-4\,a^2bcd+a^3d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3d^2x^5+2\,(12\,b^3cd+ab^2d^2)x^3+3\,(8\,b^3c^2+4\,ab^2}{48\,b^3}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)
*x^3 + 3*(8*b^3*c^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/
48*(3*(8*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) - (8*b^3*d^2*x^5 + 2*(12*b^3*c*d + a*b^2*d^2)*x^3 + 3*(8*b^3*c
^2 + 4*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2x^5}{6}+\frac{x^3\left(\frac{ad^2}{6}+2bcd\right)}{4b}+\frac{x\left(2acd-\frac{3a\left(\frac{ad^2}{6}+2bcd\right)}{4b}+bc^2\right)}{2b}\right)+\left(ac^2-\frac{a\left(2acd-\frac{3a\left(\frac{ad^2}{6}+2bcd\right)}{4b}+bc^2\right)}{2b}\right)\left(\left(\begin{cases} \frac{\log \cdots}{\cdots} \\ \frac{x\log \cdots}{\sqrt{\cdots}} \end{cases}\right.\right. \\ \sqrt{a}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**2,x)

[Out] Piecewise((sqrt(a + b*x**2)*(d**2*x**5/6 + x**3*(a*d**2/6 + 2*b*c*d)/(4*b) + x*(2*a*c*d - 3*a*(a*d**2/6 + 2*b*c*d)/(4*b) + b*c**2)/(2*b)) + (a*c**2 - a*(2*a*c*d - 3*a*(a*d**2/6 + 2*b*c*d)/(4*b) + b*c**2)/(2*b))*Piecewise((log (2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x **2), True)), Ne(b, 0)), (sqrt(a)*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), Tr ue))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx = \frac{\left(bx^2+a\right)^{\frac{3}{2}}d^2x^3}{6\,b}+\frac{1}{2}\sqrt{bx^2+a}c^2x+\frac{\left(bx^2+a\right)^{\frac{3}{2}}cdx}{2\,b} - \frac{\sqrt{bx^2+a}acdx}{4\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}ad^2x}{8\,b^2}+\frac{\sqrt{bx^2+a}a^2d^2x}{16\,b^2} + \frac{ac^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}-\frac{a^2cd\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{4\,b^{\frac{3}{2}}}+\frac{a^3d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="maxima")

[Out] 1/6*(b*x^2 + a)^(3/2)*d^2*x^3/b + 1/2*sqrt(b*x^2 + a)*c^2*x + 1/2*(b*x^2 + a)^(3/2)*c*d*x/b - 1/4*sqrt(b*x^2 + a)*a*c*d*x/b - 1/8*(b*x^2 + a)^(3/2)*a* d^2*x/b^2 + 1/16*sqrt(b*x^2 + a)*a^2*d^2*x/b^2 + 1/2*a*c^2*arcsinh(b*x/sqrt (a*b))/sqrt(b) - 1/4*a^2*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/16*a^3*d^2* arcsinh(b*x/sqrt(a*b))/b^(5/2)

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx$$
$$=\frac{1}{48}\left(2\left(4\,d^2x^2+\frac{12\,b^4cd+ab^3d^2}{b^4}\right)x^2+\frac{3\left(8\,b^4c^2+4\,ab^3cd-a^2b^2d^2\right)}{b^4}\right)\sqrt{bx^2+a}x$$
$$-\frac{\left(8\,ab^2c^2-4\,a^2bcd+a^3d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^2*x^2 + (12*b^4*c*d + a*b^3*d^2)/b^4)*x^2 + 3*(8*b^4*c^2 + 4*a
*b^3*c*d - a^2*b^2*d^2)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(8*a*b^2*c^2 - 4*a^2*
b*c*d + a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^2\,dx=\int \sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^2\,dx$$

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^2, x)
```

3.47 $\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx=\frac{(4bc-ad)x\sqrt{a+bx^2}}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}+\frac{a(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/4*d*x*(b*x^2+a)^(3/2)/b+1/8*a*(-a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)+1/8*(-a*d+4*b*c)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {396, 201, 223, 212}

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx=\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-ad)}{8b^{3/2}}+\frac{x\sqrt{a+bx^2}(4bc-ad)}{8b}+\frac{dx\left(a+bx^2\right)^{3/2}}{4b}$$

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] ((4*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b) + (d*x*(a + b*x^2)^(3/2))/(4*b) + (
a*(4*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{3/2}}{4b} - \frac{(-4bc+ad)\int \sqrt{a+bx^2}\,dx}{4b} \\
&= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx(a+bx^2)^{3/2}}{4b} + \frac{(a(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b} \\
&= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx(a+bx^2)^{3/2}}{4b} + \frac{(a(4bc-ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b} \\
&= \frac{(4bc-ad)x\sqrt{a+bx^2}}{8b} + \frac{dx(a+bx^2)^{3/2}}{4b} + \frac{a(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx = \frac{x\sqrt{a+bx^2}(4bc+ad+2bdx^2)}{8b} + \frac{a(-4bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(4*b*c + a*d + 2*b*d*x^2))/(8*b) + (a*(-4*b*c + a*d)*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result	size
risch	$\frac{x\left(2bd\,x^{2}+ad+4bc\right)\sqrt{b\,x^{2}+a}}{8b}-\frac{a\left(ad-4bc\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{3}{2}}}$	62
pseudoelliptic	$-\frac{\left(a^{2}d-4abc\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\,\left(\left(2d\,x^{2}+4c\right)b^{\frac{3}{2}}+ad\sqrt{b}\right)}{8b^{\frac{3}{2}}}$	69
default	$c\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)$	98

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(2*b*d*x^2+a*d+4*b*c)*(b*x^2+a)^(1/2)/b-1/8*a*(a*d-4*b*c)/b^(3/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.82

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx = \left[-\frac{(4\,abc-a^2d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{16\,b^2},\ -\frac{(4\,abc-a^2d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2dx^3+(4\,b^2c+abd)x)\sqrt{bx^2+a}}{8\,b^2}\right]$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/16*((4*a*b*c - a^2*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(2*b^2*d*x^3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/8*(
(4*a*b*c - a^2*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d*x^
3 + (4*b^2*c + a*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{dx^3}{4}+\frac{x\left(\frac{ad}{4}+bc\right)}{2b}\right)+\left(ac-\frac{a\left(\frac{ad}{4}+bc\right)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(cx+\frac{dx^3}{3}\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(d*x**3/4 + x*(a*d/4 + b*c)/(2*b)) + (a*c - a*(
a*d/4 + b*c)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(c*x +
d*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.93

$$\int \sqrt{a+bx^2}(c+dx^2)\,dx = \frac{1}{2}\sqrt{bx^2+a}cx+\frac{(bx^2+a)^{\frac{3}{2}}dx}{4b}-\frac{\sqrt{bx^2+a}adx}{8b}+\frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}-\frac{a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*c*x + 1/4*(b*x^2 + a)^(3/2)*d*x/b - 1/8*sqrt(b*x^2 + a)
*a*d*x/b + 1/2*a*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^2*d*arcsinh(b*x/s
qrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx = \frac{1}{8}\,\sqrt{bx^2+a}\left(2\,dx^2+\frac{4\,b^2c+abd}{b^2}\right)x - \frac{\left(4\,abc-a^2d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d*x^2 + (4*b^2*c + a*b*d)/b^2)*x - 1/8*(4*a*b*c - a^
2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)\,dx = \int \sqrt{b\,x^2+a}\,\left(d\,x^2+c\right)\,dx$$

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2), x)
```

3.48 $\int \sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}x\sqrt{a+bx^2} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*a*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/2*x*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 223, 212}

$$\int \sqrt{a+bx^2}\,dx = \frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} + \frac{1}{2}x\sqrt{a+bx^2}$$

```
[In] Int[Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\int\frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}x\sqrt{a+bx^2}+\frac{1}{2}a\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{2}x\sqrt{a+bx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int\sqrt{a+bx^2}\,dx=\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

```
[In] Integrate[Sqrt[a + b*x^2],x]
[Out] (x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,x\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a}{2\sqrt{b}}$	40

```
[In] int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \sqrt{a+bx^2}\,dx$$
$$= \left[\frac{2\sqrt{bx^2+a}bx + a\sqrt{b}\log\left(-2\,bx^2 - 2\sqrt{bx^2+a}\sqrt{b}x - a\right)}{4\,b}, \frac{\sqrt{bx^2+a}bx - a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}\,dx = \frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

```
[In] integrate((b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}x + \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

```
[In] integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx^2}\,dx = \frac{x\sqrt{b\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

```
[In] int((a + b*x^2)^(1/2),x)
```

```
[Out] (x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2)
)
```

3.49 $\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d-arctanh(x*(-a*d+b*c)^(1/2)/c^(
1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/d/c^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {399, 223, 212, 385, 214}

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx=\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}-\frac{\sqrt{bc-ad}\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}$$

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d - (Sqrt[b*c - a*d]*ArcTanh
[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{d} \\
&= \frac{b\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d} - \frac{(bc-ad)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d} \\
&= \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d} - \frac{\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = -\frac{\frac{\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2),x]
```

```
[Out] -(((Sqrt[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2)
)/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/d)
```

Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

method	result
pseudoelliptic	$\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)\sqrt{(ad-bc)c}-\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)ad+\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)bc}{d\sqrt{(ad-bc)c}}$
default	$-\frac{\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}-\frac{\sqrt{b}\,\sqrt{-cd}\,\ln\left(\frac{-\frac{b\sqrt{-cd}}{d}+b\left(x+\frac{\sqrt{-cd}}{d}\right)}{\sqrt{b}}+\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\right.}}{d}}{2\sqrt{-cd}}$

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] (b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*((a*d-b*c)*c)^(1/2)-arctan(c*(b
*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))*a*d+arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b
*c)*c)^(1/2))*b*c)/d/((a*d-b*c)*c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 596, normalized size of antiderivative = 7.27

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

$$= \left[\frac{2\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)+\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2-4\,(ac^2x+(2\right.}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}\right.$$

$$-\frac{4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2-4\,(ac^2x+(2\,bc^2-acd)x^3)\sqrt{bx^2+a}\right.}{d^2x^4+2\,cdx^2+c^2}\right.}{4\,d}$$

$$\left.-\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2\,((b^2c-abd)x^3+(abc-a^2d)x)}\right)}{2\,d}\right]$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + sqrt((b*c
 - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*
c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*
```

```
sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d, -1/4*(4*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*
b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x
 + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2
*c*d*x^2 + c^2)))/d, 1/2*(sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^
2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
 - a^2*d)*x)) + sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/d,
 -1/2*(2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(-(b*c - a*d)/c)
*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/
((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c),x)
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2+a}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt{bx^2+a}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{c+dx^2}\,dx$$

$$= \begin{cases} \frac{\sqrt{-b}\,\mathrm{asin}\left(x\,\sqrt{-\frac{b}{a}}\right)}{c} & \text{if } ((a+b\,c=0\wedge d=-1)\vee a\,d=b\,c)\wedge b<0 \\ \frac{\sqrt{b}\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{d}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)\sqrt{a\,d-b\,c}}{\sqrt{c}\,d} & \text{if } a\neq 0\wedge(((a+b\,c\neq 0\vee d\neq -1)\wedge a\,d\neq b\,c)\vee\neg b< \\ \int \frac{\sqrt{b\,x^2+a}}{d\,x^2+c}\,dx & \text{if } ((((a+b\,c=0\wedge d=-1)\vee a\,d=b\,c)\wedge b<0)\vee a= \end{cases}$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2),x)
```

```
[Out] piecewise((a + b*c == 0 & d == -1 | a*d == b*c) & b < 0, ((-b)^(1/2)*asin(x
*(-b/a)^(1/2)))/c, a ~= 0 & ((a + b*c ~= 0 | d ~= -1) & a*d ~= b*c | ~b < 0
), (b^(1/2)*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/d + (atan((x*(a*d - b*c
)^(1/2))/(c^(1/2)*(a + b*x^2)^(1/2)))*(a*d - b*c)^(1/2))/(c^(1/2)*d), ((a +
 b*c == 0 & d == -1 | a*d == b*c) & b < 0 | a == 0) & ((a + b*c ~= 0 | d ~=
 -1) & a*d ~= b*c | ~b < 0), int((a + b*x^2)^(1/2)/(c + d*x^2), x))
```

3.50 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2} dx = \frac{x\sqrt{a+bx^2}}{2c(c+dx^2)} + \frac{a\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}$$

```
[Out] 1/2*a*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c
)^(1/2)+1/2*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {386, 385, 214}

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2} dx = \frac{a\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}} + \frac{x\sqrt{a+bx^2}}{2c(c+dx^2)}$$

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c*(c + d*x^2)) + (a*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*Sqrt[b*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)}+\frac{a\int\frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2c}\\ &= \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)}+\frac{a\text{Subst}\left(\int\frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2c}\\ &= \frac{x\sqrt{a+bx^2}}{2c\,(c+dx^2)}+\frac{a\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.21

$$\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx=\frac{x\sqrt{a+bx^2}}{2c^2+2cdx^2}-\frac{a\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}\sqrt{-bc+ad}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^2,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*c^2 + 2*c*d*x^2) - (a*ArcTan[(-(d*x*Sqrt[a + b*x^2])
 + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(2*c^(3/2)*Sqrt[-(b*
c) + a*d])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{\frac{x\sqrt{b\,x^{2}+a}}{d\,x^{2}+c}-\frac{a\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2c}$	69
default	Expression too large to display	1945

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(x*(b*x^2+a)^(1/2)/(d*x^2+c)-a/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)
^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(66) = 132$.

Time = 0.32 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.50

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$$
$$= \left[\frac{4\,(bc^2-acd)\sqrt{bx^2+a}x+(adx^2+ac)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ac}{d^2x^4+2\,cdx^2+c^2}\right.}{8\,(bc^4-ac^3d+(bc^3d-ac^2d^2)x^2)}\right.$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x + (a*d*x^2 + a*c)*sqrt(b*c^2 - a*
c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b*c^4 - a*c^3*d + (b*c^3*d - a*c^2*
d^2)*x^2), 1/4*(2*(b*c^2 - a*c*d)*sqrt(b*x^2 + a)*x - (a*d*x^2 + a*c)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^4 -
 a*c^3*d + (b*c^3*d - a*c^2*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(66) = 132.

Time = 0.89 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx = -\frac{a\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}c} + \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d+a^2\sqrt{b}d}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a^2d\right)cd}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
[Out] -1/2*a*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)
/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c) + (2*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*b^(3/2)*c - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d +
 a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)*c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^2}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^2, x)
```

3.51 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)\,(c+dx^2)^2} + \frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)\,(c+dx^2)} + \frac{a(4bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}$$

```
[Out] -1/4*d*x*(b*x^2+a)^(3/2)/c/(-a*d+b*c)/(d*x^2+c)^2+1/8*a*(-3*a*d+4*b*c)*arct
anh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(3/2)+1/
8*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {390, 386, 385, 214}

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx = \frac{a(4bc-3ad)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}} + \frac{x\sqrt{a+bx^2}(4bc-3ad)}{8c^2\,(c+dx^2)\,(bc-ad)} - \frac{dx(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2\,(bc-ad)}$$

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] -1/4*(d*x*(a + b*x^2)^(3/2))/(c*(b*c - a*d)*(c + d*x^2)^2) + ((4*b*c - 3*a*
d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)*(c + d*x^2)) + (a*(4*b*c - 3*a*d)*
```

```
ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*c^(5/2)*(b*c - a
*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2}+\frac{(4bc-3ad)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{4c(bc-ad)}\\
&= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)}+\frac{(a(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{8c^2(bc-ad)}\\
&= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)}\\
&\quad+\frac{(a(4bc-3ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8c^2(bc-ad)}\\
&= -\frac{dx(a+bx^2)^{3/2}}{4c(bc-ad)(c+dx^2)^2}+\frac{(4bc-3ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)(c+dx^2)}+\frac{a(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1268 vs. $2(149) = 298$.

Time = 9.42 (sec) , antiderivative size = 1268, normalized size of antiderivative = 8.51

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$$

$$= \frac{\sqrt{c}\Big(-a^8d^2x(5c+3dx^2)+8192b^{15/2}c^3x^{12}\Big(-\sqrt{b}x+\sqrt{a+bx^2}\Big)+a^7\Big(bdx(43c^2-345cdx^2-216d^2x^4)+\sqrt{b}d\sqrt{a+bx^2}\,(-3c^2+59cdx^2+36d^2x^4)\Big)-2048ab}{}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^3,x]
```

```
[Out] ((Sqrt[c]*(-(a^8*d^2*x*(5*c + 3*d*x^2)) + 8192*b^(15/2)*c^3*x^12*(-(Sqrt[b]
*x) + Sqrt[a + b*x^2]) + a^7*(b*d*x*(43*c^2 - 345*c*d*x^2 - 216*d^2*x^4) +
Sqrt[b]*d*Sqrt[a + b*x^2]*(-3*c^2 + 59*c*d*x^2 + 36*d^2*x^4)) - 2048*a*b^(1
3/2)*c*x^10*(Sqrt[a + b*x^2]*(-11*c^2 + 10*c*d*x^2 + 4*d^2*x^4) + Sqrt[b]*(
13*c^2*x - 10*c*d*x^3 - 4*d^2*x^5)) + 512*a^2*b^(11/2)*x^8*(Sqrt[a + b*x^2]
*(45*c^3 - 106*c^2*d*x^2 - 22*c*d^2*x^4 + 12*d^3*x^6) + Sqrt[b]*(-65*c^3*x
+ 126*c^2*d*x^3 + 30*c*d^2*x^5 - 12*d^3*x^7)) + 256*a^3*b^(9/2)*x^6*(Sqrt[a
 + b*x^2]*(42*c^3 - 207*c^2*d*x^2 + 19*c*d^2*x^4 + 60*d^3*x^6) - Sqrt[b]*(7
8*c^3*x - 303*c^2*d*x^3 + c*d^2*x^5 + 72*d^3*x^7)) + 56*a^5*b^(5/2)*x^2*(Sq
rt[a + b*x^2]*(3*c^3 - 80*c^2*d*x^2 + 130*c*d^2*x^4 + 96*d^3*x^6) - Sqrt[b]
*(13*c^3*x - 216*c^2*d*x^3 + 238*c*d^2*x^5 + 192*d^3*x^7)) + 64*a^4*b^(7/2)
*x^4*(Sqrt[a + b*x^2]*(35*c^3 - 364*c^2*d*x^2 + 220*c*d^2*x^4 + 216*d^3*x^6
) - Sqrt[b]*(91*c^3*x - 692*c^2*d*x^3 + 272*c*d^2*x^5 + 324*d^3*x^7)) + 2*a
^6*(b^(3/2)*Sqrt[a + b*x^2]*(c^3 - 150*c^2*d*x^2 + 646*c*d^2*x^4 + 420*d^3*
x^6) - b^2*(13*c^3*x - 690*c^2*d*x^3 + 1846*c*d^2*x^5 + 1260*d^3*x^7))))/(d
*(-(b*c) + a*d)*(c + d*x^2)^2*(1024*a*b^5*x^10*(13*Sqrt[b]*x - 11*Sqrt[a +
b*x^2]) + 1280*a^2*b^4*x^8*(13*Sqrt[b]*x - 9*Sqrt[a + b*x^2]) + 768*a^3*b^3
*x^6*(13*Sqrt[b]*x - 7*Sqrt[a + b*x^2]) + 224*a^4*b^2*x^4*(13*Sqrt[b]*x - 5
*Sqrt[a + b*x^2]) + 28*a^5*b*x^2*(13*Sqrt[b]*x - 3*Sqrt[a + b*x^2]) + 4096*
b^6*x^12*(Sqrt[b]*x - Sqrt[a + b*x^2]) - a^6*(-13*Sqrt[b]*x + Sqrt[a + b*x^
2]))) + (21*a*b*c*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sq
rt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d)^(3/2) - (24*b^2*c^2*ArcTan[(-(d*
x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(d
*(-(b*c) + a*d)^(3/2)) - (3*a*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c +
 d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[-(b*c) + a*d] - (20*a*b*c*ArcT
anh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d]
)])/(b*c - a*d)^(3/2) + (24*b^2*c^2*ArcTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[
b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(d*(b*c - a*d)^(3/2)))/(8*c^(5/
2))
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{b\,x^{2}+a}\,\left(3a\,d^{2}x^{2}-2bcd\,x^{2}+5acd-4b\,c^{2}\right)x}{a\left(d\,x^{2}+c\right)^{2}}+\frac{(3ad-4bc)\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}\right)}{8(ad-bc)c^{2}}$	120
default	Expression too large to display	4113

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a/(a*d-b*c)/c^2*(-(b*x^2+a)^(1/2)/a*(3*a*d^2*x^2-2*b*c*d*x^2+5*a*c*d-4
*b*c^2)*x/(d*x^2+c)^2+(3*a*d-4*b*c)/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^
(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(129) = 258.

Time = 0.35 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx$$
$$= \left[\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^4+2\,(4\,abc^2d-3\,a^2cd^2)x^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a\cdots}{\cdots}\right.}{32\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^{5}\cdots}\right.$$
$$-\frac{(4\,abc^3-3\,a^2c^2d+(4\,abcd^2-3\,a^2d^3)x^4+2\,(4\,abc^2d-3\,a^2cd^2)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-\cdots}{2\,((b^2c^2-abcd)x^{\cdots}}\right.}{16\,(b^2c^7-2\,abc^6d+a^2c^5d^2+(b^2c^5d^2-2\,abc^4d^3+\cdots}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*
c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d +
 a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*
x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^
2)) + 4*((2*b^2*c^3*d - 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a
*b*c^3*d + 5*a^2*c^2*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*
c^5*d^2 + (b^2*c^5*d^2 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d -
2*a*b*c^5*d^2 + a^2*c^4*d^3)*x^2), -1/16*((4*a*b*c^3 - 3*a^2*c^2*d + (4*a*b
*c*d^2 - 3*a^2*d^3)*x^4 + 2*(4*a*b*c^2*d - 3*a^2*c*d^2)*x^2)*sqrt(-b*c^2 +
a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2
 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3*d
- 5*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (4*b^2*c^4 - 9*a*b*c^3*d + 5*a^2*c^2*d
```

```
^2)*x)*sqrt(b*x^2 + a))/(b^2*c^7 - 2*a*b*c^6*d + a^2*c^5*d^2 + (b^2*c^5*d^2
 - 2*a*b*c^4*d^3 + a^2*c^3*d^4)*x^4 + 2*(b^2*c^6*d - 2*a*b*c^5*d^2 + a^2*c^
4*d^3)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx = \int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx = \int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^3}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(129) = 258$.

Time = 1.83 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^3}\,dx = -\frac{\left(4\,ab^{\frac{3}{2}}c-3\,a^2\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}(bc^3-ac^2d)} - \frac{4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^2-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}d^3-16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4b^{\frac{7}{2}}c^3+40\left(\sqrt{b}x\right.}{}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
[Out] -1/8*(4*a*b^(3/2)*c - 3*a^2*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*(b*c^3 - a*c^2*d)) - 1/4*(4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d
^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*sqrt(b)*d^3 - 16*(sqrt(b)*x - sq
```

```
rt(b*x^2 + a))^4*b^(7/2)*c^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)
*c^2*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2
*a^2*b^(5/2)*c^2*d + 28*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 -
 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^3 - 2*a^4*b^(3/2)*c*d^2 +
3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2*(b*c
^3*d - a*c^2*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^3}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^3}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^3, x)
```

3.52 $\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 208

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx &= \frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\,(c+dx^2)^2} \\ &\quad + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\,(c+dx^2)} \\ &\quad + \frac{a(8b^2c^2-12abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}\end{aligned}$$

```
[Out] 1/16*a*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/
(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(5/2)+1/6*x*(b*x^2+a)^(1/2)/c/(d*x^2+c)
^3+1/24*(-5*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/48*(-
5*a*d+2*b*c)*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {423, 541, 12, 385, 214}

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx &= \frac{a(5a^2d^2-12abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}} \\ &\quad + \frac{x\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{48c^3\,(c+dx^2)\,(bc-ad)^2} \\ &\quad + \frac{x\sqrt{a+bx^2}(4bc-5ad)}{24c^2\,(c+dx^2)^2\,(bc-ad)} + \frac{x\sqrt{a+bx^2}}{6c\,(c+dx^2)^3}\end{aligned}$$

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(6*c*(c + d*x^2)^3) + ((4*b*c - 5*a*d)*x*Sqrt[a + b*x^2
])/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + ((2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*
Sqrt[a + b*x^2])/(48*c^3*(b*c - a*d)^2*(c + d*x^2)) + (a*(8*b^2*c^2 - 12*a*
b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/
(16*c^(7/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} - \frac{\int \frac{-5a-4bx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{6c}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} - \frac{\int \frac{-a(16bc-15ad)-2b(4bc-5ad)x^2}{\sqrt{a+bx^2}(c+dx^2)^2}dx}{24c^2(bc-ad)}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} - \frac{\int -\frac{3a(8b^2c^2-12abcd+5a^2d^2)}{\sqrt{a+bx^2}(c+dx^2)}dx}{48c^3(bc-ad)^2}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{(a(8b^2c^2-12abcd+5a^2d^2))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}dx}{16c^3(bc-ad)^2}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{(a(8b^2c^2-12abcd+5a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16c^3(bc-ad)^2}$$

$$= \frac{x\sqrt{a+bx^2}}{6c\left(c+dx^2\right)^3} + \frac{(4bc-5ad)x\sqrt{a+bx^2}}{24c^2(bc-ad)\left(c+dx^2\right)^2} + \frac{(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{48c^3(bc-ad)^2\left(c+dx^2\right)} + \frac{a(8b^2c^2-12abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 10.64 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^4}dx$$

$$= \frac{x\sqrt{a+bx^2}\left((bc-ad)\left(8b^2c^2(3c^2+3cdx^2+d^2x^4)-2abcd(30c^2+35cdx^2+13d^2x^4)+a^2d^2(33c^2+40cdx^2\right.\right.}{48c^3(bc-ad)^3\left(c+dx^2\right)^3}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((b*c - a*d)*(8*b^2*c^2*(3*c^2 + 3*c*d*x^2 + d^2*x^4) -
2*a*b*c*d*(30*c^2 + 35*c*d*x^2 + 13*d^2*x^4) + a^2*d^2*(33*c^2 + 40*c*d*x^2
 + 15*d^2*x^4)) + (3*a*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Sqrt[((b*c - a*
d)*x^2)/(c*(a + b*x^2))]*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a
 + b*x^2))]])/x^2))/(48*c^3*(b*c - a*d)^3*(c + d*x^2)^3)
```

Maple [A] (verified)

Time = 8.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$\frac{11x\sqrt{b\,x^2+a}\left(\frac{8b^2c^4}{11}-\frac{20bd\left(-\frac{2b\,x^2}{5}+a\right)c^3}{11}+\left(-\frac{4b\,x^2}{33}+a\right)(-2b\,x^2+a)d^2c^2+\frac{40x^2d^3\left(-\frac{13b\,x^2}{20}+a\right)ac}{33}+\frac{5a^2d^4x^4}{11}\right)\sqrt{(ad-bc)c}-}{16\sqrt{(ad-bc)c}\,(d\,x^2+c)^3(ad-bc)^2c^3}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/((a*d-b*c)*c)^(1/2)*(11*x*(b*x^2+a)^(1/2)*(8/11*b^2*c^4-20/11*b*d*(-2/
5*b*x^2+a)*c^3+(-4/33*b*x^2+a)*(-2*b*x^2+a)*d^2*c^2+40/33*x^2*d^3*(-13/20*b
*x^2+a)*a*c+5/11*a^2*d^4*x^4)*((a*d-b*c)*c)^(1/2)-5*(a^2*d^2-12/5*a*b*c*d+8
/5*b^2*c^2)*(d*x^2+c)^3*a*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/
(d*x^2+c)^3/(a*d-b*c)^2/c^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(184) = 368.

Time = 0.64 (sec) , antiderivative size = 1220, normalized size of antiderivative = 5.87

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a*b^2*c^5 - 12*a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3
- 12*a^2*b*c*d^4 + 5*a^3*d^5)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 +
 5*a^3*c*d^4)*x^4 + 3*(8*a*b^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^
2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4*d^2
 - 34*a*b^2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*
d - 47*a*b^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^
6 - 28*a*b^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))
/(b^3*c^10 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 -
 3*a*b^2*c^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*
a*b^2*c^7*d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2
*c^8*d^2 + 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2), -1/96*(3*(8*a*b^2*c^5 - 12*
a^2*b*c^4*d + 5*a^3*c^3*d^2 + (8*a*b^2*c^2*d^3 - 12*a^2*b*c*d^4 + 5*a^3*d^5
)*x^6 + 3*(8*a*b^2*c^3*d^2 - 12*a^2*b*c^2*d^3 + 5*a^3*c*d^4)*x^4 + 3*(8*a*b
^2*c^4*d - 12*a^2*b*c^3*d^2 + 5*a^3*c^2*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arct
an(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2
```

```
*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4*d^2 - 34*a*b^
2*c^3*d^3 + 41*a^2*b*c^2*d^4 - 15*a^3*c*d^5)*x^5 + 2*(12*b^3*c^5*d - 47*a*b
^2*c^4*d^2 + 55*a^2*b*c^3*d^3 - 20*a^3*c^2*d^4)*x^3 + 3*(8*b^3*c^6 - 28*a*b
^2*c^5*d + 31*a^2*b*c^4*d^2 - 11*a^3*c^3*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^10
 - 3*a*b^2*c^9*d + 3*a^2*b*c^8*d^2 - a^3*c^7*d^3 + (b^3*c^7*d^3 - 3*a*b^2*c
^6*d^4 + 3*a^2*b*c^5*d^5 - a^3*c^4*d^6)*x^6 + 3*(b^3*c^8*d^2 - 3*a*b^2*c^7*
d^3 + 3*a^2*b*c^6*d^4 - a^3*c^5*d^5)*x^4 + 3*(b^3*c^9*d - 3*a*b^2*c^8*d^2 +
 3*a^2*b*c^7*d^3 - a^3*c^6*d^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^4}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 958 vs. $2(184) = 368$.

Time = 1.48 (sec) , antiderivative size = 958, normalized size of antiderivative = 4.61

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = -\frac{\left(8\,ab^{\frac{5}{2}}c^2 - 12\,a^2b^{\frac{3}{2}}cd + 5\,a^3\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,(b^2c^5 - 2\,abc^4d + a^2c^3d^2)\sqrt{-b^2c^2+abcd}} - \frac{24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{5}{2}}c^2d^3 - 36\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}cd^4 + 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5 +}{}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^4,x, algorithm="giac")
```

```
[Out] -1/16*(8*a*b^(5/2)*c^2 - 12*a^2*b^(3/2)*c*d + 5*a^3*sqrt(b)*d^2)*arctan(1/2
*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d)
)/((b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1/24*(
24*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(5/2)*c^2*d^3 - 36*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^2*b^(3/2)*c*d^4 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^
3*sqrt(b)*d^5 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 480
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*d^3 + 330*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*
sqrt(b)*d^5 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 + 1216*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d - 2016*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*c^3*d^2 + 1736*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 - 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 + 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 384*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 1392*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/
2)*c^3*d^2 - 1608*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 + 780
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(
b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7
/2)*c^3*d^2 + 336*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 - 300
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^7*sqrt(b)*d^5 - 8*a^6*b^(5/2)*c^2*d^3 + 26*a^7*b^(3/2)*c*d^4
 - 15*a^8*sqrt(b)*d^5)/((b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*((sqrt(b)
*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^4}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{\left(d\,x^2+c\right)}^4}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^4,x)
```

```
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^4, x)
```

3.53 $\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 272

$$\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx = \frac{3a(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)(8b^2c^2-2abcd+a^2d^2)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)^2}{10b} + \frac{3a^2(4bc-ad)(8b^2c^2-2abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

```
[Out] 1/128*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^3+1/16
0*d*(5*a^2*d^2-20*a*b*c*d+36*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^3+1/80*d*(-5*a*d+
14*b*c)*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b^2+1/10*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)^
2/b+3/256*a^2*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/
(b*x^2+a)^(1/2))/b^(7/2)+3/256*a*(-a*d+4*b*c)*(a^2*d^2-2*a*b*c*d+8*b^2*c^2)
*x*(b*x^2+a)^(1/2)/b^3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{427, 542, 396, 201, 223, 212\}$

$$\int (a + bx^2)^{3/2} (c + dx^2)^3 \, dx = \frac{3a^2 \text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) (4bc - ad) (a^2d^2 - 2abcd + 8b^2c^2)}{256b^{7/2}} + \frac{dx(a + bx^2)^{5/2} (5a^2d^2 - 20abcd + 36b^2c^2)}{160b^3} + \frac{x(a + bx^2)^{3/2} (4bc - ad) (a^2d^2 - 2abcd + 8b^2c^2)}{128b^3} + \frac{3ax\sqrt{a + bx^2}(4bc - ad) (a^2d^2 - 2abcd + 8b^2c^2)}{256b^3} + \frac{dx(a + bx^2)^{5/2} (c + dx^2) (14bc - 5ad)}{80b^2} + \frac{dx(a + bx^2)^{5/2} (c + dx^2)^2}{10b}$$

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (3*a*(4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(25
6*b^3) + ((4*b*c - a*d)*(8*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x*(a + b*x^2)^(3/
2))/(128*b^3) + (d*(36*b^2*c^2 - 20*a*b*c*d + 5*a^2*d^2)*x*(a + b*x^2)^(5/2
))/(160*b^3) + (d*(14*b*c - 5*a*d)*x*(a + b*x^2)^(5/2)*(c + d*x^2))/(80*b^2
) + (d*x*(a + b*x^2)^(5/2)*(c + d*x^2)^2)/(10*b) + (3*a^2*(4*b*c - a*d)*(8*
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b
^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} \\
&\quad + \frac{\int (a+bx^2)^{3/2}\,(c+dx^2)\,(c(10bc-ad)+d(14bc-5ad)x^2)\;dx}{10b} \\
&= \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} \\
&\quad + \frac{\int (a+bx^2)^{3/2}\,(c(80b^2c^2-22abcd+5a^2d^2)+3d(36b^2c^2-20abcd+5a^2d^2)\,x^2)\;dx}{80b^2} \\
&= \frac{d(36b^2c^2-20abcd+5a^2d^2)\,x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} \\
&\quad + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} + \frac{((4bc-ad)\,(8b^2c^2-2abcd+a^2d^2))\int (a+bx^2)^{3/2}\;dx}{32b^3}
\end{aligned}
$$

$$= \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)\,x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} + \frac{(3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right))\int\sqrt{a+bx^2}\,dx}{128b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)\,x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} + \frac{(3a^2(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right))\int\frac{1}{\sqrt{a+bx^2}}\,dx}{256b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)\,x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} + \frac{(3a^2(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right))\,\mathrm{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{256b^3}$$

$$= \frac{3a(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x\sqrt{a+bx^2}}{256b^3} + \frac{(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)x(a+bx^2)^{3/2}}{128b^3} + \frac{d(36b^2c^2-20abcd+5a^2d^2)\,x(a+bx^2)^{5/2}}{160b^3} + \frac{d(14bc-5ad)x(a+bx^2)^{5/2}\,(c+dx^2)}{80b^2} + \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)^2}{10b} + \frac{3a^2(4bc-ad)\left(8b^2c^2-2abcd+a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.83

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(15a^4d^3-10a^3bd^2(9c+dx^2)+4a^2b^2d(60c^2+15cdx^2+2d^2x^4)+32b^4x^2(10c^3+2$$

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(15*a^4*d^3 - 10*a^3*b*d^2*(9*c + d*x^2) + 4*a^2
*b^2*d*(60*c^2 + 15*c*d*x^2 + 2*d^2*x^4) + 32*b^4*x^2*(10*c^3 + 20*c^2*d*x^
2 + 15*c*d^2*x^4 + 4*d^3*x^6) + 16*a*b^3*(50*c^3 + 70*c^2*d*x^2 + 45*c*d^2*
x^4 + 11*d^3*x^6)) + 15*a^2*(-32*b^3*c^3 + 16*a*b^2*c^2*d - 6*a^2*b*c*d^2 +
 a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1280*b^(7/2))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{3\left(\left(a^{5} d^{3}-6 a^{4} b c\, d^{2}+16 a^{3} b^{2} c^{2} d-32 a^{2} b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{x \sqrt{b}}\right)-x\left(\frac{160\left(\frac{11}{50} d^{3} x^{6}+\frac{9}{10} c\, d^{2} x^{4}+\frac{7}{5} c^{2} d\, x^{2}+c^{3}\right) a\, b^{\frac{7}{2}}}{3}+\frac{64 x^{2}\left(\frac{2}{5} d^{3}\right.}{}\right.\right.}{256 b^{\frac{7}{2}}}$
risch	$\frac{x\left(128 b^{4} d^{3} x^{8}+176 a\, b^{3} d^{3} x^{6}+480 b^{4} c\, d^{2} x^{6}+8 a^{2} b^{2} d^{3} x^{4}+720 a\, b^{3} c\, d^{2} x^{4}+640 b^{4} c^{2} d\, x^{4}-10 a^{3} b\, d^{3} x^{2}+60 a^{2} b^{2} c\, d^{2} x^{2}+1120 a\, b^{3} c^{2} d\right.}{1280 b^{3}}$
default	$c^{3}\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x \sqrt{b\, x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\, x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)+d^{3}\left(\frac{x^{5}\left(b\, x^{2}+a\right)^{\frac{5}{2}}}{10 b}-\frac{a\left(\frac{x^{3}\left(b\, x^{2}+a\right)^{\frac{5}{2}}}{8 b}-\frac{3 a\left(\frac{x\left(b\, x^{2}+a\right.}{6 b}\right.}{}\right.}{}\right.$

[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -3/256/b^(7/2)*((a^5*d^3-6*a^4*b*c*d^2+16*a^3*b^2*c^2*d-32*a^2*b^3*c^3)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(160/3*(11/50*d^3*x^6+9/10*c*d^2*x^4+7/5*c^2*d*x^2+c^3)*a*b^(7/2)+64/3*x^2*(2/5*d^3*x^6+3/2*c*d^2*x^4+2*c^2*d*x^2+c^3)*b^(9/2)+((8/15*d^2*x^4+4*c*d*x^2+16*c^2)*b^(5/2)+((-2/3*d*x^2-6*c)*b^(3/2)+a*d*b^(1/2))*d*a)*d*a^2)*(b*x^2+a)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.85

$$\int (a+bx^2)^{3/2} (c+dx^2)^3 \, dx = \left[-\frac{15\,(32\,a^2b^3c^3 - 16\,a^3b^2c^2d + 6\,a^4bcd^2 - a^5d^3)\sqrt{b}\log\left(-2\,bx^2 + 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) - 2\,(128\right.$$

$$-\frac{15\,(32\,a^2b^3c^3 - 16\,a^3b^2c^2d + 6\,a^4bcd^2 - a^5d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (128\,b^5d^3x^9 + 16\,(30\,b^5cd^2 + 11\,ab^4d^3)}{}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(128*b^5*d^3*x^
9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^5*c^2*d + 90*a*b^4*c*d^2
 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^2*d + 6*a^2*b^3*c*d^2 -
a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^2*d - 18*a^3*b^2*c*d^2 +
 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/1280*(15*(32*a^2*b^3*c^3 - 16*a^3
*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (128*b^5*d^3*x^9 + 16*(30*b^5*c*d^2 + 11*a*b^4*d^3)*x^7 + 8*(80*b^
5*c^2*d + 90*a*b^4*c*d^2 + a^2*b^3*d^3)*x^5 + 10*(32*b^5*c^3 + 112*a*b^4*c^
2*d + 6*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^3 + 5*(160*a*b^4*c^3 + 48*a^2*b^3*c^
2*d - 18*a^3*b^2*c*d^2 + 3*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 529, normalized size of antiderivative = 1.94

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bd^3x^9}{10}+\frac{x^7\cdot\left(\frac{11abd^3}{10}+3b^2cd^2\right)}{8b}+\frac{x^5\left(a^2d^3+6abcd^2-\frac{7a\left(\frac{11abd^3}{10}+3b^2cd^2\right)}{8b}+3b^2c^2d\right)}{6b}+\frac{x^3\cdot\left(3a^2cd^2+6abc^2\right.}{}\right. \\ a^{\frac{3}{2}}\left(c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**3,x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(b*d**3*x**9/10 + x**7*(11*a*b*d**3/10 + 3*b**2
*c*d**2)/(8*b) + x**5*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a*b*d**3/10 + 3*b
**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + x**3*(3*a**2*c*d**2 + 6*a*b*c**2
*d - 5*a*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a*b*d**3/10 + 3*b**2*c*d**2)/(
8*b) + 3*b**2*c**2*d)/(6*b) + b**2*c**3)/(4*b) + x*(3*a**2*c**2*d + 2*a*b*c
**3 - 3*a*(3*a**2*c*d**2 + 6*a*b*c**2*d - 5*a*(a**2*d**3 + 6*a*b*c*d**2 - 7
*a*(11*a*b*d**3/10 + 3*b**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + b**2*c**
3)/(4*b))/(2*b)) + (a**2*c**3 - a*(3*a**2*c**2*d + 2*a*b*c**3 - 3*a*(3*a**2
*c*d**2 + 6*a*b*c**2*d - 5*a*(a**2*d**3 + 6*a*b*c*d**2 - 7*a*(11*a*b*d**3/1
0 + 3*b**2*c*d**2)/(8*b) + 3*b**2*c**2*d)/(6*b) + b**2*c**3)/(4*b))/(2*b))*
Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*l
og(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(c**3*x + c**2*d*x**3 + 3*
c*d**2*x**5/5 + d**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.34

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \frac{(bx^2+a)^{\frac{5}{2}}d^3x^5}{10\,b} + \frac{3\,(bx^2+a)^{\frac{5}{2}}cd^2x^3}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}ad^3x^3}{16\,b^2} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}c^3x + \frac{3}{8}\,\sqrt{bx^2+a}ac^3x + \frac{(bx^2+a)^{\frac{5}{2}}c^2dx}{2\,b} - \frac{(bx^2+a)^{\frac{3}{2}}ac^2dx}{8\,b} - \frac{3\,\sqrt{bx^2+a}a^2c^2dx}{16\,b} - \frac{3\,(bx^2+a)^{\frac{5}{2}}acd^2x}{16\,b^2} + \frac{3\,(bx^2+a)^{\frac{3}{2}}a^2cd^2x}{64\,b^2} + \frac{9\,\sqrt{bx^2+a}a^3cd^2x}{128\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}a^2d^3x}{32\,b^3} - \frac{(bx^2+a)^{\frac{3}{2}}a^3d^3x}{128\,b^3} - \frac{3\,\sqrt{bx^2+a}a^4d^3x}{256\,b^3} + \frac{3\,a^2c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{3\,a^3c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{9\,a^4cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} - \frac{3\,a^5d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(5/2)*d^3*x^5/b + 3/8*(b*x^2 + a)^(5/2)*c*d^2*x^3/b - 1/16
*(b*x^2 + a)^(5/2)*a*d^3*x^3/b^2 + 1/4*(b*x^2 + a)^(3/2)*c^3*x + 3/8*sqrt(b
*x^2 + a)*a*c^3*x + 1/2*(b*x^2 + a)^(5/2)*c^2*d*x/b - 1/8*(b*x^2 + a)^(3/2)
*a*c^2*d*x/b - 3/16*sqrt(b*x^2 + a)*a^2*c^2*d*x/b - 3/16*(b*x^2 + a)^(5/2)*
a*c*d^2*x/b^2 + 3/64*(b*x^2 + a)^(3/2)*a^2*c*d^2*x/b^2 + 9/128*sqrt(b*x^2 +
 a)*a^3*c*d^2*x/b^2 + 1/32*(b*x^2 + a)^(5/2)*a^2*d^3*x/b^3 - 1/128*(b*x^2 +
 a)^(3/2)*a^3*d^3*x/b^3 - 3/256*sqrt(b*x^2 + a)*a^4*d^3*x/b^3 + 3/8*a^2*c^3
*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 3/16*a^3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(
3/2) + 9/128*a^4*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 3/256*a^5*d^3*arcsi
nh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.96

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \frac{1}{1280}\left(2\left(4\left(2\left(8\,bd^3x^2 + \frac{30\,b^9cd^2+11\,ab^8d^3}{b^8}\right)x^2 + \frac{80\,b^9c^2d+90\,ab^8cd^2+a^2b^7d^3}{b^8}\right)x^2 + \frac{5\,(32\right.\right.\right. - \frac{3\,(32\,a^2b^3c^3-16\,a^3b^2c^2d+6\,a^4bcd^2-a^5d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/1280*(2*(4*(2*(8*b*d^3*x^2 + (30*b^9*c*d^2 + 11*a*b^8*d^3)/b^8)*x^2 + (80
*b^9*c^2*d + 90*a*b^8*c*d^2 + a^2*b^7*d^3)/b^8)*x^2 + 5*(32*b^9*c^3 + 112*a
*b^8*c^2*d + 6*a^2*b^7*c*d^2 - a^3*b^6*d^3)/b^8)*x^2 + 5*(160*a*b^8*c^3 + 4
8*a^2*b^7*c^2*d - 18*a^3*b^6*c*d^2 + 3*a^4*b^5*d^3)/b^8)*sqrt(b*x^2 + a)*x
- 3/256*(32*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 6*a^4*b*c*d^2 - a^5*d^3)*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^3 \, dx = \int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)^3 \,dx$$

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^3, x)
```

3.54 $\int (a+bx^2)^{3/2} (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 196

$$\begin{aligned}\int (a+bx^2)^{3/2} (c+dx^2)^2 \, dx &= \frac{a(48b^2c^2-16abcd+3a^2d^2)\, x\sqrt{a+bx^2}}{128b^2} \\ &+ \frac{(48b^2c^2-16abcd+3a^2d^2)\, x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} \\ &+ \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{a^2(48b^2c^2-16abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}\end{aligned}$$

```
[Out] 1/192*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/48*d*(-3*a*
d+10*b*c)*x*(b*x^2+a)^(5/2)/b^2+1/8*d*x*(b*x^2+a)^(5/2)*(d*x^2+c)/b+1/128*a
^2*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+1/128*a*(3*a^2*d^2-16*a*b*c*d+48*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {427, 396, 201, 223, 212}

$$\begin{aligned}\int (a+bx^2)^{3/2} (c+dx^2)^2 \, dx &= \frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2d^2-16abcd+48b^2c^2)}{128b^{5/2}} \\ &+ \frac{x(a+bx^2)^{3/2}(3a^2d^2-16abcd+48b^2c^2)}{192b^2} + \frac{ax\sqrt{a+bx^2}(3a^2d^2-16abcd+48b^2c^2)}{128b^2} \\ &+ \frac{dx(a+bx^2)^{5/2}(10bc-3ad)}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b}\end{aligned}$$

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (a*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(128*b^2) + ((4
8*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(192*b^2) + (d*(10
*b*c - 3*a*d)*x*(a + b*x^2)^(5/2))/(48*b^2) + (d*x*(a + b*x^2)^(5/2)*(c + d
*x^2))/(8*b) + (a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^2)^{5/2}\,(c+dx^2)}{8b} + \frac{\int (a+bx^2)^{3/2}\,(c(8bc-ad)+d(10bc-3ad)x^2)\,dx}{8b}$$

$$= \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} - \frac{(ad(10bc-3ad)-6bc(8bc-ad))\int (a+bx^2)^{3/2}\,dx}{48b^2}$$

$$= \frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{(a(48b^2c^2-16abcd+3a^2d^2))\int \sqrt{a+bx^2}\,dx}{64b^2}$$

$$= \frac{a(48b^2c^2-16abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{128b^2} + \frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{(a^2(48b^2c^2-16abcd+3a^2d^2))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b^2}$$

$$= \frac{a(48b^2c^2-16abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{128b^2} + \frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{(a^2(48b^2c^2-16abcd+3a^2d^2))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b^2}$$

$$= \frac{a(48b^2c^2-16abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{128b^2} + \frac{(48b^2c^2-16abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{192b^2} + \frac{d(10bc-3ad)x(a+bx^2)^{5/2}}{48b^2} + \frac{dx(a+bx^2)^{5/2}(c+dx^2)}{8b} + \frac{a^2(48b^2c^2-16abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int (a+bx^2)^{3/2}(c+dx^2)^2\,dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-9a^3d^2+6a^2bd(8c+dx^2)+16b^3x^2(6c^2+8cdx^2+3d^2x^4)+8ab^2(30c^2+28cdx^2}{384b^{5/2}}$$

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-9*a^3*d^2 + 6*a^2*b*d*(8*c + d*x^2) + 16*b^3*x
^2*(6*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + 8*a*b^2*(30*c^2 + 28*c*d*x^2 + 9*d^2*x
^4)) - 3*a^2*(48*b^2*c^2 - 16*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/(384*b^(5/2))
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{\frac{3a^2\left(a^2d^2-\frac{16}{3}abcd+16b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{128}-\frac{3x\sqrt{b\,x^2+a}\left(-\frac{80\left(\frac{3}{10}d^2x^4+\frac{14}{15}cd\,x^2+c^2\right)a\,b^{\frac{5}{2}}}{3}-\frac{32x^2\left(\frac{1}{2}d^2x^4+\frac{4}{3}cd\,x^2+c^2\right)b^{\frac{7}{2}}}{3}+\left(\right.}{128}}{b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-48b^3d^2x^6-72a\,b^2d^2x^4-128b^3dc\,x^4-6x^2a^2b\,d^2-224x^2a\,b^2cd-96x^2b^3c^2+9a^3d^2-48a^2bcd-240a\,b^2c^2\right)\sqrt{b\,x^2+a}}{384b^2}+$
default	$c^2\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d^2\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)}{4}\right.}{}\right.}{}\right.$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/128/b^(5/2)*(a^2*(a^2*d^2-16/3*a*b*c*d+16*b^2*c^2)*arctanh((b*x^2+a)^(1/2
)/x/b^(1/2))-x*(b*x^2+a)^(1/2)*(-80/3*(3/10*d^2*x^4+14/15*c*d*x^2+c^2)*a*b^
(5/2)-32/3*x^2*(1/2*d^2*x^4+4/3*c*d*x^2+c^2)*b^(7/2)+(2/3*(-d*x^2-8*c)*b^(3
/2)+a*d*b^(1/2))*d*a^2))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.76

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \left[\frac{3\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(48\,b^4d^2x^7+8\left(1\right.\right.}{}\right.$$

$$-\frac{3\left(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(48\,b^4d^2x^7+8\left(16\,b^4cd+9\,ab^3d^2\right)x^5+2\left(48\,b^4c^2\right.\right.}{384\,b^3}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*
a*b^3*d^2)*x^5 + 2*(48*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80
*a*b^3*c^2 + 16*a^2*b^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/384*
(3*(48*a^2*b^2*c^2 - 16*a^3*b*c*d + 3*a^4*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - (48*b^4*d^2*x^7 + 8*(16*b^4*c*d + 9*a*b^3*d^2)*x^5 + 2*(4
8*b^4*c^2 + 112*a*b^3*c*d + 3*a^2*b^2*d^2)*x^3 + 3*(80*a*b^3*c^2 + 16*a^2*b
^2*c*d - 3*a^3*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.68

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bd^2x^7}{8}+\frac{x^5\cdot\left(\frac{9abd^2}{8}+2b^2cd\right)}{6b}+\frac{x^3\left(a^2d^2+4abcd-\frac{5a\left(\frac{9abd^2}{8}+2b^2cd\right)}{6b}+b^2c^2\right)}{4b}+\frac{x\left(2a^2cd+2abc^2-\frac{3a\left(a^2d^2+\right.}{}\right.}{}\right. \\ a^{\frac{3}{2}}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(b*d**2*x**7/8 + x**5*(9*a*b*d**2/8 + 2*b**2*c*
d)/(6*b) + x**3*(a**2*d**2 + 4*a*b*c*d - 5*a*(9*a*b*d**2/8 + 2*b**2*c*d)/(6
*b) + b**2*c**2)/(4*b) + x*(2*a**2*c*d + 2*a*b*c**2 - 3*a*(a**2*d**2 + 4*a*
b*c*d - 5*a*(9*a*b*d**2/8 + 2*b**2*c*d)/(6*b) + b**2*c**2)/(4*b))/(2*b)) +
(a**2*c**2 - a*(2*a**2*c*d + 2*a*b*c**2 - 3*a*(a**2*d**2 + 4*a*b*c*d - 5*a*
(9*a*b*d**2/8 + 2*b**2*c*d)/(6*b) + b**2*c**2)/(4*b))/(2*b))*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True)), Ne(b, 0)), (a**(3/2)*(c**2*x + 2*c*d*x**3/3 + d**2*x**5/5), T
rue))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.16

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \frac{(bx^2+a)^{\frac{5}{2}} d^2 x^3}{8\,b} + \frac{1}{4}\,(bx^2+a)^{\frac{3}{2}} c^2 x + \frac{3}{8}\,\sqrt{bx^2+a}\,a c^2 x + \frac{(bx^2+a)^{\frac{5}{2}} c d x}{3\,b} - \frac{(bx^2+a)^{\frac{3}{2}} a c d x}{12\,b} - \frac{\sqrt{bx^2+a}\,a^2 c d x}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}} a d^2 x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}} a^2 d^2 x}{64\,b^2} + \frac{3\,\sqrt{bx^2+a}\,a^3 d^2 x}{128\,b^2} + \frac{3\,a^2 c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{a^3 c d \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{3\,a^4 d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*d^2*x^3/b + 1/4*(b*x^2 + a)^(3/2)*c^2*x + 3/8*sqrt(b*
x^2 + a)*a*c^2*x + 1/3*(b*x^2 + a)^(5/2)*c*d*x/b - 1/12*(b*x^2 + a)^(3/2)*a
*c*d*x/b - 1/8*sqrt(b*x^2 + a)*a^2*c*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*d^2*x
/b^2 + 1/64*(b*x^2 + a)^(3/2)*a^2*d^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*d^2
*x/b^2 + 3/8*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/8*a^3*c*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 3/128*a^4*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \frac{1}{384}\left(2\left(4\left(6\,bd^2x^2 + \frac{16\,b^7cd+9\,ab^6d^2}{b^6}\right)x^2 + \frac{48\,b^7c^2+112\,ab^6cd+3\,a^2b^5d^2}{b^6}\right)x^2 + \frac{3\,(80\,ab^6c^2 \cdots\right. - \frac{(48\,a^2b^2c^2-16\,a^3bcd+3\,a^4d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b*d^2*x^2 + (16*b^7*c*d + 9*a*b^6*d^2)/b^6)*x^2 + (48*b^7*c^
2 + 112*a*b^6*c*d + 3*a^2*b^5*d^2)/b^6)*x^2 + 3*(80*a*b^6*c^2 + 16*a^2*b^5*
c*d - 3*a^3*b^4*d^2)/b^6)*sqrt(b*x^2 + a)*x - 1/128*(48*a^2*b^2*c^2 - 16*a^
3*b*c*d + 3*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^{3/2} \left(d\,x^2+c\right)^2 \, dx$$

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^2, x)
```

3.55 $\int (a+bx^2)^{3/2}\,(c+dx^2)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (a+bx^2)^{3/2}\,(c+dx^2)\,dx = \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] 1/24*(-a*d+6*b*c)*x*(b*x^2+a)^(3/2)/b+1/6*d*x*(b*x^2+a)^(5/2)/b+1/16*a^2*(-
a*d+6*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/16*a*(-a*d+6*b*c)*x
*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {396, 201, 223, 212}

$$\int (a+bx^2)^{3/2}\,(c+dx^2)\,dx = \frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc-ad)}{16b^{3/2}} + \frac{x(a+bx^2)^{3/2}\,(6bc-ad)}{24b} + \frac{ax\sqrt{a+bx^2}(6bc-ad)}{16b} + \frac{dx(a+bx^2)^{5/2}}{6b}$$

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (a*(6*b*c - a*d)*x*Sqrt[a + b*x^2])/(16*b) + ((6*b*c - a*d)*x*(a + b*x^2)^(
3/2))/(24*b) + (d*x*(a + b*x^2)^(5/2))/(6*b) + (a^2*(6*b*c - a*d)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{5/2}}{6b} - \frac{(-6bc+ad)\int (a+bx^2)^{3/2}\,dx}{6b} \\
&= \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{(a(6bc-ad))\int \sqrt{a+bx^2}\,dx}{8b} \\
&= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} \\
&\quad + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{(a^2(6bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b} \\
&= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} \\
&\quad + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{(a^2(6bc-ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b} \\
&= \frac{a(6bc-ad)x\sqrt{a+bx^2}}{16b} + \frac{(6bc-ad)x(a+bx^2)^{3/2}}{24b} \\
&\quad + \frac{dx(a+bx^2)^{5/2}}{6b} + \frac{a^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)\, dx = \frac{x\sqrt{a+bx^2}(30abc+3a^2d+12b^2cx^2+14abdx^2+8b^2dx^4)}{48b} + \frac{a^2(-6bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(30*a*b*c + 3*a^2*d + 12*b^2*c*x^2 + 14*a*b*d*x^2 + 8*b^
2*d*x^4))/(48*b) + (a^2*(-6*b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(16*b^(3/2))
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\left(8b^2d\,x^4+14x^2abd+12b^2c\,x^2+3a^2d+30abc\right)\sqrt{b\,x^2+a}}{48b}-\frac{a^2(ad-6bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16b^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{(a^3d-6a^2bc)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^2+a}\left(10\left(\frac{7d\,x^2}{15}+c\right)a\,b^{\frac{3}{2}}+\left(\frac{8}{3}d\,x^4+4c\,x^2\right)b^{\frac{5}{2}}+a^2d\sqrt{b}\right)}{16b^{\frac{3}{2}}}$
default	$c\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+d\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\,\mathrm{l}}{}\right.}{}\right.}{6b}\right.$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48/b*x*(8*b^2*d*x^4+14*a*b*d*x^2+12*b^2*c*x^2+3*a^2*d+30*a*b*c)*(b*x^2+a)
^(1/2)-1/16*a^2*(a*d-6*b*c)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.78

$$\int (a+bx^2)^{3/2}(c+dx^2)\,dx = \left[-\frac{3\,(6\,a^2bc-a^3d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,b^3dx^5+2\,(6\,b^3c+7\,ab^2d)x^3+3\,(10\,ab^2c+a^2bd)x)\sqrt{bx^2+a}}{96\,b^2}, -\frac{3\,(6\,a^2bc-a^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3dx^5+2\,(6\,b^3c+7\,ab^2d)x^3+3\,(10\,ab^2c+a^2bd)x)\sqrt{bx^2+a}}{48\,b^2}\right]$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(6*a^2*b*c - a^3*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) - 2*(8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x^3 + 3*(10*a*b^2*c +
 a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2, -1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*d*x^5 + 2*(6*b^3*c + 7*a*b^2*d)*x
^3 + 3*(10*a*b^2*c + a^2*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.48

$$\int (a+bx^2)^{3/2}(c+dx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{bdx^5}{6}+\frac{x^3\cdot\left(\frac{7abd}{6}+b^2c\right)}{4b}+\frac{x\left(a^2d+2abc-\frac{3a\left(\frac{7abd}{6}+b^2c\right)}{4b}\right)}{2b}\right)+\left(a^2c-\frac{a\left(a^2d+2abc-\frac{3a\left(\frac{7abd}{6}+b^2c\right)}{4b}\right)}{2b}\right) \\ a^{\frac{3}{2}}\left(cx+\frac{dx^3}{3}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(b*d*x**5/6 + x**3*(7*a*b*d/6 + b**2*c)/(4*b) +
 x*(a**2*d + 2*a*b*c - 3*a*(7*a*b*d/6 + b**2*c)/(4*b))/(2*b)) + (a**2*c - a
*(a**2*d + 2*a*b*c - 3*a*(7*a*b*d/6 + b**2*c)/(4*b))/(2*b))*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True)), Ne(b, 0)), (a**(3/2)*(c*x + d*x**3/3), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int (a+bx^2)^{3/2}(c+dx^2)\,dx = \frac{1}{4}(bx^2+a)^{\frac{3}{2}}cx + \frac{3}{8}\sqrt{bx^2+a}acx + \frac{(bx^2+a)^{\frac{5}{2}}dx}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}adx}{24\,b} - \frac{\sqrt{bx^2+a}a^2dx}{16\,b} + \frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*c*x + 3/8*sqrt(b*x^2 + a)*a*c*x + 1/6*(b*x^2 + a)^(5/
2)*d*x/b - 1/24*(b*x^2 + a)^(3/2)*a*d*x/b - 1/16*sqrt(b*x^2 + a)*a^2*d*x/b
+ 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/16*a^3*d*arcsinh(b*x/sqrt(a*
b))/b^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^{3/2}(c+dx^2)\,dx = \frac{1}{48}\left(2\left(4\,bdx^2 + \frac{6\,b^5c+7\,ab^4d}{b^4}\right)x^2 + \frac{3\,(10\,ab^4c+a^2b^3d)}{b^4}\right)\sqrt{bx^2+a}x - \frac{(6\,a^2bc-a^3d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*d*x^2 + (6*b^5*c + 7*a*b^4*d)/b^4)*x^2 + 3*(10*a*b^4*c + a^2*b
^3*d)/b^4)*sqrt(b*x^2 + a)*x - 1/16*(6*a^2*b*c - a^3*d)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)\,dx=\int \left(b\,x^2+a\right)^{3/2}\,\left(d\,x^2+c\right)\,dx$$

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2),x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2), x)
```

3.56 $\int (a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (a+bx^2)^{3/2}\,dx = \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x(a+bx^2)^{3/2}+\frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[Out] 1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+3/
8*a*x*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 223, 212}

$$\int (a+bx^2)^{3/2}\,dx = \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x(a+bx^2)^{3/2}$$

```
[In] Int[(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + b*x^2])/8 + (x*(a + b*x^2)^(3/2))/4 + (3*a^2*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{4}(3a)\int\sqrt{a+bx^2}\,dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\int\frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{1}{8}\left(3a^2\right)\text{Subst}\left(\int\frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{3}{8}ax\sqrt{a+bx^2}+\frac{1}{4}x\left(a+bx^2\right)^{3/2}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{8}x\sqrt{a+bx^2}\left(5a+2bx^2\right)-\frac{3a^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

```
[In] Integrate[(a + b*x^2)^(3/2),x]
[Out] (x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(8*Sqrt[b])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x\left(2b\,x^{2}+5a\right)\sqrt{b\,x^{2}+a}}{8}+\frac{3a^{2}\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}$	52
pseudoelliptic	$\frac{2b^{\frac{3}{2}}\sqrt{b\,x^{2}+a}\,x^{3}+5ax\sqrt{b}\,\sqrt{b\,x^{2}+a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{2}}{8\sqrt{b}}$	62

```
[In] int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)^(1/2)+3/8*a^2*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a+bx^2)^{3/2}\,dx=\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{16\,b},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{8\,b}\right]$$

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \left(a+bx^2\right)^{3/2}\, dx = \frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8} + \frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4} + \frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

```
[In] integrate((b*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 + 3
*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a+bx^2\right)^{3/2}\, dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\sqrt{bx^2+a}ax + \frac{3\,a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sqr
t(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{3/2}\, dx = \frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x - \frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

```
[In] integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/2}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{3/2}\,{{}}_2{\mathrm{F}}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

```
[In] int((a + b*x^2)^(3/2),x)
[Out] (x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(3/2)
```

3.57 $\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \frac{bx\sqrt{a+bx^2}}{2d} - \frac{\sqrt{b}(2bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}$$

```
[Out] -1/2*(-3*a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^2+(-a*d+b*
c)^(3/2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^2/c^(1/2)+1/
2*b*x*(b*x^2+a)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {427, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \frac{(bc-ad)^{3/2}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-3ad)}{2d^2} + \frac{bx\sqrt{a+bx^2}}{2d}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*x*Sqrt[a + b*x^2])/(2*d) - (Sqrt[b]*(2*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(2*d^2) + ((b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/
(Sqrt[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx\sqrt{a+bx^2}}{2d} + \frac{\int \frac{-a(bc-2ad)-b(2bc-3ad)x^2}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2d}\\ &= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{d^2}\end{aligned}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{(b(2bc-3ad))\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^2\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d^2}$$

$$= \frac{bx\sqrt{a+bx^2}}{2d} - \frac{\sqrt{b}(2bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^2} + \frac{(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^2}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \frac{bdx\sqrt{a+bx^2} - \frac{2(-bc+ad)^{3/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}} + \sqrt{b}(2bc-3ad)\log\left(-\sqrt{b}x+\sqrt{a}\right.}{2d^2}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2),x]
```

```
[Out] (b*d*x*Sqrt[a + b*x^2] - (2*(-(b*c) + a*d)^(3/2)*ArcTan[(-(d*x*Sqrt[a + b*x
^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/Sqrt[c] + Sqrt[b
]*(2*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*d^2)
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{(ad-bc)^2\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\left(\left(b^{\frac{3}{2}}c-\frac{3ad\sqrt{b}}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\frac{bdx\sqrt{b\,x^2+a}}{2}\right)\sqrt{(ad-bc)c}}{\sqrt{(ad-bc)c}\,d^2}$
risch	$\frac{bx\sqrt{b\,x^2+a}}{2d}+\frac{\frac{\sqrt{b}\,(3ad-2bc)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{d}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\frac{\frac{2ad-2bc}{d}-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right.}}{x+\frac{\sqrt{-cd}}{d}}\right.}{\sqrt{-cd}\,d\sqrt{\frac{ad-bc}{d}}}}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -((a*d-b*c)^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((b^(3/2)*c-3
/2*a*d*b^(1/2))*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-1/2*b*d*x*(b*x^2+a)^(1/2
))*((a*d-b*c)*c)^(1/2))/((a*d-b*c)*c)^(1/2)/d^2
```

Fricas [A] (verification not implemented)

none

Time = 0.38 (sec) , antiderivative size = 721, normalized size of antiderivative = 6.38

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \left[\frac{2\sqrt{bx^2+a}bdx - (2bc-3ad)\sqrt{b}\log\left(-2bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right) - (bc-ad)\sqrt{b}}{4d}\right.$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) - (b*c - a*d)*sqrt((b*c - a*d)/c)*log(((8*b^2*
c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 -
4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d
^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 + a)*b*d*x + 2*(2*b*c -
3*a*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt((b*c
- a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c
^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*s
qrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/d^2, 1/4*(2*sqrt(b*x^2 +
a)*b*d*x - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c
- a^2*d)*x)) - (2*b*c - 3*a*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(b)*x - a))/d^2, 1/2*(sqrt(b*x^2 + a)*b*d*x + (2*b*c - 3*a*d)*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (b*c - a*d)*sqrt(-(b*c - a*d)/c)*arctan(1
/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c -
 a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/d^2]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd(c+dx^2)} + \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}(2bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}$$

```
[Out] b^(3/2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^2-1/2*(a*d+2*b*c)*arctanh(x*(-
a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(3/2)/d^2-1/2*(-
a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {424, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \frac{b^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}(ad+2bc)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2} - \frac{x\sqrt{a+bx^2}(bc-ad)}{2cd(c+dx^2)}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)) + (b^(3/2)*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^2 - (Sqrt[b*c - a*d]*(2*b*c + a*d)*ArcTanh[(
Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{\int \frac{a(bc+ad)+2b^2cx^2}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^2 \int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^2} - \frac{((bc-ad)(2bc+ad)) \int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{((bc-ad)(2bc+ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{2cd\,(c+dx^2)} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{\sqrt{bc-ad}(2bc+ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^2}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \frac{\frac{d(-bc+ad)x\sqrt{a+bx^2}}{c(c+dx^2)} - \frac{\sqrt{-bc+ad}(2bc+ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}} - 2b^{3/2}\log\left(-\sqrt{b}x+\sqrt{a+\right.}{2d^2}$$

[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^2,x]

[Out] ((d*(-(b*c) + a*d)*x*Sqrt[a + b*x^2])/(c*(c + d*x^2)) - (Sqrt[-(b*c) + a*d]
*(2*b*c + a*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[
c]*Sqrt[-(b*c) + a*d])])/c^(3/2) - 2*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/(2*d^2)

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.12

method	result
pseudoelliptic	$\frac{-(d\,x^2+c)(ad+2bc)(ad-bc)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+\sqrt{(ad-bc)c}\left(2c\,b^{\frac{3}{2}}(d\,x^2+c)\,\text{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)+dx\sqrt{b\,x^2+a}\,(ad-bc)\right.}{2\sqrt{(ad-bc)c}\,d^2c(d\,x^2+c)}$
default	Expression too large to display

[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/((a*d-b*c)*c)^(1/2)*(-(d*x^2+c)*(a*d+2*b*c)*(a*d-b*c)*arctan(c*(b*x^2+a
)^(1/2)/x/((a*d-b*c)*c)^(1/2))+((a*d-b*c)*c)^(1/2)*(2*c*b^(3/2)*(d*x^2+c)*a
rctanh((b*x^2+a)^(1/2)/x/b^(1/2))+d*x*(b*x^2+a)^(1/2)*(a*d-b*c)))/d^2/c/(d*
x^2+c)

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 907, normalized size of antiderivative = 6.92

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \left[-\frac{4\,(bcd-ad^2)\sqrt{bx^2+a}x-4\,(bcdx^2+bc^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-(\right.$$

$$-\frac{4\,(bcd-ad^2)\sqrt{bx^2+a}x+8\,(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{\frac{bc-ad}{c}}\mathrm{lo}}{8\,(cd^3x^2+c^2d^2)}$$

$$-\frac{2\,(bcd-ad^2)\sqrt{bx^2+a}x-(2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}\sqrt{-\frac{bc-ad}{c}}}{2\,((b^2c-abd)x^3+(abc-a^2d)x)}\right)-}{4\,(cd^3x^2+c^2d^2)}$$

$$-\frac{2\,(bcd-ad^2)\sqrt{bx^2+a}x+4\,(bcdx^2+bc^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,bc^2+acd+(2\,bcd+ad^2)x^2)\sqrt{-\frac{bc-ad}{c}}}{4\,(cd^3x^2+c^2d^2)}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x - 4*(b*c*d*x^2 + b*c^2)*sqrt(b)*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*b*c^2 + a*c*d + (2*b*c
*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)
*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*
c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2))
)/(c*d^3*x^2 + c^2*d^2), -1/8*(4*(b*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 8*(b*c
*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*
c*d + (2*b*c*d + a*d^2)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*
d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (
2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d
*x^2 + c^2)))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b*c*d - a*d^2)*sqrt(b*x^2 + a
)*x - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-(b*c - a*d)/c)*arctan
(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c
 - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) - 2*(b*c*d*x^2 + b*c^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(c*d^3*x^2 + c^2*d^2), -1/4*(2*(b
*c*d - a*d^2)*sqrt(b*x^2 + a)*x + 4*(b*c*d*x^2 + b*c^2)*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (2*b*c^2 + a*c*d + (2*b*c*d + a*d^2)*x^2)*sqrt(-
(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(
b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)))/(c*d^3*x^2 + c^2*
d^2)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(109) = 218$.

Time = 0.30 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.42

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = -\frac{b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^2} + \frac{\left(2\,b^{\frac{5}{2}}c^2 - ab^{\frac{3}{2}}cd - a^2\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d + 2\,bc - ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}cd^2} - \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{5}{2}}c^2 - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ab^{\frac{3}{2}}cd + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^2\sqrt{b}d^2 + a^2b^{\frac{3}{2}}cd - a^3\sqrt{}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc - 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ad + a^2 d\right)cd^2}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
[Out] -1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^2 + 1/2*(2*b^(5/2)*c^2
- a*b^(3/2)*c*d - a^2*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a)
)^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c*
d^2) - (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(5/2)*c^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^(3/2)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*sqrt(b)*
d^2 + a^2*b^(3/2)*c*d - a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*
d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))
^2*a*d + a^2*d)*c*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^2, x)
```

3.59 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3} dx = \frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)} + \frac{3a^2 \operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}$$

```
[Out] 1/4*x*(b*x^2+a)^(3/2)/c/(d*x^2+c)^2+3/8*a^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(1
/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(1/2)+3/8*a*x*(b*x^2+a)^(1/2)/c^2/(
d*x^2+c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {386, 385, 214}

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3} dx = \frac{3a^2 \operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}} + \frac{3ax\sqrt{a+bx^2}}{8c^2(c+dx^2)} + \frac{x(a+bx^2)^{3/2}}{4c(c+dx^2)^2}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^(3/2))/(4*c*(c + d*x^2)^2) + (3*a*x*Sqrt[a + b*x^2])/(8*c^2*
(c + d*x^2)) + (3*a^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])
])/(8*c^(5/2)*Sqrt[b*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{(3a)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{4c} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{(3a^2)\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{8c^2} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2} \\
&= \frac{x(a+bx^2)^{3/2}}{4c\,(c+dx^2)^2} + \frac{3ax\sqrt{a+bx^2}}{8c^2\,(c+dx^2)} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] (warning: unable to verify)

Time = 10.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\,(5ac+2bcx^2+3adx^2)}{(c+dx^2)\sqrt{1+\frac{dx^2}{c}}} + \frac{3a\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{d}{c}\right)x^2}}{\sqrt{1+\frac{dx^2}{c}}}\right)}{\sqrt{\frac{(-bc+ad)x^2}{ac}}}\right)}{8c^3\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^3,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(5*a*c + 2*b*c*x
^2 + 3*a*d*x^2))/((c + d*x^2)*Sqrt[1 + (d*x^2)/c]) + (3*a*ArcSin[Sqrt[(-(b/
a) + d/c)*x^2]/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(8*
c^3*Sqrt[1 + (b*x^2)/a])
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$-\frac{a^{2}\left(-\frac{\sqrt{b\,x^{2}+a}\,\left(3ad\,x^{2}+2cb\,x^{2}+5ac\right)x}{a^{2}\left(d\,x^{2}+c\right)^{2}}+\frac{3\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}\right)}{8c^{2}}$	94
default	Expression too large to display	6921

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*a^2/c^2*(-(b*x^2+a)^(1/2)*(3*a*d*x^2+2*b*c*x^2+5*a*c)*x/a^2/(d*x^2+c)^
2+3/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time = 0.33 (sec) , antiderivative size = 526, normalized size of antiderivative = 4.65

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \left[\frac{3\,(a^2d^2x^4+2\,a^2cdx^2+a^2c^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+\cdots}{d^2x^4+2\,cdx^2+\cdots}\right.}{32\,(bc^6-ac^5d+(bc^4d^2\cdots}\right.$$

$$-\frac{3\,(a^2d^2x^4+2\,a^2cdx^2+a^2c^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((2\,b^2c^3+abc^2d-3\cdots}{16\,(bc^6-ac^5d+(bc^4d^2-ac^3d^3)x^4+2\,(bc^5d-ac^4d^2)x^2)}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a^2*d^2*x^4 + 2*a^2*c*d*x^2 + a^2*c^2)*sqrt(b*c^2 - a*c*d)*log(((
8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*
x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d
^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 +
 5*(a*b*c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2
- a*c^3*d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2), -1/16*(3*(a^2*d^2*x^4 + 2*
a^2*c*d*x^2 + a^2*c^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)
*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*
c^2 - a^2*c*d)*x)) - 2*((2*b^2*c^3 + a*b*c^2*d - 3*a^2*c*d^2)*x^3 + 5*(a*b*
c^3 - a^2*c^2*d)*x)*sqrt(b*x^2 + a))/(b*c^6 - a*c^5*d + (b*c^4*d^2 - a*c^3*
d^3)*x^4 + 2*(b*c^5*d - a*c^4*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^3}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^3}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(93) = 186$.

Time = 1.67 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.99

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = -\frac{3\,a^2\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}c^2} + \frac{8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}}c^2d-3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a^2\sqrt{b}d^3+16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{7}{2}}c^3+8\left(\sqrt{b}x-\sqrt{b\ldots}\right.}{}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
[Out] -3/8*a^2*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*
d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^2) + 1/4*(8*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*a^2*sqrt(b)*d^3 + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3 + 8*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d - 18*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^2*b^(3/2)*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*sqrt(b)*d^3
+ 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d + 16*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^3*b^(3/2)*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*s
qrt(b)*d^3 + 2*a^4*b^(3/2)*c*d^2 + 3*a^5*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b
*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a*d + a^2*d)^2*c^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^3}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^3, x)
```

3.60 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)\,(c+dx^2)^3}+\frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)\,(c+dx^2)^2}+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\,(c+dx^2)}+\frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}$$

```
[Out] -1/6*d*x*(b*x^2+a)^(5/2)/c/(-a*d+b*c)/(d*x^2+c)^3+1/24*(-5*a*d+6*b*c)*x*(b*
x^2+a)^(3/2)/c^2/(-a*d+b*c)/(d*x^2+c)^2+1/16*a^2*(-5*a*d+6*b*c)*arctanh(x*(
-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-a*d+b*c)^(3/2)+1/16*a*(-
5*a*d+6*b*c)*x*(b*x^2+a)^(1/2)/c^3/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {390, 386, 385, 214}

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \frac{a^2(6bc-5ad)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}+\frac{ax\sqrt{a+bx^2}(6bc-5ad)}{16c^3\,(c+dx^2)\,(bc-ad)}+\frac{x(a+bx^2)^{3/2}\,(6bc-5ad)}{24c^2\,(c+dx^2)^2\,(bc-ad)}-\frac{dx(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3\,(bc-ad)}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] -1/6*(d*x*(a + b*x^2)^(5/2))/(c*(b*c - a*d)*(c + d*x^2)^3) + ((6*b*c - 5*a*
d)*x*(a + b*x^2)^(3/2))/(24*c^2*(b*c - a*d)*(c + d*x^2)^2) + (a*(6*b*c - 5*
a*d)*x*Sqrt[a + b*x^2])/(16*c^3*(b*c - a*d)*(c + d*x^2)) + (a^2*(6*b*c - 5*
a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*(b
*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)\,(c+dx^2)^3}+\frac{(6bc-5ad)\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx}{6c(bc-ad)}\\
&= -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)\,(c+dx^2)^3}+\frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)\,(c+dx^2)^2}+\frac{(a(6bc-5ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{8c^2(bc-ad)}\\
&= -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)\,(c+dx^2)^3}+\frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)\,(c+dx^2)^2}\\
&\quad+\frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)\,(c+dx^2)}+\frac{(a^2(6bc-5ad))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{16c^3(bc-ad)}
\end{aligned}
$$

$$= -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)(c+dx^2)^3} + \frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)(c+dx^2)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)(c+dx^2)} + \frac{(a^2(6bc-5ad))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16c^3(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{5/2}}{6c(bc-ad)(c+dx^2)^3} + \frac{(6bc-5ad)x(a+bx^2)^{3/2}}{24c^2(bc-ad)(c+dx^2)^2} + \frac{a(6bc-5ad)x\sqrt{a+bx^2}}{16c^3(bc-ad)(c+dx^2)} + \frac{a^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 10.51 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \frac{ax\left(1+\frac{bx^2}{a}\right)\left(c(a+bx^2)\left(-4b^2c^2x^2(3c+dx^2)-2abc(15c^2+11cdx^2+4d^2x^4)+a^2d(33\right.\right.}{48c^4(-bc+ad)(a+bx^2)^{3/2}(}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^4,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(a + b*x^2)*(-4*b^2*c^2*x^2*(3*c + d*x^2) - 2*a*b*c
*(15*c^2 + 11*c*d*x^2 + 4*d^2*x^4) + a^2*d*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^
4)) + (3*a^2*(-6*b*c + 5*a*d)*(c + d*x^2)^3*ArcTanh[Sqrt[((b*c - a*d)*x^2)/
(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(48*c^4*(-(b*c
) + a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^3)
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-11x\sqrt{bx^2+a}\left(-\frac{10\left(\frac{2bx^2}{5}+a\right)bc^3}{11}+d\left(-\frac{4}{33}b^2x^4-\frac{2}{3}abx^2+a^2\right)c^2+\frac{40x^2\left(-\frac{bx^2}{5}+a\right)d^2ac}{33}+\frac{5a^2d^3x^4}{11}\right)\sqrt{(ad-bc)c}+5(dx^2+c}{16\sqrt{(ad-bc)c}\,(ad-bc)c^3(dx^2+c)^3}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/((a*d-b*c)*c)^(1/2)*(-11*x*(b*x^2+a)^(1/2)*(-10/11*(2/5*b*x^2+a)*b*c^
3+d*(-4/33*b^2*x^4-2/3*a*b*x^2+a^2)*c^2+40/33*x^2*(-1/5*b*x^2+a)*d^2*a*c+5/
```

```
11*a^2*d^3*x^4)*((a*d-b*c)*c)^(1/2)+5*(d*x^2+c)^3*(a*d-6/5*b*c)*a^2*arctan(
c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(a*d-b*c)/c^3/(d*x^2+c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(175) = 350$.

Time = 0.44 (sec) , antiderivative size = 972, normalized size of antiderivative = 4.88

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \left[\frac{3\,(6\,a^2bc^4-5\,a^3c^3d+(6\,a^2bcd^3-5\,a^3d^4)x^6+3\,(6\,a^2bc^2d^2-5\,a^3cd^3)x^4+3\,(6\,a^2bc^3d\right.$$

$$-\frac{3\,(6\,a^2bc^4-5\,a^3c^3d+(6\,a^2bcd^3-5\,a^3d^4)x^6+3\,(6\,a^2bc^2d^2-5\,a^3cd^3)x^4+3\,(6\,a^2bc^3d-5\,a^3c^2d^2)x^2)\sqrt{-bc^2}}{96\,(b^2c^9-2\,abc^8d}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^3*d^4)*x^6 + 3*
(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a^3*c^2*d^2)*x^2
)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2
+ 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2
- a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((4*b^3*c^4*d +
4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a
*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*
a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^9 - 2*a*b*c^8*d +
a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*
d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2
*c^6*d^3)*x^2), -1/96*(3*(6*a^2*b*c^4 - 5*a^3*c^3*d + (6*a^2*b*c*d^3 - 5*a^
3*d^4)*x^6 + 3*(6*a^2*b*c^2*d^2 - 5*a^3*c*d^3)*x^4 + 3*(6*a^2*b*c^3*d - 5*a
^3*c^2*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b
*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) - 2*((4*b^3*c^4*d + 4*a*b^2*c^3*d^2 - 23*a^2*b*c^2*d^3 + 15*a^
3*c*d^4)*x^5 + 2*(6*b^3*c^5 + 5*a*b^2*c^4*d - 31*a^2*b*c^3*d^2 + 20*a^3*c^2
*d^3)*x^3 + 3*(10*a*b^2*c^5 - 21*a^2*b*c^4*d + 11*a^3*c^3*d^2)*x)*sqrt(b*x^
2 + a))/(b^2*c^9 - 2*a*b*c^8*d + a^2*c^7*d^2 + (b^2*c^6*d^3 - 2*a*b*c^5*d^4
 + a^2*c^4*d^5)*x^6 + 3*(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4)*x^4 + 3
*(b^2*c^8*d - 2*a*b*c^7*d^2 + a^2*c^6*d^3)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**4,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^4}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. 2(175) = 350.

Time = 1.34 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.62

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^4}\,dx = -\frac{\left(6\,a^2b^{\frac{3}{2}}c-5\,a^3\sqrt{b}d\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,(bc^4-ac^3d)\sqrt{-b^2c^2+abcd}}$$
$$-\frac{18\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}cd^4-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5-96\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{9}{2}}c^4d+96\,(\sqrt{}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^4,x, algorithm="giac")
[Out] -1/16*(6*a^2*b^(3/2)*c - 5*a^3*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*x
^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^4 - a*c^3*d)*sq
rt(-b^2*c^2 + a*b*c*d)) - 1/24*(18*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(
3/2)*c*d^4 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*sqrt(b)*d^5 - 96*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*a*b^(7/2)*c^3*d^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*c^2*
d^3 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x
 - sqrt(b*x^2 + a))^8*a^4*sqrt(b)*d^5 - 128*(sqrt(b)*x - sqrt(b*x^2 + a))^6
```

```
*b^(11/2)*c^5 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 720*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2)*c^3*d^2 - 968*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*a^3*b^(5/2)*c^2*d^3 + 620*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*
b^(3/2)*c*d^4 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 - 96*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c^4*d - 288*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^3*b^(7/2)*c^3*d^2 + 864*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^
(5/2)*c^2*d^3 - 600*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150
*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*sqrt(b)*d^5 - 48*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^4*b^(7/2)*c^3*d^2 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(
5/2)*c^2*d^3 + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*sqrt(b)*d^5 - 4*a^6*b^(5/2)*c^2*d^3 - 8*
a^7*b^(3/2)*c*d^4 + 15*a^8*sqrt(b)*d^5)/((b*c^4*d^2 - a*c^3*d^3)*((sqrt(b)*
x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^4}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}}{{\left(d\,x^2+c\right)}^4}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^4,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^4, x)
```

3.61 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 300

$$
\begin{aligned}
\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = & -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\,(c+dx^2)^4} \\
& + \frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\,(c+dx^2)^3} + \frac{(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}}{192c^3d(bc-ad)\,(c+dx^2)^2} \\
& + \frac{(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\,(c+dx^2)} \\
& + \frac{a^2(48b^2c^2-80abcd+35a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}
\end{aligned}
$$

```
[Out] 1/128*a^2*(35*a^2*d^2-80*a*b*c*d+48*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(
1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(5/2)-1/8*(-a*d+b*c)*x*(b*x^2+a)^(
1/2)/c/d/(d*x^2+c)^4+1/48*(7*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/d/(d*x^2+c)^3
+1/192*(-35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c^3/d/(-a*d+b*c
)/(d*x^2+c)^2+1/384*(105*a^3*d^3-170*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3*c^3)
*x*(b*x^2+a)^(1/2)/c^4/d/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {424, 541, 12, 385, 214}

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \frac{a^2(35a^2d^2-80abcd+48b^2c^2)\,\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}} + \frac{x\sqrt{a+bx^2}(-35a^2d^2+24abcd+8b^2c^2)}{192c^3d\,(c+dx^2)^2\,(bc-ad)} + \frac{x\sqrt{a+bx^2}(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3)}{384c^4d\,(c+dx^2)\,(bc-ad)^2} + \frac{x\sqrt{a+bx^2}(7ad+2bc)}{48c^2d\,(c+dx^2)^3} - \frac{x\sqrt{a+bx^2}(bc-ad)}{8cd\,(c+dx^2)^4}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] -1/8*((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*(c + d*x^2)^4) + ((2*b*c + 7*a*d)
*x*Sqrt[a + b*x^2])/(48*c^2*d*(c + d*x^2)^3) + ((8*b^2*c^2 + 24*a*b*c*d - 3
5*a^2*d^2)*x*Sqrt[a + b*x^2])/(192*c^3*d*(b*c - a*d)*(c + d*x^2)^2) + ((16*
b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sqrt[a + b*x^2]
)/(384*c^4*d*(b*c - a*d)^2*(c + d*x^2)) + (a^2*(48*b^2*c^2 - 80*a*b*c*d + 3
5*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(128*c^(
9/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{\int \frac{a(bc+7ad)+2b(bc+3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^4}\,dx}{8cd}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}+\frac{\int \frac{a(bc-ad)(4bc+35ad)+4b(bc-ad)(2bc+7ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx}{48c^2d(bc-ad)}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}\\
&\quad+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}\\
&\quad+\frac{\int \frac{a(bc-ad)\left(8b^2c^2+100abcd-105a^2d^2\right)+2b(bc-ad)\left(8b^2c^2+24abcd-35a^2d^2\right)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{192c^3d(bc-ad)^2}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}\\
&\quad+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}\\
&\quad+\frac{\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\left(c+dx^2\right)}\\
&\quad+\frac{\int \frac{3a^2d(bc-ad)\left(48b^2c^2-80abcd+35a^2d^2\right)}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{384c^4d(bc-ad)^3}
\end{aligned}
$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}$$
$$+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}$$
$$+\frac{\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\left(c+dx^2\right)}$$
$$+\frac{\left(a^2\left(48b^2c^2-80abcd+35a^2d^2\right)\right)\int\frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{128c^4(bc-ad)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}$$
$$+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}$$
$$+\frac{\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\left(c+dx^2\right)}$$
$$+\frac{\left(a^2\left(48b^2c^2-80abcd+35a^2d^2\right)\right)\mathrm{Subst}\left(\int\frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{128c^4(bc-ad)^2}$$

$$= -\frac{(bc-ad)x\sqrt{a+bx^2}}{8cd\left(c+dx^2\right)^4}+\frac{(2bc+7ad)x\sqrt{a+bx^2}}{48c^2d\left(c+dx^2\right)^3}$$
$$+\frac{\left(8b^2c^2+24abcd-35a^2d^2\right)x\sqrt{a+bx^2}}{192c^3d(bc-ad)\left(c+dx^2\right)^2}$$
$$+\frac{\left(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3\right)x\sqrt{a+bx^2}}{384c^4d(bc-ad)^2\left(c+dx^2\right)}$$
$$+\frac{a^2\left(48b^2c^2-80abcd+35a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 10.91 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.02

$$\int\frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx=\frac{ax\left(1+\frac{bx^2}{a}\right)\left(c\left(a+bx^2\right)\left(16b^3c^3x^2\left(6c^2+4cdx^2+d^2x^4\right)+8ab^2c^2\left(30c^3+26c^2dx^2+19\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^5,x]
```

```
[Out] (a*x*(1 + (b*x^2)/a)*(c*(a + b*x^2)*(16*b^3*c^3*x^2*(6*c^2 + 4*c*d*x^2 + d^
2*x^4) + 8*a*b^2*c^2*(30*c^3 + 26*c^2*d*x^2 + 19*c*d^2*x^4 + 5*d^3*x^6) - 2
```

```
*a^2*b*c*d*(264*c^3 + 421*c^2*d*x^2 + 314*c*d^2*x^4 + 85*d^3*x^6) + a^3*d^2
*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3*x^6)) + (3*a^2*(48*b^2*
c^2 - 80*a*b*c*d + 35*a^2*d^2)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^5*(b*c
 - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)^4)
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{93x\Big(d^2\left(\frac{35}{93}d^3x^6+\frac{385}{279}c\,d^2x^4+\frac{511}{279}c^2d\,x^2+c^3\right)a^3-\frac{176bd\left(\frac{85}{264}d^3x^6+\frac{157}{132}c\,d^2x^4+\frac{421}{264}c^2d\,x^2+c^3\right)c\,a^2}{93}+\frac{80b^2\left(\frac{1}{6}d^3x^6+\frac{19}{30}c\,d^2x^4+\frac{13}{15}c^2}{93}}{128\sqrt{(ad-bc)c}\,(}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/((a*d-b*c)*c)^(1/2)*(93*x*(d^2*(35/93*d^3*x^6+385/279*c*d^2*x^4+511/2
79*c^2*d*x^2+c^3)*a^3-176/93*b*d*(85/264*d^3*x^6+157/132*c*d^2*x^4+421/264*
c^2*d*x^2+c^3)*c*a^2+80/93*b^2*(1/6*d^3*x^6+19/30*c*d^2*x^4+13/15*c^2*d*x^2
+c^3)*c^2*a+32/93*x^2*(1/6*d^2*x^4+2/3*c*d*x^2+c^2)*b^3*c^3)*((a*d-b*c)*c)^
(1/2)*(b*x^2+a)^(1/2)-35*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*(d*x^2+c)^4*a
^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(d*x^2+c)^4/(a*d-b*c)^2
/c^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(272) = 544.

Time = 1.37 (sec) , antiderivative size = 1604, normalized size of antiderivative = 5.35

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\left(c+dx^2\right)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(48*a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*
c^2*d^4 - 80*a^3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3
*b*c^2*d^4 + 35*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 +
 35*a^4*c^2*d^4)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*
d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 +
a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqr
t(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4
*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 - 105
*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3 +
```

```
1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*d -
 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(80*a
*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sqrt(
b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3 + (
b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(b^3
*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3*c^
10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c^11
*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2), -1/768*(3*(48*
a^2*b^2*c^6 - 80*a^3*b*c^5*d + 35*a^4*c^4*d^2 + (48*a^2*b^2*c^2*d^4 - 80*a^
3*b*c*d^5 + 35*a^4*d^6)*x^8 + 4*(48*a^2*b^2*c^3*d^3 - 80*a^3*b*c^2*d^4 + 35
*a^4*c*d^5)*x^6 + 6*(48*a^2*b^2*c^4*d^2 - 80*a^3*b*c^3*d^3 + 35*a^4*c^2*d^4
)*x^4 + 4*(48*a^2*b^2*c^5*d - 80*a^3*b*c^4*d^2 + 35*a^4*c^3*d^3)*x^2)*sqrt(
-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*s
qrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*
b^4*c^5*d^2 + 24*a*b^3*c^4*d^3 - 210*a^2*b^2*c^3*d^4 + 275*a^3*b*c^2*d^5 -
105*a^4*c*d^6)*x^7 + (64*b^4*c^6*d + 88*a*b^3*c^5*d^2 - 780*a^2*b^2*c^4*d^3
 + 1013*a^3*b*c^3*d^4 - 385*a^4*c^2*d^5)*x^5 + (96*b^4*c^7 + 112*a*b^3*c^6*
d - 1050*a^2*b^2*c^5*d^2 + 1353*a^3*b*c^4*d^3 - 511*a^4*c^3*d^4)*x^3 + 3*(8
0*a*b^3*c^7 - 256*a^2*b^2*c^6*d + 269*a^3*b*c^5*d^2 - 93*a^4*c^4*d^3)*x)*sq
rt(b*x^2 + a))/(b^3*c^12 - 3*a*b^2*c^11*d + 3*a^2*b*c^10*d^2 - a^3*c^9*d^3
+ (b^3*c^8*d^4 - 3*a*b^2*c^7*d^5 + 3*a^2*b*c^6*d^6 - a^3*c^5*d^7)*x^8 + 4*(
b^3*c^9*d^3 - 3*a*b^2*c^8*d^4 + 3*a^2*b*c^7*d^5 - a^3*c^6*d^6)*x^6 + 6*(b^3
*c^10*d^2 - 3*a*b^2*c^9*d^3 + 3*a^2*b*c^8*d^4 - a^3*c^7*d^5)*x^4 + 4*(b^3*c
^11*d - 3*a*b^2*c^10*d^2 + 3*a^2*b*c^9*d^3 - a^3*c^8*d^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**5,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^5}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1557 vs. $2(272) = 544$.

Time = 4.02 (sec) , antiderivative size = 1557, normalized size of antiderivative = 5.19

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^5,x, algorithm="giac")
```

```
[Out] -1/128*(48*a^2*b^(5/2)*c^2 - 80*a^3*b^(3/2)*c*d + 35*a^4*sqrt(b)*d^2)*arcta
n(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b
*c*d))/((b^2*c^6 - 2*a*b*c^5*d + a^2*c^4*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) - 1
/192*(144*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^2*b^(5/2)*c^2*d^5 - 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^14*a^3*b^(3/2)*c*d^6 + 105*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*a^4*sqrt(b)*d^7 + 2016*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2
)*c^3*d^4 - 4368*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(5/2)*c^2*d^5 + 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6 - 735*(sqrt(b)*x - sqr
t(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^
(13/2)*c^6*d + 4096*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*b^(11/2)*c^5*d^2 + 7
936*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2)*c^4*d^3 - 26624*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3*d^4 + 26944*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^4*b^(5/2)*c^2*d^5 - 12320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b
^(3/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/2)*c^7 - 1024*(sqrt(b)*x - sqrt(b*x^
2 + a))^8*a*b^(13/2)*c^6*d + 27392*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(1
1/2)*c^5*d^2 - 65920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 +
81680*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(7/2)*c^3*d^4 - 58840*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d^5 + 22750*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*a^6*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b
)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(13/2)*c^6*d - 8192*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(11/2)*c^5*d^2 + 47104*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^4*b^(9/2)*c^4*d^3 - 74240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a
^5*b^(7/2)*c^3*d^4 + 56416*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*
d^5 - 22400*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 + 3675*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*a^8*sqrt(b)*d^7 - 1536*(sqrt(b)*x - sqrt(b*x^2 +
a))^4*a^4*b^(11/2)*c^5*d^2 - 2304*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/
2)*c^4*d^3 + 17696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 - 23
152*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(5/2)*c^2*d^5 + 11690*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^8*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^9*sqrt(b)*d^7 - 256*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^
3 - 512*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(7/2)*c^3*d^4 + 2896*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*a^8*b^(5/2)*c^2*d^5 - 2800*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^9*b^(3/2)*c*d^6 + 735*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b
)*d^7 - 16*a^8*b^(7/2)*c^3*d^4 - 40*a^9*b^(5/2)*c^2*d^5 + 170*a^10*b^(3/2)*
```

```
c*d^6 - 105*a^11*sqrt(b)*d^7)/((b^2*c^6*d^2 - 2*a*b*c^5*d^3 + a^2*c^4*d^4)*
((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^5}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^5}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^5,x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^5, x)
```

3.62 $\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 349

$$
\begin{aligned}
\int (a &+ bx^2)^{5/2} (c+dx^2)^3 \, dx = \frac{a^2(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x\sqrt{a+bx^2}}{1024b^3} \\
&+ \frac{a(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x(a+bx^2)^{3/2}}{1536b^3} \\
&+ \frac{(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,x(a+bx^2)^{5/2}}{1920b^3} \\
&+ \frac{d(152b^2c^2-68abcd+15a^2d^2)\,x(a+bx^2)^{7/2}}{960b^3} \\
&+ \frac{d(16bc-5ad)x(a+bx^2)^{7/2}(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)^2}{12b} \\
&+ \frac{a^3(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3)\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}
\end{aligned}
$$

```
[Out] 1/1536*a*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2+a
)^(3/2)/b^3+1/1920*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*
x*(b*x^2+a)^(5/2)/b^3+1/960*d*(15*a^2*d^2-68*a*b*c*d+152*b^2*c^2)*x*(b*x^2+
a)^(7/2)/b^3+1/120*d*(-5*a*d+16*b*c)*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b^2+1/12*d
*x*(b*x^2+a)^(7/2)*(d*x^2+c)^2/b+1/1024*a^3*(-5*a^3*d^3+36*a^2*b*c*d^2-120*
a*b^2*c^2*d+320*b^3*c^3)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/1024*
a^2*(-5*a^3*d^3+36*a^2*b*c*d^2-120*a*b^2*c^2*d+320*b^3*c^3)*x*(b*x^2+a)^(1/
2)/b^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {427, 542, 396, 201, 223, 212}

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3\,dx=\frac{dx\left(a+bx^2\right)^{7/2}\left(15a^2d^2-68abcd+152b^2c^2\right)}{960b^3}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1024b^{7/2}}+\frac{x\left(a+bx^2\right)^{5/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1920b^3}+\frac{ax\left(a+bx^2\right)^{3/2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1536b^3}+\frac{a^2x\sqrt{a+bx^2}\left(-5a^3d^3+36a^2bcd^2-120ab^2c^2d+320b^3c^3\right)}{1024b^3}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)\left(16bc-5ad\right)}{120b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}$$

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (a^2*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*Sqrt[a
+ b*x^2])/(1024*b^3) + (a*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*b*c*d^2 -
 5*a^3*d^3)*x*(a + b*x^2)^(3/2))/(1536*b^3) + ((320*b^3*c^3 - 120*a*b^2*c^2
*d + 36*a^2*b*c*d^2 - 5*a^3*d^3)*x*(a + b*x^2)^(5/2))/(1920*b^3) + (d*(152*
b^2*c^2 - 68*a*b*c*d + 15*a^2*d^2)*x*(a + b*x^2)^(7/2))/(960*b^3) + (d*(16*
b*c - 5*a*d)*x*(a + b*x^2)^(7/2)*(c + d*x^2))/(120*b^2) + (d*x*(a + b*x^2)^
(7/2)*(c + d*x^2)^2)/(12*b) + (a^3*(320*b^3*c^3 - 120*a*b^2*c^2*d + 36*a^2*
b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(7/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} \\
&\quad+\frac{\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\left(c(12bc-ad)+d(16bc-5ad)x^2\right)\,dx}{12b} \\
&= \frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{120b^2}+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b} \\
&\quad+\frac{\int \left(a+bx^2\right)^{5/2}\left(c\left(120b^2c^2-26abcd+5a^2d^2\right)+d\left(152b^2c^2-68abcd+15a^2d^2\right)x^2\right)\,dx}{120b^2} \\
&= \frac{d\left(152b^2c^2-68abcd+15a^2d^2\right)x\left(a+bx^2\right)^{7/2}}{960b^3}+\frac{d(16bc-5ad)x\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)}{120b^2} \\
&\quad+\frac{dx\left(a+bx^2\right)^{7/2}\left(c+dx^2\right)^2}{12b}+\frac{\left(320b^3c^3-120ab^2c^2d+36a^2bcd^2-5a^3d^3\right)\int \left(a+bx^2\right)^{5/2}\,dx}{320b^3}
\end{aligned}
$$

$$= \frac{(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x(a+bx^2)^{5/2}}{1920b^3}$$
$$+ \frac{d(152b^2c^2 - 68abcd + 15a^2d^2)\,x(a+bx^2)^{7/2}}{960b^3}$$
$$+ \frac{d(16bc - 5ad)x(a+bx^2)^{7/2}\,(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\,(c+dx^2)^2}{12b}$$
$$+ \frac{(a(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3))\int (a+bx^2)^{3/2}\,dx}{384b^3}$$

$$= \frac{a(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x(a+bx^2)^{3/2}}{1536b^3}$$
$$+ \frac{(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x(a+bx^2)^{5/2}}{1920b^3}$$
$$+ \frac{d(152b^2c^2 - 68abcd + 15a^2d^2)\,x(a+bx^2)^{7/2}}{960b^3}$$
$$+ \frac{d(16bc - 5ad)x(a+bx^2)^{7/2}\,(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\,(c+dx^2)^2}{12b}$$
$$+ \frac{(a^2(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3))\int \sqrt{a+bx^2}\,dx}{512b^3}$$

$$= \frac{a^2(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x\sqrt{a+bx^2}}{1024b^3}$$
$$+ \frac{a(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x(a+bx^2)^{3/2}}{1536b^3}$$
$$+ \frac{(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\,x(a+bx^2)^{5/2}}{1920b^3}$$
$$+ \frac{d(152b^2c^2 - 68abcd + 15a^2d^2)\,x(a+bx^2)^{7/2}}{960b^3}$$
$$+ \frac{d(16bc - 5ad)x(a+bx^2)^{7/2}\,(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\,(c+dx^2)^2}{12b}$$
$$+ \frac{(a^3(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{1024b^3}$$

$$= \frac{a^2(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x(a+bx^2)^{3/2}}{1536b^3} + \frac{(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x(a+bx^2)^{5/2}}{1920b^3} + \frac{d(152b^2c^2 - 68abcd + 15a^2d^2)\, x(a+bx^2)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x(a+bx^2)^{7/2}\,(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\,(c+dx^2)^2}{12b} + \frac{(a^3(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{1024b^3}$$

$$= \frac{a^2(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x\sqrt{a+bx^2}}{1024b^3} + \frac{a(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x(a+bx^2)^{3/2}}{1536b^3} + \frac{(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\, x(a+bx^2)^{5/2}}{1920b^3} + \frac{d(152b^2c^2 - 68abcd + 15a^2d^2)\, x(a+bx^2)^{7/2}}{960b^3} + \frac{d(16bc-5ad)x(a+bx^2)^{7/2}\,(c+dx^2)}{120b^2} + \frac{dx(a+bx^2)^{7/2}\,(c+dx^2)^2}{12b} + \frac{a^3(320b^3c^3 - 120ab^2c^2d + 36a^2bcd^2 - 5a^3d^3)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3\, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(75a^5d^3 - 10a^4bd^2(54c+5dx^2) + 40a^3b^2d(45c^2+9cdx^2+d^2x^4) + 128b^5x^4(20c^3 +$$

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^3,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(75*a^5*d^3 - 10*a^4*b*d^2*(54*c + 5*d*x^2) + 40
*a^3*b^2*d*(45*c^2 + 9*c*d*x^2 + d^2*x^4) + 128*b^5*x^4*(20*c^3 + 45*c^2*d*
x^2 + 36*c*d^2*x^4 + 10*d^3*x^6) + 48*a^2*b^3*(220*c^3 + 295*c^2*d*x^2 + 18
6*c*d^2*x^4 + 45*d^3*x^6) + 64*a*b^4*x^2*(130*c^3 + 255*c^2*d*x^2 + 189*c*d
^2*x^4 + 50*d^3*x^6)) + 15*a^3*(-320*b^3*c^3 + 120*a*b^2*c^2*d - 36*a^2*b*c
*d^2 + 5*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(15360*b^(7/2))
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$-\frac{5\left(a^{3}\left(a^{3} d^{3}-\frac{36}{5} a^{2} b c\, d^{2}+24 a\, b^{2} c^{2} d-64 b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{x \sqrt{b}}\right)-x\left(\frac{512 x^{4}\left(\frac{1}{2} d^{3} x^{6}+\frac{9}{5} c\, d^{2} x^{4}+\frac{9}{4} c^{2} d\, x^{2}+c^{3}\right) b^{\frac{11}{2}}}{15}+\left(\frac{704 a\left(\frac{9}{44}\right.}{}\right.\right.\right.}{}$
risch	$\frac{x\left(1280 b^{5} d^{3} x^{10}+3200 a\, b^{4} d^{3} x^{8}+4608 b^{5} d^{2} c\, x^{8}+2160 a^{2} b^{3} d^{3} x^{6}+12096 a\, b^{4} c\, d^{2} x^{6}+5760 b^{5} c^{2} d\, x^{6}+40 a^{3} b^{2} d^{3} x^{4}+8928 a^{2} b^{3} c\, d^{2} x^{4}\right.}{}$
default	$c^{3}\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a\left(\frac{x\left(b\, x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x \sqrt{b\, x^{2}+a}}{2}+\frac{a \ln\left(x \sqrt{b}+\sqrt{b\, x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)+d^{3}\left(\frac{x^{5}\left(b\, x^{2}+a\right)^{\frac{7}{2}}}{12 b}-\frac{5 a\left(\frac{x^{3}\left(b\, x^{2}+\right.}{10 b}\right.}{}\right.$

```
[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -5/1024*(a^3*(a^3*d^3-36/5*a^2*b*c*d^2+24*a*b^2*c^2*d-64*b^3*c^3)*arctanh((
b*x^2+a)^(1/2)/x/b^(1/2))-x*(512/15*x^4*(1/2*d^3*x^6+9/5*c*d^2*x^4+9/4*c^2*
d*x^2+c^3)*b^(11/2)+(704/5*a*(9/44*d^3*x^6+93/110*c*d^2*x^4+59/44*c^2*d*x^2
+c^3)*b^(7/2)+1664/15*x^2*(5/13*d^3*x^6+189/130*c*d^2*x^4+51/26*c^2*d*x^2+c
^3)*b^(9/2)+((8/15*d^2*x^4+24/5*c*d*x^2+24*c^2)*b^(5/2)+((-2/3*d*x^2-36/5*c
)*b^(3/2)+a*d*b^(1/2))*d*a)*d*a^2)*a)*(b*x^2+a)^(1/2))/b^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx = \left[-\frac{15\,(320\,a^3b^3c^3-120\,a^4b^2c^2d+36\,a^5bcd^2-5\,a^6d^3)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-\ldots}{\ldots}\right.$$

$$-\frac{15\,(320\,a^3b^3c^3-120\,a^4b^2c^2d+36\,a^5bcd^2-5\,a^6d^3)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(1280\,b^6d^3x^{11}+128\,(36\,b^6cd^2+\ldots}{\ldots}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6
*d^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1280*b^6
*d^3*x^11 + 128*(36*b^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*
a*b^5*c*d^2 + 15*a^2*b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 111
6*a^2*b^4*c*d^2 + 5*a^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2
*d + 36*a^3*b^3*c*d^2 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*
b^3*c^2*d - 36*a^4*b^2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/153
60*(15*(320*a^3*b^3*c^3 - 120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (1280*b^6*d^3*x^11 + 128*(36*b
^6*c*d^2 + 25*a*b^5*d^3)*x^9 + 144*(40*b^6*c^2*d + 84*a*b^5*c*d^2 + 15*a^2*
b^4*d^3)*x^7 + 8*(320*b^6*c^3 + 2040*a*b^5*c^2*d + 1116*a^2*b^4*c*d^2 + 5*a
^3*b^3*d^3)*x^5 + 10*(832*a*b^5*c^3 + 1416*a^2*b^4*c^2*d + 36*a^3*b^3*c*d^2
 - 5*a^4*b^2*d^3)*x^3 + 15*(704*a^2*b^4*c^3 + 120*a^3*b^3*c^2*d - 36*a^4*b^
2*c*d^2 + 5*a^5*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(352) = 704$.

Time = 0.66 (sec) , antiderivative size = 823, normalized size of antiderivative = 2.36

$$\int (a+bx^2)^{5/2} (c+dx^2)^3 \, dx = \begin{cases} \sqrt{a+bx^2} \left(\frac{b^2d^3x^{11}}{12} + \frac{x^9 \cdot \left(\frac{25ab^2d^3}{12}+3b^3cd^2\right)}{10b} + \frac{x^7 \cdot \left(3a^2bd^3+9ab^2cd^2-\frac{9a\left(\frac{25ab^2d^3}{12}+3b^3cd^2\right)}{10b}+3b^3c^2d\right)}{8b} + \frac{x^5 \left(a^3d^3+9 \right.}{} \right. \\ a^{\frac{5}{2}} \left(c^3x + c^2dx^3 + \frac{3cd^2x^5}{5} + \frac{d^3x^7}{7}\right) \end{cases}$$

[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**3,x)

```
[Out] Piecewise((sqrt(a + b*x**2)*(b**2*d**3*x**11/12 + x**9*(25*a*b**2*d**3/12 +
 3*b**3*c*d**2)/(10*b) + x**7*(3*a**2*b*d**3 + 9*a*b**2*c*d**2 - 9*a*(25*a*
b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(8*b) + x**5*(a**3*d*
*3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a*b**2*c*d*
*2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(8*b)
+ b**3*c**3)/(6*b) + x**3*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2*c**3
- 5*a*(a**3*d**3 + 9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 +
 9*a*b**2*c*d**2 - 9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*
c**2*d)/(8*b) + b**3*c**3)/(6*b))/(4*b) + x*(3*a**3*c**2*d + 3*a**2*b*c**3
- 3*a*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2*c**3 - 5*a*(a**3*d**3 + 9
*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a*b**2*c*d**2 - 9
*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(8*b) + b**3
*c**3)/(6*b))/(4*b))/(2*b)) + (a**3*c**3 - a*(3*a**3*c**2*d + 3*a**2*b*c**3
 - 3*a*(3*a**3*c*d**2 + 9*a**2*b*c**2*d + 3*a*b**2*c**3 - 5*a*(a**3*d**3 +
9*a**2*b*c*d**2 + 9*a*b**2*c**2*d - 7*a*(3*a**2*b*d**3 + 9*a*b**2*c*d**2 -
9*a*(25*a*b**2*d**3/12 + 3*b**3*c*d**2)/(10*b) + 3*b**3*c**2*d)/(8*b) + b**
3*c**3)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*
b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/
2)*(c**3*x + c**2*d*x**3 + 3*c*d**2*x**5/5 + d**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.28

$$
\begin{aligned}
\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3\,dx &= \frac{(bx^2+a)^{\frac{7}{2}}d^3x^5}{12\,b} + \frac{3\,(bx^2+a)^{\frac{7}{2}}cd^2x^3}{10\,b} \\
&\quad - \frac{(bx^2+a)^{\frac{7}{2}}ad^3x^3}{24\,b^2} + \frac{1}{6}\,(bx^2+a)^{\frac{5}{2}}c^3x + \frac{5}{24}\,(bx^2+a)^{\frac{3}{2}}ac^3x + \frac{5}{16}\,\sqrt{bx^2+a}a^2c^3x \\
&\quad + \frac{3\,(bx^2+a)^{\frac{7}{2}}c^2dx}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}ac^2dx}{16\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2c^2dx}{64\,b} \\
&\quad - \frac{15\,\sqrt{bx^2+a}a^3c^2dx}{128\,b} - \frac{9\,(bx^2+a)^{\frac{7}{2}}acd^2x}{80\,b^2} + \frac{3\,(bx^2+a)^{\frac{5}{2}}a^2cd^2x}{160\,b^2} \\
&\quad + \frac{3\,(bx^2+a)^{\frac{3}{2}}a^3cd^2x}{128\,b^2} + \frac{9\,\sqrt{bx^2+a}a^4cd^2x}{256\,b^2} + \frac{(bx^2+a)^{\frac{7}{2}}a^2d^3x}{64\,b^3} - \frac{(bx^2+a)^{\frac{5}{2}}a^3d^3x}{384\,b^3} \\
&\quad - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^4d^3x}{1536\,b^3} - \frac{5\,\sqrt{bx^2+a}a^5d^3x}{1024\,b^3} + \frac{5\,a^3c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} \\
&\quad - \frac{15\,a^4c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{9\,a^5cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}} - \frac{5\,a^6d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{7}{2}}}
\end{aligned}
$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(b*x^2 + a)^(7/2)*d^3*x^5/b + 3/10*(b*x^2 + a)^(7/2)*c*d^2*x^3/b - 1/2
4*(b*x^2 + a)^(7/2)*a*d^3*x^3/b^2 + 1/6*(b*x^2 + a)^(5/2)*c^3*x + 5/24*(b*x
^2 + a)^(3/2)*a*c^3*x + 5/16*sqrt(b*x^2 + a)*a^2*c^3*x + 3/8*(b*x^2 + a)^(7
/2)*c^2*d*x/b - 1/16*(b*x^2 + a)^(5/2)*a*c^2*d*x/b - 5/64*(b*x^2 + a)^(3/2)
*a^2*c^2*d*x/b - 15/128*sqrt(b*x^2 + a)*a^3*c^2*d*x/b - 9/80*(b*x^2 + a)^(7
/2)*a*c*d^2*x/b^2 + 3/160*(b*x^2 + a)^(5/2)*a^2*c*d^2*x/b^2 + 3/128*(b*x^2
+ a)^(3/2)*a^3*c*d^2*x/b^2 + 9/256*sqrt(b*x^2 + a)*a^4*c*d^2*x/b^2 + 1/64*(
b*x^2 + a)^(7/2)*a^2*d^3*x/b^3 - 1/384*(b*x^2 + a)^(5/2)*a^3*d^3*x/b^3 - 5/
1536*(b*x^2 + a)^(3/2)*a^4*d^3*x/b^3 - 5/1024*sqrt(b*x^2 + a)*a^5*d^3*x/b^3
 + 5/16*a^3*c^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 15/128*a^4*c^2*d*arcsinh(b
*x/sqrt(a*b))/b^(3/2) + 9/256*a^5*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/
1024*a^6*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \frac{1}{15360} \left(2 \left(4 \left(2 \left(8 \left(10 \, b^2 d^3 x^2 + \frac{36 \, b^{12} c d^2 + 25 \, a b^{11} d^3}{b^{10}}\right) x^2 + \frac{9 \left(40 \, b^{12} c^2 d + 84 \, a b^{11} c d^2 + 15 \, a^2 b^{10} d^3\right)}{b^{10}} \cdots - \frac{\left(320 \, a^3 b^3 c^3 - 120 \, a^4 b^2 c^2 d + 36 \, a^5 b c d^2 - 5 \, a^6 d^3\right) \log\left(\left|-\sqrt{b} x + \sqrt{b x^2 + a}\right|\right)}{1024 \, b^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] 1/15360*(2*(4*(2*(8*(10*b^2*d^3*x^2 + (36*b^12*c*d^2 + 25*a*b^11*d^3)/b^10)
*x^2 + 9*(40*b^12*c^2*d + 84*a*b^11*c*d^2 + 15*a^2*b^10*d^3)/b^10)*x^2 + (3
20*b^12*c^3 + 2040*a*b^11*c^2*d + 1116*a^2*b^10*c*d^2 + 5*a^3*b^9*d^3)/b^10
)*x^2 + 5*(832*a*b^11*c^3 + 1416*a^2*b^10*c^2*d + 36*a^3*b^9*c*d^2 - 5*a^4*
b^8*d^3)/b^10)*x^2 + 15*(704*a^2*b^10*c^3 + 120*a^3*b^9*c^2*d - 36*a^4*b^8*
c*d^2 + 5*a^5*b^7*d^3)/b^10)*sqrt(b*x^2 + a)*x - 1/1024*(320*a^3*b^3*c^3 -
120*a^4*b^2*c^2*d + 36*a^5*b*c*d^2 - 5*a^6*d^3)*log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^3 \, dx = \int \left(b\,x^2+a\right)^{5/2} \left(d\,x^2+c\right)^3 \,dx$$

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^3, x)
```

3.63 $\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 241

$$\begin{aligned}\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx = {} & \frac{a^2(80b^2c^2-20abcd+3a^2d^2)\, x\sqrt{a+bx^2}}{256b^2} \\ & + \frac{a(80b^2c^2-20abcd+3a^2d^2)\, x(a+bx^2)^{3/2}}{384b^2} \\ & + \frac{(80b^2c^2-20abcd+3a^2d^2)\, x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} \\ & + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{a^3(80b^2c^2-20abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}\end{aligned}$$

```
[Out] 1/384*a*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(3/2)/b^2+1/480*(3*a^
2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(5/2)/b^2+3/80*d*(-a*d+4*b*c)*x*(b
*x^2+a)^(7/2)/b^2+1/10*d*x*(b*x^2+a)^(7/2)*(d*x^2+c)/b+1/256*a^3*(3*a^2*d^2
-20*a*b*c*d+80*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/256*a^
2*(3*a^2*d^2-20*a*b*c*d+80*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

= {427, 396, 201, 223, 212}

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx = \frac{x(a+bx^2)^{5/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{480b^2} + \frac{ax(a+bx^2)^{3/2}\left(3a^2d^2-20abcd+80b^2c^2\right)}{384b^2} + \frac{a^2x\sqrt{a+bx^2}(3a^2d^2-20abcd+80b^2c^2)}{256b^2} + \frac{a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^2-20abcd+80b^2c^2\right)}{256b^{5/2}} + \frac{3dx(a+bx^2)^{7/2}(4bc-ad)}{80b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b}$$

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (a^2*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(256*b^2) + (
a*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(3/2))/(384*b^2) + ((
80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*x*(a + b*x^2)^(5/2))/(480*b^2) + (3*d*
(4*b*c - a*d)*x*(a + b*x^2)^(7/2))/(80*b^2) + (d*x*(a + b*x^2)^(7/2)*(c + d
*x^2))/(10*b) + (a^3*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(256*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{\int (a+bx^2)^{5/2}\left(c(10bc-ad)+3d(4bc-ad)x^2\right)\,dx}{10b} \\
&= \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} \\
&\quad - \frac{(3ad(4bc-ad)-8bc(10bc-ad))\int (a+bx^2)^{5/2}\,dx}{80b^2} \\
&= \frac{(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} \\
&\quad + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{(a(80b^2c^2-20abcd+3a^2d^2))\int (a+bx^2)^{3/2}\,dx}{96b^2} \\
&= \frac{a(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{384b^2} \\
&\quad + \frac{(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} \\
&\quad + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{(a^2(80b^2c^2-20abcd+3a^2d^2))\int \sqrt{a+bx^2}\,dx}{128b^2} \\
&= \frac{a^2(80b^2c^2-20abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{256b^2} + \frac{a(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{3/2}}{384b^2} \\
&\quad + \frac{(80b^2c^2-20abcd+3a^2d^2)\,x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc-ad)x(a+bx^2)^{7/2}}{80b^2} \\
&\quad + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{(a^3(80b^2c^2-20abcd+3a^2d^2))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{256b^2}
\end{aligned}
$$

$$= \frac{a^2 (80b^2c^2 - 20abcd + 3a^2d^2)\, x\sqrt{a+bx^2}}{256b^2} + \frac{a(80b^2c^2 - 20abcd + 3a^2d^2)\, x(a+bx^2)^{3/2}}{384b^2} + \frac{(80b^2c^2 - 20abcd + 3a^2d^2)\, x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc - ad)x(a+bx^2)^{7/2}}{80b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{(a^3(80b^2c^2 - 20abcd + 3a^2d^2))\, \mathrm{Subst}\left(\int \frac{1}{1-bx^2}\, dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{256b^2}$$

$$= \frac{a^2 (80b^2c^2 - 20abcd + 3a^2d^2)\, x\sqrt{a+bx^2}}{256b^2} + \frac{a(80b^2c^2 - 20abcd + 3a^2d^2)\, x(a+bx^2)^{3/2}}{384b^2} + \frac{(80b^2c^2 - 20abcd + 3a^2d^2)\, x(a+bx^2)^{5/2}}{480b^2} + \frac{3d(4bc - ad)x(a+bx^2)^{7/2}}{80b^2} + \frac{dx(a+bx^2)^{7/2}(c+dx^2)}{10b} + \frac{a^3(80b^2c^2 - 20abcd + 3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79

$$\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx = \frac{\sqrt{b}x\sqrt{a+bx^2}(-45a^4d^2 + 30a^3bd(10c+dx^2) + 64b^4x^4(10c^2 + 15cdx^2 + 6d^2x^4) + 16ab^3x^2(130c^2$$

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2)^2,x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-45*a^4*d^2 + 30*a^3*b*d*(10*c + d*x^2) + 64*b^
4*x^4*(10*c^2 + 15*c*d*x^2 + 6*d^2*x^4) + 16*a*b^3*x^2*(130*c^2 + 170*c*d*x
^2 + 63*d^2*x^4) + 8*a^2*b^2*(330*c^2 + 295*c*d*x^2 + 93*d^2*x^4)) - 15*a^3
*(80*b^2*c^2 - 20*a*b*c*d + 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/(3840*b^(5/2))
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{3a^3\left(a^2d^2-\frac{20}{3}abcd+\frac{80}{3}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{256}-\frac{3x\left(-\frac{176\left(\frac{31}{110}d^2x^4+\frac{59}{66}cd\,x^2+c^2\right)a^2b^{\frac{5}{2}}}{3}-\frac{416x^2\left(\frac{63}{130}d^2x^4+\frac{17}{13}cd\,x^2+c^2\right)a\,b^{\frac{7}{2}}}{9}-\frac{128}{}\right.}{256}}{b^{\frac{5}{2}}}$
risch	$-\frac{x\left(-384b^4d^2x^8-1008a\,b^3d^2x^6-960b^4cd\,x^6-744a^2b^2d^2x^4-2720a\,b^3cd\,x^4-640b^4c^2x^4-30a^3b\,d^2x^2-2360a^2b^2cd\,x^2-2080a\,b\right.}{3840b^2}$
default	$c^2\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d^2\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right.}{8b}\right.}{}\right.$

[In] int((b*x^2+a)^(5/2)*(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] 3/256*(a^3*(a^2*d^2-20/3*a*b*c*d+80/3*b^2*c^2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(-176/3*(31/110*d^2*x^4+59/66*c*d*x^2+c^2)*a^2*b^(5/2)-416/9*x^2*(63/130*d^2*x^4+17/13*c*d*x^2+c^2)*a*b^(7/2)-128/9*x^4*(3/5*d^2*x^4+3/2*c*d*x^2+c^2)*b^(9/2)+(2/3*(-d*x^2-10*c)*b^(3/2)+a*d*b^(1/2))*d*a^3)*(b*x^2+a)^(1/2))/b^(5/2)

Fricas [A] (verification not implemented)

none

Time = 0.36 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2} (c+dx^2)^2 \, dx = \left[\frac{15 (80 a^3b^2c^2 - 20 a^4bcd + 3 a^5d^2)\sqrt{b}\log\left(-2 bx^2 - 2 \sqrt{bx^2+a}\sqrt{b}x - a\right) + 2 (384 b^5d^2x^9 + 4\ldots}{\ldots} \right.$$

$$-\frac{15 (80 a^3b^2c^2 - 20 a^4bcd + 3 a^5d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (384 b^5d^2x^9 + 48 (20 b^5cd + 21 ab^4d^2)x^7 + 8 (80\ldots}{\ldots}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(b)*log(-2*b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*d^2*x^9 + 48*(20*b^5*c*d
+ 21*a*b^4*d^2)*x^7 + 8*(80*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 +
 10*(208*a*b^4*c^2 + 236*a^2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3
*c^2 + 20*a^3*b^2*c*d - 3*a^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3, -1/3840*(15*(
80*a^3*b^2*c^2 - 20*a^4*b*c*d + 3*a^5*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - (384*b^5*d^2*x^9 + 48*(20*b^5*c*d + 21*a*b^4*d^2)*x^7 + 8*(80
*b^5*c^2 + 340*a*b^4*c*d + 93*a^2*b^3*d^2)*x^5 + 10*(208*a*b^4*c^2 + 236*a^
2*b^3*c*d + 3*a^3*b^2*d^2)*x^3 + 15*(176*a^2*b^3*c^2 + 20*a^3*b^2*c*d - 3*a
^4*b*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(236) = 472$.

Time = 0.58 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.16

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right)^2 \, dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{b^2d^2x^9}{10} + \frac{x^7\cdot\left(\frac{21ab^2d^2}{10}+2b^3cd\right)}{8b} + \frac{x^5\cdot\left(3a^2bd^2+6ab^2cd-\frac{7a\left(\frac{21ab^2d^2}{10}+2b^3cd\right)}{8b}+b^3c^2\right)}{6b} + \frac{x^3\left(a^3d^2+6a^2bcd+\right.}{} \right. \\ a^{\frac{5}{2}}\left(c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c)**2,x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(b**2*d**2*x**9/10 + x**7*(21*a*b**2*d**2/10 +
2*b**3*c*d)/(8*b) + x**5*(3*a**2*b*d**2 + 6*a*b**2*c*d - 7*a*(21*a*b**2*d**
2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b) + x**3*(a**3*d**2 + 6*a**2*b*c*
d + 3*a*b**2*c**2 - 5*a*(3*a**2*b*d**2 + 6*a*b**2*c*d - 7*a*(21*a*b**2*d**2
/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b))/(4*b) + x*(2*a**3*c*d + 3*a**2*
b*c**2 - 3*a*(a**3*d**2 + 6*a**2*b*c*d + 3*a*b**2*c**2 - 5*a*(3*a**2*b*d**2
 + 6*a*b**2*c*d - 7*a*(21*a*b**2*d**2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(
6*b))/(4*b))/(2*b)) + (a**3*c**2 - a*(2*a**3*c*d + 3*a**2*b*c**2 - 3*a*(a**
3*d**2 + 6*a**2*b*c*d + 3*a*b**2*c**2 - 5*a*(3*a**2*b*d**2 + 6*a*b**2*c*d -
 7*a*(21*a*b**2*d**2/10 + 2*b**3*c*d)/(8*b) + b**3*c**2)/(6*b))/(4*b))/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (
x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(c**2*x + 2*c*d*x**3/3
+ d**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.19

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx = \frac{\left(bx^2+a\right)^{\frac{7}{2}}d^2x^3}{10\,b}+\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}c^2x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ac^2x+\frac{5}{16}\sqrt{bx^2+a}a^2c^2x+\frac{\left(bx^2+a\right)^{\frac{7}{2}}cdx}{4\,b}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}acdx}{24\,b}-\frac{5\left(bx^2+a\right)^{\frac{3}{2}}a^2cdx}{96\,b}-\frac{5\sqrt{bx^2+a}a^3cdx}{64\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}ad^2x}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}a^2d^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}a^3d^2x}{128\,b^2}+\frac{3\sqrt{bx^2+a}a^4d^2x}{256\,b^2}+\frac{5\,a^3c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-\frac{5\,a^4cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{64\,b^{\frac{3}{2}}}+\frac{3\,a^5d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*d^2*x^3/b + 1/6*(b*x^2 + a)^(5/2)*c^2*x + 5/24*(b*x^
2 + a)^(3/2)*a*c^2*x + 5/16*sqrt(b*x^2 + a)*a^2*c^2*x + 1/4*(b*x^2 + a)^(7/
2)*c*d*x/b - 1/24*(b*x^2 + a)^(5/2)*a*c*d*x/b - 5/96*(b*x^2 + a)^(3/2)*a^2*
c*d*x/b - 5/64*sqrt(b*x^2 + a)*a^3*c*d*x/b - 3/80*(b*x^2 + a)^(7/2)*a*d^2*x
/b^2 + 1/160*(b*x^2 + a)^(5/2)*a^2*d^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*
d^2*x/b^2 + 3/256*sqrt(b*x^2 + a)*a^4*d^2*x/b^2 + 5/16*a^3*c^2*arcsinh(b*x/
sqrt(a*b))/sqrt(b) - 5/64*a^4*c*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/256*a^
5*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.92

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx = \frac{1}{3840}\left(2\left(4\left(6\left(8\,b^2d^2x^2+\frac{20\,b^{10}cd+21\,ab^9d^2}{b^8}\right)x^2+\frac{80\,b^{10}c^2+340\,ab^9cd+93\,a^2b^8d^2}{b^8}\right)x^2+\right.\right.\right. \cdots - \frac{\left(80\,a^3b^2c^2-20\,a^4bcd+3\,a^5d^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*b^2*d^2*x^2 + (20*b^10*c*d + 21*a*b^9*d^2)/b^8)*x^2 + (8
0*b^10*c^2 + 340*a*b^9*c*d + 93*a^2*b^8*d^2)/b^8)*x^2 + 5*(208*a*b^9*c^2 +
236*a^2*b^8*c*d + 3*a^3*b^7*d^2)/b^8)*x^2 + 15*(176*a^2*b^8*c^2 + 20*a^3*b^
7*c*d - 3*a^4*b^6*d^2)/b^8)*sqrt(b*x^2 + a)*x - 1/256*(80*a^3*b^2*c^2 - 20*
a^4*b*c*d + 3*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2\,dx=\int \left(b\,x^2+a\right)^{5/2}\left(d\,x^2+c\right)^2\,dx$$

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2)^2, x)
```

3.64 $\int (a+bx^2)^{5/2} (c+dx^2)\, dx$

Optimal result

Integrand size = 19, antiderivative size = 149

$$\int (a+bx^2)^{5/2} (c+dx^2)\, dx = \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{5a^3(8bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

```
[Out] 5/192*a*(-a*d+8*b*c)*x*(b*x^2+a)^(3/2)/b+1/48*(-a*d+8*b*c)*x*(b*x^2+a)^(5/2
)/b+1/8*d*x*(b*x^2+a)^(7/2)/b+5/128*a^3*(-a*d+8*b*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(3/2)+5/128*a^2*(-a*d+8*b*c)*x*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {396, 201, 223, 212}

$$\int (a+bx^2)^{5/2} (c+dx^2)\, dx = \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(8bc-ad)}{128b^{3/2}} + \frac{5a^2x\sqrt{a+bx^2}(8bc-ad)}{128b} + \frac{x(a+bx^2)^{5/2}(8bc-ad)}{48b} + \frac{5ax(a+bx^2)^{3/2}(8bc-ad)}{192b} + \frac{dx(a+bx^2)^{7/2}}{8b}$$

```
[In] Int[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (5*a^2*(8*b*c - a*d)*x*Sqrt[a + b*x^2])/(128*b) + (5*a*(8*b*c - a*d)*x*(a +
 b*x^2)^(3/2))/(192*b) + ((8*b*c - a*d)*x*(a + b*x^2)^(5/2))/(48*b) + (d*x*
```

```
(a + b*x^2)^(7/2))/(8*b) + (5*a^3*(8*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{7/2}}{8b} - \frac{(-8bc+ad)\int (a+bx^2)^{5/2}\,dx}{8b} \\
&= \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(5a(8bc-ad))\int (a+bx^2)^{3/2}\,dx}{48b} \\
&= \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} \\
&\quad + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(5a^2(8bc-ad))\int \sqrt{a+bx^2}\,dx}{64b} \\
&= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} \\
&\quad + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(5a^3(8bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{128b}
\end{aligned}
$$

$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{(5a^3(8bc-ad))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128b}$$

$$= \frac{5a^2(8bc-ad)x\sqrt{a+bx^2}}{128b} + \frac{5a(8bc-ad)x(a+bx^2)^{3/2}}{192b} + \frac{(8bc-ad)x(a+bx^2)^{5/2}}{48b} + \frac{dx(a+bx^2)^{7/2}}{8b} + \frac{5a^3(8bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.83

$$\int (a+bx^2)^{5/2}(c+dx^2)\,dx = \frac{x\sqrt{a+bx^2}(264a^2bc+15a^3d+208ab^2cx^2+118a^2bdx^2+64b^3cx^4+136ab^2dx^4+48b^3dx^6)}{384b} + \frac{5a^3(-8bc+ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{128b^{3/2}}$$

```
[In] Integrate[(a + b*x^2)^(5/2)*(c + d*x^2),x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(264*a^2*b*c + 15*a^3*d + 208*a*b^2*c*x^2 + 118*a^2*b*d*
x^2 + 64*b^3*c*x^4 + 136*a*b^2*d*x^4 + 48*b^3*d*x^6))/(384*b) + (5*a^3*(-8*
b*c + a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{5\left(\left(a^{4}d-8a^{3}bc\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\sqrt{b\,x^{2}+a}\,\left(\frac{88\left(\frac{59d\,x^{2}}{132}+c\right)a^{2}b^{\frac{3}{2}}}{5}+\frac{208x^{2}\left(\frac{17d\,x^{2}}{26}+c\right)a\,b^{\frac{5}{2}}}{15}+\frac{64x^{4}\left(\frac{3d\,x^{2}}{4}+c\right)b^{\frac{7}{2}}}{15}+a^{3}d\sqrt{\right.}{128b^{\frac{3}{2}}}$
risch	$\frac{x\left(48b^{3}d\,x^{6}+136a\,b^{2}d\,x^{4}+64b^{3}c\,x^{4}+118a^{2}bd\,x^{2}+208a\,b^{2}c\,x^{2}+15a^{3}d+264a^{2}bc\right)\sqrt{b\,x^{2}+a}}{384b}-\frac{5a^{3}\left(ad-8bc\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}$
default	$c\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+d\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\right.}{}\right.$

[In] int((b*x^2+a)^(5/2)*(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] -5/128*((a^4*d-8*a^3*b*c)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(b*x^2+a)^(1/2)*(88/5*(59/132*d*x^2+c)*a^2*b^(3/2)+208/15*x^2*(17/26*d*x^2+c)*a*b^(5/2)+64/15*x^4*(3/4*d*x^2+c)*b^(7/2)+a^3*d*b^(1/2)))/b^(3/2)

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.74

$$\int \left(a+bx^2\right)^{5/2}\left(c+dx^2\right)\,dx=\left[-\frac{15\,(8\,a^3bc-a^4d)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(48\,b^4dx^7+8\,(8\,b^4c+17\,ab^3d)x^5}{768\,b^2}\right.$$

$$-\frac{15\,(8\,a^3bc-a^4d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,b^4dx^7+8\,(8\,b^4c+17\,ab^3d)x^5+2\,(104\,ab^3c+59\,a^2b^2d)x^3+3\,(}{384\,b^2}$$

[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="fricas")

```
[Out] [-1/768*(15*(8*a^3*b*c - a^4*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) - 2*(48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^
3*c + 59*a^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/
b^2, -1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) - (48*b^4*d*x^7 + 8*(8*b^4*c + 17*a*b^3*d)*x^5 + 2*(104*a*b^3*c + 59*a
^2*b^2*d)*x^3 + 3*(88*a^2*b^2*c + 5*a^3*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(134) = 268$.

Time = 0.42 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.87

$$\int (a+bx^2)^{5/2}(c+dx^2)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{b^2dx^7}{8} + \frac{x^5\cdot\left(\frac{17ab^2d}{8}+b^3c\right)}{6b} + \frac{x^3\cdot\left(3a^2bd+3ab^2c-\frac{5a\left(\frac{17ab^2d}{8}+b^3c\right)}{6b}\right)}{4b} + \frac{x\left(a^3d+3a^2bc-\frac{3a\left(3a^2bd+3ab^2c-\cdots\right)}{4b}\right)}{2b}\right. \\ a^{\frac{5}{2}}\left(cx+\frac{dx^3}{3}\right) \end{cases}$$

```
[In] integrate((b*x**2+a)**(5/2)*(d*x**2+c),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(b**2*d*x**7/8 + x**5*(17*a*b**2*d/8 + b**3*c)/
(6*b) + x**3*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17*a*b**2*d/8 + b**3*c)/(6*b))
/(4*b) + x*(a**3*d + 3*a**2*b*c - 3*a*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17*a*
b**2*d/8 + b**3*c)/(6*b))/(4*b))/(2*b)) + (a**3*c - a*(a**3*d + 3*a**2*b*c
- 3*a*(3*a**2*b*d + 3*a*b**2*c - 5*a*(17*a*b**2*d/8 + b**3*c)/(6*b))/(4*b))
/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(5/2)*(c*x + d*x**3/3),
True))
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) \, dx = \frac{1}{6} \left(bx^2+a\right)^{\frac{5}{2}} cx + \frac{5}{24} \left(bx^2+a\right)^{\frac{3}{2}} acx + \frac{5}{16} \sqrt{bx^2+a} a^2 cx + \frac{\left(bx^2+a\right)^{\frac{7}{2}} dx}{8\,b} - \frac{\left(bx^2+a\right)^{\frac{5}{2}} adx}{48\,b} - \frac{5 \left(bx^2+a\right)^{\frac{3}{2}} a^2 dx}{192\,b} - \frac{5 \sqrt{bx^2+a} a^3 dx}{128\,b} + \frac{5\,a^3 c \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - \frac{5\,a^4 d \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*c*x + 5/24*(b*x^2 + a)^(3/2)*a*c*x + 5/16*sqrt(b*x^2
+ a)*a^2*c*x + 1/8*(b*x^2 + a)^(7/2)*d*x/b - 1/48*(b*x^2 + a)^(5/2)*a*d*x/b
 - 5/192*(b*x^2 + a)^(3/2)*a^2*d*x/b - 5/128*sqrt(b*x^2 + a)*a^3*d*x/b + 5/
16*a^3*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 5/128*a^4*d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) \, dx = \frac{1}{384} \left(2 \left(4 \left(6\,b^2 dx^2 + \frac{8\,b^8 c + 17\,ab^7 d}{b^6}\right) x^2 + \frac{104\,ab^7 c + 59\,a^2 b^6 d}{b^6}\right) x^2 + \frac{3 \left(88\,a^2 b^6 c + 5\,a^3 b^5 d\right)}{b^6}\right) \sqrt{} - \frac{5 \left(8\,a^3 bc - a^4 d\right) \log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}$$

```
[In] integrate((b*x^2+a)^(5/2)*(d*x^2+c),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*b^2*d*x^2 + (8*b^8*c + 17*a*b^7*d)/b^6)*x^2 + (104*a*b^7*c +
 59*a^2*b^6*d)/b^6)*x^2 + 3*(88*a^2*b^6*c + 5*a^3*b^5*d)/b^6)*sqrt(b*x^2 +
a)*x - 5/128*(8*a^3*b*c - a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{5/2} \left(c+dx^2\right) \, dx = \int \left(b\,x^2+a\right)^{5/2} \left(d\,x^2+c\right) \, dx$$

```
[In] int((a + b*x^2)^(5/2)*(c + d*x^2),x)
[Out] int((a + b*x^2)^(5/2)*(c + d*x^2), x)
```

3.65 $\int (a+bx^2)^{5/2}\, dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int (a+bx^2)^{5/2}\, dx = \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2} + \frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

```
[Out] 5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+5/16*a^3*arctanh(x*b^(1/2)/(
b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {201, 223, 212}

$$\int (a+bx^2)^{5/2}\, dx = \frac{5a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}} + \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2}$$

```
[In] Int[(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + b*x^2])/16 + (5*a*x*(a + b*x^2)^(3/2))/24 + (x*(a + b*x^2
)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{6}(5a)\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \sqrt{a+bx^2}\,dx\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{1}{16}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{5}{16}a^2x\sqrt{a+bx^2}+\frac{5}{24}ax\left(a+bx^2\right)^{3/2}+\frac{1}{6}x\left(a+bx^2\right)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$
\int \left(a+bx^2\right)^{5/2}\,dx=\frac{1}{48}\sqrt{a+bx^2}\left(33a^2x+26abx^3+8b^2x^5\right)-\frac{5a^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}
$$

```
[In] Integrate[(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x(8b^2x^4+26ab\,x^2+33a^2)\sqrt{b\,x^2+a}}{48}+\frac{5a^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16\sqrt{b}}$	59
pseudoelliptic	$\frac{5\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a^3}{16\sqrt{b}}+\frac{11\left(\frac{8b^{\frac{5}{2}}x^4}{33}+\frac{26x^2a\,b^{\frac{3}{2}}}{33}+\sqrt{b}\,a^2\right)x\sqrt{b\,x^2+a}}{16\sqrt{b}}$	67
default	$\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}$	68

```
[In] int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*x*(8*b^2*x^4+26*a*b*x^2+33*a^2)*(b*x^2+a)^(1/2)+5/16*a^3*ln(x*b^(1/2)+
(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2}\,dx=\left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b},\ -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 + 33*
a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

```
[In] integrate((b*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a)
/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqrt(
a))/(16*sqrt(b))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a)
*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{5/2}\,dx=-\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

```
[In] integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*x
^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

```
[In] int((a + b*x^2)^(5/2),x)
```

```
[Out] (x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
1)^(5/2)
```

3.66 $\int \frac{(a+bx^2)^{5/2}}{c+dx^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2} dx = -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d} + \frac{\sqrt{b}(8b^2c^2-20abcd+15a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3} - \frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}$$

```
[Out] 1/4*b*x*(b*x^2+a)^(3/2)/d+1/8*(15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)*arctanh(x*b
^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)/d^3-(-a*d+b*c)^(5/2)*arctanh(x*(-a*d+b*c)^(
1/2)/c^(1/2)/(b*x^2+a)^(1/2))/d^3/c^(1/2)-1/8*b*(-7*a*d+4*b*c)*x*(b*x^2+a)^
(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {427, 542, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2} dx = \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(15a^2d^2-20abcd+8b^2c^2)}{8d^3} - \frac{(bc-ad)^{5/2}\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3} - \frac{bx\sqrt{a+bx^2}(4bc-7ad)}{8d^2} + \frac{bx(a+bx^2)^{3/2}}{4d}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2),x]
```

```
[Out] -1/8*(b*(4*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/d^2 + (b*x*(a + b*x^2)^(3/2))/(4
*d) + (Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sq
```

```
rt[a + b*x^2]])/(8*d^3) - ((b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(S
qrt[c]*Sqrt[a + b*x^2]])])/(Sqrt[c]*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\left(a+bx^2\right)^{3/2}}{4d}+\frac{\int \frac{\sqrt{a+bx^2}\left(-a(bc-4ad)-b(4bc-7ad)x^2\right)}{c+dx^2}\,dx}{4d}\\
&= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}+\frac{\int \frac{a\left(4b^2c^2-9abcd+8a^2d^2\right)+b\left(8b^2c^2-20abcd+15a^2d^2\right)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8d^2}\\
&= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}-\frac{(bc-ad)^3\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{d^3}\\
&\quad+\frac{\left(b\left(8b^2c^2-20abcd+15a^2d^2\right)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8d^3}\\
&= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}\\
&\quad-\frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d^3}\\
&\quad+\frac{\left(b\left(8b^2c^2-20abcd+15a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8d^3}\\
&= -\frac{b(4bc-7ad)x\sqrt{a+bx^2}}{8d^2}+\frac{bx\left(a+bx^2\right)^{3/2}}{4d}\\
&\quad+\frac{\sqrt{b}\left(8b^2c^2-20abcd+15a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8d^3}\\
&\quad-\frac{(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2\right)^{5/2}}{c+dx^2}\,dx=\frac{bdx\sqrt{a+bx^2}\left(-4bc+9ad+2bdx^2\right)-\frac{8(-bc+ad)^{5/2}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}}-\sqrt{b}(8b^2c^2}{8d^3}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2),x]
[Out] (b*d*x*Sqrt[a + b*x^2]*(-4*b*c + 9*a*d + 2*b*d*x^2) - (8*(-(b*c) + a*d)^(5/
2)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*
c) + a*d])])/Sqrt[c] - Sqrt[b]*(8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*d^3)
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{\frac{b\left(-d\sqrt{b\,x^2+a}\,\left(2bd\,x^2+9ad-4bc\right)x-\frac{\left(15a^2d^2-20abcd+8b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{4}+\frac{2(ad-bc)^3\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2d^3}$
risch	$\frac{bx\left(2bd\,x^2+9ad-4bc\right)\sqrt{b\,x^2+a}}{8d^2}+\frac{\frac{\sqrt{b}\left(15a^2d^2-20abcd+8b^2c^2\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{d}-\frac{\left(-4a^3d^3+12a^2bc\,d^2-12a\,b^2c^2d+4b^3c^3\right)\ln\left(\frac{2a\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	Expression too large to display

[In] int((b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)

[Out] -1/2/d^3*(1/4*b*(-d*(b*x^2+a)^(1/2)*(2*b*d*x^2+9*a*d-4*b*c)*x-(15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)/b^(1/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2)))+2*(a*d-b*c)^3/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.84 (sec) , antiderivative size = 935, normalized size of antiderivative = 5.96

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \left[\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+4\,(b^2c^2-2\,abcd\cdots}{\cdots}\right.$$

$$-\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{\frac{bc-ad}{c}}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x\cdots}{\cdots}\right.}{8\,d^3}$$

$$-\frac{(8\,b^2c^2-20\,abcd+15\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-4\,(b^2c^2-2\,abcd+a^2d^2)\sqrt{-\frac{bc-ad}{c}}\arctan\left(\frac{((2\,bc-ad)x^2+a\cdots}{2\,((b^2c-abd)\cdots}\right.}{8\,d^3}$$

[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")

[Out] [1/16*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2

```
- 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt
((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 2*(2*b^2*d^2*x^3 - (4*b^2*c
*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15
*a^2*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 2*(b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*
d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
 (2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, 1/16*(8*
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a
*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 +
(a*b*c - a^2*d)*x)) + (8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2)*sqrt(b)*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 - (4*b^2*c*d -
9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3, -1/8*((8*b^2*c^2 - 20*a*b*c*d + 15*a^2*
d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 4*(b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) -
(2*b^2*d^2*x^3 - (4*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x^2 + a))/d^3]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2),x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2), x)
```

3.67 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 175

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2} \, dx = \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} - \frac{b^{3/2}(4bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(bc-ad)^{3/2}(4bc+ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}$$

```
[Out] -1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)-1/2*b^(3/2)*(-5*a*d+4*b*c)*
arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/d^3+1/2*(-a*d+b*c)^(3/2)*(a*d+4*b*c)*arc
tanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/d^3+1/2*b*(-a*d+2*
b*c)*x*(b*x^2+a)^(1/2)/c/d^2
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {424, 542, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2} \, dx = -\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-5ad)}{2d^3} + \frac{(bc-ad)^{3/2}(ad+4bc)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3} + \frac{bx\sqrt{a+bx^2}(2bc-ad)}{2cd^2} - \frac{x(a+bx^2)^{3/2}(bc-ad)}{2cd\,(c+dx^2)}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] (b*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(2*c*d^2) - ((b*c - a*d)*x*(a + b*x^2)^
(3/2))/(2*c*d*(c + d*x^2)) - (b^(3/2)*(4*b*c - 5*a*d)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*d^3) + ((b*c - a*d)^(3/2)*(4*b*c + a*d)*ArcTanh[(Sqrt[b
*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
```

```
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} + \frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+ad)+2b(2bc-ad)x^2\right)}{c+dx^2}\,dx}{2cd} \\
&= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} + \frac{\int \frac{-2a\left(2b^2c^2-2abcd-a^2d^2\right)-2b^2c(4bc-5ad)x^2}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{4cd^2} \\
&= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} \\
&\quad - \frac{\left(b^2(4bc-5ad)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2d^3} + \frac{\left((bc-ad)^2(4bc+ad)\right)\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2cd^3} \\
&= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} \\
&\quad - \frac{\left(b^2(4bc-5ad)\right)\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2d^3} \\
&\quad + \frac{\left((bc-ad)^2(4bc+ad)\right)\operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2cd^3} \\
&= \frac{b(2bc-ad)x\sqrt{a+bx^2}}{2cd^2} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{2cd\,(c+dx^2)} \\
&\quad - \frac{b^{3/2}(4bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(bc-ad)^{3/2}(4bc+ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \frac{\frac{dx\sqrt{a+bx^2}\left(-2abcd+a^2d^2+b^2c(2c+dx^2)\right)}{c(c+dx^2)} + \frac{\sqrt{-bc+ad}\left(4b^2c^2-3abcd-a^2d^2\right)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{3/2}} +}{2d^3}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^2,x]
```

```
[Out] ((d*x*Sqrt[a + b*x^2]*(-2*a*b*c*d + a^2*d^2 + b^2*c*(2*c + d*x^2)))/(c*(c +
 d*x^2)) + (Sqrt[-(b*c) + a*d]*(4*b^2*c^2 - 3*a*b*c*d - a^2*d^2)*ArcTan[(-(
```

```
d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/
c^(3/2) + b^(3/2)*(4*b*c - 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*d
^3)
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{2\left(\frac{\left(d\,x^2+c\right)(ad+4bc)(ad-bc)^2\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{4}+\left(\left(d\,x^2+c\right)\left(b^{\frac{5}{2}}c-\frac{5ad\,b^{\frac{3}{2}}}{4}\right)c\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\frac{x\sqrt{b\,x^2+a}\,d\left(2b^2c^2-2\right.}{}\right.\right.}{\sqrt{(ad-bc)c}\,d^3c(d\,x^2+c)}$
risch	Expression too large to display
default	Expression too large to display

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/4*(d*x^2+c)*(a*d+4*b*c)*(a*d-b*c)^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-
b*c)*c)^(1/2))+((d*x^2+c)*(b^(5/2)*c-5/4*a*d*b^(3/2))*c*arctanh((b*x^2+a)^(
1/2)/x/b^(1/2))-1/4*x*(b*x^2+a)^(1/2)*d*(2*b^2*c^2-2*(-1/2*b*x^2+a)*b*d*c+a
^2*d^2))*((a*d-b*c)*c)^(1/2))/((a*d-b*c)*c)^(1/2)/d^3/c/(d*x^2+c)
```

Fricas [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 1236, normalized size of antiderivative = 7.06

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (4*b^2*c^3 - 3*a*b*c^2*
d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)
/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3
*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b
*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/8*(
4*(4*b^2*c^3 - 5*a*b*c^2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*
b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2
 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(
a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*
```

```
x^4 + 2*c*d*x^2 + c^2)) + 4*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), -1/4*((4*b^2*c^3 - 3*a*b
*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2 - a^2*d^3)*x^2)*sqrt(-(b*c
- a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c -
 a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + (4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) - 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*x)*sqrt(b*x^2 + a))/(c*d^4*x^2 + c^2*d^3), 1/4*(2*(4*b^2*c^3 - 5*a*b*c^
2*d + (4*b^2*c^2*d - 5*a*b*c*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) - (4*b^2*c^3 - 3*a*b*c^2*d - a^2*c*d^2 + (4*b^2*c^2*d - 3*a*b*c*d^2
 - a^2*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)
*x)) + 2*(b^2*c*d^2*x^3 + (2*b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*sqrt(b*x
^2 + a))/(c*d^4*x^2 + c^2*d^3)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(147) = 294$.

Time = 0.32 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \frac{\sqrt{bx^2+a}b^2x}{2\,d^2} + \frac{\left(4\,b^{\frac{5}{2}}c - 5\,ab^{\frac{3}{2}}d\right)\log\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2\right)}{4\,d^3}$$
$$- \frac{\left(4\,b^{\frac{7}{2}}c^3 - 7\,ab^{\frac{5}{2}}c^2d + 2\,a^2b^{\frac{3}{2}}cd^2 + a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\sqrt{-b^2c^2+abcd}cd^3}$$
$$+ \frac{2\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 b^{\frac{7}{2}}c^3 - 5\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 ab^{\frac{5}{2}}c^2d + 4\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 a^2b^{\frac{3}{2}}cd^2 - \left(\sqrt{b}x - \sqrt{bx\right.}{\left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4 d + 4\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 bc - 2\left(\sqrt{b}x - \sqrt{bx^2+a}\right)\right.}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*b^2*x/d^2 + 1/4*(4*b^(5/2)*c - 5*a*b^(3/2)*d)*log((sqrt
(b)*x - sqrt(b*x^2 + a))^2)/d^3 - 1/2*(4*b^(7/2)*c^3 - 7*a*b^(5/2)*c^2*d +
2*a^2*b^(3/2)*c*d^2 + a^3*sqrt(b)*d^3)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2
+ a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d
)*c*d^3) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(7/2)*c^3 - 5*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*b^(5/2)*c^2*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*
b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*d^3 + a^2*b^(5/
2)*c^2*d - 2*a^3*b^(3/2)*c*d^2 + a^4*sqrt(b)*d^3)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)*c*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^2,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^2, x)
```

3.68 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = -\frac{(bc-ad)x(a+bx^2)^{3/2}}{4cd\,(c+dx^2)^2} - \frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\,(c+dx^2)} + \frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} - \frac{\sqrt{bc-ad}\,(8b^2c^2+4abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}$$

```
[Out] -1/4*(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^2+b^(5/2)*arctanh(x*b^(1/2)
/(b*x^2+a)^(1/2))/d^3-1/8*(3*a^2*d^2+4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b
*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))*(-a*d+b*c)^(1/2)/c^(5/2)/d^3-1/8*(-a*d+b
*c)*(3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/c^2/d^2/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {424, 540, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = -\frac{\sqrt{bc-ad}\,(3a^2d^2+4abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3} + \frac{b^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} - \frac{x\sqrt{a+bx^2}(bc-ad)(3ad+4bc)}{8c^2d^2\,(c+dx^2)} - \frac{x(a+bx^2)^{3/2}\,(bc-ad)}{4cd\,(c+dx^2)^2}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

```
[Out] -1/4*((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*(c + d*x^2)^2) - ((b*c - a*d)*(
4*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*c^2*d^2*(c + d*x^2)) + (b^(5/2)*ArcTan
h[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d^3 - (Sqrt[b*c - a*d]*(8*b^2*c^2 + 4*a*b*c
*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(8*
c^(5/2)*d^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}+\frac{\int \frac{\sqrt{a+bx^2}\left(a(bc+3ad)+4b^2cx^2\right)}{\left(c+dx^2\right)^2}\,dx}{4cd}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}-\frac{\int \frac{-a\left(4b^2c^2+ad(bc+3ad)\right)-8b^3c^2x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2d^2}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}\\
&\quad+\frac{b^3\int \frac{1}{\sqrt{a+bx^2}}\,dx}{d^3}-\frac{\left((bc-ad)\left(8b^2c^2+4abcd+3a^2d^2\right)\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2d^3}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}\\
&\quad+\frac{b^3\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{d^3}\\
&\quad-\frac{\left((bc-ad)\left(8b^2c^2+4abcd+3a^2d^2\right)\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8c^2d^3}\\
&= -\frac{(bc-ad)x\left(a+bx^2\right)^{3/2}}{4cd\left(c+dx^2\right)^2}-\frac{(bc-ad)(4bc+3ad)x\sqrt{a+bx^2}}{8c^2d^2\left(c+dx^2\right)}\\
&\quad+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3}-\frac{\sqrt{bc-ad}\left(8b^2c^2+4abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}d^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.30

$$
\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^3}\,dx =
$$

$$
-\frac{\frac{d(bc-ad)x\sqrt{a+bx^2}\left(2bc\left(2c+3dx^2\right)+ad\left(5c+3dx^2\right)\right)}{c^2\left(c+dx^2\right)^2}+\frac{3(-4bc+ad)^2\sqrt{-bc+ad}\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{c^{5/2}}-\frac{4b(10bc-7ad)\sqrt{bc-ad}\,\text{arc}}{}}{8d^3}
$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^3,x]
```

[Out] -1/8*((d*(b*c - a*d)*x*Sqrt[a + b*x^2]*(2*b*c*(2*c + 3*d*x^2) + a*d*(5*c + 3*d*x^2)))/(c^2*(c + d*x^2)^2) + (3*(-4*b*c + a*d)^2*Sqrt[-(b*c) + a*d]*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/c^(5/2) - (4*b*(10*b*c - 7*a*d)*Sqrt[b*c - a*d]*ArcTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/c^(3/2) + 8*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^3

Maple [A] (verified)

Time = 2.65 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{3\Bigg((dx^2+c)^2(a^2d^2+\frac{4}{3}abcd+\frac{8}{3}b^2c^2)(ad-bc)\arctan\left(\frac{c\sqrt{bx^2+a}}{x\sqrt{(ad-bc)c}}\right)-\frac{5\sqrt{(ad-bc)c}\,\Big(\frac{8c^2b^{\frac{5}{2}}(dx^2+c)^2\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)}{5}+xd\big(}{3}}{8\sqrt{(ad-bc)c}\,d^3(dx^2+c)^2c^2}$
default	Expression too large to display

[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)

[Out] -3/8*((d*x^2+c)^2*(a^2*d^2+4/3*a*b*c*d+8/3*b^2*c^2)*(a*d-b*c)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*((a*d-b*c)*c)^(1/2)*(8/5*c^2*b^(5/2)*(d*x^2+c)^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))+x*d*(4/5*b*c^2+d*(6/5*b*x^2+a)*c+3/5*a*d^2*x^2)*(b*x^2+a)^(1/2)*(a*d-b*c)))/((a*d-b*c)*c)^(1/2)/d^3/(d*x^2+c)^2/c^2

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(168) = 336.

Time = 0.50 (sec) , antiderivative size = 1517, normalized size of antiderivative = 7.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \text{Too large to display}$$

[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")

[Out] [1/32*(16*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^

```
3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d
^4*x^2 + c^4*d^3), -1/32*(32*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^2*c^4 + 4*a*b*c^3*d + 3*
a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*
d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt((b*c - a*d)/c)*log(((8*b^2*c^2 -
 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 - 4*(a*
c^2*x + (2*b*c^2 - a*c*d)*x^3)*sqrt(b*x^2 + a)*sqrt((b*c - a*d)/c))/(d^2*x^
4 + 2*c*d*x^2 + c^2)) + 4*(3*(2*b^2*c^2*d^2 - a*b*c*d^3 - a^2*d^4)*x^3 + (4
*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/(c^2*d^5*x^4 +
2*c^3*d^4*x^2 + c^4*d^3), 1/16*((8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 +
(8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*
d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2
+ a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c -
 a^2*d)*x)) + 8*(b^2*c^2*d^2*x^4 + 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(b)*log(-
2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(3*(2*b^2*c^2*d^2 - a*b*c*d^
3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*sqrt(b*x^2
+ a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3), -1/16*(16*(b^2*c^2*d^2*x^4 +
 2*b^2*c^3*d*x^2 + b^2*c^4)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
8*b^2*c^4 + 4*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 + 4*a*b*c*d^3 + 3*
a^2*d^4)*x^4 + 2*(8*b^2*c^3*d + 4*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*sqrt(-(b*
c - a*d)/c)*arctan(1/2*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(-(b*c
 - a*d)/c)/((b^2*c - a*b*d)*x^3 + (a*b*c - a^2*d)*x)) + 2*(3*(2*b^2*c^2*d^2
 - a*b*c*d^3 - a^2*d^4)*x^3 + (4*b^2*c^3*d + a*b*c^2*d^2 - 5*a^2*c*d^3)*x)*
sqrt(b*x^2 + a))/(c^2*d^5*x^4 + 2*c^3*d^4*x^2 + c^4*d^3)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^3}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^3}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(168) = 336$.

Time = 0.32 (sec) , antiderivative size = 659, normalized size of antiderivative = 3.40

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = -\frac{b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,d^3} + \frac{\left(8\,b^{\frac{7}{2}}c^3-4\,ab^{\frac{5}{2}}c^2d-a^2b^{\frac{3}{2}}cd^2-3\,a^3\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,\sqrt{-b^2c^2+abcd}c^2d^3} - \frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{7}{2}}c^3d-20\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}c^2d^2+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x-\right.}{}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/d^3 + 1/8*(8*b^(7/2)*c^3
- 4*a*b^(5/2)*c^2*d - a^2*b^(3/2)*c*d^2 - 3*a^3*sqrt(b)*d^3)*arctan(1/2*((s
qrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/(s
qrt(-b^2*c^2 + a*b*c*d)*c^2*d^3) - 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
b^(7/2)*c^3*d - 20*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2)*c^2*d^2 + (sqr
t(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^3*sqrt(b)*d^4 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(9/2)*c^4 - 7
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c^3*d + 18*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2)*c^2*d^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*d^4 + 32*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c^3*d - 28*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*a^3*b^(5/2)*c^2*d^2 - 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*
c*d^3 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*d^4 + 6*a^4*b^(5/2)*c
^2*d^2 - 3*a^5*b^(3/2)*c*d^3 - 3*a^6*sqrt(b)*d^4)/(((sqrt(b)*x - sqrt(b*x^2
 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a*d + a^2*d)^2*c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^3}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^3}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^3,x)
```

```
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^3, x)
```

3.69 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 144

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4} dx = \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)} + \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}$$

```
[Out] 1/6*x*(b*x^2+a)^(5/2)/c/(d*x^2+c)^3+5/24*a*x*(b*x^2+a)^(3/2)/c^2/(d*x^2+c)^
2+5/16*a^3*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(7/2)/(-a*
d+b*c)^(1/2)+5/16*a^2*x*(b*x^2+a)^(1/2)/c^3/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {386, 385, 214}

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4} dx = \frac{5a^3\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}} + \frac{5a^2x\sqrt{a+bx^2}}{16c^3(c+dx^2)} + \frac{5ax(a+bx^2)^{3/2}}{24c^2(c+dx^2)^2} + \frac{x(a+bx^2)^{5/2}}{6c(c+dx^2)^3}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*(a + b*x^2)^(5/2))/(6*c*(c + d*x^2)^3) + (5*a*x*(a + b*x^2)^(3/2))/(24*c
^2*(c + d*x^2)^2) + (5*a^2*x*Sqrt[a + b*x^2])/(16*c^3*(c + d*x^2)) + (5*a^3
```

```
*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(16*c^(7/2)*Sqrt[b
*c - a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{(5a)\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx}{6c}\\
&= \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{(5a^2)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{8c^2}\\
&= \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)}+\frac{(5a^3)\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{16c^3}\\
&= \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)}+\frac{(5a^3)\operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{16c^3}\\
&= \frac{x(a+bx^2)^{5/2}}{6c\,(c+dx^2)^3}+\frac{5ax(a+bx^2)^{3/2}}{24c^2\,(c+dx^2)^2}+\frac{5a^2x\sqrt{a+bx^2}}{16c^3\,(c+dx^2)}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{16c^{7/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] (warning: unable to verify)

Time = 10.60 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.40

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \frac{x\sqrt{a+bx^2}\left(\frac{\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}(8b^2c^2x^4+2abcx^2(13c+5dx^2)+a^2(33c^2+40cdx^2+15d^2x^4))}{(c+dx^2)^2\sqrt{1+\frac{dx^2}{c}}}+\frac{15a^2\arcsin\left(\frac{\sqrt{\left(-\frac{b}{a}+\frac{d}{c}\right)}}{\sqrt{1+\frac{dx^2}{c}}}\right.}{\sqrt{\frac{(-bc+ad)x^2}{ac}}}\right.}{48c^4\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^4,x]
```

```
[Out] (x*Sqrt[a + b*x^2]*((Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*(8*b^2*c^2*x^4 +
 2*a*b*c*x^2*(13*c + 5*d*x^2) + a^2*(33*c^2 + 40*c*d*x^2 + 15*d^2*x^4)))/((
c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]) + (15*a^2*ArcSin[Sqrt[(-(b/a) + d/c)*x^2]
/Sqrt[1 + (d*x^2)/c]])/Sqrt[((-(b*c) + a*d)*x^2)/(a*c)]))/(48*c^4*Sqrt[1 +
(b*x^2)/a])
```

Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{-33x\sqrt{b\,x^2+a}\left(\left(\frac{8}{33}b^2x^4+\frac{26}{33}ab\,x^2+a^2\right)c^2+\frac{40x^2\left(\frac{b\,x^2}{4}+a\right)dac}{33}+\frac{5a^2d^2x^4}{11}\right)\sqrt{(ad-bc)c}+15a^3\left(d\,x^2+c\right)^3\arctan\left(\frac{c\sqrt{b\,x^2+}}{x\sqrt{(ad-b}}\right.}{48\sqrt{(ad-bc)c}\,c^3\left(d\,x^2+c\right)^3}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48/((a*d-b*c)*c)^(1/2)*(-33*x*(b*x^2+a)^(1/2)*((8/33*b^2*x^4+26/33*a*b*x
^2+a^2)*c^2+40/33*x^2*(1/4*b*x^2+a)*d*a*c+5/11*a^2*d^2*x^4)*((a*d-b*c)*c)^(
1/2)+15*a^3*(d*x^2+c)^3*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/c^
3/(d*x^2+c)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(120) = 240$.

Time = 0.37 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.90

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \left[\frac{15\,(a^3d^3x^6+3\,a^3cd^2x^4+3\,a^3c^2dx^2+a^3c^3)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{15\,(a^3d^3x^6+3\,a^3cd^2x^4+3\,a^3c^2dx^2+a^3c^3)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((8\,b^3c^4\cdots}{96\,(bc^8-ac^7d+(bc^5d^3-ac^4d^4)x^6+3\cdots}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="fricas")
```

```
[Out] [1/192*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt
(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4
*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*
d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((8*b^3*c^4 + 2*a*b^2*
c^3*d + 5*a^2*b*c^2*d^2 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3
*d - 20*a^3*c^2*d^2)*x^3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(
b*c^8 - a*c^7*d + (b*c^5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x
^4 + 3*(b*c^7*d - a*c^6*d^2)*x^2), -1/96*(15*(a^3*d^3*x^6 + 3*a^3*c*d^2*x^4
 + 3*a^3*c^2*d*x^2 + a^3*c^3)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 +
 a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3
+ (a*b*c^2 - a^2*c*d)*x)) - 2*((8*b^3*c^4 + 2*a*b^2*c^3*d + 5*a^2*b*c^2*d^2
 - 15*a^3*c*d^3)*x^5 + 2*(13*a*b^2*c^4 + 7*a^2*b*c^3*d - 20*a^3*c^2*d^2)*x^
3 + 33*(a^2*b*c^4 - a^3*c^3*d)*x)*sqrt(b*x^2 + a))/(b*c^8 - a*c^7*d + (b*c^
5*d^3 - a*c^4*d^4)*x^6 + 3*(b*c^6*d^2 - a*c^5*d^3)*x^4 + 3*(b*c^7*d - a*c^6
*d^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^4}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(120) = 240.

Time = 1.31 (sec) , antiderivative size = 846, normalized size of antiderivative = 5.88

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = -\frac{5\,a^3\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{16\,\sqrt{-b^2c^2+abcd}c^3} + \frac{48\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}c^3d^2-15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3\sqrt{b}d^5+192\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8b^{\frac{9}{2}}c^4d+48\left(\right.}{}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^4,x, algorithm="giac")
[Out] -5/16*a^3*sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a
*d)/sqrt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*c^3) + 1/24*(48*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*c^3*d^2 - 15*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*a^3*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c^4*
d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*c^3*d^2 - 150*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*a^3*b^(3/2)*c*d^4 + 75*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^4*sqrt(b)*d^5 + 256*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(11/2)*c^5 - 64*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c^4*d + 288*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*a^2*b^(7/2)*c^3*d^2 - 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/
2)*c^2*d^3 + 440*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*c*d^4 - 150*(s
qrt(b)*x - sqrt(b*x^2 + a))^6*a^5*sqrt(b)*d^5 + 192*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(9/2)*c^4*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2)
*c^3*d^2 + 360*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*c^2*d^3 - 420*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*c*d^4 + 150*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a^6*sqrt(b)*d^5 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)
*c^3*d^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*c^2*d^3 + 120*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*c*d^4 - 75*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^7*sqrt(b)*d^5 + 8*a^6*b^(5/2)*c^2*d^3 + 10*a^7*b^(3/2)*c*d^4 + 1
5*a^8*sqrt(b)*d^5)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^3*c^3*
d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^4}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^4,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^4, x)
```

3.70 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx = -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)(c+dx^2)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)(c+dx^2)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)(c+dx^2)^2} + \frac{5a^2(8bc-7ad)x\sqrt{a+bx^2}}{128c^4(bc-ad)(c+dx^2)} + \frac{5a^3(8bc-7ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}$$

```
[Out] -1/8*d*x*(b*x^2+a)^(7/2)/c/(-a*d+b*c)/(d*x^2+c)^4+1/48*(-7*a*d+8*b*c)*x*(b*
x^2+a)^(5/2)/c^2/(-a*d+b*c)/(d*x^2+c)^3+5/192*a*(-7*a*d+8*b*c)*x*(b*x^2+a)^
(3/2)/c^3/(-a*d+b*c)/(d*x^2+c)^2+5/128*a^3*(-7*a*d+8*b*c)*arctanh(x*(-a*d+b
*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(9/2)/(-a*d+b*c)^(3/2)+5/128*a^2*(-7*a
*d+8*b*c)*x*(b*x^2+a)^(1/2)/c^4/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {390, 386, 385, 214}

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5} \, dx = \frac{5a^3(8bc-7ad)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}} + \frac{5a^2x\sqrt{a+bx^2}(8bc-7ad)}{128c^4(c+dx^2)(bc-ad)} + \frac{5ax(a+bx^2)^{3/2}(8bc-7ad)}{192c^3(c+dx^2)^2(bc-ad)} + \frac{x(a+bx^2)^{5/2}(8bc-7ad)}{48c^2(c+dx^2)^3(bc-ad)} - \frac{dx(a+bx^2)^{7/2}}{8c(c+dx^2)^4(bc-ad)}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] -1/8*(d*x*(a + b*x^2)^(7/2))/(c*(b*c - a*d)*(c + d*x^2)^4) + ((8*b*c - 7*a*
d)*x*(a + b*x^2)^(5/2))/(48*c^2*(b*c - a*d)*(c + d*x^2)^3) + (5*a*(8*b*c -
7*a*d)*x*(a + b*x^2)^(3/2))/(192*c^3*(b*c - a*d)*(c + d*x^2)^2) + (5*a^2*(8
*b*c - 7*a*d)*x*Sqrt[a + b*x^2])/(128*c^4*(b*c - a*d)*(c + d*x^2)) + (5*a^3
*(8*b*c - 7*a*d)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(1
28*c^(9/2)*(b*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4}+\frac{(8bc-7ad)\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^4}\,dx}{8c(bc-ad)} \\
&= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4}+\frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3}+\frac{(5a(8bc-7ad))\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^3}\,dx}{48c^2(bc-ad)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} \\
&\quad + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} + \frac{(5a^2(8bc-7ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^2}\,dx}{64c^3(bc-ad)} \\
&= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} \\
&\quad + \frac{5a^2(8bc-7ad)x\sqrt{a+bx^2}}{128c^4(bc-ad)\left(c+dx^2\right)} + \frac{(5a^3(8bc-7ad))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{128c^4(bc-ad)} \\
&= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} \\
&\quad + \frac{5a^2(8bc-7ad)x\sqrt{a+bx^2}}{128c^4(bc-ad)\left(c+dx^2\right)} + \frac{(5a^3(8bc-7ad))\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{128c^4(bc-ad)} \\
&= -\frac{dx(a+bx^2)^{7/2}}{8c(bc-ad)\left(c+dx^2\right)^4} + \frac{(8bc-7ad)x(a+bx^2)^{5/2}}{48c^2(bc-ad)\left(c+dx^2\right)^3} + \frac{5a(8bc-7ad)x(a+bx^2)^{3/2}}{192c^3(bc-ad)\left(c+dx^2\right)^2} \\
&\quad + \frac{5a^2(8bc-7ad)x\sqrt{a+bx^2}}{128c^4(bc-ad)\left(c+dx^2\right)} + \frac{5a^3(8bc-7ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{128c^{9/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.65 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.04

$$
\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5}\,dx = \frac{x\left(c(a+bx^2)\left(-16b^3c^3x^4(4c+dx^2) - 8ab^2c^2x^2(26c^2+11cdx^2+3d^2x^4) - 2a^2bc(132c^3\right.\right.}{}
$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^5,x]
```

```
[Out] (x*(c*(a + b*x^2)*(-16*b^3*c^3*x^4*(4*c + d*x^2) - 8*a*b^2*c^2*x^2*(26*c^2
+ 11*c*d*x^2 + 3*d^2*x^4) - 2*a^2*b*c*(132*c^3 + 129*c^2*d*x^2 + 94*c*d^2*x
^4 + 25*d^3*x^6) + a^3*d*(279*c^3 + 511*c^2*d*x^2 + 385*c*d^2*x^4 + 105*d^3
*x^6)) + (15*a^3*(-8*b*c + 7*a*d)*(c + d*x^2)^4*ArcTanh[Sqrt[((b*c - a*d)*x
^2)/(c*(a + b*x^2))]])/Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]))/(384*c^5*(
-(b*c) + a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^4)
```

Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{93x\sqrt{(ad-bc)c}\,\Big(d(\frac{35}{93}d^3x^6+\frac{385}{279}c\,d^2x^4+\frac{511}{279}c^2d\,x^2+c^3)a^3-\frac{88b(\frac{25}{132}d^3x^6+\frac{47}{66}c\,d^2x^4+\frac{43}{44}c^2d\,x^2+c^3)c\,a^2}{93}-\frac{208x^2b^2(\frac{3}{26}d^2x^4+\frac{11}{26}c}{279}}{128\sqrt{(ad-bc)c}\,(ad-bc)c^4(d\,x^2+c}$
default	Expression too large to display

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/((a*d-b*c)*c)^(1/2)*(93*x*((a*d-b*c)*c)^(1/2)*(d*(35/93*d^3*x^6+385/2
79*c*d^2*x^4+511/279*c^2*d*x^2+c^3)*a^3-88/93*b*(25/132*d^3*x^6+47/66*c*d^2
*x^4+43/44*c^2*d*x^2+c^3)*c*a^2-208/279*x^2*b^2*(3/26*d^2*x^4+11/26*c*d*x^2
+c^2)*c^2*a-64/279*x^4*b^3*(1/4*d*x^2+c)*c^3)*(b*x^2+a)^(1/2)-35*(d*x^2+c)^
4*(a*d-8/7*b*c)*a^3*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))/(a*d-b
*c)/c^4/(d*x^2+c)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(221) = 442$.

Time = 0.79 (sec) , antiderivative size = 1258, normalized size of antiderivative = 5.05

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4 - 7*a^4*d^5)*x^8 +
4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d^2 - 7*a^4*c^2*d^3)
*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b
^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2
 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*
x^4 + 2*c*d*x^2 + c^2)) + 4*((16*b^4*c^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c
^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^
5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208
*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 +
3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d + 93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(
b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2 + (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*
c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c
^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 +
 a^2*c^8*d^3)*x^2), -1/768*(15*(8*a^3*b*c^5 - 7*a^4*c^4*d + (8*a^3*b*c*d^4
- 7*a^4*d^5)*x^8 + 4*(8*a^3*b*c^2*d^3 - 7*a^4*c*d^4)*x^6 + 6*(8*a^3*b*c^3*d
^2 - 7*a^4*c^2*d^3)*x^4 + 4*(8*a^3*b*c^4*d - 7*a^4*c^3*d^2)*x^2)*sqrt(-b*c^
```

```
2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b
*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^4*c
^5*d + 8*a*b^3*c^4*d^2 + 26*a^2*b^2*c^3*d^3 - 155*a^3*b*c^2*d^4 + 105*a^4*c
*d^5)*x^7 + (64*b^4*c^6 + 24*a*b^3*c^5*d + 100*a^2*b^2*c^4*d^2 - 573*a^3*b*
c^3*d^3 + 385*a^4*c^2*d^4)*x^5 + (208*a*b^3*c^6 + 50*a^2*b^2*c^5*d - 769*a^
3*b*c^4*d^2 + 511*a^4*c^3*d^3)*x^3 + 3*(88*a^2*b^2*c^6 - 181*a^3*b*c^5*d +
93*a^4*c^4*d^2)*x)*sqrt(b*x^2 + a))/(b^2*c^11 - 2*a*b*c^10*d + a^2*c^9*d^2
+ (b^2*c^7*d^4 - 2*a*b*c^6*d^5 + a^2*c^5*d^6)*x^8 + 4*(b^2*c^8*d^3 - 2*a*b*
c^7*d^4 + a^2*c^6*d^5)*x^6 + 6*(b^2*c^9*d^2 - 2*a*b*c^8*d^3 + a^2*c^7*d^4)*
x^4 + 4*(b^2*c^10*d - 2*a*b*c^9*d^2 + a^2*c^8*d^3)*x^2)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^5}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**5,x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**5, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^5}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(221) = 442$.

Time = 3.77 (sec) , antiderivative size = 1448, normalized size of antiderivative = 5.82

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^5}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^5,x, algorithm="giac")
[Out] -5/128*(8*a^3*b^(3/2)*c - 7*a^4*sqrt(b)*d)*arctan(1/2*((sqrt(b)*x - sqrt(b*
x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b*c^5 - a*c^4*d)*s
```

```
qrt(-b^2*c^2 + a*b*c*d)) - 1/192*(120*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*
b^(3/2)*c*d^6 - 105*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*sqrt(b)*d^7 - 768*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(11/2)*c^5*d^2 + 768*(sqrt(b)*x - sqrt(b
*x^2 + a))^12*a*b^(9/2)*c^4*d^3 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3
*b^(5/2)*c^2*d^5 - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(3/2)*c*d^6
+ 735*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^5*sqrt(b)*d^7 - 2048*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*b^(13/2)*c^6*d + 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
a^2*b^(9/2)*c^4*d^3 + 8320*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2)*c^3
*d^4 - 15600*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(5/2)*c^2*d^5 + 9800*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^5*b^(3/2)*c*d^6 - 2205*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*a^6*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(15/
2)*c^7 + 1024*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(13/2)*c^6*d - 4864*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(11/2)*c^5*d^2 + 21888*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*a^3*b^(9/2)*c^4*d^3 - 38000*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^
4*b^(7/2)*c^3*d^4 + 37400*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^5*b^(5/2)*c^2*d
^5 - 18550*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(3/2)*c*d^6 + 3675*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*a^7*sqrt(b)*d^7 - 2048*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a^2*b^(13/2)*c^6*d - 9472*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(9/2)*
c^4*d^3 + 32896*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2)*c^3*d^4 - 35376
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(5/2)*c^2*d^5 + 18200*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a^7*b^(3/2)*c*d^6 - 3675*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^8*sqrt(b)*d^7 - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(11/2)*c^5*d^2
- 1536*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/2)*c^4*d^3 - 2944*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*a^6*b^(7/2)*c^3*d^4 + 12528*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*a^7*b^(5/2)*c^2*d^5 - 9170*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^8*b^(3
/2)*c*d^6 + 2205*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*sqrt(b)*d^7 - 256*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2)*c^4*d^3 - 256*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*a^7*b^(7/2)*c^3*d^4 - 608*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^8*b^
(5/2)*c^2*d^5 + 1960*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^9*b^(3/2)*c*d^6 - 73
5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*sqrt(b)*d^7 - 16*a^8*b^(7/2)*c^3*d^4
 - 24*a^9*b^(5/2)*c^2*d^5 - 50*a^10*b^(3/2)*c*d^6 + 105*a^11*sqrt(b)*d^7)/(
(b*c^5*d^3 - a*c^4*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x -
 sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{\left(c+dx^2\right)^5}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{5/2}}{{\left(d\,x^2+c\right)}^5} \,d x$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^5,x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^5, x)
```

3.71 $\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 30

$$\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx = -\arcsin(x) + \sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

```
[Out] -arcsin(x)+arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {399, 222, 385, 209}

$$\int \frac{\sqrt{1-x^2}}{1+x^2} \, dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) - \arcsin(x)$$

```
[In] Int[Sqrt[1 - x^2]/(1 + x^2),x]
```

```
[Out] -ArcSin[x] + Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx - \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\sin^{-1}(x) + 2\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right) \\
&= -\sin^{-1}(x) + \sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = \sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right) + 2\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

```
[In] Integrate[Sqrt[1 - x^2]/(1 + x^2),x]
[Out] Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]] + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result
default	$-\arcsin\left(x\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}\,x}{x^{2}-1}\right)$
pseudoelliptic	$\arctan\left(\frac{\sqrt{-x^{2}+1}}{x}\right)-\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^{2}+1}}{2x}\right)$
trager	$\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z\right.}{}\right.}{}$

```
[In] int((-x^2+1)^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -arcsin(x)-2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx=-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^{2}+1}}{2\,x}\right)+2\,\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) + 2*arctan((sqrt(-x^2 + 1) -
1)/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx=\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^{2}+1}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = \int \frac{\sqrt{-x^2+1}}{x^2+1}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/(x^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(24) = 48$.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = -\frac{1}{2}\pi \mathrm{sgn}(x) + \frac{1}{2}\sqrt{2}\left(\pi \mathrm{sgn}(x) + 2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right) - \arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1),x, algorithm="giac")
[Out] -1/2*pi*sgn(x) + 1/2*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x
^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - arctan(-1/2*x*((sqrt(-x^2
+ 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx = -\mathrm{asin}(x) + \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i}}{2} - \frac{\sqrt{2}\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i}}{2}$$

```
[In] int((1 - x^2)^(1/2)/(x^2 + 1),x)
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/2 - asin(x) - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i
)/(x + 1i))*1i)/2
```

3.72 $\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 27

$$\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx = \operatorname{arcsinh}(x) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)$$

```
[Out] arcsinh(x)-arctanh(x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {399, 221, 385, 213}

$$\int \frac{\sqrt{1+x^2}}{-1+x^2} \, dx = \operatorname{arcsinh}(x) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}x}{\sqrt{x^2+1}}\right)$$

```
[In] Int[Sqrt[1 + x^2]/(-1 + x^2),x]
```

```
[Out] ArcSinh[x] - Sqrt[2]*ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^2]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\int \frac{1}{(-1+x^2)\sqrt{1+x^2}}\,dx+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \sinh^{-1}(x)+2\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx,x,\frac{x}{\sqrt{1+x^2}}\right)\\ &= \sinh^{-1}(x)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx=-\sqrt{2}\text{arctanh}\left(\frac{1-x^2+x\sqrt{1+x^2}}{\sqrt{2}}\right)-\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[Sqrt[1 + x^2]/(-1 + x^2),x]
[Out] -(Sqrt[2]*ArcTanh[(1 - x^2 + x*Sqrt[1 + x^2])/Sqrt[2]]) - Log[-x + Sqrt[1 +
 x^2]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(21) = 42.

Time = 2.43 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

method	result
pseudoelliptic	$-\frac{\ln\left(\frac{-x+\sqrt{x^2+1}}{x}\right)}{2}+\frac{\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)}{2}-\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+1}}{2x}\right)$
trager	$\ln\left(x+\sqrt{x^2+1}\right)-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x^2+4\sqrt{x^2+1}\,x+\operatorname{RootOf}\left(\_Z^2-2\right)}{(-1+x)(1+x)}\right)}{2}$
default	$\frac{\sqrt{(-1+x)^2+2x}}{2}+\operatorname{arcsinh}(x)-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2x)\sqrt{2}}{4\sqrt{(-1+x)^2+2x}}\right)}{2}-\frac{\sqrt{(1+x)^2-2x}}{2}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2x)\sqrt{2}}{4\sqrt{(1+x)^2-2x}}\right)}{2}$

```
[In] int((x^2+1)^(1/2)/(x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*ln((-x+(x^2+1)^(1/2))/x)+1/2*ln((x+(x^2+1)^(1/2))/x)-2^(1/2)*arctanh(1
/2*2^(1/2)*(x^2+1)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(21) = 42$.

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{9x^2-2\sqrt{2}(3x^2+1)-2\sqrt{x^2+1}\left(3\sqrt{2}x-4x\right)+3}{x^2-1}\right) - \log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 1) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 3)/(x^2 - 1)) - log(-x + sqrt(x^2 + 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \int \frac{\sqrt{x^2+1}}{(x-1)(x+1)}\,dx$$

```
[In] integrate((x**2+1)**(1/2)/(x**2-1),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/((x - 1)*(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = -\frac{1}{2}\sqrt{2}\,\mathrm{arsinh}\left(\frac{2x}{|2x+2|}-\frac{2}{|2x+2|}\right) - \frac{1}{2}\sqrt{2}\,\mathrm{arsinh}\left(\frac{2x}{|2x-2|}+\frac{2}{|2x-2|}\right) + \mathrm{arsinh}\,(x)$$

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="maxima")
[Out] -1/2*sqrt(2)*arcsinh(2*x/abs(2*x + 2) - 2/abs(2*x + 2)) - 1/2*sqrt(2)*arcsi
nh(2*x/abs(2*x - 2) + 2/abs(2*x - 2)) + arcsinh(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.59

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|2\left(x-\sqrt{x^2+1}\right)^2-4\sqrt{2}-6\right|}{\left|2\left(x-\sqrt{x^2+1}\right)^2+4\sqrt{2}-6\right|}\right) - \log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2+1)^(1/2)/(x^2-1),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(abs(2*(x - sqrt(x^2 + 1))^2 - 4*sqrt(2) - 6)/abs(2*(x - sq
rt(x^2 + 1))^2 + 4*sqrt(2) - 6)) - log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.19

$$\int \frac{\sqrt{1+x^2}}{-1+x^2}\,dx = \mathrm{asinh}(x) + \frac{\sqrt{2}\left(\ln(x-1)-\ln\left(x+\sqrt{2}\sqrt{x^2+1}+1\right)\right)}{2} - \frac{\sqrt{2}\left(\ln(x+1)-\ln\left(\sqrt{2}\sqrt{x^2+1}-x+1\right)\right)}{2}$$

```
[In] int((x^2 + 1)^(1/2)/(x^2 - 1),x)
[Out] asinh(x) + (2^(1/2)*(log(x - 1) - log(x + 2^(1/2)*(x^2 + 1)^(1/2) + 1)))/2
- (2^(1/2)*(log(x + 1) - log(2^(1/2)*(x^2 + 1)^(1/2) - x + 1)))/2
```

3.73 $\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx = -\frac{\arcsin(x)}{2} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

```
[Out] -1/2*arcsin(x)-1/2*arctanh(x/(-x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {399, 222, 385, 213}

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2} \, dx = -\frac{\arcsin(x)}{2} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

```
[In] Int[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
```

```
[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\right)+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}\,(-1+2x^2)}\,dx\\
&= -\frac{1}{2}\sin^{-1}(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\
&= -\frac{1}{2}\sin^{-1}(x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = \arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

```
[In] Integrate[Sqrt[1 - x^2]/(-1 + 2*x^2),x]
[Out] ArcTan[Sqrt[1 - x^2]/(1 + x)] - ArcTanh[x/Sqrt[1 - x^2]]/2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(19) = 38$.

Time = 2.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

method	result
pseudoelliptic	$\frac{\ln\left(\frac{-x+\sqrt{-x^2+1}}{x}\right)}{4}-\frac{\ln\left(\frac{x+\sqrt{-x^2+1}}{x}\right)}{4}+\frac{\arctan\left(\frac{\sqrt{-x^2+1}}{x}\right)}{2}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}-\frac{\ln\left(-\frac{2x\sqrt{-x^2+1}+1}{2x^2-1}\right)}{4}$
default	$\frac{\sqrt{2}\left(\frac{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}{4}-\frac{\sqrt{2}\arcsin(x)}{4}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{\left(1-\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x-\frac{\sqrt{2}}{2}\right)^2-4\left(x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}\right)}{4}\right)}{2}-\frac{\sqrt{2}\left(\sqrt{-4\left(x+\frac{\sqrt{2}}{\ldots}\right.}\right.}{\ldots}$

[In] int((-x^2+1)^(1/2)/(2*x^2-1),x,method=_RETURNVERBOSE)

[Out] 1/4*ln((-x+(-x^2+1)^(1/2))/x)-1/4*ln((x+(-x^2+1)^(1/2))/x)+1/2*arctan((-x^2+1)^(1/2)/x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(19) = 38$.

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.96

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)+\frac{1}{4}\log\left(-\frac{x^2+\sqrt{-x^2+1}(x+1)-x-1}{x^2}\right) - \frac{1}{4}\log\left(-\frac{x^2-\sqrt{-x^2+1}(x-1)+x-1}{x^2}\right)$$

[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="fricas")

[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{2x^2-1}\,dx$$

[In] integrate((-x**2+1)**(1/2)/(2*x**2-1),x)

[Out] Integral(sqrt(-(x - 1)*(x + 1))/(2*x**2 - 1), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.40

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = -\frac{1}{8}\sqrt{2}\left(2\sqrt{2}\arcsin(x) - \sqrt{2}\log\left(\frac{1}{4}\sqrt{2} + \frac{\sqrt{2}\sqrt{-x^2+1}}{|4x+2\sqrt{2}|} + \frac{1}{|4x+2\sqrt{2}|}\right) + \sqrt{2}\log\left(-\frac{1}{4}\sqrt{2} + \frac{\sqrt{2}\sqrt{-x^2}}{|4x-2}\right.\right.$$

[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="maxima")

[Out] -1/8*sqrt(2)*(2*sqrt(2)*arcsin(x) - sqrt(2)*log(1/4*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x + 2*sqrt(2)) + 1/abs(4*x + 2*sqrt(2))) + sqrt(2)*log(-1/4*sqrt(2) + sqrt(2)*sqrt(-x^2 + 1)/abs(4*x - 2*sqrt(2)) + 1/abs(4*x - 2*sqrt(2))))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.72

$$\begin{aligned}\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = &-\frac{1}{4}\pi\mathrm{sgn}(x) - \frac{1}{2}\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)\\ &-\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}+2\right|\right)\\ &+\frac{1}{4}\log\left(\left|-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}-2\right|\right)\end{aligned}$$

[In] integrate((-x^2+1)^(1/2)/(2*x^2-1),x, algorithm="giac")

[Out] -1/4*pi*sgn(x) - 1/2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)) - 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x + 2)) + 1/4*log(abs(-x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x - 2))

Mupad [B] (verification not implemented)

Time = 5.00 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.40

$$\int \frac{\sqrt{1-x^2}}{-1+2x^2}\,dx = -\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}-1\right)\,1\mathrm{i}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\frac{\sqrt{2}}{2}}\right)}{4}+\frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}\,x}{2}+1\right)\,1\mathrm{i}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\frac{\sqrt{2}}{2}}\right)}{4}-\frac{\mathrm{asin}(x)}{2}$$

```
[In] int((1 - x^2)^(1/2)/(2*x^2 - 1),x)
```

```
[Out] log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2)^(1/2)*1i)/(x + 2^(1/2)/2))/
4 - log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x - 2^(1/2)/
2))/4 - asin(x)/2
```

3.74 $\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{d(44b^2c^2-44abcd+15a^2d^2)\,x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{(2bc-ad)\,(8b^2c^2-8abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

```
[Out] 1/16*(-a*d+2*b*c)*(5*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))/b^(7/2)+1/48*d*(15*a^2*d^2-44*a*b*c*d+44*b^2*c^2)*x*(b*x^2+a)^(1/
2)/b^3+5/24*d*(-a*d+2*b*c)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/b^2+1/6*d*x*(d*x^2+c
)^2*(b*x^2+a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {427, 542, 396, 223, 212}

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)\,(5a^2d^2-8abcd+8b^2c^2)}{16b^{7/2}} + \frac{dx\sqrt{a+bx^2}(15a^2d^2-44abcd+44b^2c^2)}{48b^3} + \frac{5dx\sqrt{a+bx^2}(c+dx^2)\,(2bc-ad)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b}$$

```
[In] Int[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (d*(44*b^2*c^2 - 44*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(48*b^3) + (5*
d*(2*b*c - a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(24*b^2) + (d*x*Sqrt[a + b*x
^2]*(c + d*x^2)^2)/(6*b) + ((2*b*c - a*d)*(8*b^2*c^2 - 8*a*b*c*d + 5*a^2*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{\int \frac{(c+dx^2)\left(c(6bc-ad)+5d(2bc-ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{\int \frac{c(24b^2c^2-14abcd+5a^2d^2)+d(44b^2c^2-44abcd+15a^2d^2)x^2}{\sqrt{a+bx^2}}dx}{24b^2}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{((2bc-ad)(8b^2c^2-8abcd+5a^2d^2))\int \frac{1}{\sqrt{a+bx^2}}dx}{16b^3}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{((2bc-ad)(8b^2c^2-8abcd+5a^2d^2))\operatorname{Subst}\left(\int \frac{1}{1-bx^2}dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^3}$$

$$= \frac{d(44b^2c^2-44abcd+15a^2d^2)x\sqrt{a+bx^2}}{48b^3} + \frac{5d(2bc-ad)x\sqrt{a+bx^2}(c+dx^2)}{24b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)^2}{6b} + \frac{(2bc-ad)(8b^2c^2-8abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}dx$$
$$= \frac{\sqrt{b}dx\sqrt{a+bx^2}(15a^2d^2-2abd(27c+5dx^2)+4b^2(18c^2+9cdx^2+2d^2x^4))+(-48b^3c^3+72ab^2c^2d-54a^2bc}{48b^{7/2}}$$

```
[In] Integrate[(c + d*x^2)^3/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[a + b*x^2]*(15*a^2*d^2 - 2*a*b*d*(27*c + 5*d*x^2) + 4*b^2
*(18*c^2 + 9*c*d*x^2 + 2*d^2*x^4)) + (-48*b^3*c^3 + 72*a*b^2*c^2*d - 54*a^2
*b*c*d^2 + 15*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(48*b^(7/2))
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$-\frac{5\left((ad-2bc)\left(a^2d^2-\frac{8}{5}abcd+\frac{8}{5}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-\left(\left(\frac{8}{15}d^2x^4+\frac{12}{5}cd\,x^2+\frac{24}{5}c^2\right)b^{\frac{5}{2}}+\left(\left(-\frac{2d\,x^2}{3}-\frac{18c}{5}\right)b^{\frac{3}{2}}+ad\sqrt{b}\right)da \cdots\right.\right.}{16b^{\frac{7}{2}}}$
risch	$\frac{dx\left(8b^2d^2x^4-10x^2ab\,d^2+36x^2b^2cd+15a^2d^2-54abcd+72b^2c^2\right)\sqrt{b\,x^2+a}}{48b^3}-\frac{\left(5a^3d^3-18a^2bc\,d^2+24a\,b^2c^2d-16b^3c^3\right)\ln\left(x\sqrt{b}\cdots\right.}{16b^{\frac{7}{2}}}$
default	$\frac{c^3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^3\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+3c\,d^2\left(\cdots\right.$

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/16*((a*d-2*b*c)*(a^2*d^2-8/5*a*b*c*d+8/5*b^2*c^2)*arctanh((b*x^2+a)^(1/2
)/x/b^(1/2))-((8/15*d^2*x^4+12/5*c*d*x^2+24/5*c^2)*b^(5/2)+((-2/3*d*x^2-18/
5*c)*b^(3/2)+a*d*b^(1/2))*d*a)*x*(b*x^2+a)^(1/2)*d)/b^(7/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.78

$$\int\frac{\left(c+dx^2\right)^3}{\sqrt{a+bx^2}}\,dx$$

$$=\left[-\frac{3\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(8\,b^3d^3x^5+2\,(\cdots\right.}{96\,b^4}\right.$$

$$-\frac{3\left(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(8\,b^3d^3x^5+2\left(18\,b^3cd^2-5\,ab^2d^3\right)x\cdots\right.}{48\,b^4}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b
)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*d^3*x^5 + 2*(1
8*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2*c*d^2 + 5*a^2*b
*d^3)*x)*sqrt(b*x^2 + a))/b^4, -1/48*(3*(16*b^3*c^3 - 24*a*b^2*c^2*d + 18*a
^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^
3*d^3*x^5 + 2*(18*b^3*c*d^2 - 5*a*b^2*d^3)*x^3 + 3*(24*b^3*c^2*d - 18*a*b^2
*c*d^2 + 5*a^2*b*d^3)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^3x^5}{6b}+\frac{x^3\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b}+\frac{x\left(-\frac{3a\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b}+3c^2d\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3a\left(-\frac{5ad^3}{6b}+3cd^2\right)}{4b}+3c^2d\right)}{2b}+c^3\right)\left(\left(\begin{cases} \frac{\log}{} \\ \frac{x\,\mathrm{l}}{} \end{cases}\right.\right. \\ \frac{c^3x+c^2dx^3+\frac{3cd^2x^5}{5}+\frac{d^3x^7}{7}}{\sqrt{a}} \end{cases}$$

[In] integrate((d*x**2+c)**3/(b*x**2+a)**(1/2),x)

[Out] Piecewise((sqrt(a + b*x**2)*(d**3*x**5/(6*b) + x**3*(-5*a*d**3/(6*b) + 3*c*
d**2)/(4*b) + x*(-3*a*(-5*a*d**3/(6*b) + 3*c*d**2)/(4*b) + 3*c**2*d)/(2*b))
 + (-a*(-3*a*(-5*a*d**3/(6*b) + 3*c*d**2)/(4*b) + 3*c**2*d)/(2*b) + c**3)*P
iecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c**3*x + c**2*d*x**3 + 3*c*d**2*x**
5/5 + d**3*x**7/7)/sqrt(a), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^3x^5}{6\,b}+\frac{3\,\sqrt{bx^2+a}cd^2x^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}ad^3x^3}{24\,b^2}+\frac{3\,\sqrt{bx^2+a}c^2dx}{2\,b}-\frac{9\,\sqrt{bx^2+a}acd^2x}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}a^2d^3x}{16\,b^3}+\frac{c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{3\,ac^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{9\,a^2cd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] 1/6*sqrt(b*x^2 + a)*d^3*x^5/b + 3/4*sqrt(b*x^2 + a)*c*d^2*x^3/b - 5/24*sqrt
(b*x^2 + a)*a*d^3*x^3/b^2 + 3/2*sqrt(b*x^2 + a)*c^2*d*x/b - 9/8*sqrt(b*x^2
+ a)*a*c*d^2*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*d^3*x/b^3 + c^3*arcsinh(b*x/s
qrt(a*b))/sqrt(b) - 3/2*a*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 9/8*a^2*c*
d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*d^3*arcsinh(b*x/sqrt(a*b))/b^
(7/2)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{48}\left(2\left(\frac{4\,d^3x^2}{b}+\frac{18\,b^4cd^2-5\,ab^3d^3}{b^5}\right)x^2+\frac{3\,(24\,b^4c^2d-18\,ab^3cd^2+5\,a^2b^2d^3)}{b^5}\right)\sqrt{bx^2+a}\,x$$
$$-\frac{(16\,b^3c^3-24\,ab^2c^2d+18\,a^2bcd^2-5\,a^3d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*d^3*x^2/b + (18*b^4*c*d^2 - 5*a*b^3*d^3)/b^5)*x^2 + 3*(24*b^4*c^
2*d - 18*a*b^3*c*d^2 + 5*a^2*b^2*d^3)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3
*c^3 - 24*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{\sqrt{a+bx^2}}\,dx=\int \frac{{\left(d\,x^2+c\right)}^3}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(1/2), x)
```

3.75 $\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5
/2)+3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2+1/4*d*x*(d*x^2+c)*(b*x^2+a)^(1
/2)/b
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {427, 396, 223, 212}

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2d^2-8abcd+8b^2c^2)}{8b^{5/2}} + \frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b}$$

```
[In] Int[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (3*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(8*b^2) + (d*x*Sqrt[a + b*x^2]*(c + d
*x^2))/(4*b) + ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} + \frac{\int \frac{c(4bc-ad)+3d(2bc-ad)x^2}{\sqrt{a+bx^2}}\,dx}{4b} \\
&= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} \\
&\quad - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^2} \\
&= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} \\
&\quad - \frac{(3ad(2bc-ad)-2bc(4bc-ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^2} \\
&= \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8b^2} + \frac{dx\sqrt{a+bx^2}(c+dx^2)}{4b} + \frac{(8b^2c^2-8abcd+3a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}(8bc-3ad+2bdx^2)}{8b^2} + \frac{(-8b^2c^2+8abcd-3a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

```
[In] Integrate[(c + d*x^2)^2/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(8*b*c - 3*a*d + 2*b*d*x^2))/(8*b^2) + ((-8*b^2*c^2 +
8*a*b*c*d - 3*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{dx\left(-2bd\,x^2+3ad-8bc\right)\sqrt{b\,x^2+a}}{8b^2}+\frac{\left(3a^2d^2-8abcd+8b^2c^2\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{\frac{3\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}-\frac{3x\sqrt{b\,x^2+a}\left(\frac{2\left(-d\,x^2-4c\right)b^{\frac{3}{2}}}{3}+ad\sqrt{b}\right)d}{8}}{b^{\frac{5}{2}}}$
default	$\frac{c^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^2\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+2cd\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{\right.}\right.}{}\right.$

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*d*x*(-2*b*d*x^2+3*a*d-8*b*c)*(b*x^2+a)^(1/2)/b^2+1/8*(3*a^2*d^2-8*a*b*
c*d+8*b^2*c^2)/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.78

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx}}{16\,b^3}\right.$$
$$\left.-\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2d^2x^3+(8\,b^2cd-3\,abd^2)x)\sqrt{bx^2+a}}{8\,b^3}\right]$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)*sqr
t(b*x^2 + a))/b^3, -1/8*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*d^2*x^3 + (8*b^2*c*d - 3*a*b*d^2)*x)
*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{d^2x^3}{4b}+\frac{x\left(-\frac{3ad^2}{4b}+2cd\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3ad^2}{4b}+2cd\right)}{2b}+c^2\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) & \text{for } b \neq \\ \frac{c^2x+\frac{2cdx^3}{3}+\frac{d^2x^5}{5}}{\sqrt{a}} & \text{otherw} \end{cases}$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)*(d**2*x**3/(4*b) + x*(-3*a*d**2/(4*b) + 2*c*d)/
(2*b)) + (-a*(-3*a*d**2/(4*b) + 2*c*d)/(2*b) + c**2)*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tr
ue)), Ne(b, 0)), ((c**2*x + 2*c*d*x**3/3 + d**2*x**5/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}d^2x^3}{4\,b} + \frac{\sqrt{bx^2+a}cdx}{b} - \frac{3\,\sqrt{bx^2+a}ad^2x}{8\,b^2} + \frac{c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{acd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{3\,a^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*d^2*x^3/b + sqrt(b*x^2 + a)*c*d*x/b - 3/8*sqrt(b*x^2 +
a)*a*d^2*x/b^2 + c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - a*c*d*arcsinh(b*x/sqr
t(a*b))/b^(3/2) + 3/8*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \frac{1}{8}\sqrt{bx^2+a}\left(\frac{2\,d^2x^2}{b} + \frac{8\,b^2cd-3\,abd^2}{b^3}\right)x - \frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*d^2*x^2/b + (8*b^2*c*d - 3*a*b*d^2)/b^3)*x - 1/8*(8*
b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{{\left(d\,x^2+c\right)}^2}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(1/2), x)
```

3.76 $\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx=\frac{dx\sqrt{a+bx^2}}{2b}+\frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*d*x*(b*x^2+
a)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {396, 223, 212}

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx=\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc-ad)}{2b^{3/2}}+\frac{dx\sqrt{a+bx^2}}{2b}$$

```
[In] Int[(c + d*x^2)/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b} \\
&= \frac{dx\sqrt{a+bx^2}}{2b} - \frac{(-2bc+ad)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b} \\
&= \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{2b} + \frac{(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/Sqrt[a + b*x^2],x]
[Out] (d*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*c - a*d)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
+ Sqrt[a + b*x^2])])/b^(3/2)
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{dx\sqrt{b\,x^{2}+a}}{2b}-\frac{\left(ad-2bc\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	47
default	$\frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	63
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,dx\sqrt{b}-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)ad+2\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)bc}{2b^{\frac{3}{2}}}$	64

```
[In] int((d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d*x*(b*x^2+a)^(1/2)/b-1/2*(a*d-2*b*c)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1
/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.95

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}bdx-(2\,bc-ad)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2},\frac{\sqrt{bx^2+a}bdx-(2\,bc-ad)\sqrt{-b}\,\mathrm{arc}}{2\,b^2}\right.$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*d*x - (2*b*c - a*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx=\begin{cases}\left(-\frac{ad}{2b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)+\frac{dx\sqrt{a+bx^2}}{2b} & \text{for}\ b\neq 0\\ \frac{cx+\frac{dx^3}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise(((-a*d/(2*b) + c)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b
*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + d*x*sqrt(a + b*x**
2)/(2*b), Ne(b, 0)), ((c*x + d*x**3/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}dx}{2\,b} + \frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d*arcs
inh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}dx}{2\,b} - \frac{(2\,bc-ad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*d*x/b - 1/2*(2*b*c - a*d)*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{c+dx^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{d\,x^3+3\,c\,x}{3\,\sqrt{a}} & \text{if}\ \ b=0 \\ \frac{c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{d\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

```
[In] int((c + d*x^2)/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*c*x + d*x^3)/(3*a^(1/2)), b ~= 0, (c*log(b^(1/2)*x + (
a + b*x^2)^(1/2)))/b^(1/2) - (a*d*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(
2*b^(3/2)) + (d*x*(a + b*x^2)^(1/2))/(2*b))
```

3.77 $\int \frac{1}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {223, 212}

$$\int \frac{1}{\sqrt{a+bx^2}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Int[1/Sqrt[a + b*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[In] Integrate[1/Sqrt[a + b*x^2],x]
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

```
[In] int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

```
[In] integrate(1/(b*x**2+a)**(1/2),x)
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

```
[In] integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b
)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx=\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

```
[In] int(1/(a + b*x^2)^(1/2),x)
[Out] log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

3.78 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

```
[Out] arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(1/2)/(-a*d+b*c)^(1/2
)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {385, 214}

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])]/(Sqrt[c]*Sqrt[b*c -
a*d])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = -\frac{\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}\sqrt{-bc+ad}}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)),x]
[Out] -(ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c
) + a*d])]/(Sqrt[c]*Sqrt[-(b*c) + a*d]))
```

Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}$
default	$\frac{\ln\left(\frac{\frac{2ad-2bc}{d}-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,\sqrt{\frac{ad-bc}{d}}} - \ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\sqrt{\right.}\right.}{d}\right.$

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
[Out] -1/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(39) = 78.

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \left[\frac{\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,abc^2-3\,a^2cd)x^2+4\,((2\,bc-ad)x^3+acx)\sqrt{bc^2-acd}\sqrt{bx^2+a}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,\sqrt{bc^2-acd}},\ -\frac{\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)}{2\,(bc^2-acd)}\right]$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 -
3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x
^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2))/sqrt(b*c^2 - a*c*d), -1/2*sqrt(-b*c^2
 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*
x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x))/(b*c^2 - a*c*d)
]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}}$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c),x, algorithm="giac")
[Out] -sqrt(b)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(
-b^2*c^2 + a*b*c*d))/sqrt(-b^2*c^2 + a*b*c*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx = \begin{cases} \frac{\mathrm{atan}\left(\frac{x\,\sqrt{a\,d-b\,c}}{\sqrt{c}\,\sqrt{b\,x^2+a}}\right)}{\sqrt{c\,(a\,d-b\,c)}} & \text{if } 0<a\,d-b\,c \\ \frac{\ln\left(\frac{\sqrt{c\,(b\,x^2+a)}+x\,\sqrt{b\,c-a\,d}}{\sqrt{c\,(b\,x^2+a)}-x\,\sqrt{b\,c-a\,d}}\right)}{2\,\sqrt{-c\,(a\,d-b\,c)}} & \text{if } a\,d-b\,c<0 \\ \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)}\,dx & \text{if } a\,d-b\,c\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)),x)
[Out] piecewise(0 < a*d - b*c, atan((x*(a*d - b*c)^(1/2))/(c^(1/2)*(a + b*x^2)^(1
/2)))/(c*(a*d - b*c))^(1/2), a*d - b*c < 0, log(((c*(a + b*x^2))^(1/2) + x*
(- a*d + b*c)^(1/2))/((c*(a + b*x^2))^(1/2) - x*(- a*d + b*c)^(1/2)))/(2*(-
c*(a*d - b*c))^(1/2)), ~in(a*d - b*c, 'real') | a*d == b*c, int(1/((a + b*x
^2)^(1/2)*(c + d*x^2)), x))
```

3.79 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\,(c+dx^2)} + \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-a*d+2*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2
)/(-a*d+b*c)^(3/2)-1/2*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 385, 214}

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \frac{(2bc-ad)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}} - \frac{dx\sqrt{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Arc
Tanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)
^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\int\frac{1}{\sqrt{a+bx^2}\,(c+dx^2)}\,dx}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\mathrm{Subst}\left(\int\frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$
\begin{aligned}
&\int\frac{1}{\sqrt{a+bx^2}\,\left(c+dx^2\right)^2}\,dx\\
&= -\frac{dx\sqrt{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{(2bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{3/2}}
\end{aligned}
$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)) + ((2*b*c - a*d)*Arc
Tan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a
*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(3/2))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\frac{d\sqrt{b x^2+a}\,x}{d x^2+c}-\frac{(ad-2bc)\arctan\left(\frac{c\sqrt{b x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{\sqrt{(ad-bc)c}}}{2(ad-bc)c}$
default	$-\frac{-\frac{d\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}{(ad-bc)\left(x-\frac{\sqrt{-cd}}{d}\right)}+\frac{b\sqrt{-cd}\ln\left(\frac{\frac{2ad-2bc}{d}+\frac{2b\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{\frac{ad-bc}{d}}\sqrt{\left(x-\frac{\sqrt{-cd}}{d}\right)^2 b+}}{x-\frac{\sqrt{-cd}}{d}}\right.}{(ad-bc)\sqrt{\frac{ad-bc}{d}}}}{4dc}$

[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)

[Out] 1/2/(a*d-b*c)/c*(d*(b*x^2+a)^(1/2)*x/(d*x^2+c)-(a*d-2*b*c)/((a*d-b*c)*c)^(1/2)*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(85) = 170$.

Time = 0.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 4.58

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$
$$= \left[-\frac{4\,(bc^2d-acd^2)\sqrt{bx^2+a}x-(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2}{}\right.}{8\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x}\right.$$
$$-\frac{2\,(bc^2d-acd^2)\sqrt{bx^2+ax}+(2\,bc^2-acd+(2\,bcd-ad^2)x^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}((2\,bc-ad)x^2+}{2\,((b^2c^2-abcd)x^3+(abc^2-}\right.}{4\,(b^2c^5-2\,abc^4d+a^2c^3d^2+(b^2c^4d-2\,abc^3d^2+a^2c^2d^3)x^2)}$$

[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] [-1/8*(4*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x - (2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2), -1/4*(2*(b*c^2*d - a*c*d^2)*sqrt(b*x^2 + a)*x + (2*b*c^2 - a*c*d + (2*b*c*d - a*d^2)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x^2)]

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**2,x)
[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(85) = 170$.

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.40

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx$$
$$= \frac{1}{2}b^{\frac{3}{2}}\left(\frac{(2\,bc-ad)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-abcd)\sqrt{-b^2c^2+abcd}} - \frac{2\left(2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-\left(\cdots\right.\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc\cdots\right.}\right.$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^2,x, algorithm="giac")
[Out] 1/2*b^(3/2)*((2*b*c - a*d)*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d +
 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - a*b*c*d)*sqrt(-b^2*c^2
+ a*b*c*d)) - 2*(2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - (sqrt(b)*x - sqrt(
b*x^2 + a))^2*a*d + a^2*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d
)*(b^2*c^2 - a*b*c*d)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^2), x)
```

3.80 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^3} dx = -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)(c+dx^2)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2(c+dx^2)} + \frac{(8b^2c^2-8abcd+3a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}$$

```
[Out] 1/8*(3*a^2*d^2-8*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x
^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(5/2)-1/4*d*x*(b*x^2+a)^(1/2)/c/(-a*d+b*c)/
(d*x^2+c)^2-3/8*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^3} dx = \frac{(3a^2d^2-8abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}} - \frac{3dx\sqrt{a+bx^2}(2bc-ad)}{8c^2(c+dx^2)(bc-ad)^2} - \frac{dx\sqrt{a+bx^2}}{4c(c+dx^2)^2(bc-ad)}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x*Sqrt[a + b*x^2])/(c*(b*c - a*d)*(c + d*x^2)^2) - (3*d*(2*b*c - a*
d)*x*Sqrt[a + b*x^2])/(8*c^2*(b*c - a*d)^2*(c + d*x^2)) + ((8*b^2*c^2 - 8*a
```

```
*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])
/(8*c^(5/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-2bdx^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2}-\frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)}+\frac{\int \frac{8b^2c^2-8abcd+3a^2d^2}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)^2}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)} \\
&\quad + \frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{8c^2(bc-ad)^2} \\
&= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)} \\
&\quad + \frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2(bc-ad)^2} \\
&= -\frac{dx\sqrt{a+bx^2}}{4c(bc-ad)\left(c+dx^2\right)^2} - \frac{3d(2bc-ad)x\sqrt{a+bx^2}}{8c^2(bc-ad)^2\left(c+dx^2\right)} \\
&\quad + \frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

$$
\begin{aligned}
\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)^3}\,dx &= \frac{dx\sqrt{a+bx^2}\left(-2bc\left(4c+3dx^2\right)+ad\left(5c+3dx^2\right)\right)}{8c^2(bc-ad)^2\left(c+dx^2\right)^2} \\
&\quad - \frac{\left(8b^2c^2-8abcd+3a^2d^2\right)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{8c^{5/2}(-bc+ad)^{5/2}}
\end{aligned}
$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^3),x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*(-2*b*c*(4*c + 3*d*x^2) + a*d*(5*c + 3*d*x^2)))/(8*c^2
*(b*c - a*d)^2*(c + d*x^2)^2) - ((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*ArcTan
[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d]
)])/(8*c^(5/2)*(-(b*c) + a*d)^(5/2))
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

method	result	size
pseudoelliptic	$\frac{-\frac{3\left(d\,x^2+c\right)^2\left(a^2d^2-\frac{8}{3}abcd+\frac{8}{3}b^2c^2\right)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)}{8}+\frac{5x\left(-\frac{8b\,c^2}{5}+d\left(-\frac{6b\,x^2}{5}+a\right)c+\frac{3a\,d^2x^2}{5}\right)\sqrt{b\,x^2+a}\,d\sqrt{(ad-bc)c}}{8}}{\sqrt{(ad-bc)c}\,(ad-bc)^2c^2(d\,x^2+c)^2}$	149
default	Expression too large to display	1843

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8/((a*d-b*c)*c)^(1/2)*(-3/5*(d*x^2+c)^2*(a^2*d^2-8/3*a*b*c*d+8/3*b^2*c^2)
*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+x*(-8/5*b*c^2+d*(-6/5*b*x^
2+a)*c+3/5*a*d^2*x^2)*(b*x^2+a)^(1/2)*d*((a*d-b*c)*c)^(1/2))/(a*d-b*c)^2/c^
2/(d*x^2+c)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(143) = 286$.

Time = 0.49 (sec) , antiderivative size = 864, normalized size of antiderivative = 5.30

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx$$
$$= \left[\frac{(8\,b^2c^4 - 8\,abc^3d + 3\,a^2c^2d^2 + (8\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(8\,b^2c^3d - 8\,abc^2d^2 + 3\,a^2cd^3)x^2)\sqrt{bc^2}}{32\,(b^3c^8 - 3\,ab^2c^7d + 3\,a^2bc^6d^2 - a^3}\right.$$
$$-\frac{(8\,b^2c^4 - 8\,abc^3d + 3\,a^2c^2d^2 + (8\,b^2c^2d^2 - 8\,abcd^3 + 3\,a^2d^4)x^4 + 2\,(8\,b^2c^3d - 8\,abc^2d^2 + 3\,a^2cd^3)x^2)\sqrt{-b}}{16\,(b^3c^8 - 3\,ab^2c^7d + 3\,a^2bc^6d^2 - a^3c^5d^3 + (b^3c^6d^2 -}$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/32*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2
*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a
*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*(3*(2*b^2*c^3*d^2 -
 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c^3*d^2 + 5*a^2*c^2
*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*
c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x
^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x^2),
-1/16*((8*b^2*c^4 - 8*a*b*c^3*d + 3*a^2*c^2*d^2 + (8*b^2*c^2*d^2 - 8*a*b*c*
d^3 + 3*a^2*d^4)*x^4 + 2*(8*b^2*c^3*d - 8*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^2)*s
qrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*
c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*(
3*(2*b^2*c^3*d^2 - 3*a*b*c^2*d^3 + a^2*c*d^4)*x^3 + (8*b^2*c^4*d - 13*a*b*c
^3*d^2 + 5*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^
2*b*c^6*d^2 - a^3*c^5*d^3 + (b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^
4 - a^3*c^3*d^5)*x^4 + 2*(b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a
^3*c^4*d^4)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^3}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 538 vs. 2(143) = 286.

Time = 1.74 (sec) , antiderivative size = 538, normalized size of antiderivative = 3.30

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = -\frac{1}{8}b^{\frac{5}{2}}\left(\frac{(8\,b^2c^2-8\,abcd+3\,a^2d^2)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{(b^4c^4-2\,ab^3c^3d+a^2b^2c^2d^2)\sqrt{-b^2c^2+abcd}}+\frac{2\left(8\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^2c^2d-8\,(\right.}{}\right.$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^3,x, algorithm="giac")
[Out] -1/8*b^(5/2)*((8*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*arctan(1/2*((sqrt(b)*x -
sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^4 - 2
*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(-b^2*c^2 + a*b*c*d)) + 2*(8*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*b^2*c^2*d - 8*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b*c*d
^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*d^3 + 48*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*b^3*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^2*c^2*d + 42*(sqr
t(b)*x - sqrt(b*x^2 + a))^4*a^2*b*c*d^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^3*d^3 + 40*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^2*c^2*d - 40*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*a^3*b*c*d^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*d
^3 + 6*a^4*b*c*d^2 - 3*a^5*d^3)/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2
)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*
c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^3}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^3}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^3), x)
```

3.81 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx &= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4}\\ &\quad-\frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3}\\ &\quad-\frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2}+\frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}}\\ &\quad+\frac{d(64b^3c^3-144ab^2c^2d+120a^2bcd^2-35a^3d^3)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}\end{aligned}$$

```
[Out] 1/16*d*(-35*a^3*d^3+120*a^2*b*c*d^2-144*a*b^2*c^2*d+64*b^3*c^3)*arctanh(x*b
^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(1/2
)-1/48*d*(-105*a^3*d^3+290*a^2*b*c*d^2-248*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2
+a)^(1/2)/a/b^4-1/24*d*(35*a^2*d^2-64*a*b*c*d+24*b^2*c^2)*x*(d*x^2+c)*(b*x^
2+a)^(1/2)/a/b^3-1/6*d*(-7*a*d+6*b*c)*x*(d*x^2+c)^2*(b*x^2+a)^(1/2)/a/b^2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{424, 542, 396, 223, 212\}$

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = -\frac{dx\sqrt{a+bx^2}(c+dx^2)\left(35a^2d^2-64abcd+24b^2c^2\right)}{24ab^3} + \frac{d\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-35a^3d^3+120a^2bcd^2-144ab^2c^2d+64b^3c^3\right)}{16b^{9/2}} - \frac{dx\sqrt{a+bx^2}(-105a^3d^3+290a^2bcd^2-248ab^2c^2d+48b^3c^3)}{48ab^4} - \frac{dx\sqrt{a+bx^2}(c+dx^2)^2\,(6bc-7ad)}{6ab^2} + \frac{x(c+dx^2)^3\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/48*(d*(48*b^3*c^3 - 248*a*b^2*c^2*d + 290*a^2*b*c*d^2 - 105*a^3*d^3)*x*S
qrt[a + b*x^2])/(a*b^4) - (d*(24*b^2*c^2 - 64*a*b*c*d + 35*a^2*d^2)*x*Sqrt[
a + b*x^2]*(c + d*x^2))/(24*a*b^3) - (d*(6*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*(
c + d*x^2)^2)/(6*a*b^2) + ((b*c - a*d)*x*(c + d*x^2)^3)/(a*b*Sqrt[a + b*x^2
]) + (d*(64*b^3*c^3 - 144*a*b^2*c^2*d + 120*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)^2(acd-d(6bc-7ad)x^2)}{\sqrt{a+bx^2}}\,dx}{ab} \\
&= -\frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2} + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{\int \frac{(c+dx^2)(acd(12bc-7ad)-d(24b^2c^2-64abcd+35a^2d^2)x^2)}{\sqrt{a+bx^2}}\,dx}{6ab^2} \\
&= -\frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3} - \frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2} \\
&\quad + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd(72b^2c^2-92abcd+35a^2d^2)-d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)x^2}{\sqrt{a+bx^2}}\,dx}{24ab^3} \\
&= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} \\
&\quad - \frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3} \\
&\quad - \frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2} + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{(d(64b^3c^3-144ab^2c^2d+120a^2bcd^2-35a^3d^3))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{16b^4} \\
&= -\frac{d(48b^3c^3-248ab^2c^2d+290a^2bcd^2-105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} \\
&\quad - \frac{d(24b^2c^2-64abcd+35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3} \\
&\quad - \frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2} + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{(d(64b^3c^3-144ab^2c^2d+120a^2bcd^2-35a^3d^3))\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{16b^4}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{d(48b^3c^3 - 248ab^2c^2d + 290a^2bcd^2 - 105a^3d^3)\,x\sqrt{a+bx^2}}{48ab^4} \\
&\quad - \frac{d(24b^2c^2 - 64abcd + 35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{24ab^3} \\
&\quad - \frac{d(6bc-7ad)x\sqrt{a+bx^2}(c+dx^2)^2}{6ab^2} + \frac{(bc-ad)x(c+dx^2)^3}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{d(64b^3c^3 - 144ab^2c^2d + 120a^2bcd^2 - 35a^3d^3)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.77

$$
\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{b}x\left(48b^4c^4+105a^4d^4+5a^3bd^3\left(-72c+7dx^2\right)-2a^2b^2d^2\left(-216c^2+60cdx^2+7d^2x^4\right)+8ab^3d\left(-24c^3+18c^2dx^2+6cd^2x^4+d^3 \right.\right.}{a\sqrt{a+bx^2}}}{48b^{9/2}}
$$

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(3/2),x]
```

```
[Out] ((Sqrt[b]*x*(48*b^4*c^4 + 105*a^4*d^4 + 5*a^3*b*d^3*(-72*c + 7*d*x^2) - 2*a
^2*b^2*d^2*(-216*c^2 + 60*c*d*x^2 + 7*d^2*x^4) + 8*a*b^3*d*(-24*c^3 + 18*c^
2*d*x^2 + 6*c*d^2*x^4 + d^3*x^6)))/(a*Sqrt[a + b*x^2]) + 3*d*(-64*b^3*c^3 +
 144*a*b^2*c^2*d - 120*a^2*b*c*d^2 + 35*a^3*d^3)*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(48*b^(9/2))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{35\left(\sqrt{b\,x^{2}+a}\,a d\left(a^{3} d^{3}-\frac{24}{7} a^{2} b c\,d^{2}+\frac{144}{35} a\,b^{2} c^{2} d-\frac{64}{35} b^{3} c^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\left(-\frac{64 d\left(-\frac{1}{24} d^{3} x^{6}-\frac{1}{4} c\,d^{2} x^{4}-\frac{3}{4} c^{2} d\,x^{2}+c^{3}\right) a\,b}{35}\right.\right.}{16 b^{\frac{9}{2}} \sqrt{b\,x^{2}+a}\,a}$
risch	$\frac{x\,d^{2}\left(8 b^{2} d^{2} x^{4}-22 x^{2} a b\,d^{2}+48 x^{2} b^{2} c d+57 a^{2} d^{2}-168 a b c d+144 b^{2} c^{2}\right) \sqrt{b\,x^{2}+a}}{48 b^{4}}-\frac{\frac{19 a^{3} d^{4} x}{\sqrt{b\,x^{2}+a}}-\frac{16 b^{4} c^{4} x}{a\sqrt{b\,x^{2}+a}}-\frac{56 a^{2} b c\,d^{3} x}{\sqrt{b\,x^{2}+a}}+\frac{48 a\,b^{2} c^{2} d^{2} x}{\sqrt{b\,x^{2}+a}}}{}$
default	$\frac{c^{4} x}{a\sqrt{b\,x^{2}+a}}+d^{4}\left(\frac{x^{7}}{6 b\sqrt{b\,x^{2}+a}}-\frac{7 a\left(\frac{x^{5}}{4 b\sqrt{b\,x^{2}+a}}-\frac{5 a\left(\frac{x^{3}}{2 b\sqrt{b\,x^{2}+a}}-\frac{3 a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2 b}\right)}{4 b}\right)}{6 b}\right)+4 c\,d^{3}$

```
[In] int((d*x^2+c)^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -35/16*((b*x^2+a)^(1/2)*a*d*(a^3*d^3-24/7*a^2*b*c*d^2+144/35*a*b^2*c^2*d-64
/35*b^3*c^3)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(-64/35*d*(-1/24*d^3*x^6-
1/4*c*d^2*x^4-3/4*c^2*d*x^2+c^3)*a*b^(7/2)+144/35*d^2*(-7/216*d^2*x^4-5/18*
c*d*x^2+c^2)*a^2*b^(5/2)-24/7*d^3*(-7/72*d*x^2+c)*a^3*b^(3/2)+b^(1/2)*a^4*d
^4+16/35*b^(9/2)*c^4))/b^(9/2)/(b*x^2+a)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{3\,(64\,a^2b^3c^3d-144\,a^3b^2c^2d^2+120\,a^4bcd^3-35\,a^5d^4+(64\,ab^4c^3d-144\,a^2b^3c^2d^2+12}{}\right.$$

$$-\frac{3\,(64\,a^2b^3c^3d-144\,a^3b^2c^2d^2+120\,a^4bcd^3-35\,a^5d^4+(64\,ab^4c^3d-144\,a^2b^3c^2d^2+120\,a^3b^2cd^3-35\,a^4bd^4)x}{}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2 + 120*a^4*b*c*d^3 - 35*a^
5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 120*a^3*b^2*c*d^3 - 35*a^4*
```

```
b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*
a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a*b^4*c^2*d^2
 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64*a*b^4*c^3*d
 + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*sqrt(b*x^2 +
a))/(a*b^6*x^2 + a^2*b^5), -1/48*(3*(64*a^2*b^3*c^3*d - 144*a^3*b^2*c^2*d^2
 + 120*a^4*b*c*d^3 - 35*a^5*d^4 + (64*a*b^4*c^3*d - 144*a^2*b^3*c^2*d^2 + 1
20*a^3*b^2*c*d^3 - 35*a^4*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - (8*a*b^4*d^4*x^7 + 2*(24*a*b^4*c*d^3 - 7*a^2*b^3*d^4)*x^5 + (144*a
*b^4*c^2*d^2 - 120*a^2*b^3*c*d^3 + 35*a^3*b^2*d^4)*x^3 + 3*(16*b^5*c^4 - 64
*a*b^4*c^3*d + 144*a^2*b^3*c^2*d^2 - 120*a^3*b^2*c*d^3 + 35*a^4*b*d^4)*x)*s
qrt(b*x^2 + a))/(a*b^6*x^2 + a^2*b^5)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^4}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx &= \frac{d^4x^7}{6\sqrt{bx^2+a}b} + \frac{cd^3x^5}{\sqrt{bx^2+a}b} - \frac{7\,ad^4x^5}{24\sqrt{bx^2+a}b^2} \\ &+ \frac{3\,c^2d^2x^3}{\sqrt{bx^2+a}b} - \frac{5\,acd^3x^3}{2\sqrt{bx^2+a}b^2} + \frac{35\,a^2d^4x^3}{48\sqrt{bx^2+a}b^3} + \frac{c^4x}{\sqrt{bx^2+a}a} - \frac{4\,c^3dx}{\sqrt{bx^2+a}b} \\ &+ \frac{9\,ac^2d^2x}{\sqrt{bx^2+a}b^2} - \frac{15\,a^2cd^3x}{2\sqrt{bx^2+a}b^3} + \frac{35\,a^3d^4x}{16\sqrt{bx^2+a}b^4} + \frac{4\,c^3d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} \\ &- \frac{9\,ac^2d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} + \frac{15\,a^2cd^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}} - \frac{35\,a^3d^4\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{9}{2}}}\end{aligned}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*d^4*x^7/(sqrt(b*x^2 + a)*b) + c*d^3*x^5/(sqrt(b*x^2 + a)*b) - 7/24*a*d^
4*x^5/(sqrt(b*x^2 + a)*b^2) + 3*c^2*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/2*a*c*d
^3*x^3/(sqrt(b*x^2 + a)*b^2) + 35/48*a^2*d^4*x^3/(sqrt(b*x^2 + a)*b^3) + c^
4*x/(sqrt(b*x^2 + a)*a) - 4*c^3*d*x/(sqrt(b*x^2 + a)*b) + 9*a*c^2*d^2*x/(sq
```

```
rt(b*x^2 + a)*b^2) - 15/2*a^2*c*d^3*x/(sqrt(b*x^2 + a)*b^3) + 35/16*a^3*d^4
*x/(sqrt(b*x^2 + a)*b^4) + 4*c^3*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9*a*c^2
*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 15/2*a^2*c*d^3*arcsinh(b*x/sqrt(a*b))
/b^(7/2) - 35/16*a^3*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(2\left(\frac{4\,d^4x^2}{b}+\frac{24\,ab^6cd^3-7\,a^2b^5d^4}{ab^7}\right)x^2+\frac{144\,ab^6c^2d^2-120\,a^2b^5cd^3+35\,a^3b^4d^4}{ab^7}\right)x^2+\frac{3\,(16\,b^7c^4-64\,ab^6c^3d+14\cdots}{}\right.}{48\,\sqrt{bx^2+a}}$$
$$-\frac{(64\,b^3c^3d-144\,ab^2c^2d^2+120\,a^2bcd^3-35\,a^3d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{9}{2}}}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/48*((2*(4*d^4*x^2/b + (24*a*b^6*c*d^3 - 7*a^2*b^5*d^4)/(a*b^7))*x^2 + (14
4*a*b^6*c^2*d^2 - 120*a^2*b^5*c*d^3 + 35*a^3*b^4*d^4)/(a*b^7))*x^2 + 3*(16*
b^7*c^4 - 64*a*b^6*c^3*d + 144*a^2*b^5*c^2*d^2 - 120*a^3*b^4*c*d^3 + 35*a^4
*b^3*d^4)/(a*b^7))*x/sqrt(b*x^2 + a) - 1/16*(64*b^3*c^3*d - 144*a*b^2*c^2*d
^2 + 120*a^2*b*c*d^3 - 35*a^3*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^4}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^4/(a + b*x^2)^(3/2), x)
```

3.82 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3} - \frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2} + \frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} + \frac{3d(8b^2c^2-12abcd+5a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

```
[Out] 3/8*d*(5*a^2*d^2-12*a*b*c*d+8*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(7/2)+(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(1/2)-1/8*d*(-5*a*d+2*b*c)*(-
3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)/a/b^3-1/4*d*(-5*a*d+4*b*c)*x*(d*x^2+c)*(b*x^
2+a)^(1/2)/a/b^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {424, 542, 396, 223, 212}

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{3d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(5a^2d^2-12abcd+8b^2c^2)}{8b^{7/2}} - \frac{dx\sqrt{a+bx^2}(2bc-5ad)(4bc-3ad)}{8ab^3} - \frac{dx\sqrt{a+bx^2}(c+dx^2)\,(4bc-5ad)}{4ab^2} + \frac{x(c+dx^2)^2\,(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/8*(d*(2*b*c - 5*a*d)*(4*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(a*b^3) - (d*(4*
b*c - 5*a*d)*x*Sqrt[a + b*x^2]*(c + d*x^2))/(4*a*b^2) + ((b*c - a*d)*x*(c +
 d*x^2)^2)/(a*b*Sqrt[a + b*x^2]) + (3*d*(8*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2
)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} + \frac{\int \frac{(c+dx^2)\left(acd-d(4bc-5ad)x^2\right)}{\sqrt{a+bx^2}}\,dx}{ab}$$

$$
\begin{aligned}
&= -\frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2}+\frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}} \\
&\quad +\frac{\int \frac{acd(8bc-5ad)-d(2bc-5ad)(4bc-3ad)x^2}{\sqrt{a+bx^2}}dx}{4ab^2} \\
&= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3}-\frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2} \\
&\quad +\frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}}+\frac{(3d(8b^2c^2-12abcd+5a^2d^2))\int \frac{1}{\sqrt{a+bx^2}}dx}{8b^3} \\
&= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3}-\frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2} \\
&\quad +\frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}}+\frac{(3d(8b^2c^2-12abcd+5a^2d^2))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{8b^3} \\
&= -\frac{d(2bc-5ad)(4bc-3ad)x\sqrt{a+bx^2}}{8ab^3}-\frac{d(4bc-5ad)x\sqrt{a+bx^2}(c+dx^2)}{4ab^2} \\
&\quad +\frac{(bc-ad)x(c+dx^2)^2}{ab\sqrt{a+bx^2}}+\frac{3d(8b^2c^2-12abcd+5a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$
\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{x(8b^3c^3-15a^3d^3+a^2bd^2(36c-5dx^2)+2ab^2d(-12c^2+6cdx^2+d^2x^4))}{8ab^3\sqrt{a+bx^2}} - \frac{3d(8b^2c^2-12abcd+5a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{7/2}}
$$

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(8*b^3*c^3 - 15*a^3*d^3 + a^2*b*d^2*(36*c - 5*d*x^2) + 2*a*b^2*d*(-12*c^
2 + 6*c*d*x^2 + d^2*x^4)))/(8*a*b^3*Sqrt[a + b*x^2]) - (3*d*(8*b^2*c^2 - 12
*a*b*c*d + 5*a^2*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{\frac{15\sqrt{b\,x^2+a}\,a d\left(a^2 d^2-\frac{12}{5} a b c d+\frac{8}{5} b^2 c^2\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}-\frac{15x\left(\frac{8d\left(-\frac{1}{12}d^2x^4-\frac{1}{2}c d\,x^2+c^2\right)a\,b^{\frac{5}{2}}}{5}-\frac{12\left(-\frac{5d\,x^2}{36}+c\right)d^2a^2b^{\frac{3}{2}}}{5}+\sqrt{b}\,a^3d^3-$
risch	$-\frac{x\,d^2\left(-2bd\,x^2+7ad-12bc\right)\sqrt{b\,x^2+a}}{8b^3}+\frac{\frac{7a^2d^3x}{\sqrt{b\,x^2+a}}+\frac{8b^3c^3x}{a\sqrt{b\,x^2+a}}-\frac{12abc\,d^2x}{\sqrt{b\,x^2+a}}+\left(15a^2b\,d^3-36a\,b^2c\,d^2+24b^3c^2d\right)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln}{}\right.}{8b^3}$
default	$\frac{c^3x}{a\sqrt{b\,x^2+a}}+d^3\left(\frac{x^5}{4b\sqrt{b\,x^2+a}}-\frac{5a\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)}{4b}\right)+3c\,d^2\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\right.$

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 15/8/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)*a*d*(a^2*d^2-12/5*a*b*c*d+8/5*b^2*c^2
)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(8/5*d*(-1/12*d^2*x^4-1/2*c*d*x^2+c^
2)*a*b^(5/2)-12/5*(-5/36*d*x^2+c)*d^2*a^2*b^(3/2)+b^(1/2)*a^3*d^3-8/15*b^(7
/2)*c^3))/b^(7/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.46

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \left[\frac{3\left(8\,a^2b^2c^2d-12\,a^3bcd^2+5\,a^4d^3+(8\,ab^3c^2d-12\,a^2b^2cd^2+5\,a^3bd^3)x^2\right)\sqrt{b}\log\left(-2\,bx^2\right.}{}\right.$$

$$-\frac{3\left(8\,a^2b^2c^2d-12\,a^3bcd^2+5\,a^4d^3+(8\,ab^3c^2d-12\,a^2b^2cd^2+5\,a^3bd^3)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,ab^3d^3x^5}{8\,(ab^5x^2+a^2b^4)}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2 - 5*a^2*b^2*d^3)*x^
3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15*a^3*b*d^3)*x)*sqrt(
b*x^2 + a))/(a*b^5*x^2 + a^2*b^4), -1/8*(3*(8*a^2*b^2*c^2*d - 12*a^3*b*c*d^
```

```
2 + 5*a^4*d^3 + (8*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^3*d^3*x^5 + (12*a*b^3*c*d^2
 - 5*a^2*b^2*d^3)*x^3 + (8*b^4*c^3 - 24*a*b^3*c^2*d + 36*a^2*b^2*c*d^2 - 15
*a^3*b*d^3)*x)*sqrt(b*x^2 + a))/(a*b^5*x^2 + a^2*b^4)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^3}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(3/2),x)
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.17

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx &= \frac{d^3x^5}{4\sqrt{bx^2+a}b} + \frac{3\,cd^2x^3}{2\sqrt{bx^2+a}b} - \frac{5\,ad^3x^3}{8\sqrt{bx^2+a}b^2} \\ &+ \frac{c^3x}{\sqrt{bx^2+a}a} - \frac{3\,c^2dx}{\sqrt{bx^2+a}b} + \frac{9\,acd^2x}{2\sqrt{bx^2+a}b^2} - \frac{15\,a^2d^3x}{8\sqrt{bx^2+a}b^3} \\ &+ \frac{3\,c^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{9\,acd^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{15\,a^2d^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}}\end{aligned}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] 1/4*d^3*x^5/(sqrt(b*x^2 + a)*b) + 3/2*c*d^2*x^3/(sqrt(b*x^2 + a)*b) - 5/8*a
*d^3*x^3/(sqrt(b*x^2 + a)*b^2) + c^3*x/(sqrt(b*x^2 + a)*a) - 3*c^2*d*x/(sqr
t(b*x^2 + a)*b) + 9/2*a*c*d^2*x/(sqrt(b*x^2 + a)*b^2) - 15/8*a^2*d^3*x/(sqr
t(b*x^2 + a)*b^3) + 3*c^2*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 9/2*a*c*d^2*ar
csinh(b*x/sqrt(a*b))/b^(5/2) + 15/8*a^2*d^3*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,d^3x^2}{b}+\frac{12\,ab^4cd^2-5\,a^2b^3d^3}{ab^5}\right)x^2+\frac{8\,b^5c^3-24\,ab^4c^2d+36\,a^2b^3cd^2-15\,a^3b^2d^3}{ab^5}\right)x}{8\,\sqrt{bx^2+a}} - \frac{3\,(8\,b^2c^2d-12\,abcd^2+5\,a^2d^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((2*d^3*x^2/b + (12*a*b^4*c*d^2 - 5*a^2*b^3*d^3)/(a*b^5))*x^2 + (8*b^5*
c^3 - 24*a*b^4*c^2*d + 36*a^2*b^3*c*d^2 - 15*a^3*b^2*d^3)/(a*b^5))*x/sqrt(b
*x^2 + a) - 3/8*(8*b^2*c^2*d - 12*a*b*c*d^2 + 5*a^2*d^3)*log(abs(-sqrt(b)*x
 + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(3/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(3/2), x)
```

3.83 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx=\frac{(bc-ad)^2x}{ab^2\sqrt{a+bx^2}}+\frac{d^2x\sqrt{a+bx^2}}{2b^2}+\frac{d(4bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] 1/2*d*(-3*a*d+4*b*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+(-a*d+b*c)^
2*x/a/b^2/(b*x^2+a)^(1/2)+1/2*d^2*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 396, 223, 212}

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx=\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc-3ad)}{2b^{5/2}}-\frac{dx\sqrt{a+bx^2}(2bc-3ad)}{2ab^2}+\frac{x(c+dx^2)(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]
```

```
[Out] -1/2*(d*(2*b*c - 3*a*d)*x*Sqrt[a + b*x^2])/(a*b^2) + ((b*c - a*d)*x*(c + d*
x^2))/(a*b*Sqrt[a + b*x^2]) + (d*(4*b*c - 3*a*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}}+\frac{\int \frac{acd-d(2bc-3ad)x^2}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}}+\frac{(d(4bc-3ad))\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}}\\
&\quad+\frac{(d(4bc-3ad))\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\
&= -\frac{d(2bc-3ad)x\sqrt{a+bx^2}}{2ab^2}+\frac{(bc-ad)x(c+dx^2)}{ab\sqrt{a+bx^2}}+\frac{d(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{x(2b^2c^2-4abcd+3a^2d^2+abd^2x^2)}{2ab^2\sqrt{a+bx^2}} - \frac{d(4bc-3ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(3/2),x]

[Out] (x*(2*b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 + a*b*d^2*x^2))/(2*a*b^2*Sqrt[a + b*x^2]) - (d*(4*b*c - 3*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{b\,x^2+a}\,a d\left(a d-\frac{4 b c}{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-x\left(-\frac{4\left(-\frac{d\,x^2}{4}+c\right)d a\,b^{\frac{3}{2}}}{3}+\sqrt{b}\,a^2d^2+\frac{2b^{\frac{5}{2}}c^2}{3}\right)\right)}{2\sqrt{b\,x^2+a}\,b^{\frac{5}{2}}a}$
risch	$\frac{d^2x\sqrt{b\,x^2+a}}{2b^2}-\frac{\frac{a\,d^2x}{\sqrt{b\,x^2+a}}-\frac{2b^2c^2x}{a\sqrt{b\,x^2+a}}+(3abd^2-4b^2cd)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b^2}$
default	$\frac{c^2x}{a\sqrt{b\,x^2+a}}+d^2\left(\frac{x^3}{2b\sqrt{b\,x^2+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+2cd\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+\right.}{b^{\frac{3}{2}}}$

[In] int((d*x^2+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)

[Out] -3/2/(b*x^2+a)^(1/2)/b^(5/2)*((b*x^2+a)^(1/2)*a*d*(a*d-4/3*b*c)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(-4/3*(-1/4*d*x^2+c)*d*a*b^(3/2)+b^(1/2)*a^2*d^2+2/3*b^(5/2)*c^2))/a

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.07

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^2)\sqrt{b}\log\left(-2bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)-2}{4(ab^4x^2+a^2b^3)}\right.$$

$$-\frac{(4a^2bcd-3a^3d^2+(4ab^2cd-3a^2bd^2)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(ab^2d^2x^3+(2b^3c^2-4ab^2cd+3a^2bd^2)x}{2(ab^4x^2+a^2b^3)}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(b)*
log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(a*b^2*d^2*x^3 + (2*b^3
*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
, -1/2*((4*a^2*b*c*d - 3*a^3*d^2 + (4*a*b^2*c*d - 3*a^2*b*d^2)*x^2)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (a*b^2*d^2*x^3 + (2*b^3*c^2 - 4*a*b^
2*c*d + 3*a^2*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{d^2x^3}{2\sqrt{bx^2+a}b} + \frac{c^2x}{\sqrt{bx^2+a}a} - \frac{2cdx}{\sqrt{bx^2+a}b} + \frac{3ad^2x}{2\sqrt{bx^2+a}b^2} + \frac{2cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{3ad^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*d^2*x^3/(sqrt(b*x^2 + a)*b) + c^2*x/(sqrt(b*x^2 + a)*a) - 2*c*d*x/(sqrt
(b*x^2 + a)*b) + 3/2*a*d^2*x/(sqrt(b*x^2 + a)*b^2) + 2*c*d*arcsinh(b*x/sqrt
(a*b))/b^(3/2) - 3/2*a*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{d^2x^2}{b} + \frac{2\,b^3c^2-4\,ab^2cd+3\,a^2bd^2}{ab^3}\right)x}{2\,\sqrt{bx^2+a}} - \frac{(4\,bcd-3\,ad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] 1/2*(d^2*x^2/b + (2*b^3*c^2 - 4*a*b^2*c*d + 3*a^2*b*d^2)/(a*b^3))*x/sqrt(b*
x^2 + a) - 1/2*(4*b*c*d - 3*a*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(3/2),x)
[Out] int((c + d*x^2)^2/(a + b*x^2)^(3/2), x)
```

3.84 $\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] d*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-a*d+b*c)*x/a/b/(b*x^2+a)^(1/
2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 223, 212}

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}+\frac{x(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*Sqrt[a + b*x^2]) + (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= \frac{(bc-ad)x}{ab\sqrt{a+bx^2}}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx=\frac{bcx-adx}{ab\sqrt{a+bx^2}}-\frac{d\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(3/2),x]
[Out] (b*c*x - a*d*x)/(a*b*Sqrt[a + b*x^2]) - (d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/b^(3/2)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{cx}{a\sqrt{b\,x^{2}+a}}+d\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	55
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)ad\sqrt{b\,x^{2}+a}-adx\sqrt{b}+b^{\frac{3}{2}}cx}{b^{\frac{3}{2}}\sqrt{b\,x^{2}+a}\,a}$	61

```
[In] int((d*x^2+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*x/a/(b*x^2+a)^(1/2)+d*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2
+a)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.09

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{2\left(b^{2}c-abd\right)\sqrt{bx^{2}+a}x+\left(abdx^{2}+a^{2}d\right)\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)}{2\left(ab^{3}x^{2}+a^{2}b^{2}\right)},\ \frac{\left(b^{2}c-\right.}{}\right.$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(b^2*c - a*b*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(a*b^3*x^2 + a^2*b^2), ((b^2*
c - a*b*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(-b)*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{c+dx^2}{\left(a+bx^2\right)^{3/2}}\,dx=d\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^{2}}{a}}}\right)+\frac{cx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^{2}}{a}}}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(3/2),x)
```

```
[Out] d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
c*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \frac{cx}{\sqrt{bx^2+a}a} - \frac{dx}{\sqrt{bx^2+a}b} + \frac{d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] c*x/(sqrt(b*x^2 + a)*a) - d*x/(sqrt(b*x^2 + a)*b) + d*arcsinh(b*x/sqrt(a*b)
)/b^(3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = -\frac{d\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}} + \frac{(bc-ad)x}{\sqrt{bx^2+a}ab}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + (b*c - a*d)*x/(sqrt(b*x
^2 + a)*a*b)
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2}{(a+bx^2)^{3/2}}\,dx = \frac{d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} + \frac{c\,x}{a\,\sqrt{b\,x^2+a}} - \frac{d\,x}{b\,\sqrt{b\,x^2+a}}$$

```
[In] int((c + d*x^2)/(a + b*x^2)^(3/2),x)
[Out] (d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) + (c*x)/(a*(a + b*x^2)^(1/2)
) - (d*x)/(b*(a + b*x^2)^(1/2))
```

3.85 $\int \frac{1}{(a+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(a+bx^2)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

```
[Out] x/a/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {197}

$$\int \frac{1}{(a+bx^2)^{3/2}} \, dx = \frac{x}{a\sqrt{a+bx^2}}$$

```
[In] Int[(a + b*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{x}{a\sqrt{a+bx^2}}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+bx^2}}$$

```
[In] Integrate[(a + b*x^2)^(-3/2),x]
[Out] x/(a*Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
pseudoelliptic	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15

```
[In] int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
[Out] x/a/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate(1/(b*x**2+a)**(3/2),x)
[Out] x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x}{\sqrt{bx^2+a}a}$$

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] x/(sqrt(b*x^2 + a)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x}{\sqrt{bx^2+a}a}$$

```
[In] integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] x/(sqrt(b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{x}{a\sqrt{b\,x^2+a}}$$

```
[In] int(1/(a + b*x^2)^(3/2),x)
[Out] x/(a*(a + b*x^2)^(1/2))
```

3.86 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)} dx = \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

```
[Out] -d*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/(-a*d+b*c)^(3/2)/c^(
1/2)+b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 385, 214}

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)} dx = \frac{bx}{a\sqrt{a+bx^2}(bc-ad)} - \frac{d\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]) - (d*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqr
t[c]*Sqrt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{bc-ad}\\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}} - \frac{d\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \frac{bx}{(abc-a^2d)\sqrt{a+bx^2}} - \frac{d\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{3/2}}$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)),x]
[Out] (b*x)/((a*b*c - a^2*d)*Sqrt[a + b*x^2]) - (d*ArcTan[(-(d*x*Sqrt[a + b*x^2])
 + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a
*d)^(3/2))
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$-\frac{d\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{(ad-bc)c}}\right)a\sqrt{b\,x^{2}+a}+bx\sqrt{(ad-bc)c}}{a(ad-bc)\sqrt{b\,x^{2}+a}\,\sqrt{(ad-bc)c}}$
default	$-\frac{\frac{d}{(ad-bc)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{ad-bc}{d}}}+\frac{2b\sqrt{-cd}\left(2b\left(x+\frac{\sqrt{-cd}}{d}\right)-\frac{2b\sqrt{-cd}}{d}\right)}{(ad-bc)\left(\frac{4b(ad-bc)}{d}+\frac{4b^{2}c}{d}\right)\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^{2}b-\frac{2b\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+\frac{a}{}}}}{2\sqrt{-cd}}$

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -(d*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))*a*(b*x^2+a)^(1/2)+b*x*(
(a*d-b*c)*c)^(1/2))/a/(a*d-b*c)/(b*x^2+a)^(1/2)/((a*d-b*c)*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(67) = 134.

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 5.58

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \left[\frac{4\,(b^2c^2-abcd)\sqrt{bx^2+a}x-(abdx^2+a^2d)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2}{}\right.}{4\,(a^2b^2c^3-2\,a^3bc^2d+a^4cd^2+(ab^3c^3-2}\right.$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x - (a*b*d*x^2 + a^2*d)*sqrt(b*
c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*
b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*
sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/(a^2*b^2*c^3 - 2*a^3*b*c^2*d
 + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2), 1/2*(2*(b^
2*c^2 - a*b*c*d)*sqrt(b*x^2 + a)*x + (a*b*d*x^2 + a^2*d)*sqrt(-b*c^2 + a*c*
d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a
)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a^2*b^2*c^3 - 2*a^3*
b*c^2*d + a^4*c*d^2 + (a*b^3*c^3 - 2*a^2*b^2*c^2*d + a^3*b*c*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = -\frac{\sqrt{b}d\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{\sqrt{-b^2c^2+abcd}(bc-ad)} + \frac{bx}{(abc-a^2d)\sqrt{bx^2+a}}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c),x, algorithm="giac")
[Out] -sqrt(b)*d*arctan(-1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sq
rt(-b^2*c^2 + a*b*c*d))/(sqrt(-b^2*c^2 + a*b*c*d)*(b*c - a*d)) + b*x/((a*b*
c - a^2*d)*sqrt(b*x^2 + a))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{d(4bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}$$

```
[Out] -1/2*d*(-a*d+4*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(
3/2)/(-a*d+b*c)^(5/2)+1/2*b*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(1/2)-
1/2*d*x/c/(-a*d+b*c)/(d*x^2+c)/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = -\frac{d(4bc-ad)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}} + \frac{bx(ad+2bc)}{2ac\sqrt{a+bx^2}(bc-ad)^2} - \frac{dx}{2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]) - (d*x)/(2*c*(b*c
 - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)) - (d*(4*b*c - a*d)*ArcTanh[(Sqrt[b*c -
 a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)}+\frac{\int \frac{2bc-ad-2bdx^2}{(a+bx^2)^{3/2}(c+dx^2)}\,dx}{2c(bc-ad)}\\
&= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)}-\frac{\int \frac{ad(4bc-ad)}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2ac(bc-ad)^2}\\
&= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}}-\frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)}-\frac{(d(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{2c(bc-ad)^2}
\end{aligned}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{(d(4bc-ad))\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{2ac(bc-ad)^2\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)} - \frac{d(4bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^2}\,dx = \frac{\frac{\sqrt{c}x(a^2d^2+abd^2x^2+2b^2c(c+dx^2))}{a(bc-ad)^2\sqrt{a+bx^2}(c+dx^2)} + \frac{d(4bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{(-bc+ad)^{5/2}}}{2c^{3/2}}$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x]
```

```
[Out] ((Sqrt[c]*x*(a^2*d^2 + a*b*d^2*x^2 + 2*b^2*c*(c + d*x^2)))/(a*(b*c - a*d)^2
*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*ArcTan[(-(d*x*Sqrt[a + b*x
^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(-(b*c) + a*d)^(
5/2))/(2*c^(3/2))
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.05

method	result	size
pseudoelliptic	$\frac{-ad\sqrt{b\,x^2+a}\,(d\,x^2+c)(ad-4bc)\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+x\sqrt{(ad-bc)c}\,(2b^2c^2+2x^2b^2cd+a\,d^2(b\,x^2+a))}{2\sqrt{b\,x^2+a}\,\sqrt{(ad-bc)c}\,c(d\,x^2+c)(ad-bc)^2a}$	150
default	Expression too large to display	1906

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-a*d*(b*x^2+a)^(1/2)*(d*x^2+c)*(a*d-4*b*c)*arctan(c*(b*x^2+a)^(1/2)/x/
((a*d-b*c)*c)^(1/2))+x*((a*d-b*c)*c)^(1/2)*(2*b^2*c^2+2*x^2*b^2*c*d+a*d^2*(
b*x^2+a)))/(b*x^2+a)^(1/2)/((a*d-b*c)*c)^(1/2)/c/(d*x^2+c)/(a*d-b*c)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(123) = 246$.

Time = 0.49 (sec) , antiderivative size = 864, normalized size of antiderivative = 6.04

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \left[-\frac{(4\,a^2bc^2d - a^3cd^2 + (4\,ab^2cd^2 - a^2bd^3)x^4 + (4\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)x^2}{8\,(a^2b^3c^6 - 3\,a^3b^2}\right.$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*((4*a^2*b*c^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c
^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4
*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4
+ 2*c*d*x^2 + c^2)) - 4*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
(2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a))
/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c^
5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*c
^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2), 1/4*((4*a^2*b*c
^2*d - a^3*c*d^2 + (4*a*b^2*c*d^2 - a^2*b*d^3)*x^4 + (4*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*
d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*
b*c^2 - a^2*c*d)*x)) + 2*((2*b^3*c^3*d - a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^3 +
 (2*b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 - a^3*c*d^3)*x)*sqrt(b*x^2 + a)
)/(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3 + (a*b^4*c
^5*d - 3*a^2*b^3*c^4*d^2 + 3*a^3*b^2*c^3*d^3 - a^4*b*c^2*d^4)*x^4 + (a*b^4*
c^6 - 2*a^2*b^3*c^5*d + 2*a^4*b*c^3*d^3 - a^5*c^2*d^4)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**2,x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(123) = 246$.

Time = 0.87 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.22

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx = \frac{b^2x}{\left(ab^2c^2-2\,a^2bcd+a^3d^2\right)\sqrt{bx^2+a}} + \frac{\left(4\,b^{\frac{3}{2}}cd-a\sqrt{b}d^2\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\sqrt{-b^2c^2+abcd}} + \frac{2\,\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}cd-\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d^2+a^2\sqrt{b}d^2}{\left(b^2c^3-2\,abc^2d+a^2cd^2\right)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4d+4\,\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2bc-2\,\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2ad+a}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/((a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*sqrt(b*x^2 + a)) + 1/2*(4*b^(3/2
)*c*d - a*sqrt(b)*d^2)*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*
c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*sqr
t(-b^2*c^2 + a*b*c*d)) + (2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d - (
sqrt(b)*x - sqrt(b*x^2 + a))^2*a*sqrt(b)*d^2 + a^2*sqrt(b)*d^2)/((b^2*c^3 -
 2*a*b*c^2*d + a^2*c*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^2), x)
```

3.88 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 225

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3(c+dx^2)} - \frac{3d(8b^2c^2-4abcd+a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}$$

```
[Out] -3/8*d*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*
x^2+a)^(1/2))/c^(5/2)/(-a*d+b*c)^(7/2)-1/4*d*x/c/(-a*d+b*c)/(d*x^2+c)^2/(b*
x^2+a)^(1/2)+1/4*b*(a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(d*x^2+c)/(b*x^2+a)^(1/2)
+1/8*d*(-a*d+4*b*c)*(3*a*d+2*b*c)*x*(b*x^2+a)^(1/2)/a/c^2/(-a*d+b*c)^3/(d*x
^2+c)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = -\frac{3d(a^2d^2-4abcd+8b^2c^2)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}} + \frac{dx\sqrt{a+bx^2}(4bc-ad)(3ad+2bc)}{8ac^2(c+dx^2)(bc-ad)^3} + \frac{bx(ad+4bc)}{4ac\sqrt{a+bx^2}(c+dx^2)(bc-ad)^2} - \frac{dx}{4c\sqrt{a+bx^2}(c+dx^2)^2(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*Sqrt[a + b*x^2]*(c + d*x^2)^2) + (b*(4*b*c + a*d)
*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a + b*x^2]*(c + d*x^2)) + (d*(4*b*c - a*d)*(2
*b*c + 3*a*d)*x*Sqrt[a + b*x^2])/(8*a*c^2*(b*c - a*d)^3*(c + d*x^2)) - (3*d
*(8*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqr
t[a + b*x^2])])/(8*c^(5/2)*(b*c - a*d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{\int \frac{4bc-3ad-4bdx^2}{(a+bx^2)^{3/2}(c+dx^2)^2}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} - \frac{\int \frac{ad(8bc-3ad)-2bd(4bc+ad)x^2}{\sqrt{a+bx^2}(c+dx^2)^2}\,dx}{4ac(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)} - \frac{\int \frac{3ad(8b^2c^2-4abcd+a^2d^2)}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{8ac^2(bc-ad)^3}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)} - \frac{(3d(8b^2c^2-4abcd+a^2d^2))\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{8c^2(bc-ad)^3}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)} - \frac{(3d(8b^2c^2-4abcd+a^2d^2))\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2(bc-ad)^3}$$

$$= -\frac{dx}{4c(bc-ad)\sqrt{a+bx^2}\,(c+dx^2)^2} + \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt{a+bx^2}\,(c+dx^2)} + \frac{d(4bc-ad)(2bc+3ad)x\sqrt{a+bx^2}}{8ac^2(bc-ad)^3\,(c+dx^2)} - \frac{3d(8b^2c^2-4abcd+a^2d^2)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{7/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 13.40 (sec) , antiderivative size = 1392, normalized size of antiderivative = 6.19

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^3}\,dx = \frac{x\left(-108045\sqrt{\frac{(bc-ad)x^2}{c(a+bx^2)}} - \frac{324135dx^2\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}}{c} - \frac{324135d^2x^4\sqrt{\frac{(bc-ad)x^2}{c\left(a+bx^2\right)}}}{c^2} - \frac{103320d^3x^{\cdots}}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x]

[Out] (x*(-108045*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))] - (324135*d*x^2*Sqrt[((
b*c - a*d)*x^2)/(c*(a + b*x^2))])/c - (324135*d^2*x^4*Sqrt[((b*c - a*d)*x^2
)/(c*(a + b*x^2))])/c^2 - (103320*d^3*x^6*Sqrt[((b*c - a*d)*x^2)/(c*(a + b*
x^2))])/c^3 + 42735*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2) + (128205*d*x
^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c + (139545*d^2*x^4*(((b*c -
a*d)*x^2)/(c*(a + b*x^2)))^(3/2))/c^2 + (46200*d^3*x^6*(((b*c - a*d)*x^2)/(
c*(a + b*x^2)))^(3/2))/c^3 - 3864*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2)
 - (4032*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c - (4032*d^2*x^4
*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^2 - (1344*d^3*x^6*(((b*c - a*
d)*x^2)/(c*(a + b*x^2)))^(5/2))/c^3 + 108045*ArcTanh[Sqrt[((b*c - a*d)*x^2)
/(c*(a + b*x^2))]] + (324135*d*x^2*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b
*x^2))]])/c + (324135*d^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2)
)]])/c^2 + (103320*d^3*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]]
)/c^3 + (8505*(b*c - a*d)^2*x^4*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^
2))]])/(c^2*(a + b*x^2)^2) + (17955*d*(b*c - a*d)^2*x^6*ArcTanh[Sqrt[((b*c
- a*d)*x^2)/(c*(a + b*x^2))]])/(c^3*(a + b*x^2)^2) + (21735*d^2*(b*c - a*d)
^2*x^8*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)^2
) + (7560*d^3*(b*c - a*d)^2*x^10*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x
^2))]])/(c^5*(a + b*x^2)^2) - (78750*(b*c - a*d)*x^2*ArcTanh[Sqrt[((b*c - a
*d)*x^2)/(c*(a + b*x^2))]])/(c*(a + b*x^2)) + (236250*d*(-(b*c) + a*d)*x^4*
ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])/(c^2*(a + b*x^2)) + (2475
90*d^2*(-(b*c) + a*d)*x^6*ArcTanh[Sqrt[((b*c - a*d)*x^2)/(c*(a + b*x^2))]])
/(c^3*(a + b*x^2)) + (80640*d^3*(-(b*c) + a*d)*x^8*ArcTanh[Sqrt[((b*c - a*d
)*x^2)/(c*(a + b*x^2))]])/(c^4*(a + b*x^2)) + 64*(((b*c - a*d)*x^2)/(c*(a +
 b*x^2)))^(9/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d
)*x^2)/(c*(a + b*x^2))] + (192*d*x^2*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9
/2)*HypergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a
 + b*x^2))])/c + (192*d^2*x^4*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hyp
ergeometricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^
2))])/c^2 + (64*d^3*x^6*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(9/2)*Hypergeom
etricPFQ[{2, 2, 2, 5/2}, {1, 1, 11/2}, ((b*c - a*d)*x^2)/(c*(a + b*x^2))])/
c^3))/(2520*c*(((b*c - a*d)*x^2)/(c*(a + b*x^2)))^(7/2)*(a + b*x^2)^(3/2)*(
c + d*x^2)^2)
```

Maple [A] (verified)

Time = 2.66 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{3\left(\sqrt{b\,x^{2}+a}\,ad\left(d\,x^{2}+c\right)^{2}\left(a^{2}d^{2}-4abcd+8b^{2}c^{2}\right)\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{\left(ad-bc\right)c}}\right)-\frac{5x\left(d^{3}\left(\frac{3d\,x^{2}}{5}+c\right)a^{3}-\frac{12\left(-\frac{d\,x^{2}}{3}+c\right)b\,d^{2}\left(\frac{3d\,x^{2}}{4}+c\right)c \cdots}{5} \cdots\right)}{\cdots}\right)}{8\sqrt{b\,x^{2}+a}\,\sqrt{\left(ad-bc\right)c}\,\left(d\,x^{2}+c\right)^{2}c^{2}\left(ad-bc\right)^{3}a}$
default	Expression too large to display

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8*((b*x^2+a)^(1/2)*a*d*(d*x^2+c)^2*(a^2*d^2-4*a*b*c*d+8*b^2*c^2)*arctan(
c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*x*(d^3*(3/5*d*x^2+c)*a^3-12/5*
(-1/3*d*x^2+c)*b*d^2*(3/4*d*x^2+c)*a^2-12/5*x^2*(5/6*d*x^2+c)*b^2*d^2*c*a-8
/5*b^3*c^2*(d*x^2+c)^2)*((a*d-b*c)*c)^(1/2))/(b*x^2+a)^(1/2)/((a*d-b*c)*c)^
(1/2)/(d*x^2+c)^2/c^2/(a*d-b*c)^3/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(201) = 402.

Time = 0.88 (sec) , antiderivative size = 1482, normalized size of antiderivative = 6.59

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x
^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x
)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) - 4*((8
*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a^3*b*c*d^5)*x^5 +
(16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a^3*b*c^2*d^4 + 3*a
^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c^4*d^2 - 17*a^3*b*
c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9 - 4*a^3*b^3*c^8*d
 + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a*b^5*c^7*d^2 - 4*a
^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 + a^5*b*c^3*d^6)*x^6
 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3 - 2*a^4*b^2*c^5*d
^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*a^2*b^4*c^8*d - 2*
a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2*a^6*c^4*d^5)*x^2)
```

```
, 1/16*(3*(8*a^2*b^2*c^4*d - 4*a^3*b*c^3*d^2 + a^4*c^2*d^3 + (8*a*b^3*c^2*d
^3 - 4*a^2*b^2*c*d^4 + a^3*b*d^5)*x^6 + (16*a*b^3*c^3*d^2 - 2*a^3*b*c*d^4 +
 a^4*d^5)*x^4 + (8*a*b^3*c^4*d + 12*a^2*b^2*c^3*d^2 - 7*a^3*b*c^2*d^3 + 2*a
^4*c*d^4)*x^2)*sqrt(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c
 - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^
2*c*d)*x)) + 2*((8*b^4*c^4*d^2 + 2*a*b^3*c^3*d^3 - 13*a^2*b^2*c^2*d^4 + 3*a
^3*b*c*d^5)*x^5 + (16*b^4*c^5*d - 4*a*b^3*c^4*d^2 - 7*a^2*b^2*c^3*d^3 - 8*a
^3*b*c^2*d^4 + 3*a^4*c*d^5)*x^3 + (8*b^4*c^6 - 8*a*b^3*c^5*d + 12*a^2*b^2*c
^4*d^2 - 17*a^3*b*c^3*d^3 + 5*a^4*c^2*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^4*c^9
 - 4*a^3*b^3*c^8*d + 6*a^4*b^2*c^7*d^2 - 4*a^5*b*c^6*d^3 + a^6*c^5*d^4 + (a
*b^5*c^7*d^2 - 4*a^2*b^4*c^6*d^3 + 6*a^3*b^3*c^5*d^4 - 4*a^4*b^2*c^4*d^5 +
a^5*b*c^3*d^6)*x^6 + (2*a*b^5*c^8*d - 7*a^2*b^4*c^7*d^2 + 8*a^3*b^3*c^6*d^3
 - 2*a^4*b^2*c^5*d^4 - 2*a^5*b*c^4*d^5 + a^6*c^3*d^6)*x^4 + (a*b^5*c^9 - 2*
a^2*b^4*c^8*d - 2*a^3*b^3*c^7*d^2 + 8*a^4*b^2*c^6*d^3 - 7*a^5*b*c^5*d^4 + 2
*a^6*c^4*d^5)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^3}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^3}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(201) = 402$.

Time = 1.21 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.86

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = \frac{b^3x}{(ab^3c^3-3\,a^2b^2c^2d+3\,a^3bcd^2-a^4d^3)\sqrt{bx^2+a}}$$
$$+\frac{3\left(8\,b^{\frac{5}{2}}c^2d-4\,ab^{\frac{3}{2}}cd^2+a^2\sqrt{b}d^3\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{8\,(b^3c^5-3\,ab^2c^4d+3\,a^2bc^3d^2-a^3c^2d^3)\sqrt{-b^2c^2+abcd}}$$
$$+\frac{16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}c^2d^2-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^3+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}d^4+80\left(\sqrt{b}\right.}{}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/((a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3)*sqrt(b*x^2 +
 a)) + 3/8*(8*b^(5/2)*c^2*d - 4*a*b^(3/2)*c*d^2 + a^2*sqrt(b)*d^3)*arctan(1
/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*
d))/((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*sqrt(-b^2*c^
2 + a*b*c*d)) + 1/4*(16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d^2 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^3 + 3*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*a^2*sqrt(b)*d^4 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7/2)*c^3
*d - 104*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^2 + 54*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
a^3*sqrt(b)*d^4 + 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c^2*d^2 -
52*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^3 + 9*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*a^4*sqrt(b)*d^4 + 10*a^4*b^(3/2)*c*d^3 - 3*a^5*sqrt(b)*d^4)/(
(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*((sqrt(b)*x - sqr
t(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b*c - 2*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^3}\,dx = \int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,{\left(d\,x^2+c\right)}^3}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3),x)
```

```
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^3), x)
```

3.89 $\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 255

$$\begin{aligned}\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx &= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} \\ &\quad -\frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3}+\frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}} \\ &\quad +\frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}}+\frac{d^2(48b^2c^2-80abcd+35a^2d^2)\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}\end{aligned}$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)^3/a/b/(b*x^2+a)^(3/2)+1/8*d^2*(35*a^2*d^2-80*a*b
*c*d+48*b^2*c^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)+1/3*(-a*d+b*c)*
(7*a*d+2*b*c)*x*(d*x^2+c)^2/a^2/b^2/(b*x^2+a)^(1/2)-1/24*d*(105*a^3*d^3-170
*a^2*b*c*d^2+40*a*b^2*c^2*d+16*b^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/b^4-1/12*d*(-
35*a^2*d^2+24*a*b*c*d+8*b^2*c^2)*x*(d*x^2+c)*(b*x^2+a)^(1/2)/a^2/b^3
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{424, 540, 542, 396, 223, 212\}$

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(35a^2d^2-80abcd+48b^2c^2)}{8b^{9/2}} + \frac{x(c+dx^2)^2(bc-ad)(7ad+2bc)}{3a^2b^2\sqrt{a+bx^2}} - \frac{dx\sqrt{a+bx^2}(c+dx^2)(-35a^2d^2+24abcd+8b^2c^2)}{12a^2b^3} - \frac{dx\sqrt{a+bx^2}(105a^3d^3-170a^2bcd^2+40ab^2c^2d+16b^3c^3)}{24a^2b^4} + \frac{x(c+dx^2)^3(bc-ad)}{3ab(a+bx^2)^{3/2}}$$

```
[In] Int[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/24*(d*(16*b^3*c^3 + 40*a*b^2*c^2*d - 170*a^2*b*c*d^2 + 105*a^3*d^3)*x*Sq
rt[a + b*x^2])/(a^2*b^4) - (d*(8*b^2*c^2 + 24*a*b*c*d - 35*a^2*d^2)*x*Sqrt[
a + b*x^2]*(c + d*x^2))/(12*a^2*b^3) + ((b*c - a*d)*(2*b*c + 7*a*d)*x*(c +
d*x^2)^2)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*x*(c + d*x^2)^3)/(3*a*
b*(a + b*x^2)^(3/2)) + (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^2)^3}{3ab\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{(c+dx^2)^2\left(c(2bc+ad)-d(4bc-7ad)x^2\right)}{(a+bx^2)^{3/2}}\,dx}{3ab}\\
&= \frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)^3}{3ab\left(a+bx^2\right)^{3/2}}\\
&\quad -\frac{\int \frac{(c+dx^2)\left(acd(4bc-7ad)+d(8b^2c^2+24abcd-35a^2d^2)x^2\right)}{\sqrt{a+bx^2}}\,dx}{3a^2b^2}\\
&= -\frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3}+\frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}}\\
&\quad +\frac{(bc-ad)x(c+dx^2)^3}{3ab\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{acd(8b^2c^2-52abcd+35a^2d^2)+d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)x^2}{\sqrt{a+bx^2}}\,dx}{12a^2b^3}\\
&= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4}\\
&\quad -\frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3}\\
&\quad +\frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)^3}{3ab\left(a+bx^2\right)^{3/2}}\\
&\quad +\frac{\left(d^2(48b^2c^2-80abcd+35a^2d^2)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} \\
&\quad - \frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3} \\
&\quad + \frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}} \\
&\quad + \frac{(d^2(48b^2c^2-80abcd+35a^2d^2))\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8b^4} \\
&= -\frac{d(16b^3c^3+40ab^2c^2d-170a^2bcd^2+105a^3d^3)\,x\sqrt{a+bx^2}}{24a^2b^4} \\
&\quad - \frac{d(8b^2c^2+24abcd-35a^2d^2)\,x\sqrt{a+bx^2}(c+dx^2)}{12a^2b^3} \\
&\quad + \frac{(bc-ad)(2bc+7ad)x(c+dx^2)^2}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^3}{3ab\,(a+bx^2)^{3/2}} \\
&\quad + \frac{d^2(48b^2c^2-80abcd+35a^2d^2)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.79

$$
\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{x(-105a^5d^4+16b^5c^4x^2+20a^4bd^3(12c-7dx^2)+8ab^4c^3(3c+4dx^2)+a^3b^2d^2(-144c^2}{24a^2b^4\,(a+bx^2)^{3/2}}
$$

$$
-\frac{d^2(48b^2c^2-80abcd+35a^2d^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{9/2}}
$$

```
[In] Integrate[(c + d*x^2)^4/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(-105*a^5*d^4 + 16*b^5*c^4*x^2 + 20*a^4*b*d^3*(12*c - 7*d*x^2) + 8*a*b^4
*c^3*(3*c + 4*d*x^2) + a^3*b^2*d^2*(-144*c^2 + 320*c*d*x^2 - 21*d^2*x^4) +
6*a^2*b^3*d^2*x^2*(-32*c^2 + 8*c*d*x^2 + d^2*x^4)))/(24*a^2*b^4*(a + b*x^2)
^(3/2)) - (d^2*(48*b^2*c^2 - 80*a*b*c*d + 35*a^2*d^2)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/(8*b^(9/2))
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{35\left(b\,x^2+a\right)^{\frac{3}{2}} d^2\left(a^2 d^2-\frac{16}{7} a b c d+\frac{48}{35} b^2 c^2\right) a^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{8}-\frac{35 x\left(\frac{48 d^2 a^3\left(\frac{7}{48} d^2 x^4-\frac{20}{9} c d\,x^2+c^2\right) b^{\frac{5}{2}}}{35}+\frac{64 x^2\left(-\frac{1}{32} d^2 x^4-\frac{1}{4} c d\,x^2+c\right.}{35}\right.}{}}{a^2 b^{\frac{9}{2}}\left(b\,x^2+a\right)^{\frac{3}{2}}}$
default	$c^4\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+d^4\left(\frac{x^7}{4b(b\,x^2+a)^{\frac{3}{2}}}-\frac{7a\left(\frac{x^5}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}\right.}{}}{b}\right.}{2b}\right.}{4b}\right.$
risch	$-\frac{d^3 x\left(-2 b d\,x^2+11 a d-16 b c\right)\sqrt{b\,x^2+a}}{8 b^4}+\frac{\frac{d^2\left(35 a^2 d^2-80 a b c d+48 b^2 c^2\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{2\left(a^4 d^4-4 a^3 b c\,d^3+6 a^2 b^2 c^2 d^2-4 a\,b^3 c^3\right.}{}}{}$

[In] int((d*x^2+c)^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)

[Out] 35/8*((b*x^2+a)^(3/2)*d^2*(a^2*d^2-16/7*a*b*c*d+48/35*b^2*c^2)*a^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-x*(48/35*d^2*a^3*(7/48*d^2*x^4-20/9*c*d*x^2+c^2) *b^(5/2)+64/35*x^2*(-1/32*d^2*x^4-1/4*c*d*x^2+c^2)*d^2*a^2*b^(7/2)-16/7*(-7 /12*d*x^2+c)*d^3*a^4*b^(3/2)-8/35*(4/3*d*x^2+c)*c^3*a*b^(9/2)+b^(1/2)*a^5*d ^4-16/105*b^(11/2)*c^4*x^2))/b^(9/2)/(b*x^2+a)^(3/2)/a^2

Fricas [A] (verification not implemented)

none

Time = 0.40 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.68

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(48\,a^4b^2c^2d^2-80\,a^5bcd^3+35\,a^6d^4+(48\,a^2b^4c^2d^2-80\,a^3b^3cd^3+35\,a^4b^2d^4)x^4+2\,(48}{}\right.$$

$$-\frac{3\,(48\,a^4b^2c^2d^2-80\,a^5bcd^3+35\,a^6d^4+(48\,a^2b^4c^2d^2-80\,a^3b^3cd^3+35\,a^4b^2d^4)x^4+2\,(48\,a^3b^3c^2d^2-80\,a^4b^2cd}{}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*a^2*b^4*c^
2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^2*d^2 - 80
*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d
^4)*x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*
d^3 - 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^
2*c*d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 +
a^4*b^5), -1/24*(3*(48*a^4*b^2*c^2*d^2 - 80*a^5*b*c*d^3 + 35*a^6*d^4 + (48*
a^2*b^4*c^2*d^2 - 80*a^3*b^3*c*d^3 + 35*a^4*b^2*d^4)*x^4 + 2*(48*a^3*b^3*c^
2*d^2 - 80*a^4*b^2*c*d^3 + 35*a^5*b*d^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (6*a^2*b^4*d^4*x^7 + 3*(16*a^2*b^4*c*d^3 - 7*a^3*b^3*d^4)*
x^5 + 4*(4*b^6*c^4 + 8*a*b^5*c^3*d - 48*a^2*b^4*c^2*d^2 + 80*a^3*b^3*c*d^3
- 35*a^4*b^2*d^4)*x^3 + 3*(8*a*b^5*c^4 - 48*a^3*b^3*c^2*d^2 + 80*a^4*b^2*c*
d^3 - 35*a^5*b*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*
b^5)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^4}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**4/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**4/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{d^4x^7}{4\,(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,cd^3x^5}{(bx^2+a)^{\frac{3}{2}}b} - \frac{7\,ad^4x^5}{8\,(bx^2+a)^{\frac{3}{2}}b^2} - 2\,c^2d^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) + \frac{10\,acd^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b} - \frac{35\,a^2d^4x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{24\,b^2} + \frac{2\,c^4x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^4x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{4\,c^3dx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,c^3dx}{3\,\sqrt{bx^2+a}ab} - \frac{2\,c^2d^2x}{\sqrt{bx^2+a}b^2} + \frac{10\,acd^3x}{3\,\sqrt{bx^2+a}b^3} - \frac{35\,a^2d^4x}{24\,\sqrt{bx^2+a}b^4} + \frac{6\,c^2d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{10\,acd^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{7}{2}}} + \frac{35\,a^2d^4\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{9}{2}}}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*d^4*x^7/((b*x^2 + a)^(3/2)*b) + 2*c*d^3*x^5/((b*x^2 + a)^(3/2)*b) - 7/8
*a*d^4*x^5/((b*x^2 + a)^(3/2)*b^2) - 2*c^2*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*
b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 10/3*a*c*d^3*x*(3*x^2/((b*x^2 + a)^(3/2
)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b - 35/24*a^2*d^4*x*(3*x^2/((b*x^2 + a)
^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 2/3*c^4*x/(sqrt(b*x^2 + a)*a
^2) + 1/3*c^4*x/((b*x^2 + a)^(3/2)*a) - 4/3*c^3*d*x/((b*x^2 + a)^(3/2)*b) +
 4/3*c^3*d*x/(sqrt(b*x^2 + a)*a*b) - 2*c^2*d^2*x/(sqrt(b*x^2 + a)*b^2) + 10
/3*a*c*d^3*x/(sqrt(b*x^2 + a)*b^3) - 35/24*a^2*d^4*x/(sqrt(b*x^2 + a)*b^4)
+ 6*c^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 10*a*c*d^3*arcsinh(b*x/sqrt(a*
b))/b^(7/2) + 35/8*a^2*d^4*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,d^4x^2}{b} + \frac{16\,a^2b^6cd^3-7\,a^3b^5d^4}{a^2b^7}\right)x^2 + \frac{4\,(4\,b^8c^4+8\,ab^7c^3d-48\,a^2b^6c^2d^2+80\,a^3b^5cd^3-35\,a^4b^4d^4)}{a^2b^7}\right)x^2 + 3\,(8 \cdots}{24\,(bx^2+a)^{\frac{3}{2}}} - \frac{(48\,b^2c^2d^2-80\,abcd^3+35\,a^2d^4)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{9}{2}}}$$

```
[In] integrate((d*x^2+c)^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((3*(2*d^4*x^2/b + (16*a^2*b^6*c*d^3 - 7*a^3*b^5*d^4)/(a^2*b^7))*x^2 +
 4*(4*b^8*c^4 + 8*a*b^7*c^3*d - 48*a^2*b^6*c^2*d^2 + 80*a^3*b^5*c*d^3 - 35*
a^4*b^4*d^4)/(a^2*b^7))*x^2 + 3*(8*a*b^7*c^4 - 48*a^3*b^5*c^2*d^2 + 80*a^4*
b^4*c*d^3 - 35*a^5*b^3*d^4)/(a^2*b^7))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^2*c^
2*d^2 - 80*a*b*c*d^3 + 35*a^2*d^4)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^4}{(a+bx^2)^{5/2}}\,dx=\int \frac{{\left(d\,x^2+c\right)}^4}{{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

```
[In] int((c + d*x^2)^4/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^4/(a + b*x^2)^(5/2), x)
```

3.90 $\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}} \, dx = &-\frac{d(4b^2c^2+8abcd-15a^2d^2)\,x\sqrt{a+bx^2}}{6a^2b^3} \\ &+\frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}} \\ &+\frac{d^2(6bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}\end{aligned}$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)^2/a/b/(b*x^2+a)^(3/2)+1/2*d^2*(-5*a*d+6*b*c)*arc
tanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(7/2)+1/3*(-a*d+b*c)*(5*a*d+2*b*c)*x*(d*x
^2+c)/a^2/b^2/(b*x^2+a)^(1/2)-1/6*d*(-15*a^2*d^2+8*a*b*c*d+4*b^2*c^2)*x*(b*
x^2+a)^(1/2)/a^2/b^3
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {424, 540, 396, 223, 212}

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}} \, dx = &\frac{x(c+dx^2)\,(bc-ad)(5ad+2bc)}{3a^2b^2\sqrt{a+bx^2}} \\ &-\frac{dx\sqrt{a+bx^2}(-15a^2d^2+8abcd+4b^2c^2)}{6a^2b^3} \\ &+\frac{d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc-5ad)}{2b^{7/2}}+\frac{x(c+dx^2)^2\,(bc-ad)}{3ab\,(a+bx^2)^{3/2}}\end{aligned}$$

```
[In] Int[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/6*(d*(4*b^2*c^2 + 8*a*b*c*d - 15*a^2*d^2)*x*Sqrt[a + b*x^2])/(a^2*b^3) +
 ((b*c - a*d)*(2*b*c + 5*a*d)*x*(c + d*x^2))/(3*a^2*b^2*Sqrt[a + b*x^2]) +
((b*c - a*d)*x*(c + d*x^2)^2)/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*(6*b*c - 5*a
*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x(c+dx^2)^2}{3ab\,(a+bx^2)^{3/2}} + \frac{\int \frac{(c+dx^2)\left(c(2bc+ad)-d(2bc-5ad)x^2\right)}{(a+bx^2)^{3/2}}\,dx}{3ab}$$

$$
\begin{aligned}
&= \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)^2}{3ab\left(a+bx^2\right)^{3/2}} \\
&\quad - \frac{\int \frac{acd(2bc-5ad)+d\left(4b^2c^2+8abcd-15a^2d^2\right)x^2}{\sqrt{a+bx^2}}\,dx}{3a^2b^2} \\
&= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} \\
&\quad + \frac{(bc-ad)x(c+dx^2)^2}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\left(d^2(6bc-5ad)\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^3} \\
&= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} \\
&\quad + \frac{(bc-ad)x(c+dx^2)^2}{3ab\left(a+bx^2\right)^{3/2}} + \frac{\left(d^2(6bc-5ad)\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^3} \\
&= -\frac{d\left(4b^2c^2+8abcd-15a^2d^2\right)x\sqrt{a+bx^2}}{6a^2b^3} + \frac{(bc-ad)(2bc+5ad)x(c+dx^2)}{3a^2b^2\sqrt{a+bx^2}} \\
&\quad + \frac{(bc-ad)x(c+dx^2)^2}{3ab\left(a+bx^2\right)^{3/2}} + \frac{d^2(6bc-5ad)\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

$$
\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(15a^4d^3+4b^4c^3x^2+3a^2b^2d^2x^2(-8c+dx^2)+6ab^3c^2(c+dx^2)+2a^3bd^2(-9c+10dx^2)\right)}{6a^2b^3\left(a+bx^2\right)^{3/2}} + \frac{d^2(-6bc+5ad)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{7/2}}
$$

```
[In] Integrate[(c + d*x^2)^3/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*(15*a^4*d^3 + 4*b^4*c^3*x^2 + 3*a^2*b^2*d^2*x^2*(-8*c + d*x^2) + 6*a*b^3
*c^2*(c + d*x^2) + 2*a^3*b*d^2*(-9*c + 10*d*x^2)))/(6*a^2*b^3*(a + b*x^2)^(
3/2)) + (d^2*(-6*b*c + 5*a*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(7/
2))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{5\left(\left(b\,x^{2}+a\right)^{\frac{3}{2}} d^{2}\left(a d-\frac{6 b c}{5}\right) a^{2} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-x\left(-\frac{6\left(-\frac{10 d\,x^{2}}{9}+c\right) d^{2} a^{3} b^{\frac{3}{2}}}{5}-\frac{8 x^{2}\left(-\frac{d\,x^{2}}{8}+c\right) d^{2} a^{2} b^{\frac{5}{2}}}{5}+\frac{2 a\,c^{2}\left(d\,x^{2}+c\right) b^{\frac{7}{2}}}{5}\right.\right.}{2 b^{\frac{7}{2}}\left(b\,x^{2}+a\right)^{\frac{3}{2}} a^{2}}$
default	$c^{3}\left(\frac{x}{3 a(b\,x^{2}+a)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2}\sqrt{b\,x^{2}+a}}\right)+d^{3}\left(\frac{x^{5}}{2 b(b\,x^{2}+a)^{\frac{3}{2}}}-\frac{5 a\left(-\frac{x^{3}}{3 b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2 b}\right)$
risch	$\frac{d^{3} x\sqrt{b\,x^{2}+a}}{2 b^{3}}-\frac{\frac{d^{2}(5 a d-6 b c)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{\left(a^{3} d^{3}-3 a^{2} b c\,d^{2}+3 a\,b^{2} c^{2} d-b^{3} c^{3}\right)\left(-\frac{\sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} b+2\sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}}{3\sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}}-\sqrt{\right.}{2 b a}}{}$

```
[In] int((d*x^2+c)^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -5/2*((b*x^2+a)^(3/2)*d^2*(a*d-6/5*b*c)*a^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/
2))-x*(-6/5*(-10/9*d*x^2+c)*d^2*a^3*b^(3/2)-8/5*x^2*(-1/8*d*x^2+c)*d^2*a^2*
b^(5/2)+2/5*a*c^2*(d*x^2+c)*b^(7/2)+b^(1/2)*a^4*d^3+4/15*b^(9/2)*c^3*x^2))/
b^(7/2)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.83

$$\int \frac{\left(c+dx^2\right)^3}{\left(a+bx^2\right)^{5/2}}\,dx = \left[-\frac{3\,(6\,a^4bcd^2-5\,a^5d^3+(6\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(6\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{b}\log\left(\right.}{}\right.$$

$$-\frac{3\,(6\,a^4bcd^2-5\,a^5d^3+(6\,a^2b^3cd^2-5\,a^3b^2d^3)x^4+2\,(6\,a^3b^2cd^2-5\,a^4bd^3)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(3\,a^2b^{}}{6\,(a^2b^6x^4+2\,a^3}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)*x^
4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2
```

```
*d - 12*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*
d^2 + 5*a^4*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b
^4), -1/6*(3*(6*a^4*b*c*d^2 - 5*a^5*d^3 + (6*a^2*b^3*c*d^2 - 5*a^3*b^2*d^3)
*x^4 + 2*(6*a^3*b^2*c*d^2 - 5*a^4*b*d^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (3*a^2*b^3*d^3*x^5 + 2*(2*b^5*c^3 + 3*a*b^4*c^2*d - 12*a^2
*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^3 + 3*(2*a*b^4*c^3 - 6*a^3*b^2*c*d^2 + 5*a^4
*b*d^3)*x)*sqrt(b*x^2 + a))/(a^2*b^6*x^4 + 2*a^3*b^5*x^2 + a^4*b^4)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^3}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**3/(b*x**2+a)**(5/2),x)
[Out] Integral((c + d*x**2)**3/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx &= \frac{d^3x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} - cd^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) \\ &\quad + \frac{5\,ad^3x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{2\,c^3x}{3\,\sqrt{bx^2+a}a^2} + \frac{c^3x}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{c^2dx}{(bx^2+a)^{\frac{3}{2}}b} \\ &\quad + \frac{c^2dx}{\sqrt{bx^2+a}ab} - \frac{cd^2x}{\sqrt{bx^2+a}b^2} + \frac{5\,ad^3x}{6\,\sqrt{bx^2+a}b^3} + \frac{3\,cd^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{5\,ad^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}\end{aligned}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 1/2*d^3*x^5/((b*x^2 + a)^(3/2)*b) - c*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) +
2*a/((b*x^2 + a)^(3/2)*b^2)) + 5/6*a*d^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
*a/((b*x^2 + a)^(3/2)*b^2))/b + 2/3*c^3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^3*x
/((b*x^2 + a)^(3/2)*a) - c^2*d*x/((b*x^2 + a)^(3/2)*b) + c^2*d*x/(sqrt(b*x^
2 + a)*a*b) - c*d^2*x/(sqrt(b*x^2 + a)*b^2) + 5/6*a*d^3*x/(sqrt(b*x^2 + a)*
b^3) + 3*c*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/2*a*d^3*arcsinh(b*x/sqrt(
a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,d^3x^2}{b}+\frac{2\,(2\,b^6c^3+3\,ab^5c^2d-12\,a^2b^4cd^2+10\,a^3b^3d^3)}{a^2b^5}\right)x^2+\frac{3\,(2\,ab^5c^3-6\,a^3b^3cd^2+5\,a^4b^2d^3)}{a^2b^5}\right)x}{6\,(bx^2+a)^{\frac{3}{2}}} - \frac{(6\,bcd^2-5\,ad^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{7}{2}}}$$

```
[In] integrate((d*x^2+c)^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((3*d^3*x^2/b + 2*(2*b^6*c^3 + 3*a*b^5*c^2*d - 12*a^2*b^4*c*d^2 + 10*a^
3*b^3*d^3)/(a^2*b^5))*x^2 + 3*(2*a*b^5*c^3 - 6*a^3*b^3*c*d^2 + 5*a^4*b^2*d^
3)/(a^2*b^5))*x/(b*x^2 + a)^(3/2) - 1/2*(6*b*c*d^2 - 5*a*d^3)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^3}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^3}{(b\,x^2+a)^{5/2}}\,dx$$

```
[In] int((c + d*x^2)^3/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^3/(a + b*x^2)^(5/2), x)
```

3.91 $\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}} + \frac{(bc-ad)x(c+dx^2)}{3ab\,(a+bx^2)^{3/2}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] 1/3*(-a*d+b*c)*x*(d*x^2+c)/a/b/(b*x^2+a)^(3/2)+d^2*arctanh(x*b^(1/2)/(b*x^2
+a)^(1/2))/b^(5/2)+1/3*(-a*d+b*c)*(3*a*d+2*b*c)*x/a^2/b^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {424, 393, 223, 212}

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{x(bc-ad)(3ad+2bc)}{3a^2b^2\sqrt{a+bx^2}} + \frac{d^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}} + \frac{x(c+dx^2)(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

```
[In] Int[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*(2*b*c + 3*a*d)*x)/(3*a^2*b^2*Sqrt[a + b*x^2]) + ((b*c - a*d)*
x*(c + d*x^2))/(3*a*b*(a + b*x^2)^(3/2)) + (d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x(c+dx^2)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{c(2bc+ad)+3ad^2x^2}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= \frac{(bc-ad)(2bc+3ad)x}{3a^2b^2\sqrt{a+bx^2}}+\frac{(bc-ad)x(c+dx^2)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{(-bc+ad)x(3abc+3a^2d+2b^2cx^2+4abdx^2)}{3a^2b^2(a+bx^2)^{3/2}} - \frac{d^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

```
[In] Integrate[(c + d*x^2)^2/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*((-(b*c) + a*d)*x*(3*a*b*c + 3*a^2*d + 2*b^2*c*x^2 + 4*a*b*d*x^2))/(a^
2*b^2*(a + b*x^2)^(3/2)) - (d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2
)
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{a^2(b\,x^2+a)^{\frac{3}{2}} d^2 \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right) - x\left(-c\left(\frac{2d\,x^2}{3}+c\right)a\,b^{\frac{5}{2}} + \frac{4b^{\frac{3}{2}}a^2d^2x^2}{3} + \sqrt{b}\,a^3d^2 - \frac{2b^{\frac{7}{2}}c^2x^2}{3}\right)}{(b\,x^2+a)^{\frac{3}{2}}\, b^{\frac{5}{2}}\, a^2}$
default	$c^2\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}} + \frac{2x}{3a^2\sqrt{b\,x^2+a}}\right) + d^2\left(-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}} + \frac{-\frac{x}{b\sqrt{b\,x^2+a}} + \frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right) + 2cd\left(-\frac{x}{2b(b\,x^2\right.$

```
[In] int((d*x^2+c)^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*x^2+a)^(3/2)*(a^2*(b*x^2+a)^(3/2)*d^2*arctanh((b*x^2+a)^(1/2)/x/b^(1/2
))-x*(-c*(2/3*d*x^2+c)*a*b^(5/2)+4/3*b^(3/2)*a^2*d^2*x^2+b^(1/2)*a^3*d^2-2/
3*b^(7/2)*c^2*x^2))/b^(5/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.03

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,(b^4c^2+\right.}{6\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

$$-\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,(b^4c^2+ab^3cd-2\,a^2b^2d^2)x^3+3\,(ab^3c^2-a^3bd^2)}{3\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*x^4 + 2*a
^3*b^4*x^2 + a^4*b^3), -1/3*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*(b^4*c^2 + a*b^3*c*d - 2
*a^2*b^2*d^2)*x^3 + 3*(a*b^3*c^2 - a^3*b*d^2)*x)*sqrt(b*x^2 + a))/(a^2*b^5*
x^4 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^2}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**2/(b*x**2+a)**(5/2),x)
```

```
[Out] Integral((c + d*x**2)**2/(a + b*x**2)**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,d^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)+\frac{2\,c^2x}{3\,\sqrt{bx^2+a}a^2}$$
$$+\frac{c^2x}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{2\,cdx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,cdx}{3\,\sqrt{bx^2+a}ab}-\frac{d^2x}{3\,\sqrt{bx^2+a}b^2}+\frac{d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*d^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) + 2/
3*c^2*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c^2*x/((b*x^2 + a)^(3/2)*a) - 2/3*c*d*x
/((b*x^2 + a)^(3/2)*b) + 2/3*c*d*x/(sqrt(b*x^2 + a)*a*b) - 1/3*d^2*x/(sqrt(
b*x^2 + a)*b^2) + d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,(b^4c^2+ab^3cd-2\,a^2b^2d^2)x^2}{a^2b^3}+\frac{3\,(ab^3c^2-a^3bd^2)}{a^2b^3}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{d^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

```
[In] integrate((d*x^2+c)^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*(b^4*c^2 + a*b^3*c*d - 2*a^2*b^2*d^2)*x^2/(a^2*b^3) + 3*(a*b^3*c^2
 - a^3*b*d^2)/(a^2*b^3))/(b*x^2 + a)^(3/2) - d^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^2}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^2}{(b\,x^2+a)^{5/2}}\,dx$$

```
[In] int((c + d*x^2)^2/(a + b*x^2)^(5/2),x)
```

```
[Out] int((c + d*x^2)^2/(a + b*x^2)^(5/2), x)
```

3.92 $\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx=\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\,(a+bx^2)^{3/2}}$$

```
[Out] 1/3*x*(d*x^2+c)/a/(b*x^2+a)^(3/2)+2/3*c*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {386, 197}

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx=\frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\,(a+bx^2)^{3/2}}$$

```
[In] Int[(c + d*x^2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (2*c*x)/(3*a^2*Sqrt[a + b*x^2]) + (x*(c + d*x^2))/(3*a*(a + b*x^2)^(3/2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}+\frac{(2c)\int\frac{1}{(a+bx^2)^{3/2}}\,dx}{3a}\\ &= \frac{2cx}{3a^2\sqrt{a+bx^2}}+\frac{x(c+dx^2)}{3a\left(a+bx^2\right)^{3/2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int\frac{c+dx^2}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{x(3ac+2bcx^2+adx^2)}{3a^2\left(a+bx^2\right)^{3/2}}$$

```
[In] Integrate[(c + d*x^2)/(a + b*x^2)^(5/2),x]
[Out] (x*(3*a*c + 2*b*c*x^2 + a*d*x^2))/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{x\left(ad\,x^{2}+2cb\,x^{2}+3ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	34
trager	$\frac{x\left(ad\,x^{2}+2cb\,x^{2}+3ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	34
pseudoelliptic	$\frac{x\left(ad\,x^{2}+2cb\,x^{2}+3ac\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	34
default	$c\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)+d\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)$	90

```
[In] int((d*x^2+c)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(a*d*x^2+2*b*c*x^2+3*a*c)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{((2\,bc+ad)x^3+3\,acx)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*c + a*d)*x^3 + 3*a*c*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^
2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(41) = 82.

Time = 4.02 (sec) , antiderivative size = 144, normalized size of antiderivative = 3.06

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = c\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right.$$
$$\left.+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+\frac{dx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate((d*x**2+c)/(b*x**2+a)**(5/2),x)
```

```
[Out] c*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + d*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sq
rt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.45

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{2\,cx}{3\,\sqrt{bx^2+a}a^2}+\frac{cx}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{dx}{3\,(bx^2+a)^{\frac{3}{2}}b}+\frac{dx}{3\,\sqrt{bx^2+a}ab}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*c*x/(sqrt(b*x^2 + a)*a^2) + 1/3*c*x/((b*x^2 + a)^(3/2)*a) - 1/3*d*x/((b
*x^2 + a)^(3/2)*b) + 1/3*d*x/(sqrt(b*x^2 + a)*a*b)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{3\,c}{a}+\frac{(2\,b^2c+abd)x^2}{a^2b}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate((d*x^2+c)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(3*c/a + (2*b^2*c + a*b*d)*x^2/(a^2*b))/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{c+dx^2}{(a+bx^2)^{5/2}}\,dx = \frac{3\,a\,c\,x+a\,d\,x^3+2\,b\,c\,x^3}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

```
[In] int((c + d*x^2)/(a + b*x^2)^(5/2),x)
[Out] (3*a*c*x + a*d*x^3 + 2*b*c*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

3.93 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{x}{3a(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}$$

```
[In] Int[(a + b*x^2)^(-5/2),x]
```

```
[Out] x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= \frac{x}{3a\left(a+bx^2\right)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{3ax+2bx^3}{3a^2\left(a+bx^2\right)^{3/2}}$$

```
[In] Integrate[(a + b*x^2)^(-5/2),x]
[Out] (3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Maple [A] (verified)

Time = 2.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
trager	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
pseudoelliptic	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
default	$\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}$	32

```
[In] int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,bx^3+3\,ax)\sqrt{bx^2+a}}{3\,(a^2b^2x^4+2\,a^3bx^2+a^4)}$$

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

Time = 0.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

```
[In] integrate(1/(b*x**2+a)**(5/2),x)
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)
) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*
x**2/a))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{bx^2+a}a^2} + \frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

```
[In] int(1/(a + b*x^2)^(5/2),x)
[Out] (2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

3.94 $\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)} dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)} dx = \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+d^2*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)
/(b*x^2+a)^(1/2))/(-a*d+b*c)^(5/2)/c^(1/2)+1/3*b*(-5*a*d+2*b*c)*x/a^2/(-a*d
+b*c)^2/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)} dx = \frac{bx(2bc-5ad)}{3a^2\sqrt{a+bx^2}(bc-ad)^2} + \frac{d^2\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}} + \frac{bx}{3a\left(a+bx^2\right)^{3/2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (b*(2*b*c - 5*a*d)*x)/(3*a^2*(b
*c - a*d)^2*Sqrt[a + b*x^2]) + (d^2*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sq
rt[a + b*x^2])])/(Sqrt[c]*(b*c - a*d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2bc+3ad-2bdx^2}{\left(a+bx^2\right)^{3/2}(c+dx^2)}\,dx}{3a(bc-ad)} \\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{\int \frac{3a^2d^2}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{3a^2(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)}\,dx}{(bc-ad)^2}
\end{aligned}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\mathrm{Subst}\left(\int\frac{1}{c-(bc-ad)x^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{b(2bc-5ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{\sqrt{c}(bc-ad)^{5/2}}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int\frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)}\,dx=\frac{bx(-6a^2d+2b^2cx^2+ab(3c-5dx^2))}{3a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}}-\frac{d^2\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}(c+dx^2)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{\sqrt{c}(-bc+ad)^{5/2}}$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)),x]
```

```
[Out] (b*x*(-6*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 5*d*x^2)))/(3*a^2*(b*c - a*d)^2*(
a + b*x^2)^(3/2)) - (d^2*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^
2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(Sqrt[c]*(-(b*c) + a*d)^(5/2))
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

method	result	size
pseudoelliptic	$-\frac{a^2d^2\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)\left(b\,x^2+a\right)^{\frac{3}{2}}+2x\sqrt{(ad-bc)c}\,b\left(a^2d-\frac{\left(-\frac{5d\,x^2}{3}+c\right)ba}{2}-\frac{b^2c\,x^2}{3}\right)}{\left(b\,x^2+a\right)^{\frac{3}{2}}\sqrt{(ad-bc)c}\,(ad-bc)^2a^2}$	124
default	Expression too large to display	1378

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(b*x^2+a)^(3/2)*(a^2*d^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))
*(b*x^2+a)^(3/2)+2*x*((a*d-b*c)*c)^(1/2)*b*(a^2*d-1/2*(-5/3*d*x^2+c)*b*a-1/
3*b^2*c*x^2))/((a*d-b*c)*c)^(1/2)/(a*d-b*c)^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(104) = 208.

Time = 0.48 (sec) , antiderivative size = 764, normalized size of antiderivative = 6.26

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)}\,dx = \left[\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{bc^2-acd}\log\left(\frac{(8\,b^2c^2-8\,abcd+a^2d^2)x^4+a^2c^2+2\,(4\,a\cdots}{\cdots}\right)\cdots}{12\,(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+\cdots}\right.$$

$$-\frac{3\,(a^2b^2d^2x^4+2\,a^3bd^2x^2+a^4d^2)\sqrt{-bc^2+acd}\arctan\left(\frac{\sqrt{-bc^2+acd}\,((2\,bc-ad)x^2+ac)\sqrt{bx^2+a}}{2\,((b^2c^2-abcd)x^3+(abc^2-a^2cd)x)}\right)-2\,((2\,b^4c^3-7\,ab^3c^2d\cdots}{6\,(a^4b^3c^4-3\,a^5b^2c^3d+3\,a^6bc^2d^2-a^7cd^3+(a^2b^5c^4-3\,a^3b^4c^3d+3\,a^4b^3c^2d^2-a^5b^2cd^3)x^4+2\cdots}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(b*c^2 - a*c*d)*
log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2
*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 +
a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((2*b^4*c^3 - 7*a*b^3*c^2*d + 5*a^2*b^
2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x)*sqrt(b*x^
2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c*d^3 + (a^2
*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*x^4 + 2*(a^
3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x^2), -1/6*(
3*(a^2*b^2*d^2*x^4 + 2*a^3*b*d^2*x^2 + a^4*d^2)*sqrt(-b*c^2 + a*c*d)*arctan
(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt(b*x^2 + a)/((b^2*c
^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((2*b^4*c^3 - 7*a*b^3*c^2*d
 + 5*a^2*b^2*c*d^2)*x^3 + 3*(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 2*a^3*b*c*d^2)*x
)*sqrt(b*x^2 + a))/(a^4*b^3*c^4 - 3*a^5*b^2*c^3*d + 3*a^6*b*c^2*d^2 - a^7*c
*d^3 + (a^2*b^5*c^4 - 3*a^3*b^4*c^3*d + 3*a^4*b^3*c^2*d^2 - a^5*b^2*c*d^3)*
x^4 + 2*(a^3*b^4*c^4 - 3*a^4*b^3*c^3*d + 3*a^5*b^2*c^2*d^2 - a^6*b*c*d^3)*x
^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(104) = 208$.

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = -\frac{\sqrt{b}d^2\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\sqrt{-b^2c^2+abcd}}\right)}{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-b^2c^2+abcd}} + \frac{\left(\frac{(2\,b^6c^3-9\,ab^5c^2d+12\,a^2b^4cd^2-5\,a^3b^3d^3)x^2}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}+\frac{3\,(ab^5c^3-4\,a^2b^4c^2d+5\,a^3b^3cd^2-2\,a^4b^2d^3)}{a^2b^5c^4-4\,a^3b^4c^3d+6\,a^4b^3c^2d^2-4\,a^5b^2cd^3+a^6bd^4}\right)x}{3\,(bx^2+a)^{\frac{3}{2}}}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c),x, algorithm="giac")
[Out] -sqrt(b)*d^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/s
qrt(-b^2*c^2 + a*b*c*d))/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-b^2*c^2 + a
*b*c*d)) + 1/3*((2*b^6*c^3 - 9*a*b^5*c^2*d + 12*a^2*b^4*c*d^2 - 5*a^3*b^3*d
^3)*x^2/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4) + 3*(a*b^5*c^3 - 4*a^2*b^4*c^2*d + 5*a^3*b^3*c*d^2 - 2*a^4*b
^2*d^3)/(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^
3 + a^6*b*d^4))*x/(b*x^2 + a)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)), x)
```

3.95 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx &= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/2}} \\ &+ \frac{b(4b^2c^2-16abcd-3a^2d^2)\,x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} - \frac{dx}{2c(bc-ad)\,(a+bx^2)^{3/2}\,(c+dx^2)} \\ &+ \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}\end{aligned}$$

```
[Out] 1/6*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)-1/2*d*x/c/(-a*d+b*c)
/(b*x^2+a)^(3/2)/(d*x^2+c)+1/2*d^2*(-a*d+6*b*c)*arctanh(x*(-a*d+b*c)^(1/2)/
c^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(-a*d+b*c)^(7/2)+1/6*b*(-3*a^2*d^2-16*a*b*
c*d+4*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {425, 541, 12, 385, 214}

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx &= \frac{bx(-3a^2d^2-16abcd+4b^2c^2)}{6a^2c\sqrt{a+bx^2}(bc-ad)^3} \\ &+ \frac{d^2(6bc-ad)\text{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}} \\ &- \frac{dx}{2c\,(a+bx^2)^{3/2}\,(c+dx^2)\,(bc-ad)} + \frac{bx(3ad+2bc)}{6ac\,(a+bx^2)^{3/2}\,(bc-ad)^2}\end{aligned}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (b*(2*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (b*(4*b^2*c
^2 - 16*a*b*c*d - 3*a^2*d^2)*x)/(6*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]) - (
d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a*d)*A
rcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])])/(2*c^(3/2)*(b*c - a*
d)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-4bdx^2}{(a+bx^2)^{5/2}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$
\begin{aligned}
&= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} \\
&\quad - \frac{\int \frac{-4b^2c^2+12abcd-3a^2d^2-2bd(2bc+3ad)x^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}\,dx}{6ac(bc-ad)^2} \\
&= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} \\
&\quad - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{\int \frac{3a^2d^2(6bc-ad)}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{6a^2c(bc-ad)^3} \\
&= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} \\
&\quad - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{\left(d^2(6bc-ad)\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{2c(bc-ad)^3} \\
&= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} \\
&\quad - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} \\
&\quad + \frac{\left(d^2(6bc-ad)\right)\operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2c(bc-ad)^3} \\
&= \frac{b(2bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{b\left(4b^2c^2-16abcd-3a^2d^2\right)x}{6a^2c(bc-ad)^3\sqrt{a+bx^2}} \\
&\quad - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} + \frac{d^2(6bc-ad)\tanh^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{2c^{3/2}(bc-ad)^{7/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.08

$$
\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx = \frac{x\left(3a^4d^3+6a^3bd^3x^2-4b^4c^2x^2\left(c+dx^2\right)+3a^2b^2d\left(6c^2+6cdx^2+d^2x^4\right)+2ab^3c\cdots\right)}{6a^2c(-bc+ad)^3\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}
$$
$$
+ \frac{d^2(6bc-ad)\arctan\left(\frac{-dx\sqrt{a+bx^2}+\sqrt{b}\left(c+dx^2\right)}{\sqrt{c}\sqrt{-bc+ad}}\right)}{2c^{3/2}(-bc+ad)^{7/2}}
$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x]
```

```
[Out] (x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 4*b^4*c^2*x^2*(c + d*x^2) + 3*a^2*b^2*d*(
6*c^2 + 6*c*d*x^2 + d^2*x^4) + 2*a*b^3*c*(-3*c^2 + 5*c*d*x^2 + 8*d^2*x^4)))
/(6*a^2*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*(c + d*x^2)) + (d^2*(6*b*c - a
*d)*ArcTan[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b
*c) + a*d])])/(2*c^(3/2)*(-(b*c) + a*d)^(7/2))
```

Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{-(ad-6bc)(d\,x^2+c)d^2(b\,x^2+a)^{\frac{3}{2}}a^2\arctan\left(\frac{c\sqrt{b\,x^2+a}}{x\sqrt{(ad-bc)c}}\right)+x\sqrt{(ad-bc)c}\,((-\frac{4}{3}b^4x^2-2a\,b^3)c^3+6b^2d(-\frac{2}{9}b^2x^4+\frac{5}{9}ab\,x^2+a^2)}{2\sqrt{(ad-bc)c}\,(b\,x^2+a)^{\frac{3}{2}}c(d\,x^2+c)(ad-bc)^3a^2}$
default	Expression too large to display

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/((a*d-b*c)*c)^(1/2)/(b*x^2+a)^(3/2)*(-(a*d-6*b*c)*(d*x^2+c)*d^2*(b*x^2+
a)^(3/2)*a^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))+x*((a*d-b*c)*c
)^(1/2)*((-4/3*b^4*x^2-2*a*b^3)*c^3+6*b^2*d*(-2/9*b^2*x^4+5/9*a*b*x^2+a^2)*
c^2+6*x^2*b^2*d^2*a*(8/9*b*x^2+a)*c+a^2*d^3*(b*x^2+a)^2))/c/(d*x^2+c)/(a*d-
b*c)^3/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(178) = 356$.

Time = 0.95 (sec) , antiderivative size = 1440, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6
 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c
^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2
- 8*a*b*c*d + a^2*d^2)*x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((
2*b*c - a*d)*x^3 + a*c*x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2
*c*d*x^2 + c^2)) + 4*((4*b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3
+ 3*a^3*b^2*c*d^4)*x^5 + 2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 +
 6*a^3*b^2*c^2*d^3 + 3*a^4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d
+ 6*a^3*b^2*c^3*d^2 - a^4*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b
^4*c^7 - 4*a^5*b^3*c^6*d + 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^
4 + (a^2*b^6*c^6*d - 4*a^3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*
d^4 + a^6*b^2*c^2*d^5)*x^6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5
*d^2 + 8*a^5*b^3*c^4*d^3 - 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^
3*b^5*c^7 - 7*a^4*b^4*c^6*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7
*b*c^3*d^4 + a^8*c^2*d^5)*x^2), -1/12*(3*(6*a^4*b*c^2*d^2 - a^5*c*d^3 + (6*
a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^6 + (6*a^2*b^3*c^2*d^2 + 11*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*x^4 + (12*a^3*b^2*c^2*d^2 + 4*a^4*b*c*d^3 - a^5*d^4)*x^2)*sqrt
```

```
(-b*c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*
sqrt(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((4*
b^5*c^4*d - 20*a*b^4*c^3*d^2 + 13*a^2*b^3*c^2*d^3 + 3*a^3*b^2*c*d^4)*x^5 +
2*(2*b^5*c^5 - 7*a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3 + 3*a^
4*b*c*d^4)*x^3 + 3*(2*a*b^4*c^5 - 8*a^2*b^3*c^4*d + 6*a^3*b^2*c^3*d^2 - a^4
*b*c^2*d^3 + a^5*c*d^4)*x)*sqrt(b*x^2 + a))/(a^4*b^4*c^7 - 4*a^5*b^3*c^6*d
+ 6*a^6*b^2*c^5*d^2 - 4*a^7*b*c^4*d^3 + a^8*c^3*d^4 + (a^2*b^6*c^6*d - 4*a^
3*b^5*c^5*d^2 + 6*a^4*b^4*c^4*d^3 - 4*a^5*b^3*c^3*d^4 + a^6*b^2*c^2*d^5)*x^
6 + (a^2*b^6*c^7 - 2*a^3*b^5*c^6*d - 2*a^4*b^4*c^5*d^2 + 8*a^5*b^3*c^4*d^3
- 7*a^6*b^2*c^3*d^4 + 2*a^7*b*c^2*d^5)*x^4 + (2*a^3*b^5*c^7 - 7*a^4*b^4*c^6
*d + 8*a^5*b^3*c^5*d^2 - 2*a^6*b^2*c^4*d^3 - 2*a^7*b*c^3*d^4 + a^8*c^2*d^5)
*x^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}\,(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 620 vs. $2(178) = 356$.

Time = 0.89 (sec) , antiderivative size = 620, normalized size of antiderivative = 3.07

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx = \frac{\left(\frac{2\,(b^8c^4-7\,ab^7c^3d+15\,a^2b^6c^2d^2-13\,a^3b^5cd^3+4\,a^4b^4d^4)x^2}{a^2b^7c^6-6\,a^3b^6c^5d+15\,a^4b^5c^4d^2-20\,a^5b^4c^3d^3+15\,a^6b^3c^2d^4-6\,a^7b^2cd^5+a^8bd^6} + \frac{3\,(ab^7c^4-}{a^2b^7c^6-6\,a^3b^6c^5d+}\right.}{3\,(bx^2+a)^{\frac{3}{2}}}$$

$$+ \frac{\left(6\,b^{\frac{3}{2}}cd^2 - a\sqrt{b}d^3\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{2\,(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\sqrt{-b^2c^2+abcd}}$$

$$- \frac{2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 b^{\frac{3}{2}}cd^2 - \left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a\sqrt{b}d^3 + a^2\sqrt{b}d^3}{(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 d + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 bc - 2\left(\sqrt{b}x-\sqrt{bx\right.\right.}}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*(b^8*c^4 - 7*a*b^7*c^3*d + 15*a^2*b^6*c^2*d^2 - 13*a^3*b^5*c*d^3 + 4
*a^4*b^4*d^4)*x^2/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*
a^5*b^4*c^3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6) + 3*(a*
b^7*c^4 - 6*a^2*b^6*c^3*d + 12*a^3*b^5*c^2*d^2 - 10*a^4*b^4*c*d^3 + 3*a^5*b
^3*d^4)/(a^2*b^7*c^6 - 6*a^3*b^6*c^5*d + 15*a^4*b^5*c^4*d^2 - 20*a^5*b^4*c^
3*d^3 + 15*a^6*b^3*c^2*d^4 - 6*a^7*b^2*c*d^5 + a^8*b*d^6))*x/(b*x^2 + a)^(3
/2) + 1/2*(6*b^(3/2)*c*d^2 - a*sqrt(b)*d^3)*arctan(-1/2*((sqrt(b)*x - sqrt(
b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^3*c^4 - 3*a*b^
2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*sqrt(-b^2*c^2 + a*b*c*d)) - (2*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c*d^2 - (sqrt(b)*x - sqrt(b*x^2 + a))^2*
a*sqrt(b)*d^3 + a^2*sqrt(b)*d^3)/((b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^
2 - a^3*c*d^3)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{5/2}\,\left(d\,x^2+c\right)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2),x)
```

```
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^2), x)
```

3.96 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 313

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx =& -\frac{dx}{4c(bc-ad)(a+bx^2)^{3/2}(c+dx^2)^2}\\ &+\frac{b(4bc+3ad)x}{12ac(bc-ad)^2(a+bx^2)^{3/2}(c+dx^2)}+\frac{b(8b^2c^2-40abcd-3a^2d^2)\,x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\,(c+dx^2)}\\ &+\frac{d(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3)\,x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4(c+dx^2)}\\ &+\frac{d^2(48b^2c^2-16abcd+3a^2d^2)\,\mathrm{arctanh}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}\end{aligned}$$

```
[Out] -1/4*d*x/c/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^2+1/12*b*(3*a*d+4*b*c)*x/a/
c/(-a*d+b*c)^2/(b*x^2+a)^(3/2)/(d*x^2+c)+1/8*d^2*(3*a^2*d^2-16*a*b*c*d+48*b
^2*c^2)*arctanh(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(-a*d+b
*c)^(9/2)+1/12*b*(-3*a^2*d^2-40*a*b*c*d+8*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(d*
x^2+c)/(b*x^2+a)^(1/2)+1/24*d*(9*a^3*d^3-42*a^2*b*c*d^2-88*a*b^2*c^2*d+16*b
^3*c^3)*x*(b*x^2+a)^(1/2)/a^2/c^2/(-a*d+b*c)^4/(d*x^2+c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used

$= \{425, 541, 12, 385, 214\}$

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \frac{d^2(3a^2d^2-16abcd+48b^2c^2)\operatorname{arctanh}\left(\frac{x\sqrt{bc-ad}}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}} + \frac{bx(-3a^2d^2-40abcd+8b^2c^2)}{12a^2c\sqrt{a+bx^2}\,(c+dx^2)\,(bc-ad)^3} + \frac{dx\sqrt{a+bx^2}(9a^3d^3-42a^2bcd^2-88ab^2c^2d+16b^3c^3)}{24a^2c^2\,(c+dx^2)\,(bc-ad)^4} - \frac{dx}{4c\,(a+bx^2)^{3/2}\,(c+dx^2)^2\,(bc-ad)} + \frac{bx(3ad+4bc)}{12ac\,(a+bx^2)^{3/2}\,(c+dx^2)\,(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] -1/4*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(3/2)*(c + d*x^2)^2) + (b*(4*b*c + 3*
a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/2)*(c + d*x^2)) + (b*(8*b^2*c^
2 - 40*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a + b*x^2]*(c +
 d*x^2)) + (d*(16*b^3*c^3 - 88*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 9*a^3*d^3)*x*
Sqrt[a + b*x^2])/(24*a^2*c^2*(b*c - a*d)^4*(c + d*x^2)) + (d^2*(48*b^2*c^2
- 16*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x
^2])])/(8*c^(5/2)*(b*c - a*d)^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}+\frac{\int \frac{4bc-3ad-6bdx^2}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^2}\,dx}{4c(bc-ad)} \\
&= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}+\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} \\
&\quad -\frac{\int \frac{-8b^2c^2+24abcd-9a^2d^2-4bd(4bc+3ad)x^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}\,dx}{12ac(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}+\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} \\
&\quad +\frac{b\left(8b^2c^2-40abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\left(c+dx^2\right)}+\frac{\int \frac{ad\left(8b^2c^2+36abcd-9a^2d^2\right)+2bd\left(8b^2c^2-40abcd-3a^2d^2\right)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^2}\,dx}{12a^2c(bc-ad)^3} \\
&= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2}+\frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)} \\
&\quad +\frac{b\left(8b^2c^2-40abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\left(c+dx^2\right)} \\
&\quad +\frac{d\left(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3\right)x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4\left(c+dx^2\right)} \\
&\quad +\frac{\int \frac{3a^2d^2\left(48b^2c^2-16abcd+3a^2d^2\right)}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{24a^2c^2(bc-ad)^4}
\end{aligned}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}$$
$$+ \frac{b\left(8b^2c^2-40abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\left(c+dx^2\right)}$$
$$+ \frac{d\left(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3\right)x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4\left(c+dx^2\right)}$$
$$+ \frac{\left(d^2\left(48b^2c^2-16abcd+3a^2d^2\right)\right)\int \frac{1}{\sqrt{a+bx^2}\left(c+dx^2\right)}\,dx}{8c^2(bc-ad)^4}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}$$
$$+ \frac{b\left(8b^2c^2-40abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\left(c+dx^2\right)}$$
$$+ \frac{d\left(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3\right)x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4\left(c+dx^2\right)}$$
$$+ \frac{\left(d^2\left(48b^2c^2-16abcd+3a^2d^2\right)\right)\operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{8c^2(bc-ad)^4}$$

$$= -\frac{dx}{4c(bc-ad)\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^2} + \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)}$$
$$+ \frac{b\left(8b^2c^2-40abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a+bx^2}\left(c+dx^2\right)}$$
$$+ \frac{d\left(16b^3c^3-88ab^2c^2d-42a^2bcd^2+9a^3d^3\right)x\sqrt{a+bx^2}}{24a^2c^2(bc-ad)^4\left(c+dx^2\right)}$$
$$+ \frac{d^2\left(48b^2c^2-16abcd+3a^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)}{8c^{5/2}(bc-ad)^{9/2}}$$

Mathematica [A] (verified)

Time = 3.26 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx = \frac{\sqrt{c}x\left(16b^5c^3x^2\left(c+dx^2\right)^2+8ab^4c^2\left(3c-11dx^2\right)\left(c+dx^2\right)^2+3a^5d^4\left(5c+3dx^2\right)+3a^3b^2d^3x^2\left(-32c^2-23cdx^2+\right.\right.}{a^2(bc-ad)^4\left(a+bx^2\right)^{3/2}(c}$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x]
```

```
[Out] ((Sqrt[c]*x*(16*b^5*c^3*x^2*(c + d*x^2)^2 + 8*a*b^4*c^2*(3*c - 11*d*x^2)*(c
 + d*x^2)^2 + 3*a^5*d^4*(5*c + 3*d*x^2) + 3*a^3*b^2*d^3*x^2*(-32*c^2 - 23*c
*d*x^2 + 3*d^2*x^4) + 6*a^4*b*d^3*(-8*c^2 - 2*c*d*x^2 + 3*d^2*x^4) - 6*a^2*
b^3*c*d*(16*c^3 + 32*c^2*d*x^2 + 24*c*d^2*x^4 + 7*d^3*x^6)))/(a^2*(b*c - a*
```

```
d)^4*(a + b*x^2)^(3/2)*(c + d*x^2)^2) - (9*d^2*(-4*b*c + a*d)^2*ArcTan[(-(d
*x*Sqrt[a + b*x^2]) + Sqrt[b]*(c + d*x^2))/(Sqrt[c]*Sqrt[-(b*c) + a*d])])/(
-(b*c) + a*d)^(9/2) + (24*a*b*c*d^3*ArcTanh[(-(d*x*Sqrt[a + b*x^2]) + Sqrt[
b]*(c + d*x^2))/(Sqrt[c]*Sqrt[b*c - a*d])])/(b*c - a*d)^(9/2))/(24*c^(5/2))
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{3\left(\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)^{2}\left(a^{2}d^{2}-\frac{16}{3}abcd+16b^{2}c^{2}\right)d^{2}a^{2}\arctan\left(\frac{c\sqrt{b\,x^{2}+a}}{x\sqrt{\left(ad-bc\right)c}}\right)-\frac{5x\sqrt{\left(ad-bc\right)c}\left(\frac{3a^{3}x^{2}\left(b\,x^{2}+a\right)^{2}d^{5}}{5}+\left(b\,x^{2}+a\right)^{2}\left(\right.\right.}{}\right.}{8\left(b\,x^{2}+a\right)}$
default	Expression too large to display

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -3/8/(b*x^2+a)^(3/2)*((b*x^2+a)^(3/2)*(d*x^2+c)^2*(a^2*d^2-16/3*a*b*c*d+16*
b^2*c^2)*d^2*a^2*arctan(c*(b*x^2+a)^(1/2)/x/((a*d-b*c)*c)^(1/2))-5/3*x*((a*
d-b*c)*c)^(1/2)*(3/5*a^3*x^2*(b*x^2+a)^2*d^5+(b*x^2+a)^2*(-14/5*b*x^2+a)*c*
a^2*d^4-16/5*(11/6*b^3*x^6+3*a*b^2*x^4+2*a^2*b*x^2+a^3)*b*c^2*a*d^3-64/5*x^
2*b^3*(-1/12*b^2*x^4+19/24*a*b*x^2+a^2)*c^3*d^2-32/5*b^3*(-1/3*b^2*x^4+5/12
*a*b*x^2+a^2)*c^4*d+8/5*b^4*(2/3*b*x^2+a)*c^5))/((a*d-b*c)*c)^(1/2)/(d*x^2+
c)^2/c^2/(a*d-b*c)^4/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(285) = 570$.

Time = 2.76 (sec) , antiderivative size = 2250, normalized size of antiderivative = 7.19

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\left(c+dx^2\right)^3}\,dx=\text{Too large to display}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(48*a^4*b^2*c^4*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b
^4*c^2*d^4 - 16*a^3*b^3*c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3
+ 32*a^3*b^3*c^2*d^4 - 13*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^
4*d^2 + 176*a^3*b^3*c^3*d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^
6)*x^4 + 2*(48*a^3*b^3*c^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*
a^6*c*d^5)*x^2)*sqrt(b*c^2 - a*c*d)*log(((8*b^2*c^2 - 8*a*b*c*d + a^2*d^2)*
x^4 + a^2*c^2 + 2*(4*a*b*c^2 - 3*a^2*c*d)*x^2 + 4*((2*b*c - a*d)*x^3 + a*c*
```

```
x)*sqrt(b*c^2 - a*c*d)*sqrt(b*x^2 + a))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + 4*((
16*b^6*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^
5 - 9*a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^
4*d^3 + 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16
*b^6*c^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a
^4*b^2*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*
a^2*b^4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4
- 5*a^6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a
^6*b^3*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*
b^7*c^8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 +
 5*a^6*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^
8*d^2 + 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c
^3*d^7)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^
4*c^7*d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^
3*d^7)*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*
b^2*c^6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2), -1/48*(3*(48*a^4*b^2*c^4
*d^2 - 16*a^5*b*c^3*d^3 + 3*a^6*c^2*d^4 + (48*a^2*b^4*c^2*d^4 - 16*a^3*b^3*
c*d^5 + 3*a^4*b^2*d^6)*x^8 + 2*(48*a^2*b^4*c^3*d^3 + 32*a^3*b^3*c^2*d^4 - 1
3*a^4*b^2*c*d^5 + 3*a^5*b*d^6)*x^6 + (48*a^2*b^4*c^4*d^2 + 176*a^3*b^3*c^3*
d^3 - 13*a^4*b^2*c^2*d^4 - 4*a^5*b*c*d^5 + 3*a^6*d^6)*x^4 + 2*(48*a^3*b^3*c
^4*d^2 + 32*a^4*b^2*c^3*d^3 - 13*a^5*b*c^2*d^4 + 3*a^6*c*d^5)*x^2)*sqrt(-b*
c^2 + a*c*d)*arctan(1/2*sqrt(-b*c^2 + a*c*d)*((2*b*c - a*d)*x^2 + a*c)*sqrt
(b*x^2 + a)/((b^2*c^2 - a*b*c*d)*x^3 + (a*b*c^2 - a^2*c*d)*x)) - 2*((16*b^6
*c^5*d^2 - 104*a*b^5*c^4*d^3 + 46*a^2*b^4*c^3*d^4 + 51*a^3*b^3*c^2*d^5 - 9*
a^4*b^2*c*d^6)*x^7 + (32*b^6*c^6*d - 184*a*b^5*c^5*d^2 + 8*a^2*b^4*c^4*d^3
+ 75*a^3*b^3*c^3*d^4 + 87*a^4*b^2*c^2*d^5 - 18*a^5*b*c*d^6)*x^5 + (16*b^6*c
^7 - 56*a*b^5*c^6*d - 152*a^2*b^4*c^5*d^2 + 96*a^3*b^3*c^4*d^3 + 84*a^4*b^2
*c^3*d^4 + 21*a^5*b*c^2*d^5 - 9*a^6*c*d^6)*x^3 + 3*(8*a*b^5*c^7 - 40*a^2*b^
4*c^6*d + 32*a^3*b^3*c^5*d^2 - 16*a^4*b^2*c^4*d^3 + 21*a^5*b*c^3*d^4 - 5*a^
6*c^2*d^5)*x)*sqrt(b*x^2 + a))/(a^4*b^5*c^10 - 5*a^5*b^4*c^9*d + 10*a^6*b^3
*c^8*d^2 - 10*a^7*b^2*c^7*d^3 + 5*a^8*b*c^6*d^4 - a^9*c^5*d^5 + (a^2*b^7*c^
8*d^2 - 5*a^3*b^6*c^7*d^3 + 10*a^4*b^5*c^6*d^4 - 10*a^5*b^4*c^5*d^5 + 5*a^6
*b^3*c^4*d^6 - a^7*b^2*c^3*d^7)*x^8 + 2*(a^2*b^7*c^9*d - 4*a^3*b^6*c^8*d^2
+ 5*a^4*b^5*c^7*d^3 - 5*a^6*b^3*c^5*d^5 + 4*a^7*b^2*c^4*d^6 - a^8*b*c^3*d^7
)*x^6 + (a^2*b^7*c^10 - a^3*b^6*c^9*d - 9*a^4*b^5*c^8*d^2 + 25*a^5*b^4*c^7*
d^3 - 25*a^6*b^3*c^6*d^4 + 9*a^7*b^2*c^5*d^5 + a^8*b*c^4*d^6 - a^9*c^3*d^7)
*x^4 + 2*(a^3*b^6*c^10 - 4*a^4*b^5*c^9*d + 5*a^5*b^4*c^8*d^2 - 5*a^7*b^2*c^
6*d^4 + 4*a^8*b*c^5*d^5 - a^9*c^4*d^6)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^3}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(285) = 570$.

Time = 1.72 (sec) , antiderivative size = 1010, normalized size of antiderivative = 3.23

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^3}\,dx = \frac{\left(\frac{(2\,b^{10}c^5-19\,ab^9c^4d+56\,a^2b^8c^3d^2-74\,a^3b^7c^2d^3+46\,a^4b^6cd^4-11\,a^5b^5d^5)x^2}{a^2b^9c^8-8\,a^3b^8c^7d+28\,a^4b^7c^6d^2-56\,a^5b^6c^5d^3+70\,a^6b^5c^4d^4-56\,a^7b^4c^3d^5+28\,a^8b^3c^2d^6-8\,a^9b^2cd^7+a^{10}b}\right.}{3\,(}$$
$$-\frac{\left(48\,b^{\frac{5}{2}}c^2d^2-16\,ab^{\frac{3}{2}}cd^3+3\,a^2\sqrt{b}d^4\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2d+2\,bc-ad}{2\,\sqrt{-b^2c^2+abcd}}\right)}{8\,(b^4c^6-4\,ab^3c^5d+6\,a^2b^2c^4d^2-4\,a^3bc^3d^3+a^4c^2d^4)\sqrt{-b^2c^2+abcd}}$$
$$-\frac{24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6b^{\frac{5}{2}}c^2d^3-16\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6ab^{\frac{3}{2}}cd^4+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2\sqrt{b}d^5+112\left(\sqrt{b}\right.}{}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^3,x, algorithm="giac")
[Out] 1/3*((2*b^10*c^5 - 19*a*b^9*c^4*d + 56*a^2*b^8*c^3*d^2 - 74*a^3*b^7*c^2*d^3
 + 46*a^4*b^6*c*d^4 - 11*a^5*b^5*d^5)*x^2/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d +
28*a^4*b^7*c^6*d^2 - 56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c
^3*d^5 + 28*a^8*b^3*c^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8) + 3*(a*b^9*c^5
- 8*a^2*b^8*c^4*d + 22*a^3*b^7*c^3*d^2 - 28*a^4*b^6*c^2*d^3 + 17*a^5*b^5*c*
```

```
d^4 - 4*a^6*b^4*d^5)/(a^2*b^9*c^8 - 8*a^3*b^8*c^7*d + 28*a^4*b^7*c^6*d^2 -
56*a^5*b^6*c^5*d^3 + 70*a^6*b^5*c^4*d^4 - 56*a^7*b^4*c^3*d^5 + 28*a^8*b^3*c
^2*d^6 - 8*a^9*b^2*c*d^7 + a^10*b*d^8))*x/(b*x^2 + a)^(3/2) - 1/8*(48*b^(5/
2)*c^2*d^2 - 16*a*b^(3/2)*c*d^3 + 3*a^2*sqrt(b)*d^4)*arctan(1/2*((sqrt(b)*x
 - sqrt(b*x^2 + a))^2*d + 2*b*c - a*d)/sqrt(-b^2*c^2 + a*b*c*d))/((b^4*c^6
- 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-
b^2*c^2 + a*b*c*d)) - 1/4*(24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2)*c^2*d
^3 - 16*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(3/2)*c*d^4 + 3*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*a^2*sqrt(b)*d^5 + 112*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(7
/2)*c^3*d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(5/2)*c^2*d^3 + 66*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2)*c*d^4 - 9*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^3*sqrt(b)*d^5 + 88*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2)*c
^2*d^3 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(3/2)*c*d^4 + 9*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^4*sqrt(b)*d^5 + 14*a^4*b^(3/2)*c*d^4 - 3*a^5*sqrt(
b)*d^5)/((b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4*a^3*b*c^3*d^3 + a
^4*c^2*d^4)*((sqrt(b)*x - sqrt(b*x^2 + a))^4*d + 4*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*b*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*d + a^2*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}\,(c+dx^2)^3}\,dx=\int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^3}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^3), x)
```

3.97 $\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = -\frac{dx(a+bx^2)^4}{9c(bc-ad)\,(c+dx^2)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)\,(c+dx^2)^{7/2}} + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)\,(c+dx^2)^{5/2}} + \frac{8a^2(9bc-8ad)x(a+bx^2)}{315c^4(bc-ad)\,(c+dx^2)^{3/2}} + \frac{16a^3(9bc-8ad)x}{315c^5(bc-ad)\sqrt{c+dx^2}}$$

```
[Out] -1/9*d*x*(b*x^2+a)^4/c/(-a*d+b*c)/(d*x^2+c)^(9/2)+1/63*(-8*a*d+9*b*c)*x*(b*
x^2+a)^3/c^2/(-a*d+b*c)/(d*x^2+c)^(7/2)+2/105*a*(-8*a*d+9*b*c)*x*(b*x^2+a)^
2/c^3/(-a*d+b*c)/(d*x^2+c)^(5/2)+8/315*a^2*(-8*a*d+9*b*c)*x*(b*x^2+a)/c^4/(
-a*d+b*c)/(d*x^2+c)^(3/2)+16/315*a^3*(-8*a*d+9*b*c)*x/c^5/(-a*d+b*c)/(d*x^2
+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 386, 197}

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{16a^3x(9bc-8ad)}{315c^5\sqrt{c+dx^2}(bc-ad)} + \frac{8a^2x(a+bx^2)\,(9bc-8ad)}{315c^4\,(c+dx^2)^{3/2}\,(bc-ad)} + \frac{2ax(a+bx^2)^2\,(9bc-8ad)}{105c^3\,(c+dx^2)^{5/2}\,(bc-ad)} + \frac{x(a+bx^2)^3\,(9bc-8ad)}{63c^2\,(c+dx^2)^{7/2}\,(bc-ad)} - \frac{dx(a+bx^2)^4}{9c\,(c+dx^2)^{9/2}\,(bc-ad)}$$

```
[In] Int[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] -1/9*(d*x*(a + b*x^2)^4)/(c*(b*c - a*d)*(c + d*x^2)^(9/2)) + ((9*b*c - 8*a*
d)*x*(a + b*x^2)^3)/(63*c^2*(b*c - a*d)*(c + d*x^2)^(7/2)) + (2*a*(9*b*c -
8*a*d)*x*(a + b*x^2)^2)/(105*c^3*(b*c - a*d)*(c + d*x^2)^(5/2)) + (8*a^2*(9
*b*c - 8*a*d)*x*(a + b*x^2))/(315*c^4*(b*c - a*d)*(c + d*x^2)^(3/2)) + (16*
a^3*(9*b*c - 8*a*d)*x)/(315*c^5*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx(a+bx^2)^4}{9c(bc-ad)\,(c+dx^2)^{9/2}}+\frac{(9bc-8ad)\int\frac{(a+bx^2)^3}{(c+dx^2)^{9/2}}\,dx}{9c(bc-ad)}\\
&= -\frac{dx(a+bx^2)^4}{9c(bc-ad)\,(c+dx^2)^{9/2}}+\frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)\,(c+dx^2)^{7/2}}+\frac{(2a(9bc-8ad))\int\frac{(a+bx^2)^2}{(c+dx^2)^{7/2}}\,dx}{21c^2(bc-ad)}\\
&= -\frac{dx(a+bx^2)^4}{9c(bc-ad)\,(c+dx^2)^{9/2}}+\frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)\,(c+dx^2)^{7/2}}\\
&\quad+\frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)\,(c+dx^2)^{5/2}}+\frac{(8a^2(9bc-8ad))\int\frac{a+bx^2}{(c+dx^2)^{5/2}}\,dx}{105c^3(bc-ad)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{dx(a+bx^2)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} \\
&\quad + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{8a^2(9bc-8ad)x(a+bx^2)}{315c^4(bc-ad)\left(c+dx^2\right)^{3/2}} \\
&\quad + \frac{(16a^3(9bc-8ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{315c^4(bc-ad)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{dx(a+bx^2)^4}{9c(bc-ad)\left(c+dx^2\right)^{9/2}} + \frac{(9bc-8ad)x(a+bx^2)^3}{63c^2(bc-ad)\left(c+dx^2\right)^{7/2}} \\
&\quad + \frac{2a(9bc-8ad)x(a+bx^2)^2}{105c^3(bc-ad)\left(c+dx^2\right)^{5/2}} \\
&\quad + \frac{8a^2(9bc-8ad)x(a+bx^2)}{315c^4(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{16a^3(9bc-8ad)x}{315c^5(bc-ad)\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.73

$$
\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{5b^3c^3x^7(9c+2dx^2) + 3ab^2c^2x^5(63c^2+36cdx^2+8d^2x^4) + 3a^2bcx^3(105c^3+126c^2dx^2+}{315c^5\,(c+d}
$$

```
[In] Integrate[(a + b*x^2)^3/(c + d*x^2)^(11/2),x]
```

```
[Out] (5*b^3*c^3*x^7*(9*c + 2*d*x^2) + 3*a*b^2*c^2*x^5*(63*c^2 + 36*c*d*x^2 + 8*d
^2*x^4) + 3*a^2*b*c*x^3*(105*c^3 + 126*c^2*d*x^2 + 72*c*d^2*x^4 + 16*d^3*x^
6) + a^3*(315*c^4*x + 840*c^3*d*x^3 + 1008*c^2*d^2*x^5 + 576*c*d^3*x^7 + 12
8*d^4*x^9))/(315*c^5*(c + d*x^2)^(9/2))
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{7}b^3x^6+\frac{3}{5}a\,b^2x^4+a^2b\,x^2+a^3\right)c^4+\frac{8x^2\left(\frac{1}{84}b^3x^6+\frac{9}{70}a\,b^2x^4+\frac{9}{20}a^2b\,x^2+a^3\right)d\,c^3}{3}+\frac{16x^4\left(\frac{1}{42}b^2x^4+\frac{3}{14}ab\,x^2+a^2\right)d^2a\,c^2}{5}+\frac{64x^6\left(\frac{b\,x^2}{12}\right.}{3}\right.}{(d\,x^2+c)^{\frac{9}{2}}c^5}$
gosper	$\frac{x(128a^3d^4x^8+48a^2bc\,d^3x^8+24a\,b^2c^2d^2x^8+10b^3c^3d\,x^8+576a^3c\,d^3x^6+216a^2b\,c^2d^2x^6+108a\,b^2c^3d\,x^6+45b^3c^4x^6+1008a^3c^2d^2}{315(d\,x^2+c)^{\frac{9}{2}}c^5}$
trager	$\frac{x(128a^3d^4x^8+48a^2bc\,d^3x^8+24a\,b^2c^2d^2x^8+10b^3c^3d\,x^8+576a^3c\,d^3x^6+216a^2b\,c^2d^2x^6+108a\,b^2c^3d\,x^6+45b^3c^4x^6+1008a^3c^2d^2}{315(d\,x^2+c)^{\frac{9}{2}}c^5}$
default	$a^3\left(\frac{x}{\ldots}+\frac{8x}{63c\left(d\,x^2+c\right)^{\frac{7}{2}}}+\frac{8\left(\frac{6x}{35c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{d\,x^2+c}}\right)}{7c}\right)}{9c}\right)+b^3\left(\ldots\frac{x^5}{\ldots}\right.$

```
[In] int((b*x^2+a)^3/(d*x^2+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(d*x^2+c)^(9/2)*((1/7*b^3*x^6+3/5*a*b^2*x^4+a^2*b*x^2+a^3)*c^4+8/3*x^2*(1
/84*b^3*x^6+9/70*a*b^2*x^4+9/20*a^2*b*x^2+a^3)*d*c^3+16/5*x^4*(1/42*b^2*x^4
+3/14*a*b*x^2+a^2)*d^2*a*c^2+64/35*x^6*(1/12*b*x^2+a)*d^3*a^2*c+128/315*a^3
*d^4*x^8)/c^5
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{(2\,(5\,b^3c^3d+12\,ab^2c^2d^2+24\,a^2bcd^3+64\,a^3d^4)x^9+315\,a^3c^4x+9\,(5\,b^3c^4+12\,ab^2c^3d+}{315\,(c^5d^5x^{10}+5\,c^6d^4x^8}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(2*(5*b^3*c^3*d + 12*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 64*a^3*d^4)*x^9
 + 315*a^3*c^4*x + 9*(5*b^3*c^4 + 12*a*b^2*c^3*d + 24*a^2*b*c^2*d^2 + 64*a^
3*c*d^3)*x^7 + 63*(3*a*b^2*c^4 + 6*a^2*b*c^3*d + 16*a^3*c^2*d^2)*x^5 + 105*
(3*a^2*b*c^4 + 8*a^3*c^3*d)*x^3)*sqrt(d*x^2 + c)/(c^5*d^5*x^10 + 5*c^6*d^4*
x^8 + 10*c^7*d^3*x^6 + 10*c^8*d^2*x^4 + 5*c^9*d*x^2 + c^10)
```

Sympy [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \int \frac{(a+bx^2)^3}{(c+dx^2)^{\frac{11}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)**(11/2),x)
```

```
[Out] Integral((a + b*x**2)**3/(c + d*x**2)**(11/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(204) = 408.

Time = 0.22 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.08

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = -\frac{b^3x^5}{4\,(dx^2+c)^{\frac{9}{2}}d} - \frac{5\,b^3cx^3}{24\,(dx^2+c)^{\frac{9}{2}}d^2} - \frac{ab^2x^3}{2\,(dx^2+c)^{\frac{9}{2}}d} + \frac{128\,a^3x}{315\,\sqrt{dx^2+c}c^5} + \frac{64\,a^3x}{315\,(dx^2+c)^{\frac{3}{2}}c^4} + \frac{16\,a^3x}{105\,(dx^2+c)^{\frac{5}{2}}c^3} + \frac{8\,a^3x}{63\,(dx^2+c)^{\frac{7}{2}}c^2} + \frac{a^3x}{9\,(dx^2+c)^{\frac{9}{2}}c} + \frac{b^3x}{84\,(dx^2+c)^{\frac{5}{2}}d^3} + \frac{2\,b^3x}{63\,\sqrt{dx^2+c}c^2d^3} + \frac{b^3x}{63\,(dx^2+c)^{\frac{3}{2}}cd^3} + \frac{5\,b^3cx}{504\,(dx^2+c)^{\frac{7}{2}}d^3} - \frac{5\,b^3c^2x}{72\,(dx^2+c)^{\frac{9}{2}}d^3} + \frac{ab^2x}{42\,(dx^2+c)^{\frac{7}{2}}d^2} + \frac{8\,ab^2x}{105\,\sqrt{dx^2+c}c^3d^2} + \frac{4\,ab^2x}{105\,(dx^2+c)^{\frac{3}{2}}c^2d^2} + \frac{ab^2x}{35\,(dx^2+c)^{\frac{5}{2}}cd^2} - \frac{ab^2cx}{6\,(dx^2+c)^{\frac{9}{2}}d^2} - \frac{a^2bx}{3\,(dx^2+c)^{\frac{9}{2}}d} + \frac{16\,a^2bx}{105\,\sqrt{dx^2+c}c^4d} + \frac{8\,a^2bx}{105\,(dx^2+c)^{\frac{3}{2}}c^3d} + \frac{2\,a^2bx}{35\,(dx^2+c)^{\frac{5}{2}}c^2d} + \frac{a^2bx}{21\,(dx^2+c)^{\frac{7}{2}}cd}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^3*x^5/((d*x^2 + c)^(9/2)*d) - 5/24*b^3*c*x^3/((d*x^2 + c)^(9/2)*d^2)
 - 1/2*a*b^2*x^3/((d*x^2 + c)^(9/2)*d) + 128/315*a^3*x/(sqrt(d*x^2 + c)*c^5
) + 64/315*a^3*x/((d*x^2 + c)^(3/2)*c^4) + 16/105*a^3*x/((d*x^2 + c)^(5/2)*
c^3) + 8/63*a^3*x/((d*x^2 + c)^(7/2)*c^2) + 1/9*a^3*x/((d*x^2 + c)^(9/2)*c)
 + 1/84*b^3*x/((d*x^2 + c)^(5/2)*d^3) + 2/63*b^3*x/(sqrt(d*x^2 + c)*c^2*d^3
) + 1/63*b^3*x/((d*x^2 + c)^(3/2)*c*d^3) + 5/504*b^3*c*x/((d*x^2 + c)^(7/2)
*d^3) - 5/72*b^3*c^2*x/((d*x^2 + c)^(9/2)*d^3) + 1/42*a*b^2*x/((d*x^2 + c)^
(7/2)*d^2) + 8/105*a*b^2*x/(sqrt(d*x^2 + c)*c^3*d^2) + 4/105*a*b^2*x/((d*x^
2 + c)^(3/2)*c^2*d^2) + 1/35*a*b^2*x/((d*x^2 + c)^(5/2)*c*d^2) - 1/6*a*b^2*
c*x/((d*x^2 + c)^(9/2)*d^2) - 1/3*a^2*b*x/((d*x^2 + c)^(9/2)*d) + 16/105*a^
2*b*x/(sqrt(d*x^2 + c)*c^4*d) + 8/105*a^2*b*x/((d*x^2 + c)^(3/2)*c^3*d) + 2
/35*a^2*b*x/((d*x^2 + c)^(5/2)*c^2*d) + 1/21*a^2*b*x/((d*x^2 + c)^(7/2)*c*d
)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{2\,(5\,b^3c^3d^5+12\,ab^2c^2d^6+24\,a^2bcd^7+64\,a^3d^8)x^2}{c^5d^4} + \frac{9\,(5\,b^3c^4d^4+12\,ab^2c^3d^5+24\,a^2bc^2d^6+64\,a^3cd^7)}{c^5d^4}\right) + \frac{63}{}\right.\right.\right.}{315\,(dx^2+c)^{\frac{9}{2}}}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)^(11/2),x, algorithm="giac")
```

```
[Out] 1/315*(((x^2*(2*(5*b^3*c^3*d^5 + 12*a*b^2*c^2*d^6 + 24*a^2*b*c*d^7 + 64*a^3
*d^8)*x^2/(c^5*d^4) + 9*(5*b^3*c^4*d^4 + 12*a*b^2*c^3*d^5 + 24*a^2*b*c^2*d^
6 + 64*a^3*c*d^7)/(c^5*d^4)) + 63*(3*a*b^2*c^4*d^4 + 6*a^2*b*c^3*d^5 + 16*a
^3*c^2*d^6)/(c^5*d^4))*x^2 + 105*(3*a^2*b*c^4*d^4 + 8*a^3*c^3*d^5)/(c^5*d^4
))*x^2 + 315*a^3/c)*x/(d*x^2 + c)^(9/2)
```

Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^3}{(c+dx^2)^{11/2}}\,dx = \frac{x\left(\frac{a^3}{9\,c}-\frac{c\left(\frac{c\left(\frac{b^3}{9\,d}-\frac{a\,b^2}{3\,c}\right)}{d}+\frac{a^2\,b}{3\,c}\right)}{d}\right)}{(d\,x^2+c)^{9/2}}$$
$$-\frac{x\left(\frac{b^3}{5\,d^3}-\frac{16\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-4\,b^3\,c^3}{105\,c^3\,d^3}\right)}{(d\,x^2+c)^{5/2}}$$
$$+\frac{x\left(\frac{c\left(\frac{b^3}{7\,d^2}-\frac{b^2\,(3\,a\,d-b\,c)}{7\,c\,d^2}\right)}{d}+\frac{8\,a^3\,d^3+3\,a^2\,b\,c\,d^2-3\,a\,b^2\,c^2\,d+b^3\,c^3}{63\,c^2\,d^3}\right)}{(d\,x^2+c)^{7/2}}$$
$$+\frac{x\,(64\,a^3\,d^3+24\,a^2\,b\,c\,d^2+12\,a\,b^2\,c^2\,d+5\,b^3\,c^3)}{315\,c^4\,d^3\,(d\,x^2+c)^{3/2}}$$
$$+\frac{x\,(128\,a^3\,d^3+48\,a^2\,b\,c\,d^2+24\,a\,b^2\,c^2\,d+10\,b^3\,c^3)}{315\,c^5\,d^3\,\sqrt{d\,x^2+c}}$$

```
[In] int((a + b*x^2)^3/(c + d*x^2)^(11/2),x)
```

```
[Out] (x*(a^3/(9*c) - (c*((c*(b^3/(9*d) - (a*b^2)/(3*c)))/d + (a^2*b)/(3*c)))/d))
/(c + d*x^2)^(9/2) - (x*(b^3/(5*d^3) - (16*a^3*d^3 - 4*b^3*c^3 + 3*a*b^2*c^
2*d + 6*a^2*b*c*d^2)/(105*c^3*d^3)))/(c + d*x^2)^(5/2) + (x*((c*(b^3/(7*d^2
) - (b^2*(3*a*d - b*c))/(7*c*d^2)))/d + (8*a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*
d + 3*a^2*b*c*d^2)/(63*c^2*d^3)))/(c + d*x^2)^(7/2) + (x*(64*a^3*d^3 + 5*b^
3*c^3 + 12*a*b^2*c^2*d + 24*a^2*b*c*d^2))/(315*c^4*d^3*(c + d*x^2)^(3/2)) +
 (x*(128*a^3*d^3 + 10*b^3*c^3 + 24*a*b^2*c^2*d + 48*a^2*b*c*d^2))/(315*c^5*
d^3*(c + d*x^2)^(1/2))
```

3.98 $\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = -\frac{dx(a+bx^2)^3}{7c(bc-ad)\,(c+dx^2)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\,(c+dx^2)^{5/2}} + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\,(c+dx^2)^{3/2}} + \frac{8a^2(7bc-6ad)x}{105c^4(bc-ad)\sqrt{c+dx^2}}$$

```
[Out] -1/7*d*x*(b*x^2+a)^3/c/(-a*d+b*c)/(d*x^2+c)^(7/2)+1/35*(-6*a*d+7*b*c)*x*(b*
x^2+a)^2/c^2/(-a*d+b*c)/(d*x^2+c)^(5/2)+4/105*a*(-6*a*d+7*b*c)*x*(b*x^2+a)/
c^3/(-a*d+b*c)/(d*x^2+c)^(3/2)+8/105*a^2*(-6*a*d+7*b*c)*x/c^4/(-a*d+b*c)/(d
*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {390, 386, 197}

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{8a^2x(7bc-6ad)}{105c^4\sqrt{c+dx^2}(bc-ad)} + \frac{4ax(a+bx^2)\,(7bc-6ad)}{105c^3\,(c+dx^2)^{3/2}\,(bc-ad)} + \frac{x(a+bx^2)^2\,(7bc-6ad)}{35c^2\,(c+dx^2)^{5/2}\,(bc-ad)} - \frac{dx(a+bx^2)^3}{7c\,(c+dx^2)^{7/2}\,(bc-ad)}$$

```
[In] Int[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] -1/7*(d*x*(a + b*x^2)^3)/(c*(b*c - a*d)*(c + d*x^2)^(7/2)) + ((7*b*c - 6*a*
d)*x*(a + b*x^2)^2)/(35*c^2*(b*c - a*d)*(c + d*x^2)^(5/2)) + (4*a*(7*b*c -
```

```
6*a*d)*x*(a + b*x^2))/(105*c^3*(b*c - a*d)*(c + d*x^2)^(3/2)) + (8*a^2*(7*b
*c - 6*a*d)*x)/(105*c^4*(b*c - a*d)*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)\int \frac{(a+bx^2)^2}{(c+dx^2)^{7/2}}\,dx}{7c(bc-ad)} \\
&= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{(4a(7bc-6ad))\int \frac{a+bx^2}{(c+dx^2)^{5/2}}\,dx}{35c^2(bc-ad)} \\
&= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} \\
&\quad + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{(8a^2(7bc-6ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{105c^3(bc-ad)} \\
&= -\frac{dx(a+bx^2)^3}{7c(bc-ad)\left(c+dx^2\right)^{7/2}} + \frac{(7bc-6ad)x(a+bx^2)^2}{35c^2(bc-ad)\left(c+dx^2\right)^{5/2}} \\
&\quad + \frac{4a(7bc-6ad)x(a+bx^2)}{105c^3(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{8a^2(7bc-6ad)x}{105c^4(bc-ad)\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.61

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{3b^2c^2x^5(7c+2dx^2)+2abcx^3(35c^2+28cdx^2+8d^2x^4)+3a^2(35c^3x+70c^2dx^3+56cd^2x^5}{105c^4\left(c+dx^2\right)^{7/2}}$$

```
[In] Integrate[(a + b*x^2)^2/(c + d*x^2)^(9/2),x]
```

```
[Out] (3*b^2*c^2*x^5*(7*c + 2*d*x^2) + 2*a*b*c*x^3*(35*c^2 + 28*c*d*x^2 + 8*d^2*x
^4) + 3*a^2*(35*c^3*x + 70*c^2*d*x^3 + 56*c*d^2*x^5 + 16*d^3*x^7))/(105*c^4
*(c + d*x^2)^(7/2))
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{1}{5}b^2x^4+\frac{2}{3}ab\,x^2+a^2\right)c^3+2x^2d\left(\frac{1}{35}b^2x^4+\frac{4}{15}ab\,x^2+a^2\right)c^2+\frac{8x^4d^2a\left(\frac{2b\,x^2}{21}+a\right)c}{5}+\frac{16a^2d^3x^6}{35}\right)}{\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
gosper	$\frac{x\left(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3\right)}{105\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
trager	$\frac{x\left(48a^2d^3x^6+16abc\,d^2x^6+6b^2c^2d\,x^6+168a^2c\,d^2x^4+56ab\,c^2d\,x^4+21b^2c^3x^4+210a^2c^2d\,x^2+70ab\,c^3x^2+105a^2c^3\right)}{105\left(d\,x^2+c\right)^{\frac{7}{2}}c^4}$
default	$a^2\left(\frac{x}{7c\left(d\,x^2+c\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35c\left(d\,x^2+c\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15c\left(d\,x^2+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^2\sqrt{d\,x^2+c}}\right)}{7c}}{c}\right)+b^2\left(-\frac{x^3}{4d\left(d\,x^2+c\right)^{\frac{7}{2}}}+\frac{3c\left(-\frac{x}{6d\left(d\,x^2+c\right)}\right.}{}\right.$

[In] int((b*x^2+a)^2/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] x*((1/5*b^2*x^4+2/3*a*b*x^2+a^2)*c^3+2*x^2*d*(1/35*b^2*x^4+4/15*a*b*x^2+a^2
)*c^2+8/5*x^4*d^2*a*(2/21*b*x^2+a)*c+16/35*a^2*d^3*x^6)/(d*x^2+c)^(7/2)/c^4

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx = \frac{\left(2\left(3\,b^2c^2d+8\,abcd^2+24\,a^2d^3\right)x^7+105\,a^2c^3x+7\left(3\,b^2c^3+8\,abc^2d+24\,a^2cd^2\right)x^5+70\,(\cdots}{105\left(c^4d^4x^8+4\,c^5d^3x^6+6\,c^6d^2x^4+4\,c^7dx^2+c^8\right)}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*b^2*c^2*d + 8*a*b*c*d^2 + 24*a^2*d^3)*x^7 + 105*a^2*c^3*x + 7*(
3*b^2*c^3 + 8*a*b*c^2*d + 24*a^2*c*d^2)*x^5 + 70*(a*b*c^3 + 3*a^2*c^2*d)*x^
3)*sqrt(d*x^2 + c)/(c^4*d^4*x^8 + 4*c^5*d^3*x^6 + 6*c^6*d^2*x^4 + 4*c^7*d*x
^2 + c^8)
```

Sympy [F]

$$\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx = \int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)**(9/2),x)
```

```
[Out] Integral((a + b*x**2)**2/(c + d*x**2)**(9/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.43

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2}{\left(c+dx^2\right)^{9/2}}\,dx = &-\frac{b^2x^3}{4\left(dx^2+c\right)^{\frac{7}{2}}d}+\frac{16\,a^2x}{35\sqrt{dx^2+c}\,c^4}+\frac{8\,a^2x}{35\left(dx^2+c\right)^{\frac{3}{2}}c^3}\\&+\frac{6\,a^2x}{35\left(dx^2+c\right)^{\frac{5}{2}}c^2}+\frac{a^2x}{7\left(dx^2+c\right)^{\frac{7}{2}}c}+\frac{3\,b^2x}{140\left(dx^2+c\right)^{\frac{5}{2}}d^2}+\frac{2\,b^2x}{35\sqrt{dx^2+c}\,c^2d^2}\\&+\frac{b^2x}{35\left(dx^2+c\right)^{\frac{3}{2}}cd^2}-\frac{3\,b^2cx}{28\left(dx^2+c\right)^{\frac{7}{2}}d^2}-\frac{2\,abx}{7\left(dx^2+c\right)^{\frac{7}{2}}d}\\&+\frac{16\,abx}{105\sqrt{dx^2+c}\,c^3d}+\frac{8\,abx}{105\left(dx^2+c\right)^{\frac{3}{2}}c^2d}+\frac{2\,abx}{35\left(dx^2+c\right)^{\frac{5}{2}}cd}\end{aligned}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*x^3/((d*x^2 + c)^(7/2)*d) + 16/35*a^2*x/(sqrt(d*x^2 + c)*c^4) + 8/
35*a^2*x/((d*x^2 + c)^(3/2)*c^3) + 6/35*a^2*x/((d*x^2 + c)^(5/2)*c^2) + 1/7
```

```
*a^2*x/((d*x^2 + c)^(7/2)*c) + 3/140*b^2*x/((d*x^2 + c)^(5/2)*d^2) + 2/35*b
^2*x/(sqrt(d*x^2 + c)*c^2*d^2) + 1/35*b^2*x/((d*x^2 + c)^(3/2)*c*d^2) - 3/2
8*b^2*c*x/((d*x^2 + c)^(7/2)*d^2) - 2/7*a*b*x/((d*x^2 + c)^(7/2)*d) + 16/10
5*a*b*x/(sqrt(d*x^2 + c)*c^3*d) + 8/105*a*b*x/((d*x^2 + c)^(3/2)*c^2*d) + 2
/35*a*b*x/((d*x^2 + c)^(5/2)*c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,b^2c^2d^4+8\,abcd^5+24\,a^2d^6)x^2}{c^4d^3}+\frac{7\,(3\,b^2c^3d^3+8\,abc^2d^4+24\,a^2cd^5)}{c^4d^3}\right)+\frac{70\,(abc^3d^3+3\,a^2c^2d^4)}{c^4d^3}\right)x^2+1\right.}{105\,(dx^2+c)^{\frac{7}{2}}}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*(2*(3*b^2*c^2*d^4 + 8*a*b*c*d^5 + 24*a^2*d^6)*x^2/(c^4*d^3) + 7
*(3*b^2*c^3*d^3 + 8*a*b*c^2*d^4 + 24*a^2*c*d^5)/(c^4*d^3)) + 70*(a*b*c^3*d^
3 + 3*a^2*c^2*d^4)/(c^4*d^3))*x^2 + 105*a^2/c)*x/(d*x^2 + c)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^2}{(c+dx^2)^{9/2}}\,dx = \frac{x\left(\frac{a^2}{7\,c}+\frac{c\left(\frac{b^2}{7\,d}-\frac{2\,a\,b}{7\,c}\right)}{d}\right)}{(d\,x^2+c)^{7/2}}-\frac{x\left(\frac{b^2}{5\,d^2}-\frac{6\,a^2\,d^2+2\,a\,b\,c\,d-b^2\,c^2}{35\,c^2\,d^2}\right)}{(d\,x^2+c)^{5/2}} + \frac{x\,(24\,a^2\,d^2+8\,a\,b\,c\,d+3\,b^2\,c^2)}{105\,c^3\,d^2\,(d\,x^2+c)^{3/2}}+\frac{x\,(48\,a^2\,d^2+16\,a\,b\,c\,d+6\,b^2\,c^2)}{105\,c^4\,d^2\,\sqrt{d\,x^2+c}}$$

```
[In] int((a + b*x^2)^2/(c + d*x^2)^(9/2),x)
```

```
[Out] (x*(a^2/(7*c) + (c*(b^2/(7*d) - (2*a*b)/(7*c)))/d))/(c + d*x^2)^(7/2) - (x*
(b^2/(5*d^2) - (6*a^2*d^2 - b^2*c^2 + 2*a*b*c*d)/(35*c^2*d^2)))/(c + d*x^2)
^(5/2) + (x*(24*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/(105*c^3*d^2*(c + d*x^2)^
(3/2)) + (x*(48*a^2*d^2 + 6*b^2*c^2 + 16*a*b*c*d))/(105*c^4*d^2*(c + d*x^2)
^(1/2))
```

3.99 $\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx=-\frac{(bc-ad)x}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\left(c+dx^2\right)^{3/2}}+\frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}$$

```
[Out] -1/5*(-a*d+b*c)*x/c/d/(d*x^2+c)^(5/2)+1/15*(4*a*d+b*c)*x/c^2/d/(d*x^2+c)^(3
/2)+2/15*(4*a*d+b*c)*x/c^3/d/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {393, 198, 197}

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{7/2}}\,dx=\frac{2x(4ad+bc)}{15c^3d\sqrt{c+dx^2}}+\frac{x(4ad+bc)}{15c^2d\left(c+dx^2\right)^{3/2}}-\frac{x(bc-ad)}{5cd\left(c+dx^2\right)^{5/2}}$$

```
[In] Int[(a + b*x^2)/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/5*((b*c - a*d)*x)/(c*d*(c + d*x^2)^(5/2)) + ((b*c + 4*a*d)*x)/(15*c^2*d*
(c + d*x^2)^(3/2)) + (2*(b*c + 4*a*d)*x)/(15*c^3*d*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}}+\frac{(bc+4ad)\int \frac{1}{(c+dx^2)^{5/2}}\,dx}{5cd}\\
&= -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\,(c+dx^2)^{3/2}}+\frac{(2(bc+4ad))\int \frac{1}{(c+dx^2)^{3/2}}\,dx}{15c^2d}\\
&= -\frac{(bc-ad)x}{5cd\,(c+dx^2)^{5/2}}+\frac{(bc+4ad)x}{15c^2d\,(c+dx^2)^{3/2}}+\frac{2(bc+4ad)x}{15c^3d\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx=\frac{15ac^2x+5bc^2x^3+20acdx^3+2bcdx^5+8ad^2x^5}{15c^3\,(c+dx^2)^{5/2}}$$

```
[In] Integrate[(a + b*x^2)/(c + d*x^2)^(7/2),x]
[Out] (15*a*c^2*x + 5*b*c^2*x^3 + 20*a*c*d*x^3 + 2*b*c*d*x^5 + 8*a*d^2*x^5)/(15*c
^3*(c + d*x^2)^(5/2))
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.57

method	result
pseudoelliptic	$\frac{x\left(\left(\frac{b\,x^{2}}{3}+a\right)c^{2}+\frac{4x^{2}d\left(\frac{b\,x^{2}}{10}+a\right)c}{3}+\frac{8a\,d^{2}x^{4}}{15}\right)}{\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
gosper	$\frac{x\left(8a\,d^{2}x^{4}+2bcd\,x^{4}+20acd\,x^{2}+5b\,c^{2}x^{2}+15c^{2}a\right)}{15\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
trager	$\frac{x\left(8a\,d^{2}x^{4}+2bcd\,x^{4}+20acd\,x^{2}+5b\,c^{2}x^{2}+15c^{2}a\right)}{15\left(d\,x^{2}+c\right)^{\frac{5}{2}}c^{3}}$
default	$a\left(\frac{x}{5c\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{d\,x^{2}+c}}}{c}\right)+b\left(-\frac{x}{4d\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{c\left(\frac{x}{5c\left(d\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\frac{4x}{15c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{8x}{15c^{2}\sqrt{d\,x}}}{c}\right.}{4d}\right.$

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] x/(d*x^2+c)^(5/2)*((1/3*b*x^2+a)*c^2+4/3*x^2*d*(1/10*b*x^2+a)*c+8/15*a*d^2*
x^4)/c^3
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{(2\,(bcd+4\,ad^2)x^5+15\,ac^2x+5\,(bc^2+4\,acd)x^3)\sqrt{dx^2+c}}{15\,(c^3d^3x^6+3\,c^4d^2x^4+3\,c^5dx^2+c^6)}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(b*c*d + 4*a*d^2)*x^5 + 15*a*c^2*x + 5*(b*c^2 + 4*a*c*d)*x^3)*sqrt(
d*x^2 + c)/(c^3*d^3*x^6 + 3*c^4*d^2*x^4 + 3*c^5*d*x^2 + c^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(83) = 166$.

Time = 10.37 (sec) , antiderivative size = 566, normalized size of antiderivative = 6.22

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = a\left(\frac{15c^5x}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{35c^4dx^3}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{28c^3d^2x^5}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}} + \frac{8c^2d^3x^7}{15c^{\frac{17}{2}}\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{15}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+45c^{\frac{13}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{11}{2}}d^3x^6\sqrt{1+\frac{dx^2}{c}}}\right) + b\left(\frac{5cx^3}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}} + \frac{2dx^5}{15c^{\frac{9}{2}}\sqrt{1+\frac{dx^2}{c}}+30c^{\frac{7}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}+15c^{\frac{5}{2}}d^2x^4\sqrt{1+\frac{dx^2}{c}}}\right)$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2),x)
```

```
[Out] a*(15*c**5*x/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1
+ d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3
*x**6*sqrt(1 + d*x**2/c)) + 35*c**4*d*x**3/(15*c**(17/2)*sqrt(1 + d*x**2/c)
 + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 +
 d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c)) + 28*c**3*d**2*x**5
/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(15/2)*d*x**2*sqrt(1 + d*x**2/c)
+ 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c) + 15*c**(11/2)*d**3*x**6*sqrt(1
 + d*x**2/c)) + 8*c**2*d**3*x**7/(15*c**(17/2)*sqrt(1 + d*x**2/c) + 45*c**(
15/2)*d*x**2*sqrt(1 + d*x**2/c) + 45*c**(13/2)*d**2*x**4*sqrt(1 + d*x**2/c)
 + 15*c**(11/2)*d**3*x**6*sqrt(1 + d*x**2/c))) + b*(5*c*x**3/(15*c**(9/2)*s
qrt(1 + d*x**2/c) + 30*c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**
2*x**4*sqrt(1 + d*x**2/c)) + 2*d*x**5/(15*c**(9/2)*sqrt(1 + d*x**2/c) + 30*
c**(7/2)*d*x**2*sqrt(1 + d*x**2/c) + 15*c**(5/2)*d**2*x**4*sqrt(1 + d*x**2/
c)))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{8\,ax}{15\,\sqrt{dx^2+c}c^3}+\frac{4\,ax}{15\,(dx^2+c)^{\frac{3}{2}}c^2}+\frac{ax}{5\,(dx^2+c)^{\frac{5}{2}}c} - \frac{bx}{5\,(dx^2+c)^{\frac{5}{2}}d}+\frac{2\,bx}{15\,\sqrt{dx^2+c}c^2d}+\frac{bx}{15\,(dx^2+c)^{\frac{3}{2}}cd}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*a*x/(sqrt(d*x^2 + c)*c^3) + 4/15*a*x/((d*x^2 + c)^(3/2)*c^2) + 1/5*a*x
/((d*x^2 + c)^(5/2)*c) - 1/5*b*x/((d*x^2 + c)^(5/2)*d) + 2/15*b*x/(sqrt(d*x
^2 + c)*c^2*d) + 1/15*b*x/((d*x^2 + c)^(3/2)*c*d)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{\left(x^2\left(\frac{2\,(bcd^3+4\,ad^4)x^2}{c^3d^2}+\frac{5\,(bc^2d^2+4\,acd^3)}{c^3d^2}\right)+\frac{15\,a}{c}\right)x}{15\,(dx^2+c)^{\frac{5}{2}}}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(x^2*(2*(b*c*d^3 + 4*a*d^4)*x^2/(c^3*d^2) + 5*(b*c^2*d^2 + 4*a*c*d^3)/
(c^3*d^2)) + 15*a/c)*x/(d*x^2 + c)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 4.90 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}}\,dx = \frac{8\,a\,d\,x\,(d\,x^2+c)^2-3\,b\,c^3\,x+2\,b\,c\,x\,(d\,x^2+c)^2+b\,c^2\,x\,(d\,x^2+c)+3\,a\,c^2\,d\,x+4\,a\,c\,d\,x\,(}{15\,c^3\,d\,(d\,x^2+c)^{5/2}}$$

```
[In] int((a + b*x^2)/(c + d*x^2)^(7/2),x)
```

```
[Out] (8*a*d*x*(c + d*x^2)^2 - 3*b*c^3*x + 2*b*c*x*(c + d*x^2)^2 + b*c^2*x*(c + d
*x^2) + 3*a*c^2*d*x + 4*a*c*d*x*(c + d*x^2))/(15*c^3*d*(c + d*x^2)^(5/2))
```

3.100 $\int \frac{1}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{x}{3c\,(c+dx^2)^{3/2}} + \frac{2x}{3c^2\sqrt{c+dx^2}}$$

```
[Out] 1/3*x/c/(d*x^2+c)^(3/2)+2/3*x/c^2/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {198, 197}

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{2x}{3c^2\sqrt{c+dx^2}} + \frac{x}{3c\,(c+dx^2)^{3/2}}$$

```
[In] Int[(c + d*x^2)^(-5/2),x]
```

```
[Out] x/(3*c*(c + d*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + d*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
```

```
0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{x}{3c\left(c+dx^2\right)^{3/2}}+\frac{2\int\frac{1}{\left(c+dx^2\right)^{3/2}}\,dx}{3c}\\&=\frac{x}{3c\left(c+dx^2\right)^{3/2}}+\frac{2x}{3c^2\sqrt{c+dx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int\frac{1}{\left(c+dx^2\right)^{5/2}}\,dx=\frac{3cx+2dx^3}{3c^2\left(c+dx^2\right)^{3/2}}$$

```
[In] Integrate[(c + d*x^2)^(-5/2),x]
[Out] (3*c*x + 2*d*x^3)/(3*c^2*(c + d*x^2)^(3/2))
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(2d\,x^{2}+3c\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{2}}$	26
trager	$\frac{x\left(2d\,x^{2}+3c\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{2}}$	26
pseudoelliptic	$\frac{x\left(2d\,x^{2}+3c\right)}{3\left(d\,x^{2}+c\right)^{\frac{3}{2}}c^{2}}$	26
default	$\frac{x}{3c\left(d\,x^{2}+c\right)^{\frac{3}{2}}}+\frac{2x}{3c^{2}\sqrt{d\,x^{2}+c}}$	32

```
[In] int(1/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(2*d*x^2+3*c)/(d*x^2+c)^(3/2)/c^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{\left(2\,dx^3+3\,cx\right)\sqrt{dx^2+c}}{3\left(c^2d^2x^4+2\,c^3dx^2+c^4\right)}$$

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] 1/3*(2*d*x^3 + 3*c*x)*sqrt(d*x^2 + c)/(c^2*d^2*x^4 + 2*c^3*d*x^2 + c^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(32) = 64.

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{3cx}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}+\frac{2dx^3}{3c^{\frac{7}{2}}\sqrt{1+\frac{dx^2}{c}}+3c^{\frac{5}{2}}dx^2\sqrt{1+\frac{dx^2}{c}}}$$

```
[In] integrate(1/(d*x**2+c)**(5/2),x)
[Out] 3*c*x/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*x**2/c)
) + 2*d*x**3/(3*c**(7/2)*sqrt(1 + d*x**2/c) + 3*c**(5/2)*d*x**2*sqrt(1 + d*
x**2/c))
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{\left(c+dx^2\right)^{5/2}}\,dx = \frac{2\,x}{3\sqrt{dx^2+c}c^2}+\frac{x}{3\left(dx^2+c\right)^{\frac{3}{2}}c}$$

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] 2/3*x/(sqrt(d*x^2 + c)*c^2) + 1/3*x/((d*x^2 + c)^(3/2)*c)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,dx^2}{c^2}+\frac{3}{c}\right)}{3\,(dx^2+c)^{\frac{3}{2}}}$$

```
[In] integrate(1/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] 1/3*x*(2*d*x^2/c^2 + 3/c)/(d*x^2 + c)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(c+dx^2)^{5/2}}\,dx = \frac{2\,x\,(d\,x^2+c)+c\,x}{3\,c^2\,(d\,x^2+c)^{3/2}}$$

```
[In] int(1/(c + d*x^2)^(5/2),x)
[Out] (2*x*(c + d*x^2) + c*x)/(3*c^2*(c + d*x^2)^(3/2))
```

3.101 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}} dx = -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}} + \frac{b\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}$$

```
[Out] b*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/(-a*d+b*c)^(3/2)/a^(1/
2)-d*x/c/(-a*d+b*c)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 385, 211}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}} dx = \frac{b\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}} - \frac{dx}{c\sqrt{c+dx^2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*ArcTan[(Sqrt[b*c - a*d]*x)/(S
qrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*(b*c - a*d)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad}\\
&= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{bc-ad}\\
&= -\frac{dx}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$
\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx=\frac{dx}{c(-bc+ad)\sqrt{c+dx^2}}-\frac{b\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}(bc-ad)^{3/2}}
$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (d*x)/(c*(-(b*c) + a*d)*Sqrt[c + d*x^2]) - (b*ArcTan[(a*Sqrt[d] + b*x*(Sqrt
[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c - a*d])])/(Sqrt[a]*(b*c - a*d)^
(3/2))
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

method	result
pseudoelliptic	$\frac{-bc\,\operatorname{arctanh}\left(\frac{\sqrt{dx^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)\sqrt{dx^2+c}+dx\sqrt{(ad-bc)a}}{(ad-bc)\sqrt{(ad-bc)a}\,\sqrt{dx^2+c}\,c}$
default	$-\frac{b}{(ad-bc)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}+\frac{2d\sqrt{-ab}\left(2d\left(x-\frac{\sqrt{-ab}}{b}\right)+\frac{2d\sqrt{-ab}}{b}\right)}{(ad-bc)\left(-\frac{4d(ad-bc)}{b}+\frac{4d^2a}{b}\right)\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}}}$ … $\over 2\sqrt{-ab}$

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*c*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))*(d*x^2+c)^(1/2)+d*x*
((a*d-b*c)*a)^(1/2))/(a*d-b*c)/((a*d-b*c)*a)^(1/2)/(d*x^2+c)^(1/2)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 442, normalized size of antiderivative = 5.59

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \left[-\frac{4\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8 \cdots}{\cdots}\right)}{4\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d \cdots)}\right.$$

$$\left.-\frac{2\,(abcd-a^2d^2)\sqrt{dx^2+c}\,x-(bcdx^2+bc^2)\sqrt{abc-a^2d}\arctan\left(\frac{\sqrt{abc-a^2d}\,((bc-2\,ad)x^2-ac)\sqrt{dx^2+c}}{2\,((abcd-a^2d^2)x^3+(abc^2-a^2cd)x)}\right)}{2\,(ab^2c^4-2\,a^2bc^3d+a^3c^2d^2+(ab^2c^3d-2\,a^2bc^2d^2+a^3cd^3)x^2)}\right]$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(-
a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*
a*b*c^2 - 4*a^2*c*d)*x^2 + 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*
d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a*b^2*c^4 - 2*a^2*b*c^3*
d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2), -1/2*(2
*(a*b*c*d - a^2*d^2)*sqrt(d*x^2 + c)*x - (b*c*d*x^2 + b*c^2)*sqrt(a*b*c - a
^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 +
 c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)))/(a*b^2*c^4 - 2*a^2*
b*c^3*d + a^3*c^2*d^2 + (a*b^2*c^3*d - 2*a^2*b*c^2*d^2 + a^3*c*d^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2),x)
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{d}\arctan\left(-\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2 b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}(bc-ad)} - \frac{dx}{(bc^2-acd)\sqrt{dx^2+c}}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] b*sqrt(d)*arctan(-1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqr
t(a*b*c*d - a^2*d^2))/(sqrt(a*b*c*d - a^2*d^2)*(b*c - a*d)) - d*x/((b*c^2 -
 a*c*d)*sqrt(d*x^2 + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\,(a+bx^2)}+\frac{(bc-2ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

```
[Out] 1/2*(-2*a*d+b*c)*arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(3/2)
/(-a*d+b*c)^(3/2)+1/2*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {390, 385, 211}

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\frac{(bc-2ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}+\frac{bx\sqrt{c+dx^2}}{2a\,(a+bx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*a*(b*c - a*d)*(a + b*x^2)) + ((b*c - 2*a*d)*ArcTan
[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(2*a^(3/2)*(b*c - a*d)^(3/
2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\int\frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{2a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\mathrm{Subst}\left(\int\frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{2a(bc-ad)\left(a+bx^2\right)}+\frac{(bc-2ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.22

$$\int\frac{1}{\left(a+bx^2\right)^2\sqrt{c+dx^2}}\,dx = -\frac{bx\sqrt{c+dx^2}}{2a(-bc+ad)\left(a+bx^2\right)} + \frac{(-bc+2ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{2a^{3/2}(bc-ad)^{3/2}}$$

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]),x]
```

```
[Out] -1/2*(b*x*Sqrt[c + d*x^2])/(a*(-(b*c) + a*d)*(a + b*x^2)) + ((-(b*c) + 2*a*
d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b
*c - a*d])])/(2*a^(3/2)*(b*c - a*d)^(3/2))
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{-\frac{b\sqrt{d\,x^2+c}\,x}{b\,x^2+a}+\frac{(2ad-bc)\,\mathrm{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}}{2(ad-bc)a}$
default	$-\frac{\frac{b\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{(ad-bc)\left(x-\frac{\sqrt{-ab}}{b}\right)}-\frac{d\sqrt{-ab}\,\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2 -}}{x-\frac{\sqrt{-ab}}{b}}\right.}{(ad-bc)\sqrt{-\frac{ad-bc}{b}}}}{4ba}$

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*d-b*c)/a*(-b*(d*x^2+c)^(1/2)*x/(b*x^2+a)+(2*a*d-b*c)/((a*d-b*c)*a)^(
1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(84) = 168$.

Time = 0.35 (sec) , antiderivative size = 459, normalized size of antiderivative = 4.59

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$
$$= \left[\frac{4\,(ab^2c-a^2bd)\sqrt{dx^2+c}x-(abc-2\,a^2d+(b^2c-2\,abd)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2}{}\right.}{8\,(a^3b^2c^2-2\,a^4bcd+a^5d^2+(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2)x}\right.$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x - (a*b*c - 2*a^2*d + (b^2*c -
 2*a*b*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*
x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*
x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)))/(a^3
*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2
)*x^2), 1/4*(2*(a*b^2*c - a^2*b*d)*sqrt(d*x^2 + c)*x + sqrt(a*b*c - a^2*d)*
(a*b*c - 2*a^2*d + (b^2*c - 2*a*b*d)*x^2)*arctan(1/2*sqrt(a*b*c - a^2*d)*((
b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2
 - a^2*c*d)*x))))/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (a^2*b^3*c^2 - 2*a^
3*b^2*c*d + a^4*b*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.25

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx = -\frac{1}{2}d^{\frac{3}{2}}\left(\frac{(bc-2\,ad)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{(abcd-a^2d^2)^{\frac{3}{2}}} + \frac{2\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc-2\right.}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b\right.}\right.$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] -1/2*d^(3/2)*((b*c - 2*a*d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b -
 b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/(a*b*c*d - a^2*d^2)^(3/2) + 2*((sqrt
(d)*x - sqrt(d*x^2 + c))^2*b*c - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d - b*
c^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^
2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)*(a*b*c*d - a^2*d^2))
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(b\,x^2+a\right)^2\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)), x)
```

3.103 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)\,(a+bx^2)^2} + \frac{c(3bc-4ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

```
[Out] 1/4*b*x*(d*x^2+c)^(3/2)/a/(-a*d+b*c)/(b*x^2+a)^2+1/8*c*(-4*a*d+3*b*c)*arcta
n(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(5/2)/(-a*d+b*c)^(3/2)+1/8*
(-4*a*d+3*b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {390, 386, 385, 211}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \frac{c(3bc-4ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}} + \frac{x\sqrt{c+dx^2}(3bc-4ad)}{8a^2\,(a+bx^2)\,(bc-ad)} + \frac{bx(c+dx^2)^{3/2}}{4a\,(a+bx^2)^2\,(bc-ad)}$$

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]
```

```
[Out] ((3*b*c - 4*a*d)*x*Sqrt[c + d*x^2])/(8*a^2*(b*c - a*d)*(a + b*x^2)) + (b*x*
(c + d*x^2)^(3/2))/(4*a*(b*c - a*d)*(a + b*x^2)^2) + (c*(3*b*c - 4*a*d)*Arc
```

```
Tan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*(b*c - a*d)^
(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)(a+bx^2)^2}+\frac{(3bc-4ad)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx}{4a(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)(a+bx^2)}+\frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)(a+bx^2)^2}+\frac{(c(3bc-4ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{8a^2(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)(a+bx^2)}+\frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)(a+bx^2)^2}\\
&\quad+\frac{(c(3bc-4ad))\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{8a^2(bc-ad)}\\
&= \frac{(3bc-4ad)x\sqrt{c+dx^2}}{8a^2(bc-ad)(a+bx^2)}+\frac{bx(c+dx^2)^{3/2}}{4a(bc-ad)(a+bx^2)^2}+\frac{c(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \frac{x\sqrt{c+dx^2}(-5abc+4a^2d-3b^2cx^2+2abdx^2)}{8a^2(-bc+ad)\,(a+bx^2)^2} - \frac{c(3bc-4ad)\arctan\left(\frac{a\sqrt{d}+b\sqrt{d}x^2-bx\sqrt{c+dx^2}}{\sqrt{a}\sqrt{bc-ad}}\right)}{8a^{5/2}(bc-ad)^{3/2}}$$

[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^3,x]

[Out] (x*Sqrt[c + d*x^2]*(-5*a*b*c + 4*a^2*d - 3*b^2*c*x^2 + 2*a*b*d*x^2))/(8*a^2*(-(b*c) + a*d)*(a + b*x^2)^2) - (c*(3*b*c - 4*a*d)*ArcTan[(a*Sqrt[d] + b*Sqrt[d]*x^2 - b*x*Sqrt[c + d*x^2])/(Sqrt[a]*Sqrt[b*c - a*d])])/(8*a^(5/2)*(b*c - a*d)^(3/2))

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{c\left(-\frac{\sqrt{d\,x^{2}+c}\,\left(2x^{2}abd-3b^{2}c\,x^{2}+4a^{2}d-5abc\right)x}{c\left(b\,x^{2}+a\right)^{2}}-\frac{(4ad-3bc)\,\operatorname{arctanh}\left(\frac{\sqrt{d\,x^{2}+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}\right)}{8(ad-bc)a^{2}}$	121
default	Expression too large to display	4155

[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] -1/8*c/(a*d-b*c)/a^2*(-(d*x^2+c)^(1/2)/c*(2*a*b*d*x^2-3*b^2*c*x^2+4*a^2*d-5*a*b*c)*x/(b*x^2+a)^2-(4*a*d-3*b*c)/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(129) = 258.

Time = 0.38 (sec) , antiderivative size = 698, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx$$

$$= \Bigg[-\frac{(3\,a^2bc^2-4\,a^3cd+(3\,b^3c^2-4\,ab^2cd)x^4+2\,(3\,ab^2c^2-4\,a^2bcd)x^2)\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4}{}\right.}{32\,(a^5b^2c^2-2\,a^6bcd+a^7d^2+(a}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b
^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d +
 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d
)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 +
 a^2)) - 4*((3*a*b^3*c^2 - 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^
2 - 9*a^3*b*c*d + 4*a^4*d^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d
 + a^7*d^2 + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c
^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x^2), 1/16*((3*a^2*b*c^2 - 4*a^3*c*d + (3*b
^3*c^2 - 4*a*b^2*c*d)*x^4 + 2*(3*a*b^2*c^2 - 4*a^2*b*c*d)*x^2)*sqrt(a*b*c -
 a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2
 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x)) + 2*((3*a*b^3*c^2
- 5*a^2*b^2*c*d + 2*a^3*b*d^2)*x^3 + (5*a^2*b^2*c^2 - 9*a^3*b*c*d + 4*a^4*d
^2)*x)*sqrt(d*x^2 + c))/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (a^3*b^4*c^2
 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*x^4 + 2*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*
b*d^2)*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)^3}\, dx = \int \frac{\sqrt{c + dx^2}}{(a + bx^2)^3}\, dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{c + dx^2}}{(a + bx^2)^3}\, dx = \int \frac{\sqrt{dx^2 + c}}{(bx^2 + a)^3}\, dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(129) = 258.

Time = 1.66 (sec) , antiderivative size = 487, normalized size of antiderivative = 3.27

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = -\frac{\left(3\,bc^2\sqrt{d}-4\,acd^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{8\,(a^2bc-a^3d)\sqrt{abcd-a^2d^2}}$$
$$-\frac{3\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6b^3c^2\sqrt{d}-4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^6ab^2cd^{\frac{3}{2}}-9\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4b^3c^3\sqrt{d}+30\left(\sqrt{d}\right.}{}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*b*c^2*sqrt(d) - 4*a*c*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2
+ c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^2*b*c - a^3*d)*sqrt(a
*b*c*d - a^2*d^2)) - 1/4*(3*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b^3*c^2*sqrt(d)
 - 4*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^2*c*d^(3/2) - 9*(sqrt(d)*x - sqrt(
d*x^2 + c))^4*b^3*c^3*sqrt(d) + 30*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a*b^2*c^
2*d^(3/2) - 40*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b*c*d^(5/2) + 16*(sqrt(d
)*x - sqrt(d*x^2 + c))^4*a^3*d^(7/2) + 9*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b^
3*c^4*sqrt(d) - 28*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^2*c^3*d^(3/2) + 16*(
sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b*c^2*d^(5/2) - 3*b^3*c^5*sqrt(d) + 2*a*
b^2*c^4*d^(3/2))/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(
d*x^2 + c))^2*b*c + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^2*(a^2*b
^2*c - a^3*b*d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^3}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\left(b\,x^2+a\right)^3}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^3,x)
```

```
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^3, x)
```

3.104 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx = \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)(a+bx^2)^3} + \frac{c^2(5bc-6ad)\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}$$

```
[Out] 1/24*(-6*a*d+5*b*c)*x*(d*x^2+c)^(3/2)/a^2/(-a*d+b*c)/(b*x^2+a)^2+1/6*b*x*(d
*x^2+c)^(5/2)/a/(-a*d+b*c)/(b*x^2+a)^3+1/16*c^2*(-6*a*d+5*b*c)*arctan(x*(-a
*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(7/2)/(-a*d+b*c)^(3/2)+1/16*c*(-6*
a*d+5*b*c)*x*(d*x^2+c)^(1/2)/a^3/(-a*d+b*c)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {390, 386, 385, 211}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4} dx = \frac{c^2(5bc-6ad)\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}} + \frac{cx\sqrt{c+dx^2}(5bc-6ad)}{16a^3(a+bx^2)(bc-ad)} + \frac{x(c+dx^2)^{3/2}(5bc-6ad)}{24a^2(a+bx^2)^2(bc-ad)} + \frac{bx(c+dx^2)^{5/2}}{6a(a+bx^2)^3(bc-ad)}$$

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (c*(5*b*c - 6*a*d)*x*Sqrt[c + d*x^2])/(16*a^3*(b*c - a*d)*(a + b*x^2)) + ((
5*b*c - 6*a*d)*x*(c + d*x^2)^(3/2))/(24*a^2*(b*c - a*d)*(a + b*x^2)^2) + (b
*x*(c + d*x^2)^(5/2))/(6*a*(b*c - a*d)*(a + b*x^2)^3) + (c^2*(5*b*c - 6*a*d
)*ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(16*a^(7/2)*(b*c -
 a*d)^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 386

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] - Dist[
c*(q/(a*(p + 1))), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1,
0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(5bc-6ad)\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^3}\,dx}{6a(bc-ad)} \\
&= \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(c(5bc-6ad))\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^2}\,dx}{8a^2(bc-ad)} \\
&= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)\,(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)\,(a+bx^2)^2} \\
&\quad + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)\,(a+bx^2)^3} + \frac{(c^2(5bc-6ad))\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{16a^3(bc-ad)}
\end{aligned}
$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)(a+bx^2)^3} + \frac{(c^2(5bc-6ad))\,\text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{16a^3(bc-ad)}$$

$$= \frac{c(5bc-6ad)x\sqrt{c+dx^2}}{16a^3(bc-ad)(a+bx^2)} + \frac{(5bc-6ad)x(c+dx^2)^{3/2}}{24a^2(bc-ad)(a+bx^2)^2} + \frac{bx(c+dx^2)^{5/2}}{6a(bc-ad)(a+bx^2)^3} + \frac{c^2(5bc-6ad)\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{16a^{7/2}(bc-ad)^{3/2}}$$

Mathematica [A] (verified)

Time = 15.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \frac{-\frac{\sqrt{a}x\sqrt{c+dx^2}(15b^3c^2x^4+8ab^2cx^2(5c-dx^2)-6a^3d(5c+2dx^2)+a^2b(33c^2-22cdx^2-4d^2x^4))}{(-bc+ad)(a+bx^2)^3} + \frac{3c^2(5bc-6ad)\arctan}{(bc-ad)^3}}{48a^{7/2}}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^4,x]
```

```
[Out] (-((Sqrt[a]*x*Sqrt[c + d*x^2]*(15*b^3*c^2*x^4 + 8*a*b^2*c*x^2*(5*c - d*x^2)
 - 6*a^3*d*(5*c + 2*d*x^2) + a^2*b*(33*c^2 - 22*c*d*x^2 - 4*d^2*x^4)))/((-(
b*c) + a*d)*(a + b*x^2)^3)) + (3*c^2*(5*b*c - 6*a*d)*ArcTan[(Sqrt[b*c - a*d
]*x)/(Sqrt[a]*Sqrt[c + d*x^2])])/(b*c - a*d)^(3/2))/(48*a^(7/2))
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{5\left(d\left(\frac{2dx^2}{5}+c\right)a^3-\frac{11\left(-\frac{4}{33}d^2x^4-\frac{2}{3}cdx^2+c^2\right)ba^2}{10}-\frac{4\left(-\frac{dx^2}{5}+c\right)x^2b^2ca}{3}-\frac{c^2b^3x^4}{2}\right)x\sqrt{dx^2+c}\,\sqrt{(ad-bc)a}+3(bx^2+a)^3\left(ad-\right.}{8\sqrt{(ad-bc)a}\,(ad-bc)a^3(bx^2+a)^3}$
default	Expression too large to display

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(5*(d*(2/5*d*x^2+c)*a^3-11/10*(-4/33*d^2*x^4-2/3*c*d*x^2+c^2)*b*a^2-4/3
*(-1/5*d*x^2+c)*x^2*b^2*c*a-1/2*c^2*b^3*x^4)*x*(d*x^2+c)^(1/2)*((a*d-b*c)*a
)^(1/2)+3*(b*x^2+a)^3*(a*d-5/6*b*c)*c^2*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b
*c)*a)^(1/2)))/((a*d-b*c)*a)^(1/2)/(a*d-b*c)/a^3/(b*x^2+a)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(175) = 350$.

Time = 0.48 (sec) , antiderivative size = 972, normalized size of antiderivative = 4.88

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \left[-\frac{3\,(5\,a^3bc^3 - 6\,a^4c^2d + (5\,b^4c^3 - 6\,ab^3c^2d)x^6 + 3\,(5\,ab^3c^3 - 6\,a^2b^2c^2d)x^4 + 3\,(5\,a^2b^2c^3 -}{}\right.$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^3*c^2*d)*x^6 + 3
*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6*a^3*b*c^2*d)*x^
2)*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^
2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b
*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2)) - 4*((15*a*b^4*c^
3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^4*b*d^3)*x^5 + 2*(20*a^2*b^3*c
^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5*d^3)*x^3 + 3*(11*a^3*b^2*c^3
- 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^2 + c))/(a^7*b^2*c^2 - 2*a^8*b
*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x^6 + 3*(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3*(a^6*b^3*c^2 - 2*a^7*b^2*c*d
+ a^8*b*d^2)*x^2), 1/96*(3*(5*a^3*b*c^3 - 6*a^4*c^2*d + (5*b^4*c^3 - 6*a*b^
3*c^2*d)*x^6 + 3*(5*a*b^3*c^3 - 6*a^2*b^2*c^2*d)*x^4 + 3*(5*a^2*b^2*c^3 - 6
*a^3*b*c^2*d)*x^2)*sqrt(a*b*c - a^2*d)*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c
 - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 -
a^2*c*d)*x)) + 2*((15*a*b^4*c^3 - 23*a^2*b^3*c^2*d + 4*a^3*b^2*c*d^2 + 4*a^
4*b*d^3)*x^5 + 2*(20*a^2*b^3*c^3 - 31*a^3*b^2*c^2*d + 5*a^4*b*c*d^2 + 6*a^5
*d^3)*x^3 + 3*(11*a^3*b^2*c^3 - 21*a^4*b*c^2*d + 10*a^5*c*d^2)*x)*sqrt(d*x^
2 + c))/(a^7*b^2*c^2 - 2*a^8*b*c*d + a^9*d^2 + (a^4*b^5*c^2 - 2*a^5*b^4*c*d
 + a^6*b^3*d^2)*x^6 + 3*(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2)*x^4 + 3
*(a^6*b^3*c^2 - 2*a^7*b^2*c*d + a^8*b*d^2)*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**4,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^4}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 919 vs. $2(175) = 350$.

Time = 1.26 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.62

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = -\frac{\left(5\,bc^3\sqrt{d}-6\,ac^2d^{\frac{3}{2}}\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\,\sqrt{abcd-a^2d^2}}\right)}{16\,(a^3bc-a^4d)\sqrt{abcd-a^2d^2}}$$
$$-\frac{15\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}b^5c^3\sqrt{d}-18\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{10}ab^4c^2d^{\frac{3}{2}}-75\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^{8}b^5c^4\sqrt{d}+24}{}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^4,x, algorithm="giac")
```

```
[Out] -1/16*(5*b*c^3*sqrt(d) - 6*a*c^2*d^(3/2))*arctan(1/2*((sqrt(d)*x - sqrt(d*x
^2 + c))^2*b - b*c + 2*a*d)/sqrt(a*b*c*d - a^2*d^2))/((a^3*b*c - a^4*d)*sqr
t(a*b*c*d - a^2*d^2)) - 1/24*(15*(sqrt(d)*x - sqrt(d*x^2 + c))^10*b^5*c^3*s
qrt(d) - 18*(sqrt(d)*x - sqrt(d*x^2 + c))^10*a*b^4*c^2*d^(3/2) - 75*(sqrt(d
)*x - sqrt(d*x^2 + c))^8*b^5*c^4*sqrt(d) + 240*(sqrt(d)*x - sqrt(d*x^2 + c)
)^8*a*b^4*c^3*d^(3/2) - 180*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^2*b^3*c^2*d^(
5/2) - 96*(sqrt(d)*x - sqrt(d*x^2 + c))^8*a^3*b^2*c*d^(7/2) + 96*(sqrt(d)*x
 - sqrt(d*x^2 + c))^8*a^4*b*d^(9/2) + 150*(sqrt(d)*x - sqrt(d*x^2 + c))^6*b
^5*c^5*sqrt(d) - 620*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a*b^4*c^4*d^(3/2) + 96
8*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^2*b^3*c^3*d^(5/2) - 720*(sqrt(d)*x - sq
rt(d*x^2 + c))^6*a^3*b^2*c^2*d^(7/2) + 64*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a
^4*b*c*d^(9/2) + 128*(sqrt(d)*x - sqrt(d*x^2 + c))^6*a^5*d^(11/2) - 150*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*b^5*c^6*sqrt(d) + 600*(sqrt(d)*x - sqrt(d*x^2
+ c))^4*a*b^4*c^5*d^(3/2) - 864*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^2*b^3*c^4
*d^(5/2) + 288*(sqrt(d)*x - sqrt(d*x^2 + c))^4*a^3*b^2*c^3*d^(7/2) + 96*(sq
rt(d)*x - sqrt(d*x^2 + c))^4*a^4*b*c^2*d^(9/2) + 75*(sqrt(d)*x - sqrt(d*x^2
 + c))^2*b^5*c^7*sqrt(d) - 210*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*b^4*c^6*d^
(3/2) + 72*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a^2*b^3*c^5*d^(5/2) + 48*(sqrt(d
```

```
)*x - sqrt(d*x^2 + c))^2*a^3*b^2*c^4*d^(7/2) - 15*b^5*c^8*sqrt(d) + 8*a*b^4
*c^7*d^(3/2) + 4*a^2*b^3*c^6*d^(5/2))/((a^3*b^3*c - a^4*b^2*d)*((sqrt(d)*x
- sqrt(d*x^2 + c))^4*b - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c + 4*(sqrt(d)
*x - sqrt(d*x^2 + c))^2*a*d + b*c^2)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^4}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^4}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^4,x)
```

```
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^4, x)
```

3.105 $\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{dx}{bc\sqrt{c+dx^2}}$$

```
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {21, 197}

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx = \frac{dx}{bc\sqrt{c+dx^2}}$$

```
[In] Int[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{d \int \frac{1}{(c+dx^2)^{3/2}} \, dx}{b}$$
$$= \frac{dx}{bc\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}} \, dx = \frac{dx}{bc\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(((b*c)/d + b*x^2)*Sqrt[c + d*x^2]),x]
[Out] (d*x)/(b*c*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19
default	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19
trager	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19
pseudoelliptic	$\frac{dx}{bc\sqrt{d\,x^2+c}}$	19

```
[In] int(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] d*x/b/c/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{\sqrt{dx^2+c}dx}{bcdx^2+bc^2}$$

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] sqrt(d*x^2 + c)*d*x/(b*c*d*x^2 + b*c^2)
```

Sympy [F]

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{d\int \frac{1}{c\sqrt{c+dx^2}+dx^2\sqrt{c+dx^2}}\,dx}{b}$$

```
[In] integrate(1/(b*c/d+b*x**2)/(d*x**2+c)**(1/2),x)
[Out] d*Integral(1/(c*sqrt(c + d*x**2) + d*x**2*sqrt(c + d*x**2)), x)/b
```

Maxima [F]

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+\frac{bc}{d}\right)\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + b*c/d)*sqrt(d*x^2 + c)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{dx}{\sqrt{dx^2+c}bc}$$

```
[In] integrate(1/(b*c/d+b*x^2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] d*x/(sqrt(d*x^2 + c)*b*c)
```

Mupad [B] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(\frac{bc}{d}+bx^2\right)\sqrt{c+dx^2}}\,dx=\frac{d\,x}{b\,c\,\sqrt{d\,x^2+c}}$$

```
[In] int(1/((c + d*x^2)^(1/2)*(b*x^2 + (b*c)/d)),x)
[Out] (d*x)/(b*c*(c + d*x^2)^(1/2))
```

3.106 $\int \frac{1}{\sqrt{1-x^2}(1+x^2)} dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x*2^(1/2)/(-x^2+1)^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {385, 209}

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + x^2)),x]
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+1}}{2x}\right)}{2}$	24
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-x^2+1}\,x}{x^2-1}\right)}{2}$	28
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2+2\right)x^2+4x\sqrt{-x^2+1}-\operatorname{RootOf}\left(\_Z^2+2\right)}{x^2+1}\right)}{4}$	50

```
[In] int(1/(x^2+1)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*2^(1/2)*arctan(1/2/x*2^(1/2)*(-x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2\,x}\right)$$

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \int \frac{1}{\sqrt{-(x-1)\,(x+1)}\,(x^2+1)}\,dx$$

```
[In] integrate(1/(x**2+1)/(-x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \int \frac{1}{(x^2+1)\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 1)*sqrt(-x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(20) = 40$.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.04

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{1}{4}\,\sqrt{2}\left(\pi\mathrm{sgn}(x) + 2\,\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

```
[In] integrate(1/(x^2+1)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^
2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{1}{\sqrt{1-x^2}\,(1+x^2)}\,dx = \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(-1+x\,1\mathrm{i})\,1\mathrm{i}}{2}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\mathrm{i}}\right)\,1\mathrm{i}}{4} - \frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{2}\,(1+x\,1\mathrm{i})\,1\mathrm{i}}{2}+\sqrt{1-x^2}\,1\mathrm{i}}{x+1\mathrm{i}}\right)\,1\mathrm{i}}{4}$$

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 1)),x)
```

```
[Out] (2^(1/2)*log(((2^(1/2)*(x*1i - 1)*1i)/2 - (1 - x^2)^(1/2)*1i)/(x - 1i))*1i)
/4 - (2^(1/2)*log(((2^(1/2)*(x*1i + 1)*1i)/2 + (1 - x^2)^(1/2)*1i)/(x + 1i)
)*1i)/4
```

3.107 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bc-ad}x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[Out] arctan(x*(-a*d+b*c)^(1/2)/a^(1/2)/(d*x^2+c)^(1/2))/a^(1/2)/(-a*d+b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {385, 211}

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{\arctan\left(\frac{x\sqrt{bc-ad}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[b*c - a*d]*x)/(Sqrt[a]*Sqrt[c + d*x^2])]/(Sqrt[a]*Sqrt[b*c - a
*d])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst}\left(\int \frac{1}{a-(-bc+ad)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt{a}\sqrt{c+dx^2}}\right)}{\sqrt{a}\sqrt{bc-ad}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\arctan\left(\frac{a\sqrt{d}+bx\left(\sqrt{d}x-\sqrt{c+dx^2}\right)}{\sqrt{a}\sqrt{bc-ad}}\right)}{\sqrt{a}\sqrt{bc-ad}}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]),x]
[Out] -(ArcTan[(a*Sqrt[d] + b*x*(Sqrt[d]*x - Sqrt[c + d*x^2]))/(Sqrt[a]*Sqrt[b*c
- a*d])]/(Sqrt[a]*Sqrt[b*c - a*d]))
```

Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{d\,x^2+c}\,a}{x\sqrt{(ad-bc)a}}\right)}{\sqrt{(ad-bc)a}}$
default	$-\frac{\ln\left(\frac{-\frac{2(ad-bc)}{b}+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{ad-bc}{b}}\sqrt{d\left(x-\frac{\sqrt{-ab}}{b}\right)^2+\frac{2d\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{ad-bc}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\sqrt{-\frac{ad-bc}{b}}}+$ ln $\left(-\frac{2(ad-bc)}{b}-\frac{2d\sqrt{-}\right.$

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((a*d-b*c)*a)^(1/2)*arctanh((d*x^2+c)^(1/2)/x*a/((a*d-b*c)*a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$
$$= \left[-\frac{\sqrt{-abc+a^2d}\log\left(\frac{(b^2c^2-8\,abcd+8\,a^2d^2)x^4+a^2c^2-2\,(3\,abc^2-4\,a^2cd)x^2-4\,((bc-2\,ad)x^3-acx)\sqrt{-abc+a^2d}\sqrt{dx^2+c}}{b^2x^4+2\,abx^2+a^2}\right)}{4\,(abc-a^2d)}, \frac{\arctan\left(\right.}{}\right.$$

[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-a*b*c + a^2*d)*log(((b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*x^4 + a^2*c^2 - 2*(3*a*b*c^2 - 4*a^2*c*d)*x^2 - 4*((b*c - 2*a*d)*x^3 - a*c*x)*sqrt(-a*b*c + a^2*d)*sqrt(d*x^2 + c))/(b^2*x^4 + 2*a*b*x^2 + a^2))/(a*b*c - a^2*d), 1/2*arctan(1/2*sqrt(a*b*c - a^2*d)*((b*c - 2*a*d)*x^2 - a*c)*sqrt(d*x^2 + c)/((a*b*c*d - a^2*d^2)*x^3 + (a*b*c^2 - a^2*c*d)*x))/sqrt(a*b*c - a^2*d)]

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2),x)

[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)), x)

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)), x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{d}\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2b-bc+2\,ad}{2\sqrt{abcd-a^2d^2}}\right)}{\sqrt{abcd-a^2d^2}}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*b - b*c + 2*a*d)/sqrt(
a*b*c*d - a^2*d^2))/sqrt(a*b*c*d - a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{\mathrm{atan}\left(\frac{x\sqrt{b\,c-a\,d}}{\sqrt{a}\,\sqrt{d\,x^2+c}}\right)}{\sqrt{-a\,(a\,d-b\,c)}} & \text{if } 0<b\,c-a\,d \\ \frac{\ln\left(\frac{\sqrt{a\,\left(d\,x^2+c\right)}+x\,\sqrt{a\,d-b\,c}}{\sqrt{a\,\left(d\,x^2+c\right)}-x\,\sqrt{a\,d-b\,c}}\right)}{2\,\sqrt{a\,(a\,d-b\,c)}} & \text{if } b\,c-a\,d<0 \\ \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx & \text{if } b\,c-a\,d\notin\mathbb{R}\vee a\,d=b\,c \end{cases}$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*d + b*c, atan((x*(- a*d + b*c)^(1/2))/(a^(1/2)*(c + d*x^2
)^(1/2)))/(-a*(a*d - b*c))^(1/2), - a*d + b*c < 0, log(((a*(c + d*x^2))^(1/
2) + x*(a*d - b*c)^(1/2))/((a*(c + d*x^2))^(1/2) - x*(a*d - b*c)^(1/2)))/(2
*(a*(a*d - b*c))^(1/2)), ~in(- a*d + b*c, 'real') | a*d == b*c, int(1/((a +
 b*x^2)*(c + d*x^2)^(1/2)), x))
```

3.108 $\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2x}{\sqrt{1+x^2}} + \operatorname{arcsinh}(x)$$

```
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {393, 221}

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = \operatorname{arcsinh}(x) - \frac{2x}{\sqrt{x^2+1}}$$

```
[In] Int[(-1 + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] (-2*x)/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
```

```
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2x}{\sqrt{1+x^2}}+\int\frac{1}{\sqrt{1+x^2}}\,dx\\ &= -\frac{2x}{\sqrt{1+x^2}}+\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int\frac{-1+x^2}{(1+x^2)^{3/2}}\,dx=-\frac{2x}{\sqrt{1+x^2}}-\log\left(-x+\sqrt{1+x^2}\right)$$

```
[In] Integrate[(-1 + x^2)/(1 + x^2)^(3/2),x]
[Out] (-2*x)/Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^2+1}}$	14
risch	$\operatorname{arcsinh}(x)-\frac{2x}{\sqrt{x^2+1}}$	14
trager	$-\frac{2x}{\sqrt{x^2+1}}+\ln\left(x+\sqrt{x^2+1}\right)$	22
meijerg	$-\frac{x}{\sqrt{x^2+1}}+\frac{-\frac{\sqrt{\pi}\,x}{\sqrt{x^2+1}}+\sqrt{\pi}\,\operatorname{arcsinh}(x)}{\sqrt{\pi}}$	36
pseudoelliptic	$\frac{-\ln\left(\frac{-x+\sqrt{x^2+1}}{x}\right)\sqrt{x^2+1}+\ln\left(\frac{x+\sqrt{x^2+1}}{x}\right)\sqrt{x^2+1}-4x}{2\sqrt{x^2+1}}$	61

```
[In] int((x^2-1)/(x^2+1)^(3/2),x,method=_RETURNVERBOSE)
[Out] arcsinh(x)-2*x/(x^2+1)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.93

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2\,x^2 + (x^2+1)\log\left(-x+\sqrt{x^2+1}\right) + 2\,\sqrt{x^2+1}x + 2}{x^2+1}$$

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="fricas")
[Out] -(2*x^2 + (x^2 + 1)*log(-x + sqrt(x^2 + 1)) + 2*sqrt(x^2 + 1)*x + 2)/(x^2 +
 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 2.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = \frac{x^2\operatorname{asinh}(x)}{x^2+1} - \frac{2x}{\sqrt{x^2+1}} + \frac{\operatorname{asinh}(x)}{x^2+1}$$

```
[In] integrate((x**2-1)/(x**2+1)**(3/2),x)
[Out] x**2*asinh(x)/(x**2 + 1) - 2*x/sqrt(x**2 + 1) + asinh(x)/(x**2 + 1)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2\,x}{\sqrt{x^2+1}} + \operatorname{arsinh}(x)$$

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="maxima")
[Out] -2*x/sqrt(x^2 + 1) + arcsinh(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.67

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = -\frac{2\,x}{\sqrt{x^2+1}} - \log\left(-x+\sqrt{x^2+1}\right)$$

```
[In] integrate((x^2-1)/(x^2+1)^(3/2),x, algorithm="giac")
[Out] -2*x/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int \frac{-1+x^2}{(1+x^2)^{3/2}}\,dx = \frac{\operatorname{asinh}(x)+x^2\operatorname{asinh}(x)-2\,x\,\sqrt{x^2+1}}{x^2+1}$$

```
[In] int((x^2 - 1)/(x^2 + 1)^(3/2),x)
[Out] (asinh(x) + x^2*asinh(x) - 2*x*(x^2 + 1)^(1/2))/(x^2 + 1)
```

3.109 $\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 648

$$\int (a-bx^2)^{2/3} (3a+bx^2)^3 \, dx = \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{378}{475}ax(a-bx^2)^{5/3}(3a+bx^2) - \frac{3}{25}x(a-bx^2)^{5/3}(3a+bx^2)^2 - \frac{72576a^4x}{1235\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{36288\sqrt[4]{3}}{}$$

```
[Out] 18144/1235*a^3*x*(-b*x^2+a)^(2/3)-23544/6175*a^2*x*(-b*x^2+a)^(5/3)-378/475
*a*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)-3/25*x*(-b*x^2+a)^(5/3)*(b*x^2+3*a)^2-725
76/1235*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+24192/1235*3^(3/4)*a^
(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3
^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+
a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36288/1235*3^(1/4)*a^(13/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {427, 542, 396, 201, 241, 310, 225, 1893}

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = \frac{24192\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36288\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{1235bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18144a^3x(a-bx^2)^{2/3}}{1235}-\frac{23544a^2x(a-bx^2)^{5/3}}{6175}$$
$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2$$

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (18144*a^3*x*(a - b*x^2)^(2/3))/1235 - (23544*a^2*x*(a - b*x^2)^(5/3))/6175
 - (378*a*x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/475 - (3*x*(a - b*x^2)^(5/3)*(
3*a + b*x^2)^2)/25 - (72576*a^4*x)/(1235*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36288*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^
2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))]
, -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24192*Sqrt[2]*3^(3/4)*
a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1235*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2-\frac{3\int\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\left(-78a^2b-42ab^2x^2\right)\,dx}{25b}\\
&= -\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\\
&\quad-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2+\frac{9\int\left(a-bx^2\right)^{2/3}\left(1608a^3b^2+872a^2b^3x^2\right)\,dx}{475b^2}\\
&= -\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\\
&\quad-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2+\frac{\left(42336a^3\right)\int\left(a-bx^2\right)^{2/3}\,dx}{1235}\\
&= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}\\
&\quad-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2+\frac{\left(24192a^4\right)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{1235}\\
&= \frac{18144a^3x\left(a-bx^2\right)^{2/3}}{1235}-\frac{23544a^2x\left(a-bx^2\right)^{5/3}}{6175}\\
&\quad-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2-\frac{\left(36288a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx\right.}{1235bx}
\end{aligned}$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2 + \frac{\left(36288a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)}{\sqrt{-a+}}\right.}{1235bx}$$

$$= \frac{18144a^3x(a-bx^2)^{2/3}}{1235} - \frac{23544a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{378}{475}ax\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{3}{25}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2 - \frac{72576a^4x}{1235\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 11.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.15

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx =$$

$$-\frac{3\left(-15255a^4x+3390a^3bx^3+8992a^2b^2x^5+2626ab^3x^7+247b^4x^9-40320a^4x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2}\right.}{6175\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (-3*(-15255*a^4*x + 3390*a^3*b*x^3 + 8992*a^2*b^2*x^5 + 2626*a*b^3*x^7 + 24
7*b^4*x^9 - 40320*a^4*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3
/2, (b*x^2)/a]))/(6175*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \left(-b\,x^2+a\right)^{\frac{2}{3}}\left(b\,x^2+3a\right)^3 dx$$

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
, x)
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx=27a^{\frac{11}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{7}{2}\\\frac{9}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**3,x)
[Out] 27*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a
**(5/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
+ a**(2/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)
/7
```

Maxima [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = \int \left(bx^2+3\,a\right)^3 \left(-bx^2+a\right)^{\frac{2}{3}} \, dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^3 \, dx = \int \left(a-b\,x^2\right)^{2/3} \left(b\,x^2+3\,a\right)^3 \, dx$$

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^3, x)
```

3.110 $\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 617

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)^2 \, dx = \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{252}{247}ax\left(a-bx^2\right)^{5/3} - \frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right) - \frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{}$$

```
[Out] 7776/1729*a^2*x*(-b*x^2+a)^(2/3)-252/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x^
2+a)^(5/3)*(b*x^2+3*a)-31104/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))+10368/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b
*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-
b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-15552/1729
*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a
^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*
((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(
1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 396, 201, 241, 310, 225, 1893}

$$\int (a-bx^2)^{2/3} (3a+bx^2)^2\,dx = \frac{10368\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)}{\left(1-\sqrt{3}\right)}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{7776a^2x(a-bx^2)^{2/3}}{1729}$$

$$-\frac{252}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{5/3}(3a+bx^2)$$

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (7776*a^2*x*(a - b*x^2)^(2/3))/1729 - (252*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(5/3)*(3*a + b*x^2))/19 - (31104*a^3*x)/(1729*((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))) - (15552*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (103
68*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
```

```
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{3\int\left(a-bx^2\right)^{2/3}\left(-60a^2b-28ab^2x^2\right)dx}{19b}\\
&= -\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{1}{247}\left(2592a^2\right)\int\left(a-bx^2\right)^{2/3}dx\\
&= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}\\
&\quad-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{\left(10368a^3\right)\int\frac{1}{\sqrt[3]{a-bx^2}}dx}{1729}\\
&= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}\\
&\quad-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{\left(15552a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a-bx^2}\right)}{1729bx}\\
&= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}\\
&\quad-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)+\frac{\left(15552a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}dx,x,\sqrt[3]{a-bx^2}\right)}{1729bx}-\frac{(1555
\\
&= \frac{7776a^2x\left(a-bx^2\right)^{2/3}}{1729}-\frac{252}{247}ax\left(a-bx^2\right)^{5/3}\\
&\quad-\frac{3}{19}x\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)-\frac{31104a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15552\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\right.
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.29

$$\int (a-bx^2)^{2/3} (3a+bx^2)^2 \, dx = \frac{x(a-bx^2)^{2/3} \left(21a(45a^2+10abx^2+b^2x^4)\,\text{Gamma}\left(-\frac{2}{3}\right)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right)+8\right.}{1}$$

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-2/3]*Hype
rgeometric2F1[-2/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[1/3]*Hypergeometric2F1[1/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x
+ b*x^3)^2*Gamma[1/3]*HypergeometricPFQ[{1/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]
))/(105*a*(1 - (b*x^2)/a)^(2/3)*Gamma[-2/3])
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)^{2} dx$$

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x)
```

Fricas [F]

$$\int (a-bx^2)^{2/3} (3a+bx^2)^2 \, dx = \int (bx^2+3\,a)^2(-bx^2+a)^{\frac{2}{3}} \, dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx = 9a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + 2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Maxima [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx = \int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx = \int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\,dx=\int \left(a-b\,x^2\right)^{2/3}\left(b\,x^2+3\,a\right)^2\,dx$$

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2,x)
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2)^2, x)
```

3.111 $\int (a-bx^2)^{2/3}(3a+bx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 588

$$\int (a-bx^2)^{2/3}(3a+bx^2)\,dx=\frac{18}{13}ax(a-bx^2)^{2/3} -\frac{3}{13}x(a-bx^2)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} -\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} +\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 18/13*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(5/3)-72/13*a^2*x/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))+24/13*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+
(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)-36/13*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2
+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-
I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a
```

```
)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {396, 201, 241, 310, 225, 1893}

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right) dx = \frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ldots}}\right.\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\ldots\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}$$

```
[In] Int[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (18*a*x*(a - b*x^2)^(2/3))/13 - (3*x*(a - b*x^2)^(5/3))/13 - (72*a^2*x)/(13
*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3
]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(13*b*x*Sqrt[-((a^(1/3)*(
a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
)]) + (24*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/
3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1
3*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}(42a)\int\left(a-bx^2\right)^{2/3}\,dx\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{1}{13}\left(24a^2\right)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{13bx}\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}\\
&\quad-\frac{3}{13}x\left(a-bx^2\right)^{5/3}+\frac{\left(36a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{13bx}\\
&\quad-\frac{\left(36\left(1+\sqrt{3}\right)a^{7/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{13bx}\\
&= \frac{18}{13}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{5/3}-\frac{72a^2x}{13\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}\\
&\quad-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}\right.\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\\
&\quad+\frac{24\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{13bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.66 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx=\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(-a+bx^2+\frac{14a\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

```
[In] Integrate[(a - b*x^2)^(2/3)*(3*a + b*x^2),x]
```

```
[Out] (3*x*(a - b*x^2)^(2/3)*(-a + b*x^2 + (14*a*Hypergeometric2F1[-2/3, 1/2, 3/2
, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/13
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{2}{3}} \left(b\,x^{2}+3a\right)dx$$

```
[In] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(2/3)*(b*x^2+3*a),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\,dx = \int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.11

$$\int \left(a-bx^2\right)^{2/3} \left(3a+bx^2\right)\,dx = 3a^{\frac{5}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3}$$

```
[In] integrate((-b*x**2+a)**(2/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(5/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + a**(2
/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Maxima [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx=\int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx=\int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{2}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)*(b*x^2+3*a),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\,dx=\int \left(a-b\,x^2\right)^{2/3}\left(b\,x^2+3\,a\right)\,dx$$

```
[In] int((a - b*x^2)^(2/3)*(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(2/3)*(3*a + b*x^2), x)
```

3.112 $\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 740

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \frac{3x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2^(1/3)*a^(1/6)*arctanh(x*b^(1/
2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))/b^(1/2)-1/3*2^(1/3)*a^(1/6)*
```

```
arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a^(1
/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)+1/3*2^(1/3
)*a^(1/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*3^(1/2)/b^(1/2)-3^(3/4)*a^(1/3)
*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)
))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)
+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+3/2*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x
^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

$= \{405, 241, 310, 225, 1893, 402\}$

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx =$$

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|-}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$

$$+\frac{\sqrt[3]{2}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}$$

$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (3*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)) + (2^(1/3)*a^(1/6)*ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*a^(1/6)*ArcTa
n[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sq
rt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b])
 + (2^(1/3)*a^(1/6)*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*
x^2)^(1/3)))])/Sqrt[b] + (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a
 - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^
(1/3))], -7 + 4*Sqrt[3]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (Sqrt[2]*3^(3/4)*a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/
3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elli
pticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(
```

```
1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 405

```
Int[((a_) + (b_.)*(x_)^2)^(2/3)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[1/(a + b*x^2)^(1/3), x], x] - Dist[(b*c - a*d)/d, Int[1/((a + b*x^2)
^(1/3)*(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& EqQ[b*c + 3*a*d, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
```

```
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx - \int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} + \frac{\left(3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}$$

$$= \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} - \frac{\left(3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx} + \frac{\left(3\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}$$

$$= \frac{3x}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$+ \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$- \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$- \frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.14 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.22

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \frac{9ax(a-bx^2)^{2/3}\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{(3a+bx^2)\left(9a\operatorname{AppellF1}\left(\frac{1}{2},-\frac{2}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)-2bx^2\left(\operatorname{AppellF1}\left(\frac{3}{2},-\frac{2}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right.\right.\right.}$$

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2),x]
```

```
[Out] (9*a*x*(a - b*x^2)^(2/3)*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2
)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, -2/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] - 2*b*x^2*(AppellF1[3/2, -2/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2
*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{b\,x^{2}+3a}dx$$

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{\left(a-bx^2\right)^{\frac{2}{3}}}{3a+bx^2}\,dx$$

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a),x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{\left(-bx^2+a\right)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{bx^2+3\,a}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{3a+bx^2}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{b\,x^2+3\,a}\,dx$$

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2), x)
```

3.113 $\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 584

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2} \, dx = \frac{x(a-bx^2)^{2/3}}{6a(3a+bx^2)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/6*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)-1/6*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-
3^(1/2)))+1/18*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*
x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/12*(a^(1/3
)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
```

```
)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+
a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {423, 21, 241, 310, 225, 1893}

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right.}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)}-\frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) - x/(6*a*((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt
[3]])/(4*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
-7 + 4*Sqrt[3]])/(3*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{\int \frac{-a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{6a} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{18a} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} + \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12abx} \\
&\quad - \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{12a^{2/3}bx} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{6a\left(3a+bx^2\right)} - \frac{x}{6a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} \\
&\quad - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-\right.}{4\,3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&\quad + \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right.}{3\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.15

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{6a\,(3a+bx^2)} + \frac{x\sqrt[3]{\frac{a-bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{18a\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x]
[Out] (x*(a - b*x^2)^(2/3))/(6*a*(3*a + b*x^2)) + (x*((a - b*x^2)/a)^(1/3)*Hyperg
eometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(18*a*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{2}{3}}}{(b\,x^2+3a)^2}\,dx$$

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
[Out] integral((-b*x^2 + a)^(2/3)/(b^2*x^4 + 6*a*b*x^2 + 9*a^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \int \frac{(a-bx^2)^{\frac{2}{3}}}{(3a+bx^2)^2}\,dx$$

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**2,x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^2}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^2}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^2}\,dx$$

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2,x)
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^2, x)
```

3.114 $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 818

$$\begin{aligned}
\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^3}\,dx &= \frac{x\left(a-bx^2\right)^{2/3}}{12a\left(3a+bx^2\right)^2}+\frac{x\left(a-bx^2\right)^{2/3}}{36a^2\left(3a+bx^2\right)}-\frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} \\
&+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} \\
&-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}} \\
&-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{}\right.}{24\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right.}{18\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}$$

```
[Out] 1/12*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^2+1/36*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+
3*a)-1/36*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/144*arctanh(x*b^(
```

```
1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(11/6)/b^(1/2)-1
/432*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1/2)+1/432*arctan(a^(1/
6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3
^(1/2)/b^(1/2)+1/432*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(
1/2)/b^(1/2)+1/108*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+
a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))
*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-
(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/72*(a^
(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b
*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*
x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {423, 541, 544, 241, 310, 225, 1893, 402}

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \frac{(a-bx^2)^{2/3}\,x}{36a^2\,(bx^2+3a)} - \frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{(a-bx^2)^{2/3}\,x}{12a\,(bx^2+3a)^2}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}}{24\ 3^{3/4}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{18\sqrt{2}\sqrt[4]{3}a^{5/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(12*a*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(36*a^
2*(3*a + b*x^2)) - x/(36*a^2*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) +
 ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]
) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(72*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(
216*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(
1/3)*(a - b*x^2)^(1/3)))]/(72*2^(2/3)*a^(11/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]
]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(24*3^(3/4)*a^(5/3)*b*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*
x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(18*Sqrt[2]*3^(1/4)*a^
(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
```

```
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a-bx^2)^{2/3}}{12a\left(3a+bx^2\right)^2} - \frac{\int \frac{-3a+\frac{5bx^2}{3}}{\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx}{12a}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\int \frac{16a^2b+\frac{8}{3}ab^2x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{288a^3b}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{108a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{36a}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}} - \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{72a^2bx}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\left(3a+bx^2\right)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{72a^2bx} - \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{72a^{5/3}bx}$$

$$= \frac{x(a-bx^2)^{2/3}}{12a\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{36a^2\,(3a+bx^2)} - \frac{x}{36a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{72\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{216\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{72\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-}{24\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right.}{18\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \frac{\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3} + \frac{27x\left(6-\frac{5bx^2}{a}-\frac{b^2x^4}{a^2}+\frac{18\left(3a+bx^2\right)\operatorname{A}}{9a\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\operatorname{Ap}\right.}\right.}{(3a+bx}}{972\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x]
```

```
[Out] ((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b
*x^2)/a])/a^3 + (27*x*(6 - (5*b*x^2)/a - (b^2*x^4)/a^2 + (18*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))/(3*a + b*x^2)^2)/(972*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{3}}dx$$

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{\left(a-bx^{2}\right)^{\frac{2}{3}}}{\left(3a+bx^{2}\right)^{3}}\,dx$$

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**3,x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**3, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{\left(-bx^{2}+a\right)^{\frac{2}{3}}}{\left(bx^{2}+3\,a\right)^{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^3}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^3}\,dx$$

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^3, x)
```

3.115 $\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 849

$$\int \frac{\left(a-bx^2\right)^{2/3}}{\left(3a+bx^2\right)^4}\,dx = \frac{x\left(a-bx^2\right)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x\left(a-bx^2\right)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x\left(a-bx^2\right)^{2/3}}{144a^3\left(3a+bx^2\right)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}} - \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{96\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)^3+1/54*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+
3*a)^2+1/144*x*(-b*x^2+a)^(2/3)/a^3/(b*x^2+3*a)-1/144*x/a^3/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))+7/2592*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*
(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-7/7776*arctanh(x*b^(1/2)/a^(1/2
))*2^(1/3)/a^(17/6)/b^(1/2)+7/7776*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+
a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+7/7776*arctan
(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/432*(a^(1/3)
-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1/288*(a^(1/3)-(-b*x^2+a)^(1/3))*El
lipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(
2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(
1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)
^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{423, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \frac{(a-bx^2)^{2/3}\,x}{144a^3\,(bx^2+3a)} - \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{(a-bx^2)^{2/3}\,x}{54a^2\,(bx^2+3a)^2}+\frac{(a-bx^2)^{2/3}\,x}{18a\,(bx^2+3a)^3}+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$-\frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{96\ 3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{72\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(18*a*(3*a + b*x^2)^3) + (x*(a - b*x^2)^(2/3))/(54*a^
2*(3*a + b*x^2)^2) + (x*(a - b*x^2)^(2/3))/(144*a^3*(3*a + b*x^2)) - x/(144
*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(Sqrt[3]*Sqrt
[a])/(Sqrt[b]*x)])/(1296*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + (7*ArcTan[(Sqr
t[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(1296*2^(
2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3888*2^(
2/3)*a^(17/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)
*(a - b*x^2)^(1/3)))])/(1296*2^(2/3)*a^(17/6)*Sqrt[b]) - (Sqrt[2 + Sqrt[3]]
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(96*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x
```

```
^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(72*Sqrt[2]*3^(1/4)*a^(
8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3}-\frac{\int \frac{-5a+\frac{11bx^2}{3}}{\sqrt[3]{a-bx^2}(3a+bx^2)^3}\,dx}{18a}\\
&= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3}+\frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2}+\frac{\int \frac{64a^2b-\frac{80}{3}ab^2x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx}{864a^3b}\\
&= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3}+\frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2}+\frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)}-\frac{\int \frac{-368a^3b^2-48a^2b^3x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{20736a^5b^2}\\
&= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3}+\frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2}+\frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)}\\
&\quad+\frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{432a^3}+\frac{7\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{648a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} \\
&\quad + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{1296\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{1296\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{7\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3888\;2^{2/3}a^{17/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\;2^{2/3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{288a^3bx}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x(a-bx^2)^{2/3}}{18a\,(3a+bx^2)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\,(3a+bx^2)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\,(3a+bx^2)} \\
&\quad + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{1296\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{1296\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{7\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3888\;2^{2/3}a^{17/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\;2^{2/3}a^{17/6}\sqrt{b}} \\
&\quad + \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{288a^3bx} \\
&\quad - \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{288a^{8/3}bx}
\end{aligned}
$$

$$= \frac{x(a-bx^2)^{2/3}}{18a\left(3a+bx^2\right)^3} + \frac{x(a-bx^2)^{2/3}}{54a^2\left(3a+bx^2\right)^2} + \frac{x(a-bx^2)^{2/3}}{144a^3\left(3a+bx^2\right)}$$

$$- \frac{x}{144a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$+ \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{1296\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3888\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{1296\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$- \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{96\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{\right.}{72\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \frac{x\left(\frac{9a(a-bx^2)(75a^2+26abx^2+3b^2x^4)}{(3a+bx^2)^3} + bx^2\sqrt[3]{1-\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right) + \frac{\cdots}{(3a+bx^2\cdots}\right.}{3888a^4\sqrt[3]{\cdots}}$$

```
[In] Integrate[(a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x]
```

```
[Out] (x*((9*a*(a - b*x^2)*(75*a^2 + 26*a*b*x^2 + 3*b^2*x^4))/(3*a + b*x^2)^3 + b
*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^
2)/a] + (621*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3
*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*
b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/
2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))))/(3888*a^4*(a - b*x^2)^(1/3)
)
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{2}{3}}}{\left(b\,x^{2}+3a\right)^{4}}dx$$

```
[In] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
[Out] int((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(a-bx^2)^{\frac{2}{3}}}{(3a+bx^2)^4}\,dx$$

```
[In] integrate((-b*x**2+a)**(2/3)/(b*x**2+3*a)**4,x)
[Out] Integral((a - b*x**2)**(2/3)/(3*a + b*x**2)**4, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^4}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(-bx^2+a)^{\frac{2}{3}}}{(bx^2+3\,a)^4}\,dx$$

```
[In] integrate((-b*x^2+a)^(2/3)/(b*x^2+3*a)^4,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(2/3)/(b*x^2 + 3*a)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{2/3}}{(3a+bx^2)^4}\,dx = \int \frac{(a-b\,x^2)^{2/3}}{(b\,x^2+3\,a)^4}\,dx$$

```
[In] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4,x)
[Out] int((a - b*x^2)^(2/3)/(3*a + b*x^2)^4, x)
```

3.116 $\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 668

$$\int (a-bx^2)^{5/3} (3a+bx^2)^3 \, dx = \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/3}}{14725}$$

$$-\frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2) - \frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2 - \frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{5619456}{\ldots}$$

```
[Out] 2809728/267995*a^4*x*(-b*x^2+a)^(2/3)+1404864/191425*a^3*x*(-b*x^2+a)^(5/3)
-33264/14725*a^2*x*(-b*x^2+a)^(8/3)-432/775*a*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a
)-3/31*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)^2-11238912/267995*a^5*x/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))+3746304/267995*3^(3/4)*a^(16/3)*(a^(1/3)-(-b*x^2
+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+
a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)-5619456/267995*3^(1/4)*a^(16/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*
EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)
^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2
^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {427, 542, 396, 201, 241, 310, 225, 1893}

$$\int (a-bx^2)^{5/3}(3a+bx^2)^3\,dx=\frac{3746304\sqrt{2}3^{3/4}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{}}{(1-\sqrt{}}\right.\right.}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{5619456\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{16/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{267995bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{2809728a^4x(a-bx^2)^{2/3}}{267995}+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}$$

$$-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}-\frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2)-\frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2$$

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
```

```
[Out] (2809728*a^4*x*(a - b*x^2)^(2/3))/267995 + (1404864*a^3*x*(a - b*x^2)^(5/3)
)/191425 - (33264*a^2*x*(a - b*x^2)^(8/3))/14725 - (432*a*x*(a - b*x^2)^(8/
3)*(3*a + b*x^2))/775 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2)^2)/31 - (11238
912*a^5*x)/(267995*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5619456*
3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(16/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2
/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(
267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) + (3746304*Sqrt[2]*3^(3/4)*a^(16/3)*(a^(1/3
) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x
^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(267995*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2-\frac{3\int\left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\left(-96a^2b-48ab^2x^2\right)\,dx}{31b}\\
&= -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)\\
&\quad -\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2+\frac{9\int\left(a-bx^2\right)^{5/3}\left(2544a^3b^2+1232a^2b^3x^2\right)\,dx}{775b^2}\\
&= -\frac{33264a^2x(a-bx^2)^{8/3}}{14725}-\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)\\
&\quad -\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2+\frac{(468288a^3)\int\left(a-bx^2\right)^{5/3}\,dx}{14725}\\
&= \frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}\\
&\quad -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2+\frac{(936576a^4)\int\left(a-bx^2\right)^{2/3}\,dx}{38285}\\
&= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995}+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}\\
&\quad -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2+\frac{(3746304a^5)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{267995}\\
&= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995}+\frac{1404864a^3x(a-bx^2)^{5/3}}{191425}-\frac{33264a^2x(a-bx^2)^{8/3}}{14725}\\
&\quad -\frac{432}{775}ax\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3}{31}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)^2-\frac{\left(5619456a^5\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+}}\right.}{267995bx}
\end{aligned}
$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/3}}{14725} - \frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2) - \frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2 + \frac{\left(5619456a^5\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)}{\sqrt{-a}}\right.}{267995bx}$$

$$= \frac{2809728a^4x(a-bx^2)^{2/3}}{267995} + \frac{1404864a^3x(a-bx^2)^{5/3}}{191425} - \frac{33264a^2x(a-bx^2)^{8/3}}{14725} - \frac{432}{775}ax(a-bx^2)^{8/3}(3a+bx^2) - \frac{3}{31}x(a-bx^2)^{8/3}(3a+bx^2)^2 - \frac{11238912a^5x}{267995\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.77 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3 dx = \frac{3\left(5815935a^5x - 5312355a^4bx^3 - 1675114a^3b^2x^5 + 749658a^2b^3x^7 + 378651ab^4x^9 + 43225b^5x^{11}\right.}{1339975\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x]
[Out] (3*(5815935*a^5*x - 5312355*a^4*b*x^3 - 1675114*a^3*b^2*x^5 + 749658*a^2*b^
3*x^7 + 378651*a*b^4*x^9 + 43225*b^5*x^11 + 6243840*a^5*x*(1 - (b*x^2)/a)^(
1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1339975*(a - b*x^2)^(1/
3))
```

Maple [F]

$$\int \left(-b\,x^2+a\right)^{\frac{5}{3}}\left(b\,x^2+3a\right)^3 dx$$

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b^4*x^8 + 8*a*b^3*x^6 + 18*a^2*b^2*x^4 - 27*a^4)*(-b*x^2 + a)^(2
/3), x)
```

Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=27a^{\frac{14}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)-\frac{18a^{\frac{8}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}-\frac{8a^{\frac{5}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}-\frac{a^{\frac{2}{3}}b^4x^9\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9}$$

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**3,x)
```

```
[Out] 27*a**(14/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 18*
a**(8/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
 - 8*a**(5/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)
/a)/7 - a**(2/3)*b**4*x**9*hyper((-2/3, 9/2), (11/2,), b*x**2*exp_polar(2*I
*pi)/a)/9
```

Maxima [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=\int \left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3*(-b*x^2 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^3\,dx=\int \left(a-b\,x^2\right)^{5/3}\left(b\,x^2+3\,a\right)^3\,dx$$

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^3, x)
```

3.117 $\int (a-bx^2)^{5/3} (3a+bx^2)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 637

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right)^2 \, dx = \frac{28512a^3x(a-bx^2)^{2/3}}{8645} + \frac{14256a^2x(a-bx^2)^{5/3}}{6175}$$

$$-\frac{306}{475}ax\left(a-bx^2\right)^{8/3} - \frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right) - \frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a}{}$$

```
[Out] 28512/8645*a^3*x*(-b*x^2+a)^(2/3)+14256/6175*a^2*x*(-b*x^2+a)^(5/3)-306/475
*a*x*(-b*x^2+a)^(8/3)-3/25*x*(-b*x^2+a)^(8/3)*(b*x^2+3*a)-114048/8645*a^4*x
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+38016/8645*3^(3/4)*a^(13/3)*(a^(1/
3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2)))^2)^(1/2)-57024/8645*3^(1/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(
1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*
x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)
+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 396, 201, 241, 310, 225, 1893}

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx = \frac{38016\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{28512a^3x\left(a-bx^2\right)^{2/3}}{8645}$$

$$+\frac{14256a^2x\left(a-bx^2\right)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)$$

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (28512*a^3*x*(a - b*x^2)^(2/3))/8645 + (14256*a^2*x*(a - b*x^2)^(5/3))/6175
 - (306*a*x*(a - b*x^2)^(8/3))/475 - (3*x*(a - b*x^2)^(8/3)*(3*a + b*x^2))/
25 - (114048*a^4*x)/(8645*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (5
7024*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (38016*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{3\int\left(a-bx^2\right)^{5/3}\left(-78a^2b-34ab^2x^2\right)\,dx}{25b}\\
&= -\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)+\frac{1}{475}\left(4752a^2\right)\int\left(a-bx^2\right)^{5/3}\,dx\\
&= \frac{14256a^2x(a-bx^2)^{5/3}}{6175}\\
&\quad -\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)+\frac{\left(9504a^3\right)\int\left(a-bx^2\right)^{2/3}\,dx}{1235}\\
&= \frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}\\
&\quad -\frac{306}{475}ax\left(a-bx^2\right)^{8/3}-\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)+\frac{\left(38016a^4\right)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{8645}\\
&= \frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}\\
&\quad -\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{\left(57024a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{8645bx}\\
&= \frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}\\
&\quad -\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)+\frac{\left(57024a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{8645bx}-\frac{(57024
\\
&= \frac{28512a^3x(a-bx^2)^{2/3}}{8645}+\frac{14256a^2x(a-bx^2)^{5/3}}{6175}-\frac{306}{475}ax\left(a-bx^2\right)^{8/3}\\
&\quad -\frac{3}{25}x\left(a-bx^2\right)^{8/3}\left(3a+bx^2\right)-\frac{114048a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{57024\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.27

$$\int (a-bx^2)^{5/3} (3a+bx^2)^2\,dx = \frac{x(a-bx^2)^{2/3}\left(21a(45a^2+10abx^2+b^2x^4)\,\text{Gamma}\left(-\frac{5}{3}\right)\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right)+\right.}{}$$

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x]
```

```
[Out] (x*(a - b*x^2)^(2/3)*(21*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Gamma[-5/3]*Hype
rgeometric2F1[-5/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^
2*x^4)*Gamma[-2/3]*Hypergeometric2F1[-2/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*
x + b*x^3)^2*Gamma[-2/3]*HypergeometricPFQ[{-2/3, 3/2, 2}, {1, 9/2}, (b*x^2
)/a]))/(105*(1 - (b*x^2)/a)^(2/3)*Gamma[-5/3])
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{5}{3}} \left(b\,x^{2}+3a\right)^{2} dx$$

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

```
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x)
```

Fricas [F]

$$\int (a-bx^2)^{5/3} (3a+bx^2)^2\,dx = \int \left(bx^2+3\,a\right)^2 \left(-bx^2+a\right)^{\frac{5}{3}} dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 5*a*b^2*x^4 + 3*a^2*b*x^2 - 9*a^3)*(-b*x^2 + a)^(2/3),
 x)
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.21

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx = 9a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) - a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) - a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) - \frac{a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7}$$

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a)**2,x)
```

```
[Out] 9*a**(11/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(
8/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(5/
3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a) - a**(2
/3)*b**3*x**7*hyper((-2/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7
```

Maxima [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx = \int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx = \int \left(bx^2+3\,a\right)^2\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)^2*(-b*x^2 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)^2\,dx=\int \left(a-b\,x^2\right)^{5/3}\left(b\,x^2+3\,a\right)^2\,dx$$

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2,x)
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2)^2, x)
```

3.118 $\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right) \, dx$

Optimal result

Integrand size = 22, antiderivative size = 608

$$\int \left(a-bx^2\right)^{5/3} \left(3a+bx^2\right) \, dx = \frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}$$
$$-\frac{3}{19}x\left(a-bx^2\right)^{8/3}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\ldots}{\left(\left(1-\sqrt{3}\ldots\right.\right.}}}{1729bx\sqrt{\ldots}}$$

```
[Out] 1800/1729*a^2*x*(-b*x^2+a)^(2/3)+180/247*a*x*(-b*x^2+a)^(5/3)-3/19*x*(-b*x^
2+a)^(8/3)-7200/1729*a^3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+2400/172
9*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+
a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))
*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-3600/1729*3^(1/4)*a^(10/
3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/
2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/
3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/
3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {396, 201, 241, 310, 225, 1893}

$$\int (a-bx^2)^{5/3} (3a+bx^2)\, dx = \frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{1800a^2x(a-bx^2)^{2/3}}{1729}$$
$$+\frac{180}{247}ax(a-bx^2)^{5/3}-\frac{3}{19}x(a-bx^2)^{8/3}$$

```
[In] Int[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
```

```
[Out] (1800*a^2*x*(a - b*x^2)^(2/3))/1729 + (180*a*x*(a - b*x^2)^(5/3))/247 - (3*
x*(a - b*x^2)^(8/3))/19 - (7200*a^3*x)/(1729*((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))) - (3600*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3
)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (2400*Sqrt[2]*3^(3/4
)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2
)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3]
)*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)
*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
^2)])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{1}{19}(60a)\int \left(a-bx^2\right)^{5/3}\,dx$$

$$=\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{1}{247}\left(600a^2\right)\int \left(a-bx^2\right)^{2/3}\,dx$$

$$=\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{\left(2400a^3\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{1729}$$

$$=\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}-\frac{\left(3600a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{1729bx}$$

$$=\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}+\frac{\left(3600a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{1729bx}-\frac{\left(3600\left(1+\sqrt{3}\right)a^{10/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{1729bx}$$

$$=\frac{1800a^2x\left(a-bx^2\right)^{2/3}}{1729}+\frac{180}{247}ax\left(a-bx^2\right)^{5/3}-\frac{3}{19}x\left(a-bx^2\right)^{8/3}-\frac{7200a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3600\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}+\frac{2400\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.87 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.11

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx = \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(-\left(a-bx^2\right)^2+\frac{20a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{2/3}}\right)$$

```
[In] Integrate[(a - b*x^2)^(5/3)*(3*a + b*x^2),x]
[Out] (3*x*(a - b*x^2)^(2/3)*(-(a - b*x^2)^2 + (20*a^2*Hypergeometric2F1[-5/3, 1/
2, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(2/3)))/19
```

Maple [F]

$$\int \left(-b\,x^2+a\right)^{\frac{5}{3}}\left(b\,x^2+3a\right)dx$$

```
[In] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
[Out] int((-b*x^2+a)^(5/3)*(b*x^2+3*a),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx = \int \left(bx^2+3\,a\right)\left(-bx^2+a\right)^{\frac{5}{3}}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="fricas")
[Out] integral(-(b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(-b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.16

$$\int \left(a-bx^2\right)^{5/3}\left(3a+bx^2\right)\,dx = 3a^{\frac{8}{3}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right) - \frac{2a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3} - \frac{a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\left.\begin{matrix}-\frac{2}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5}$$

```
[In] integrate((-b*x**2+a)**(5/3)*(b*x**2+3*a),x)
```

```
[Out] 3*a**(8/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) - 2*a**
(5/3)*b*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3 - a**
(2/3)*b**2*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Maxima [F]

$$\int (a-bx^2)^{5/3} (3a+bx^2) \, dx = \int (bx^2+3a)(-bx^2+a)^{\frac{5}{3}} \, dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Giac [F]

$$\int (a-bx^2)^{5/3} (3a+bx^2) \, dx = \int (bx^2+3a)(-bx^2+a)^{\frac{5}{3}} \, dx$$

```
[In] integrate((-b*x^2+a)^(5/3)*(b*x^2+3*a),x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + 3*a)*(-b*x^2 + a)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a-bx^2)^{5/3} (3a+bx^2) \, dx = \int (a-b\,x^2)^{5/3} \left(b\,x^2+3\,a\right) \, dx$$

```
[In] int((a - b*x^2)^(5/3)*(3*a + b*x^2),x)
```

```
[Out] int((a - b*x^2)^(5/3)*(3*a + b*x^2), x)
```

3.119 $\int \frac{(a-bx^2)^{5/3}}{3a+bx^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 765

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2} dx = -\frac{3}{7}x(a-bx^2)^{2/3} + \frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$-\frac{4\sqrt[3]{2}a^{7/6}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} + \frac{4\sqrt[3]{2}a^{7/6}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] -3/7*x*(-b*x^2+a)^(2/3)+96/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+4*
2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)
```

```
))/b^(1/2)-4/3*2^(1/3)*a^(7/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)+4/3*2^(1/
3)*a^(7/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1
/2))*3^(1/2)/b^(1/2)+4/3*2^(1/3)*a^(7/6)*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*
3^(1/2)/b^(1/2)-32/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((
-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2
/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3
)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+48/7*3
^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1
/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a
^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3
)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-
b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{427, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx =$$

$$-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$

$$+\frac{4\sqrt[3]{2}a^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}-\frac{4\sqrt[3]{2}a^{7/6}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

$$+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 + (96*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(Sqr
t[3]*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/
3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) - (4*2^(1/3)*a^(7/6)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) + (4*2^(1/3)*a^(7/6)*ArcTanh[(Sq
rt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] + (48*3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3)
 + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))^2)]) - (32*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2
)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))],
```

```
-7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{3\int \frac{\frac{16a^2b}{3}-\frac{32}{3}ab^2x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{7b}\\
&= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{1}{7}(32a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx+\left(16a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx\\
&= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}\\
&\quad+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}\\
&\quad-\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}\\
&\quad+\frac{\left(48a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}
\end{aligned}
$$

$$=-\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$-\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$-\frac{\left(48a\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}$$
$$+\frac{\left(48\left(1+\sqrt{3}\right)a^{4/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}$$
$$=-\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{96ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$
$$-\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}+\frac{4\sqrt[3]{2}a^{7/6}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$+\frac{48\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{32\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.21 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.30

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \frac{x\left(-32bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27\left(-a+bx^2+\frac{\cdots}{(3a+bx^2)\left(9a\,\text{AppellF}\cdots\right.}\right.\right.}{63\sqrt[3]{a-bx\cdots}}$$

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2),x]
```

```
[Out] (x*(-32*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + 27*(-a + b*x^2 + (48*a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2
)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))))/(
63*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{b\,x^{2}+3a}dx$$

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

```
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(a-bx^2)^{\frac{5}{3}}}{3a+bx^2}\,dx$$

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a),x)
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{bx^2+3\,a}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{bx^2+3\,a}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{3a+bx^2}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{b\,x^2+3\,a}\,dx$$

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2),x)
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2), x)
```

3.120 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 775

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)} - \frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$

$$+\frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$

$$-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7\right.}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 2/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)-11/3*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))-2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a
```

```
)^(1/3)))/b^(1/2)+1/3*2^(1/3)*a^(1/6)*arctanh(x*b^(1/2)/a^(1/2))/b^(1/2)-1/
3*2^(1/3)*a^(1/6)*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)
/x/b^(1/2))*3^(1/2)/b^(1/2)-1/3*2^(1/3)*a^(1/6)*arctan(3^(1/2)*a^(1/2)/x/b^
(1/2))*3^(1/2)/b^(1/2)+11/9*3^(3/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*Elli
pticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1
-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^
2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*
(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
-11/6*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/
3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/
2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(
1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {424, 544, 241, 310, 225, 1893, 402}

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7\right.}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}-\frac{\sqrt[3]{2}\sqrt[6]{a}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}}$$

$$-\frac{\sqrt[3]{2}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{\sqrt{b}}+\frac{\sqrt[3]{2}\sqrt[6]{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}}$$

$$+\frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)}-\frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (2*x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)) - (11*x)/(3*((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b
]*x)])/(Sqrt[3]*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3
) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(Sqrt[3]*Sqrt[b]) + (2^(1/3)*
a^(1/6)*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(3*Sqrt[b]) - (2^(1/3)*a^(1/6)*ArcTan
h[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/Sqrt[b] - (
11*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*3^(3
/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))^2)]) + (11*Sqrt[2]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3*3^(1/4)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
```

```
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{2x(a-bx^2)^{2/3}}{3\left(3a+bx^2\right)} + \frac{\int \frac{-2a^2b+\frac{22}{3}ab^2x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{6ab}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3\left(3a+bx^2\right)} + \frac{11}{9}\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx - (4a)\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx$$

$$= \frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}}$$
$$- \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}}$$
$$+ \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$- \frac{\left(11\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{6bx}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}}$$
$$- \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{\sqrt{3}\sqrt{b}}$$
$$+ \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{3\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}}$$
$$+ \frac{\left(11\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{6bx}$$
$$- \frac{\left(11\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{6bx}$$

$$= \frac{2x(a-bx^2)^{2/3}}{3(3a+bx^2)} - \frac{11x}{3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{\sqrt{3}\sqrt{b}} + \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3\sqrt{b}} - \frac{\sqrt[3]{2}\sqrt[6]{a}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{\sqrt{b}} - \frac{11\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{2\,3^{3/4}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{11\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{3\sqrt[4]{3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.30

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \frac{x\left(\frac{11bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a} + \frac{27\left(2a-2bx^2-\frac{9a^2\,\text{Ap}}{9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\right.}\right.}{3}\right.}{81\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x]
```

```
[Out] (x*((11*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a])/a + (27*(2*a - 2*b*x^2 - (9*a^2*AppellF1[1/2, 1/3, 1, 3/2,
(b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3
*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/
a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2
)))/(81*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{2}}dx$$

```
[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \int \frac{(a-bx^2)^{\frac{5}{3}}}{(3a+bx^2)^2}\,dx$$

```
[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**2,x)
[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^2}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^2}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^2}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{(b\,x^2+3\,a)^2}\,dx$$

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2,x)
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^2, x)
```

3.121 $\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 815

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{x}{18a\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+ \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$
$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}}$$
$$+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{3}\right)}{12\ 3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$- \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{9\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/3*x*(-b*x^2+a)^(2/3)/(b*x^2+3*a)^2-1/18*x*(-b*x^2+a)^(2/3)/a/(b*x^2+3*a)+
1/18*x/a/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/36*arctanh(x*b^(1/2)/a^(
```

```
1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(5/6)/b^(1/2)-1/108*arct
anh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/108*arctan(a^(1/6)*(a^(1/3
)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)/b^(1
/2)+1/108*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(5/6)*3^(1/2)/b^(1/2)
-1/54*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^
(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^
(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/36*(a^(1/3)-(-b*x^2
+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)
+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^
(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 815, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {424, 12, 482, 544, 241, 310, 225, 1893, 402}

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = -\frac{(a-bx^2)^{2/3}x}{18a(bx^2+3a)} + \frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{(a-bx^2)^{2/3}x}{3(bx^2+3a)^2} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$
$$-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}}$$
$$+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}}{12\ 3^{3/4}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}}{9\sqrt{2}\sqrt[4]{3}a^{2/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(3*(3*a + b*x^2)^2) - (x*(a - b*x^2)^(2/3))/(18*a*(3*
a + b*x^2)) + x/(18*a*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan
[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(18*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) + ArcT
an[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(18
*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(54*2^(2/3
)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b
*x^2)^(1/3)))]/(18*2^(2/3)*a^(5/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) -
 (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1
 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))], -7 + 4*Sqrt[3]])/(12*3^(3/4)*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
F[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(9*Sqrt[2]*3^(1/4)*a^(2/3)*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} + \frac{\int \frac{16ab^2x^2}{3\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx}{12ab}$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} + \frac{1}{9}(4b)\int \frac{x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{\int \frac{a-\frac{bx^2}{3}}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{18a}$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{1}{9}\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{54a}$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{36abx}$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{36abx} + \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{36a^{2/3}bx}$$

$$= \frac{x(a-bx^2)^{2/3}}{3(3a+bx^2)^2} - \frac{x(a-bx^2)^{2/3}}{18a(3a+bx^2)} + \frac{x}{18a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{18\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{54\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{18\ 2^{2/3}a^{5/6}\sqrt{b}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{12\ 3^{3/4}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{\ }\right.}{9\sqrt{2}\sqrt[4]{3}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.31

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \frac{-\frac{bx^3\sqrt[3]{1-\frac{bx^2}{a}}\ \mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^2} + \frac{27x\left(3a-4bx^2+\frac{b^2x^4}{a}+\frac{9a(3a+}{9a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2}\right.}{(3}}{486\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x]
```

```
[Out] (-((b*x^3*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a])/a^2) + (27*x*(3*a - 4*b*x^2 + (b^2*x^4)/a + (9*a*(3*a + b*x^2)*
AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))/(3*a + b*x^2)^2)/(486*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{5}{3}}}{\left(b\,x^{2}+3a\right)^{3}}dx$$

[In] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)

[Out] int((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \text{Timed out}$$

[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-bx^2)^{\frac{5}{3}}}{(3a+bx^2)^3}\,dx$$

[In] integrate((-b*x**2+a)**(5/3)/(b*x**2+3*a)**3,x)

[Out] Integral((a - b*x**2)**(5/3)/(3*a + b*x**2)**3, x)

Maxima [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^3}\,dx$$

[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="maxima")

[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)

Giac [F]

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{3}}}{(bx^2+3\,a)^3}\,dx$$

```
[In] integrate((-b*x^2+a)^(5/3)/(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate((-b*x^2 + a)^(5/3)/(b*x^2 + 3*a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/3}}{(3a+bx^2)^3}\,dx = \int \frac{(a-b\,x^2)^{5/3}}{(b\,x^2+3\,a)^3}\,dx$$

```
[In] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3,x)
[Out] int((a - b*x^2)^(5/3)/(3*a + b*x^2)^3, x)
```

3.122 $\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 659

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}$$

$$-\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3 - \frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{1897344}{}$$

```
[Out] -1552608/43225*a^3*x*(-b*x^2+a)^(2/3)-36288/6175*a^2*x*(-b*x^2+a)^(2/3)*(b*
x^2+3*a)-18/19*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-3/25*x*(-b*x^2+a)^(2/3)*(
b*x^2+3*a)^3-3794688/8645*a^4*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+126
4896/8645*3^(3/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+
a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-1897344/8645*3^(
1/4)*a^(13/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/
3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^
(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)
*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b
*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {427, 542, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{1264896\sqrt{2}3^{3/4}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{1897344\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{13/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{8645bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{1552608a^3x(a-bx^2)^{2/3}}{43225}$$

$$-\frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}$$

$$-\frac{18}{19}ax(a-bx^2)^{2/3}(3a+bx^2)^2-\frac{3}{25}x(a-bx^2)^{2/3}(3a+bx^2)^3$$

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (-1552608*a^3*x*(a - b*x^2)^(2/3))/43225 - (36288*a^2*x*(a - b*x^2)^(2/3)*(
3*a + b*x^2))/6175 - (18*a*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (3*x*(
a - b*x^2)^(2/3)*(3*a + b*x^2)^3)/25 - (3794688*a^4*x)/(8645*((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))) - (1897344*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(13/3
)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
 (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8645*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (
1264896*Sqrt[2]*3^(3/4)*a^(13/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(86
45*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3-\frac{3\int\frac{\left(3a+bx^2\right)^2\left(-78a^2b-50ab^2x^2\right)}{\sqrt[3]{a-bx^2}}\,dx}{25b}\\
&= -\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\\
&\quad -\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3+\frac{9\int\frac{\left(3a+bx^2\right)\left(1632a^3b^2+1344a^2b^3x^2\right)}{\sqrt[3]{a-bx^2}}\,dx}{475b^2}\\
&= -\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}-\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2\\
&\quad -\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3-\frac{27\int\frac{-25248a^4b^3-19168a^3b^4x^2}{\sqrt[3]{a-bx^2}}\,dx}{6175b^3}\\
&= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}\\
&\quad -\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3+\frac{\left(1264896a^4\right)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{8645}\\
&= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}\\
&\quad -\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3-\frac{\left(1897344a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x}}\right.}{8645bx}\\
&= -\frac{1552608a^3x\left(a-bx^2\right)^{2/3}}{43225}-\frac{36288a^2x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)}{6175}\\
&\quad -\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3+\frac{\left(1897344a^4\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{(1+\sqrt{3}}{\sqrt{-}}\right.}{8645bx}
\end{aligned}$$

$$= -\frac{1552608a^3x(a-bx^2)^{2/3}}{43225} - \frac{36288a^2x(a-bx^2)^{2/3}(3a+bx^2)}{6175}$$

$$-\frac{18}{19}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2-\frac{3}{25}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^3-\frac{3794688a^4x}{8645\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.15

$$\int \frac{\left(3a+bx^2\right)^4}{\sqrt[3]{a-bx^2}}\,dx$$
$$= \frac{3x\left(-941085a^4+727830a^3bx^2+184044a^2b^2x^4+27482ab^3x^6+1729b^4x^8+2108160a^4\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hyperge}\right.}{43225\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*x*(-941085*a^4 + 727830*a^3*b*x^2 + 184044*a^2*b^2*x^4 + 27482*a*b^3*x^6
 + 1729*b^4*x^8 + 2108160*a^4*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3,
1/2, 3/2, (b*x^2)/a]))/(43225*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(b\,x^2+3a\right)^4}{\left(-b\,x^2+a\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="fricas")

[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.25

$$\int \frac{(3a+bx^2)^4}{\sqrt[3]{a-bx^2}}\,dx = 81a^{\frac{11}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + 36a^{\frac{8}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{54a^{\frac{5}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5} + \frac{12a^{\frac{2}{3}}b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7} + \frac{b^4x^9\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{9}{2}\\ \frac{11}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{9\sqrt[3]{a}}$$

[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(1/3),x)

[Out] 81*a**(11/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 36*a**(8/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 54*a**(5/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 + 12*a**(2/3)*b**3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/7 + b**4*x**9*hyper((1/3, 9/2), (11/2,), b*x**2*exp_polar(2*I*pi)/a)/(9*a**(1/3))

Maxima [F]

$$\int \frac{(3a + bx^2)^4}{\sqrt[3]{a - bx^2}}\, dx = \int \frac{(bx^2 + 3\,a)^4}{(-bx^2 + a)^{\frac{1}{3}}}\, dx$$

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(3a + bx^2)^4}{\sqrt[3]{a - bx^2}}\, dx = \int \frac{(bx^2 + 3\,a)^4}{(-bx^2 + a)^{\frac{1}{3}}}\, dx$$

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a + bx^2)^4}{\sqrt[3]{a - bx^2}}\, dx = \int \frac{(b\,x^2 + 3\,a)^4}{(a - b\,x^2)^{1/3}}\, dx$$

```
[In] int((3*a + b*x^2)^4/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)^4/(a - b*x^2)^(1/3), x)
```

3.123 $\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 628

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}} \, dx = -\frac{15768a^2x(a-bx^2)^{2/3}}{1729} - \frac{324}{247}ax(a-bx^2)^{2/3}(3a+bx^2) - \frac{3}{19}x(a-bx^2)^{2/3}(3a+bx^2)^2 - \frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-\cdots}\right.}{\cdots}$$

```
[Out] -15768/1729*a^2*x*(-b*x^2+a)^(2/3)-324/247*a*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)
-3/19*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)^2-215136/1729*a^3*x/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2)))+71712/1729*3^(3/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+
(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)-107568/1729*3^(1/4)*a^(10/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-
(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)
)),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-
b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/
(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {427, 542, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{71712\sqrt{2}3^{3/4}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{1729bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{15768a^2x(a-bx^2)^{2/3}}{1729}$$

$$-\frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2$$

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (-15768*a^2*x*(a - b*x^2)^(2/3))/1729 - (324*a*x*(a - b*x^2)^(2/3)*(3*a + b
*x^2))/247 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2)^2)/19 - (215136*a^3*x)/(1
729*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (107568*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
 - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(1729*b*x*Sqrt[-(
(a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))^2)]) + (71712*Sqrt[2]*3^(3/4)*a^(10/3)*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(1729*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = -\frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{3\int \frac{\left(3a+bx^2\right)\left(-60a^2b-36ab^2x^2\right)}{\sqrt[3]{a-bx^2}}\,dx}{19b}$$

$$= -\frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{9\int \frac{888a^3b^2+584a^2b^3x^2}{\sqrt[3]{a-bx^2}}\,dx}{247b^2}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{\left(71712a^3\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{1729}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{\left(107568a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{1729bx}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 + \frac{\left(107568a^3\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{1729bx} - \frac{(1075}{}$$

$$= -\frac{15768a^2x\left(a-bx^2\right)^{2/3}}{1729} - \frac{324}{247}ax\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{3}{19}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)^2 - \frac{215136a^3x}{1729\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{107568\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}\right.}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.14

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \frac{3\left(-8343a^3x+7041a^2bx^3+1211ab^2x^5+91b^3x^7+23904a^3x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{1729\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(1/3),x]
```

```
[Out] (3*(-8343*a^3*x + 7041*a^2*b*x^3 + 1211*a*b^2*x^5 + 91*b^3*x^7 + 23904*a^3*
x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(1729
*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{\left(b\,x^2+3a\right)^3}{\left(-b\,x^2+a\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{\left(bx^2+3\,a\right)^3}{\left(-bx^2+a\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b*x^2 - a), x)
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.21

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = 27a^{\frac{8}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+9a^{\frac{5}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{9a^{\frac{2}{3}}b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5}+\frac{b^3x^7\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[3]{a}}$$

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(1/3),x)
[Out] 27*a**(8/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**
(5/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + 9*a**(
2/3)*b**2*x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5 + b*
*3*x**7*hyper((1/3, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/3))
```

Maxima [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{\left(b\,x^2+3\,a\right)^3}{\left(a-b\,x^2\right)^{1/3}}\,dx$$

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(1/3), x)
```

3.124 $\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 597

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{198}{91}ax(a-bx^2)^{2/3} - \frac{3}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$- \frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] -198/91*a*x*(-b*x^2+a)^(2/3)-3/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-3240/91*a^
2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1080/91*3^(3/4)*a^(7/3)*(a^(1/3
)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b
*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)
*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)-1620/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
```

*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {427, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{198}{91}ax\left(a-bx^2\right)^{2/3} - \frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]

[Out] (-198*a*x*(a - b*x^2)^(2/3))/91 - (3*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13 - (3240*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (1620*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (1080*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))

```
], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/
((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
```

```
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3\int\frac{-42a^2b-22ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{13b}\\
&= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)+\frac{1}{91}\left(1080a^2\right)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx\\
&= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}\\
&\quad -\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{\left(1620a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{91bx}\\
&= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)\\
&\quad +\frac{\left(1620a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{91bx}\\
&\quad -\frac{\left(1620\left(1+\sqrt{3}\right)a^{7/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{91bx}\\
&= -\frac{198}{91}ax\left(a-bx^2\right)^{2/3}-\frac{3}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}\\
&\quad -\frac{1620\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\cdots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}\\
&\quad +\frac{1080\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-\cdots}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 12.96 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.26

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \frac{x\sqrt[3]{1-\frac{bx^2}{a}}\left(63a(45a^2+10abx^2+b^2x^4)\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{7}{2},\frac{bx^2}{a}\right)+8bx^2(18a^2+9abx^2+b^2x^4)\,\text{Hy}\right.}{315a\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(1/3),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/3)*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*Hypergeometr
ic2F1[1/3, 1/2, 7/2, (b*x^2)/a] + 8*b*x^2*(18*a^2 + 9*a*b*x^2 + b^2*x^4)*Hy
pergeometric2F1[4/3, 3/2, 9/2, (b*x^2)/a] + 4*b*(3*a*x + b*x^3)^2*Hypergeom
etricPFQ[{4/3, 3/2, 2}, {1, 9/2}, (b*x^2)/a]))/(315*a*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{(b\,x^2+3a)^2}{(-b\,x^2+a)^{\frac{1}{3}}}dx$$

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = 9a^{\frac{5}{3}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + 2a^{\frac{2}{3}}bx^3\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right) + \frac{b^2x^5\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(1/3),x)
[Out] 9*a**(5/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + 2*a**(
2/3)*b*x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a) + b**2*x*
*5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/3))
```

Maxima [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{(b\,x^2+3\,a)^2}{(a-b\,x^2)^{1/3}}\,dx$$

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)^2/(a - b*x^2)^(1/3), x)
```

3.125 $\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 568

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$- \frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] -3/7*x*(-b*x^2+a)^(2/3)-72/7*a*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+24
/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+
a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))
*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-36/7*3^(1/4)*a^(4/3)*(a^
(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b
*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^
(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {396, 241, 310, 225, 1893}

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$$
$$= \frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right)}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{3}{7}x\left(a-bx^2\right)^{2/3}$$

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
```

```
[Out] (-3*x*(a - b*x^2)^(2/3))/7 - (72*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^
2)^(1/3))) - (36*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(
1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (24*Sqrt[2]*3^(3/4)*a^(4/3)*(a
^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
- b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a -
 b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{1}{7}(24a)\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx\\
&= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}-\frac{\left(36a\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}\\
&= -\frac{3}{7}x\left(a-bx^2\right)^{2/3}+\frac{\left(36a\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}\\
&\quad-\frac{\left(36\left(1+\sqrt{3}\right)a^{4/3}\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{7bx}
\end{aligned}
$$

$$= -\frac{3}{7}x\left(a-bx^2\right)^{2/3} - \frac{72ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$- \frac{36\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+ \frac{24\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = \frac{3x\left(-a+bx^2+8a\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{7\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(1/3),x]
[Out] (3*x*(-a + b*x^2 + 8*a*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/
2, (b*x^2)/a]))/(7*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{b\,x^2+3a}{\left(-b\,x^2+a\right)^{\frac{1}{3}}}dx$$

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="fricas")
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b*x^2 - a), x)
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = 3a^{\frac{2}{3}}x{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)+\frac{bx^3{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(1/3),x)
[Out] 3*a**(2/3)*x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a) + b*x**3
*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/3))
```

Maxima [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx = \int \frac{b\,x^2+3\,a}{{\left(a-b\,x^2\right)}^{1/3}}\,dx$$

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(1/3),x)
[Out] int((3*a + b*x^2)/(a - b*x^2)^(1/3), x)
```

3.126 $\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx &= \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} \\
&\quad - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}
\end{aligned}
$$

```
[Out] 1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a
^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/12
*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a
^(5/6)*3^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {402}

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$

$$= \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\text{Appe}\right.\right.}$$

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}\left(b\,x^{2}+3a\right)}dx$$

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\left(a-b\,x^2\right)^{1/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.127 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 787

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \frac{x(a-bx^2)^{2/3}}{24a^2(3a+bx^2)} - \frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}}$$
$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\cdots}{16\,3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\cdots}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/24*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)-1/24*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^
```

```
(1/3)))*2^(1/3)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^
(11/6)/b^(1/2)+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/
2)/x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(1/2)/
x/b^(1/2))*2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/72*(a^(1/3)-(-b*x^2+a)^(1/3))
*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a
)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b
/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*
(1-3^(1/2)))^2)^(1/2)-1/48*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a
)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*
3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/
3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{425, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$$

$$= \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right.}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\right.}{16\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$+\frac{x(a-bx^2)^{2/3}}{24a^2(3a+bx^2)}-\frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(24*a^2*(3*a + b*x^2)) - x/(24*a^2*((1 - Sqrt[3])*a^(
1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/
3)*Sqrt[3]*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(
a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcT
anh[(Sqrt[b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x
)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt
[b]) - (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*3^(3/4
)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + ((a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
```

```
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
)/(12*Sqrt[2]*3^(1/4)*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} - \frac{\int \frac{-7ab-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{24a^2b} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{72a^2} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{4a} \\
&= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} \\
&\quad - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}} \\
&\quad - \frac{\sqrt{-bx^2}\,\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{48a^2bx}
\end{aligned}
$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}$$
$$+ \frac{\sqrt{-bx^2}\,\mathrm{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{48a^2bx}$$
$$- \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{48a^{5/3}bx}$$

$$= \frac{x\left(a-bx^2\right)^{2/3}}{24a^2\left(3a+bx^2\right)} - \frac{x}{24a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$
$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}$$
$$- \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-}{16\ 3^{3/4}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+ \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right.}{12\sqrt{2}\sqrt[4]{3}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$$
$$= \frac{x\left(\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+\frac{27\left(\frac{a-bx^2}{a^2}+\frac{63\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{9a\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\right.}\right)}{3a+bx^2}\right)}{648\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3
*(b*x^2)/a])/a^3 + (27*((a - b*x^2)/a^2 + (63*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(
b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2))
)/(648*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{3}}\left(b\,x^2+3a\right)^2}dx$$

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**2,x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{1/3}\,(b\,x^2+3\,a)^2}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2),x)
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^2), x)
```

3.128 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 818

$$
\begin{aligned}
&\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx \\
&= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} \\
&\quad + \frac{5\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\ 2^{2/3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{192\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} \\
&\quad + \frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\right.}{144\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}
\end{aligned}
$$

```
[Out] 1/48*x*(-b*x^2+a)^(2/3)/a^2/(b*x^2+3*a)^2+5/288*x*(-b*x^2+a)^(2/3)/a^3/(b*x
^2+3*a)-5/288*x/a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+5/288*arctanh(x
*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/
2)-5/864*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+5/864*arctan(a
^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/
6)*3^(1/2)/b^(1/2)+5/864*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)
*3^(1/2)/b^(1/2)+5/864*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1
/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1
/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1
/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)-5/57
6*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \frac{5(a-bx^2)^{2/3}\,x}{288a^3\,(bx^2+3a)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{(a-bx^2)^{2/3}\,x}{48a^2\,(bx^2+3a)^2} + \frac{5\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\;2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\;2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\;2^{2/3}a^{17/6}\sqrt{b}}$$

$$- \frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{192\;3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

$$+ \frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\right.}{144\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(a - b*x^2)^(2/3))/(48*a^2*(3*a + b*x^2)^2) + (5*x*(a - b*x^2)^(2/3))/(2
88*a^3*(3*a + b*x^2)) - (5*x)/(288*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) + (5*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a
^(17/6)*Sqrt[b]) + (5*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2
)^(1/3)))/(Sqrt[b]*x)])/(144*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) - (5*ArcTanh
[(Sqrt[b]*x)/Sqrt[a]])/(432*2^(2/3)*a^(17/6)*Sqrt[b]) + (5*ArcTanh[(Sqrt[b]
*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(144*2^(2/3)*a^(17/6)
*Sqrt[b]) - (5*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
 (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(19
2*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (5*(a^(1/3) - (a - b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/
3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
```

```
4*Sqrt[3]])/(144*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x(a-bx^2)^{2/3}}{48a^2\left(3a+bx^2\right)^2} - \frac{\int \frac{-15ab+\frac{5b^2x^2}{3}}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}\,dx}{48a^2b} \\
&= \frac{x(a-bx^2)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{\int \frac{100a^2b^2+\frac{20}{3}ab^3x^2}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{1152a^4b^2} \\
&= \frac{x(a-bx^2)^{2/3}}{48a^2\left(3a+bx^2\right)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3\left(3a+bx^2\right)} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{864a^3} + \frac{5\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx}{72a^2}
\end{aligned}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+ \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$- \frac{\left(5\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{576a^3bx}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$+ \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$+ \frac{\left(5\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{576a^3bx}$$
$$- \frac{\left(5\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{576a^{8/3}bx}$$

$$= \frac{x(a-bx^2)^{2/3}}{48a^2(3a+bx^2)^2} + \frac{5x(a-bx^2)^{2/3}}{288a^3(3a+bx^2)} - \frac{5x}{288a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+ \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{5\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{144\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{432\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{144\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$- \frac{5\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{192\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+ \frac{5\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{\ }\right.}{144\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.31

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$$
$$= \frac{x\left(\frac{27a(a-bx^2)(21a+5bx^2)}{(3a+bx^2)^2}+5bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{6}{(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right.\right.}\right.}{7776a^4\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*((27*a*(a - b*x^2)*(21*a + 5*b*x^2))/(3*a + b*x^2)^2 + 5*b*x^2*(1 - (b*x
^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + (6075*
a^3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(
9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-Appel
lF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a])))))/(7776*a^4*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}} \left(b\,x^{2}+3a\right)^{3}}dx$$

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a)**3,x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^2+3\,a)^3(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(a-b\,x^2)^{1/3}\,(b\,x^2+3\,a)^3}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3),x)
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)^3), x)
```

3.129 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 623

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \frac{2538}{91}ax(a-bx^2)^{2/3}$$

$$+\frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2)+\frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 2538/91*a*x*(-b*x^2+a)^(2/3)+81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)+6*x*(b*x^
2+3*a)^2/(-b*x^2+a)^(1/3)+20088/91*a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2)))-6696/91*3^(3/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^
2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I
-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x
^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+10044/91*3^(1
/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)
*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x
^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {424, 542, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx =$$

$$-\frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+\frac{2538}{91}ax\left(a-bx^2\right)^{2/3}+\frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}}+\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)$$

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (2538*a*x*(a - b*x^2)^(2/3))/91 + (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/13
 + (6*x*(3*a + b*x^2)^2)/(a - b*x^2)^(1/3) + (20088*a^2*x)/(91*((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))) + (10044*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)
*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) +
(a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) -
(a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (669
6*Sqrt[2]*3^(3/4)*a^(7/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sq
rt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{(3a+bx^2)(6a^2b+18ab^2x^2)}{\sqrt[3]{a-bx^2}}\,dx}{2ab} \\
&= \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} + \frac{9\int \frac{-132a^3b^2-188a^2b^3x^2}{\sqrt[3]{a-bx^2}}\,dx}{26ab^2} \\
&= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} \\
&\quad + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} - \frac{1}{91}\left(6696a^2\right)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx \\
&= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} \\
&\quad + \frac{\left(10044a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx} \\
&= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} \\
&\quad - \frac{\left(10044a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx} \\
&\quad + \frac{\left(10044\left(1+\sqrt{3}\right)a^{7/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx}
\end{aligned}
$$

$$= \frac{2538}{91}ax\left(a-bx^2\right)^{2/3} + \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) + \frac{6x(3a+bx^2)^2}{\sqrt[3]{a-bx^2}} + \frac{20088a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{10044\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{6696\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.12

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = -\frac{3x\left(-3051a^2+132abx^2+7b^2x^4+2232a^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{91\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(4/3),x]
```

```
[Out] (-3*x*(-3051*a^2 + 132*a*b*x^2 + 7*b^2*x^4 + 2232*a^2*(1 - (b*x^2)/a)^(1/3)
*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a]))/(91*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{(b\,x^2+3a)^3}{(-b\,x^2+a)^{\frac{4}{3}}}dx$$

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="fricas")
[Out] integral((b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3)
/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(3a+bx^2)^3}{(a-bx^2)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(4/3),x)
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(4/3), x)
```

Maxima [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{4/3}}\,dx = \int \frac{(b\,x^2+3\,a)^3}{(a-b\,x^2)^{4/3}}\,dx$$

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(4/3), x)
```

3.130 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 592

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\cdots\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{108\sqrt{2}\,3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\cdots\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 45/7*x*(-b*x^2+a)^(2/3)+6*x*(b*x^2+3*a)/(-b*x^2+a)^(1/3)+324/7*a*x/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(
1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1
/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)/b/x/
(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)+162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b
*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),
2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x
```

```
^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b/x/(-a
^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {424, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = -\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$+\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}}+\frac{45}{7}x\left(a-bx^2\right)^{2/3}$$

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (45*x*(a - b*x^2)^(2/3))/7 + (6*x*(3*a + b*x^2))/(a - b*x^2)^(1/3) + (324*a
*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (162*3^(1/4)*Sqrt[2 +
 Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a
- b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)]) - (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
 - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
```

```
EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{3\int \frac{6a^2b+10ab^2x^2}{\sqrt[3]{a-bx^2}}\,dx}{2ab}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} - \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx} + \frac{\left(162\left(1+\sqrt{3}\right)a^{4/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= \frac{45}{7}x\left(a-bx^2\right)^{2/3} + \frac{6x(3a+bx^2)}{\sqrt[3]{a-bx^2}} + \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 14.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.28

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \frac{x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Gamma}\left(\frac{1}{3}\right)\left(63a(45a^2+10abx^2+b^2x^4)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{2},\frac{bx^2}{a}\right)\right.}{945}$$

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(4/3),x]
```

```
[Out] (x*(1 - (b*x^2)/a)^(1/3)*Gamma[1/3]*(63*a*(45*a^2 + 10*a*b*x^2 + b^2*x^4)*H
ypergeometric2F1[1/2, 4/3, 7/2, (b*x^2)/a] + 32*b*x^2*(18*a^2 + 9*a*b*x^2 +
 b^2*x^4)*Hypergeometric2F1[3/2, 7/3, 9/2, (b*x^2)/a] + 16*b*(3*a*x + b*x^3
)^2*HypergeometricPFQ[{3/2, 2, 7/3}, {1, 9/2}, (b*x^2)/a]))/(945*a^2*(a - b
*x^2)^(1/3)*Gamma[4/3])
```

Maple [F]

$$\int \frac{(b\,x^2+3a)^2}{(-b\,x^2+a)^{\frac{4}{3}}}dx$$

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

```
[Out] int((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 6*a*b*x^2 + 9*a^2)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*
x^2 + a^2), x)
```

Sympy [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(3a+bx^2)^2}{(a-bx^2)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(4/3),x)
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(4/3), x)
```

Maxima [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{(b\,x^2+3\,a)^2}{(a-b\,x^2)^{4/3}}\,dx$$

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)^2/(a - b*x^2)^(4/3), x)
```

3.131 $\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 561

$$\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx=\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 6*x/(-b*x^2+a)^(1/3)+9*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))-3*3^(3/4)*
a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+
3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((
a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/
3)*(1-3^(1/2)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/2*3^(1/4)*a^(1/3)*(a^(1/3)-(-b*x^
2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3
)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6
^(1/2)+1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(
1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {393, 241, 310, 225, 1893}

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx =$$

$$-\frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7\right.}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{6x}{\sqrt[3]{a-bx^2}}+\frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
```

```
[Out] (6*x)/(a - b*x^2)^(1/3) + (9*x)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))
 + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3
]])/(2*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^
(1/3) - (a - b*x^2)^(1/3))^2)]) - (3*Sqrt[2]*3^(3/4)*a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + S
qrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(
1/3))], -7 + 4*Sqrt[3]])/(b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
```

```
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{6x}{\sqrt[3]{a-bx^2}} - 3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx \\ &= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}\end{aligned}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} - \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx} + \frac{\left(9\left(1+\sqrt{3}\right)\sqrt[3]{a}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{2bx}$$

$$= \frac{6x}{\sqrt[3]{a-bx^2}} + \frac{9x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}$$

$$+ \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\Bigg(\sin^{-1}\Bigg(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{3\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\Bigg(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.09

$$\int \frac{3a+bx^2}{\left(a-bx^2\right)^{4/3}}\,dx = \frac{6x-3x\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[3]{a-bx^2}}$$

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(4/3),x]
[Out] (6*x - 3*x*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
])/(a - b*x^2)^(1/3)
```

Maple [F]

$$\int \frac{b\,x^2+3a}{\left(-b\,x^2+a\right)^{\frac{4}{3}}}dx$$

[In] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)

[Out] int((b*x^2+3*a)/(-b*x^2+a)^(4/3),x)

Fricas [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="fricas")

[Out] integral((b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)

Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.11

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \frac{3x{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[3]{a}} + \frac{bx^3{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(4/3),x)

[Out] 3*x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/3) + b*x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(4/3))

Maxima [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{4/3}}\,dx = \int \frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="maxima")

[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)

Giac [F]

$$\int \frac{3a + bx^2}{(a - bx^2)^{4/3}}\,dx = \int \frac{bx^2 + 3\,a}{(-bx^2 + a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(4/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3a + bx^2}{(a - bx^2)^{4/3}}\,dx = \int \frac{b\,x^2 + 3\,a}{(a - b\,x^2)^{4/3}}\,dx$$

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(4/3),x)
[Out] int((3*a + b*x^2)/(a - b*x^2)^(4/3), x)
```

3.132 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 776

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\,2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\,2^{2/3}a^{11/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\,2^{2/3}a^{11/6}\sqrt{b}} + \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\right.}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 3/8*x/a^2/(-b*x^2+a)^(1/3)+3/8*x/a^2/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)+1/16*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3
```

```
)/a^(11/6)/b^(1/2)-1/48*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(11/6)/b^(1/2)
+1/48*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*
2^(1/3)/a^(11/6)*3^(1/2)/b^(1/2)+1/48*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(
1/3)/a^(11/6)*3^(1/2)/b^(1/2)-1/8*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-
b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))
,2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(5/3)/b/x*2^(1/2)/(-a^
(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)
^(1/2)+3/16*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)
*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2
/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(
1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(5/3)/b/x/(-a^(1/3
)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/
2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used

$= \{425, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx =$$

$$-\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right), -7+4}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}$$

$$+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$+\frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(8*a^2*(a - b*x^2)^(1/3)) + (3*x)/(8*a^2*((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]
*a^(11/6)*Sqrt[b]) + ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)
^(1/3)))/(Sqrt[b]*x)]/(8*2^(2/3)*Sqrt[3]*a^(11/6)*Sqrt[b]) - ArcTanh[(Sqrt[
b]*x)/Sqrt[a]]/(24*2^(2/3)*a^(11/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)
*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))]/(8*2^(2/3)*a^(11/6)*Sqrt[b]) + (3*
3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(16*a^(5/3
)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))^2)]) - (3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]]
```

```
)/(4*Sqrt[2]*a^(5/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{3\int\frac{-\frac{ab}{3}-\frac{b^2x^2}{3}}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{8a^2b}\\
&= \frac{3x}{8a^2\sqrt[3]{a-bx^2}}-\frac{\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{8a^2}+\frac{\int\frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{4a}\\
&= \frac{3x}{8a^2\sqrt[3]{a-bx^2}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}}\\
&\quad+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}+\frac{\left(3\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{16a^2bx}
\end{aligned}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}} - \frac{\left(3\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{16a^2bx} + \frac{\left(3\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{16a^{5/3}bx}$$

$$= \frac{3x}{8a^2\sqrt[3]{a-bx^2}} + \frac{3x}{8a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{8\ 2^{2/3}\sqrt{3}a^{11/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24\ 2^{2/3}a^{11/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{8\ 2^{2/3}a^{11/6}\sqrt{b}}$$

$$+ \frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\cdots\right.\right.}{16a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\cdots\right.}{4\sqrt{2}a^{5/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \frac{x\left(-\frac{bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{a^3}+27\left(\frac{1}{a^2}-\frac{}{(3a+bx^2)\left(9a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3}\right.\right.}\right.\right.}{72\sqrt[3]{a-}}$$

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-((b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a])/a^3) + 27*(a^(-2) - (3*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a,
 -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*
x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))))/(72*(
a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}\left(b\,x^{2}+3a\right)}dx$$

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \int \frac{1}{(a-bx^2)^{\frac{4}{3}}\cdot(3a+bx^2)}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{4/3}\,(b\,x^2+3\,a)}\,dx$$

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)),x)
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)), x)
```

3.133 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 807

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x}{12a^3\sqrt[3]{a-bx^2}}$$
$$+\frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)}+\frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$+\frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}}{8\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}}{6\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/12*x/a^3/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)+1/12*x/
a^3/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/32*arctanh(x*b^(1/2)/a^(1/6)/
(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/96*arctanh(x
*b^(1/2)/a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/96*arctan(a^(1/6)*(a^(1/3)-2^(
1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+
1/96*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)-1/3
6*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+1/24*(a^(1/3)-(-b*x^2+a)^
(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)
+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b
*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(
-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(bx^2+3a)} + \frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{3}\right)}{8\ 3^{3/4}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x} - \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right)}{6\sqrt{2}\sqrt[4]{3}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,x}$$

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] x/(12*a^3*(a - b*x^2)^(1/3)) + x/(24*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)) +
 x/(12*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(16*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(16*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(48*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(16*2^(2/3)*a^(17/6)*Sqrt[b]) + (Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b
*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))
/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt
[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3
))], -7 + 4*Sqrt[3]])/(8*3^(3/4)*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
- b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - ((a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
```

```
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(6*Sqrt[2]*3^(1/4)*a^(8/3)*b*x*Sqrt[-((a^(
1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-7ab+\frac{5b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx}{24a^2b} \\
&= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-\frac{8}{3}a^2b^2+\frac{16}{9}ab^3x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{64a^4b^2} \\
&= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{36a^3} + \frac{\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{8a^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\ 2^{2/3}a^{17/6}\sqrt{b}} \\
&\quad + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{24a^3bx} \\
&= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} \\
&\quad + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\ 2^{2/3}a^{17/6}\sqrt{b}} \\
&\quad - \frac{\sqrt{-bx^2}\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{24a^3bx} \\
&\quad + \frac{\left(\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{24a^{8/3}bx}
\end{aligned}
$$

$$= \frac{x}{12a^3\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)} + \frac{x}{12a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{16\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{16\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$+ \frac{\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{8\ 3^{3/4}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\sqrt{\ }\right.}{6\sqrt{2}\sqrt[4]{3}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.29

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx = \frac{x\left(-2bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(7a+2bx^2+\frac{\ }{9a\,\text{AppellF1}}\right.}{\ }\right.}{648a^4\sqrt[3]{a}}$$

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-2*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a] + (27*a*(7*a + 2*b*x^2 + (9*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b
*x^2)/a, -1/3*(b*x^2)/a])/(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(
b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]
 + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2))
)/(648*a^4*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3a\right)^{2}}dx$$

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

```
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx=\int \frac{1}{(a-bx^2)^{\frac{4}{3}}\,(3a+bx^2)^2}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**2,x)
```

```
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx=\int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{4/3}\,(b\,x^2+3\,a)^2}\,dx$$

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2),x)
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^2), x)
```

3.134 $\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 849

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2}+\frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)}$$
$$-\frac{19x(a-bx^2)^{2/3}}{1152a^4(3a+bx^2)}+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{bx}}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}$$
$$-\frac{7\text{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}}+\frac{7\text{arctanh}\left(\frac{\sqrt{bx}}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{768\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{3}\right.}{576\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 1/48*x/a^2/(-b*x^2+a)^(1/3)/(b*x^2+3*a)^2+17/192*x/a^3/(-b*x^2+a)^(1/3)/(b*
x^2+3*a)-19/1152*x*(-b*x^2+a)^(2/3)/a^4/(b*x^2+3*a)+19/1152*x/a^4/(-(-b*x^2
+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+7/576*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(
1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(23/6)/b^(1/2)-7/1728*arctanh(x*b^(1/2)/a
^(1/2))*2^(1/3)/a^(23/6)/b^(1/2)+7/1728*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b
*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+7/1728*a
rctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-19/3456*(
a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))
/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(
-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2
)^(1/2)*3^(3/4)/a^(11/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+19/2304*(a^(1/3)-(-b*x^2+a)
^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3
)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-
b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/
2)+1/2*2^(1/2))*3^(1/4)/a^(11/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{(a-bx^2)^{4/3}(3a+bx^2)^3}\,dx = -\frac{19(a-bx^2)^{2/3}x}{1152a^4(bx^2+3a)} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}(bx^2+3a)}$$
$$+\frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{x}{48a^2\sqrt[3]{a-bx^2}(bx^2+3a)^2}$$
$$+\frac{7\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} + \frac{7\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}}$$
$$-\frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$+\frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\sqrt{}\right.}{768\ 3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$
$$-\frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{}\right.}{576\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] x/(48*a^2*(a - b*x^2)^(1/3)*(3*a + b*x^2)^2) + (17*x)/(192*a^3*(a - b*x^2)^
(1/3)*(3*a + b*x^2)) - (19*x*(a - b*x^2)^(2/3))/(1152*a^4*(3*a + b*x^2)) +
(19*x)/(1152*a^4*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (7*ArcTan[(
Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)])/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) + (7*
ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(288*2^(2/3)*Sqrt[3]*a^(23/6)*Sqrt[b]) - (7*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(864*2^(2/3)*a^(23/6)*Sqrt[b]) + (7*ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3)
+ 2^(1/3)*(a - b*x^2)^(1/3)))])/(288*2^(2/3)*a^(23/6)*Sqrt[b]) + (19*Sqrt[2
 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(768*3^(3/4)*a^(11/3)*b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (
a - b*x^2)^(1/3))^2)]) - (19*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) +
```

```
a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
- b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(576*Sq
rt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/(
(1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} - \frac{\int \frac{-15ab+\frac{11b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)^2}\,dx}{48a^2b}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{\int \frac{-6a^2b^2-\frac{170}{9}ab^3x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)^2}\,dx}{128a^4b^2}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)} + \frac{\int \frac{\frac{296a^3b^3}{3}-\frac{152}{9}a^2b^4x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{3072a^6b^3}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)} - \frac{19\int\frac{1}{\sqrt[3]{a-bx^2}}\,dx}{3456a^4} + \frac{7\int\frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{144a^3}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\left(19\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{2304a^4bx}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}} - \frac{\left(19\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{2304a^4bx} + \frac{\left(19\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx,x,\sqrt[3]{a-bx^2}\right)}{2304a^{11/3}bx}$$

$$= \frac{x}{48a^2\sqrt[3]{a-bx^2}\,(3a+bx^2)^2} + \frac{17x}{192a^3\sqrt[3]{a-bx^2}\,(3a+bx^2)} - \frac{19x(a-bx^2)^{2/3}}{1152a^4\,(3a+bx^2)} + \frac{19x}{1152a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} + \frac{7\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{288\ 2^{2/3}\sqrt{3}a^{23/6}\sqrt{b}} - \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{864\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{7\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{288\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{19\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{768\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{19\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\right.}{576\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.30

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^3}\,dx = \frac{x\left(-19bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(273a^2+140abx^2+19b\right.}{}\right.}{31104}$$

```
[In] Integrate[1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x]
```

```
[Out] (x*(-19*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + (27*a*(273*a^2 + 140*a*b*x^2 + 19*b^2*x^4 + (333*a^2*(3*a
+ b*x^2)*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)^2))/(31104*a^5*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}} \left(b\,x^{2}+3a\right)^{3}}dx$$

```
[In] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
[Out] int(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^3}\,dx = \int \frac{1}{\left(a-bx^{2}\right)^{\frac{4}{3}}\left(3a+bx^{2}\right)^{3}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(4/3)/(b*x**2+3*a)**3,x)
[Out] Integral(1/((a - b*x**2)**(4/3)*(3*a + b*x**2)**3), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{4/3}\,(3a+bx^2)^3}\,dx = \int \frac{1}{(bx^{2}+3\,a)^{3}(-bx^{2}+a)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx = \int \frac{1}{\left(bx^2+3\,a\right)^3\left(-bx^2+a\right)^{\frac{4}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(4/3)/(b*x^2+3*a)^3,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^3*(-b*x^2 + a)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)^3}\,dx = \int \frac{1}{\left(a-b\,x^2\right)^{4/3}\left(b\,x^2+3\,a\right)^3}\,dx$$

```
[In] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3),x)
[Out] int(1/((a - b*x^2)^(4/3)*(3*a + b*x^2)^3), x)
```

3.135 $\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 653

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

$$-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}}+\frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}}-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}-\frac{18468\sqrt[4]{3}\cdots}{\cdots}$$

```
[Out] -3240/91*a*x*(-b*x^2+a)^(2/3)-81/13*x*(-b*x^2+a)^(2/3)*(b*x^2+3*a)-9/2*x*(b
*x^2+3*a)^2/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)^3/(-b*x^2+a)^(4/3)-36936/91*
a^2*x/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+12312/91*3^(3/4)*a^(7/3)*(a^(
1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1
/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2
)))^2)^(1/2)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)-18468/91*3^(1/4)*a^(7/3)*(a^(1/3)-(-b*x^2+a)^(1
/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a
^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x
^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+
1/2*2^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^
(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {424, 540, 542, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\cdots\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{18468\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\cdots\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}}-\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}}$$
$$-\frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right)-\frac{3240}{91}ax\left(a-bx^2\right)^{2/3}$$

```
[In] Int[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3240*a*x*(a - b*x^2)^(2/3))/91 - (81*x*(a - b*x^2)^(2/3)*(3*a + b*x^2))/1
3 - (9*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2)^3)/(2*
(a - b*x^2)^(4/3)) - (36936*a^2*x)/(91*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))) - (18468*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(7/3)*(a^(1/3) - (a - b*x^2)^
(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 -
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1
- Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) + (12312*Sqrt[2]*3^(3/4)*a^(7/
3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3)
+ (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(91*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
```

```
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
```

```
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{(3a+bx^2)^2(-12a^2b+20ab^2x^2)}{(a-bx^2)^{4/3}}\,dx}{8ab}$$

$$= -\frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{9\int \frac{(3a+bx^2)(-48a^3b^2-48a^2b^3x^2)}{\sqrt[3]{a-bx^2}}\,dx}{16a^2b^2}$$

$$= -\frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{27\int \frac{768a^4b^3+640a^3b^4x^2}{\sqrt[3]{a-bx^2}}\,dx}{208a^2b^3}$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} + \frac{1}{91}(12312a^2)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{3240}{91}ax(a-bx^2)^{2/3} - \frac{81}{13}x(a-bx^2)^{2/3}(3a+bx^2) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2(a-bx^2)^{4/3}} - \frac{\left(18468a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}} + \frac{\left(18468a^2\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx} - \frac{\left(18468\left(1+\sqrt{3}\right)a^{7/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{91bx}$$

$$= -\frac{3240}{91}ax\left(a-bx^2\right)^{2/3} - \frac{81}{13}x\left(a-bx^2\right)^{2/3}\left(3a+bx^2\right) - \frac{9x(3a+bx^2)^2}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)^3}{2\left(a-bx^2\right)^{4/3}} - \frac{36936a^2x}{91\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{18468\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{12312\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-}}\right.\right.}{91bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.15

$$\int\frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = -\frac{3\left(1647a^3x-4743a^2bx^3+177ab^2x^5+7b^3x^7-4104a^2x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\right.\right.}{91\left(a-bx^2\right)^{4/3}}$$

```
[In] Integrate[(3*a + b*x^2)^4/(a - b*x^2)^(7/3),x]
```

```
[Out] (-3*(1647*a^3*x - 4743*a^2*b*x^3 + 177*a*b^2*x^5 + 7*b^3*x^7 - 4104*a^2*x*(
a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a
]))/(91*(a - b*x^2)^(4/3))
```

Maple [F]

$$\int \frac{(b\,x^2+3a)^4}{(-b\,x^2+a)^{\frac{7}{3}}}dx$$

[In] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)

[Out] int((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x)

Fricas [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="fricas")

[Out] integral(-(b^4*x^8 + 12*a*b^3*x^6 + 54*a^2*b^2*x^4 + 108*a^3*b*x^2 + 81*a^4)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)

Sympy [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^4}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

[In] integrate((b*x**2+3*a)**4/(-b*x**2+a)**(7/3),x)

[Out] Integral((3*a + b*x**2)**4/(a - b*x**2)**(7/3), x)

Maxima [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="maxima")

[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)

Giac [F]

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^4}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^4/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^4/(-b*x^2 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^4}{(a-bx^2)^{7/3}}\,dx = \int \frac{(b\,x^2+3\,a)^4}{(a-b\,x^2)^{7/3}}\,dx$$

```
[In] int((3*a + b*x^2)^4/(a - b*x^2)^(7/3),x)
[Out] int((3*a + b*x^2)^4/(a - b*x^2)^(7/3), x)
```

3.136 $\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 596

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = -\frac{27}{14}x\left(a-bx^2\right)^{2/3}+\frac{3x(3a+bx^2)^2}{2\left(a-bx^2\right)^{4/3}}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right|}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] -27/14*x*(-b*x^2+a)^(2/3)+3/2*x*(b*x^2+3*a)^2/(-b*x^2+a)^(4/3)-324/7*a*x/(-
(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+108/7*3^(3/4)*a^(4/3)*(a^(1/3)-(-b*x^
2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^
(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*2^(1/2)*((a^(2/3)+a^(1/3)*(-b*x^2
+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2
)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^
(1/2)))^2)^(1/2)-162/7*3^(1/4)*a^(4/3)*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE
((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1
/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(
```

```
-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))/b
/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/
2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {424, 21, 396, 241, 310, 225, 1893}

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{(1}{(1}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}}-\frac{27}{14}x(a-bx^2)^{2/3}-\frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

```
[In] Int[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (-27*x*(a - b*x^2)^(2/3))/14 + (3*x*(3*a + b*x^2)^2)/(2*(a - b*x^2)^(4/3))
- (324*a*x)/(7*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) - (162*3^(1/4)*
Sqrt[2 + Sqrt[3]]*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(
1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(7*b*x*Sqr
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)]) + (108*Sqrt[2]*3^(3/4)*a^(4/3)*(a^(1/3) - (a - b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 +
 4*Sqrt[3]])/(7*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sq
rt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
```

```
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{3\int \frac{(3a+bx^2)(-12a^2b+12ab^2x^2)}{(a-bx^2)^{4/3}}\,dx}{8ab}$$

$$= \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{9}{2}\int \frac{3a+bx^2}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{1}{7}(108a)\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} + \frac{\left(162a\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx} - \frac{\left(162\left(1+\sqrt{3}\right)a^{4/3}\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{7bx}$$

$$= -\frac{27}{14}x\left(a-bx^2\right)^{2/3} + \frac{3x(3a+bx^2)^2}{2(a-bx^2)^{4/3}} - \frac{324ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} - \frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ldots}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} + \frac{108\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-b\ldots}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 15.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.14

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \frac{81a^2x+90abx^3-3b^2x^5+108ax(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{7\,(a-bx^2)^{4/3}}$$

```
[In] Integrate[(3*a + b*x^2)^3/(a - b*x^2)^(7/3),x]
```

```
[Out] (81*a^2*x + 90*a*b*x^3 - 3*b^2*x^5 + 108*a*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1
/3)*Hypergeometric2F1[1/3, 1/2, 3/2, (b*x^2)/a])/(7*(a - b*x^2)^(4/3))
```

Maple [F]

$$\int \frac{(b\,x^2+3a)^3}{(-b\,x^2+a)^{\frac{7}{3}}}dx$$

```
[In] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b^3*x^6 + 9*a*b^2*x^4 + 27*a^2*b*x^2 + 27*a^3)*(-b*x^2 + a)^(2/3
)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b*x^2 - a^3), x)
```

Sympy [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^3}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x**2+3*a)**3/(-b*x**2+a)**(7/3),x)
```

```
[Out] Integral((3*a + b*x**2)**3/(a - b*x**2)**(7/3), x)
```

Maxima [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Giac [F]

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^3}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^3/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^3/(-b*x^2 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3a+bx^2)^3}{(a-bx^2)^{7/3}}\,dx = \int \frac{(b\,x^2+3\,a)^3}{(a-b\,x^2)^{7/3}}\,dx$$

```
[In] int((3*a + b*x^2)^3/(a - b*x^2)^(7/3),x)
[Out] int((3*a + b*x^2)^3/(a - b*x^2)^(7/3), x)
```

3.137 $\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 44

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}} \, dx = \frac{9x}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}}$$

```
[Out] 9/2*x/(-b*x^2+a)^(1/3)+3/2*x*(b*x^2+3*a)/(-b*x^2+a)^(4/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {424, 391}

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}} \, dx = \frac{3x(3a+bx^2)}{2(a-bx^2)^{4/3}} + \frac{9x}{2\sqrt[3]{a-bx^2}}$$

```
[In] Int[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
[Out] (9*x)/(2*(a - b*x^2)^(1/3)) + (3*x*(3*a + b*x^2))/(2*(a - b*x^2)^(4/3))
```

Rule 391

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[c*x*((a + b*x^n)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
```

```
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3x(3a+bx^2)}{2\left(a-bx^2\right)^{4/3}} - \frac{3\int \frac{-12a^2b+4ab^2x^2}{(a-bx^2)^{4/3}}\,dx}{8ab} \\
&= \frac{9x}{2\sqrt[3]{a-bx^2}} + \frac{3x(3a+bx^2)}{2\left(a-bx^2\right)^{4/3}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 15.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$
\int \frac{\left(3a+bx^2\right)^2}{\left(a-bx^2\right)^{7/3}}\,dx = \frac{9ax-3bx^3}{\left(a-bx^2\right)^{4/3}}
$$

```
[In] Integrate[(3*a + b*x^2)^2/(a - b*x^2)^(7/3),x]
[Out] (9*a*x - 3*b*x^3)/(a - b*x^2)^(4/3)
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{3x\left(-b\,x^{2}+3a\right)}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}$	24
trager	$\frac{3x\left(-b\,x^{2}+3a\right)}{\left(-b\,x^{2}+a\right)^{\frac{4}{3}}}$	24

```
[In] int((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x,method=_RETURNVERBOSE)
[Out] 3/(-b*x^2+a)^(4/3)*x*(-b*x^2+3*a)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = -\frac{3\,(bx^3-3\,ax)(-bx^2+a)^{\frac{2}{3}}}{b^2x^4-2\,abx^2+a^2}$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="fricas")
[Out] -3*(b*x^3 - 3*a*x)*(-b*x^2 + a)^(2/3)/(b^2*x^4 - 2*a*b*x^2 + a^2)
```

Sympy [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{(3a+bx^2)^2}{(a-bx^2)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x**2+3*a)**2/(-b*x**2+a)**(7/3),x)
[Out] Integral((3*a + b*x**2)**2/(a - b*x**2)**(7/3), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{3\,(bx^3-3\,ax)}{(bx^2-a)(-bx^2+a)^{\frac{1}{3}}}$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="maxima")
[Out] 3*(b*x^3 - 3*a*x)/((b*x^2 - a)*(-b*x^2 + a)^(1/3))
```

Giac [F]

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{(bx^2+3\,a)^2}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)^2/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)^2/(-b*x^2 + a)^(7/3), x)
```

Mupad [B] (verification not implemented)

Time = 4.88 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{(3a+bx^2)^2}{(a-bx^2)^{7/3}}\,dx = \frac{3\,x\,(a-b\,x^2)+6\,a\,x}{(a-b\,x^2)^{4/3}}$$

```
[In] int((3*a + b*x^2)^2/(a - b*x^2)^(7/3),x)
[Out] (3*x*(a - b*x^2) + 6*a*x)/(a - b*x^2)^(4/3)
```

3.138 $\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 590

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x}{2\left(a-bx^2\right)^{4/3}}+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4\cdots}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+\cdots}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 3/2*x/(-b*x^2+a)^(4/3)+9/4*x/a/(-b*x^2+a)^(1/3)+9/4*x/a/(-(-b*x^2+a)^(1/3)+
a^(1/3)*(1-3^(1/2)))-3/4*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^
(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^
(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1
/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/b/x*2^(1/2)/(-a^(1/3)*(a^
(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+9/
8*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2
)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3
)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)
-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {393, 205, 241, 310, 225, 1893}

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx =$$

$$-\frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+\right.}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$+\frac{9x}{4a\sqrt[3]{a-bx^2}}+\frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}+\frac{3x}{2\left(a-bx^2\right)^{4/3}}$$

```
[In] Int[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (3*x)/(2*(a - b*x^2)^(4/3)) + (9*x)/(4*a*(a - b*x^2)^(1/3)) + (9*x)/(4*a*((
1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt[3]]*(
a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a
 - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[A
rcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(8*a^(2/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/
3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) -
 (3*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^
2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2
]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(2*Sqrt[2]*a^(2/3)*b*x*S
qrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a -
 b*x^2)^(1/3))^2)])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
```

```
inator[p + 1/n] < Denominator[p])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$\text{integral} = \frac{3x}{2(a-bx^2)^{4/3}} + \frac{3}{2}\int \frac{1}{(a-bx^2)^{4/3}}\,dx$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} - \frac{3\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{4a}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} + \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{8abx}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} - \frac{\left(9\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{8abx} + \frac{\left(9\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{8a^{2/3}bx}$$

$$= \frac{3x}{2(a-bx^2)^{4/3}} + \frac{9x}{4a\sqrt[3]{a-bx^2}} + \frac{9x}{4a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\ldots\right)}{8a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}} - \frac{3\,3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7\ldots\right)}{2\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.13

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{15ax-9bx^3-3x(a-bx^2)\sqrt[3]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{4a(a-bx^2)^{4/3}}$$

```
[In] Integrate[(3*a + b*x^2)/(a - b*x^2)^(7/3),x]
```

```
[Out] (15*a*x - 9*b*x^3 - 3*x*(a - b*x^2)*(1 - (b*x^2)/a)^(1/3)*Hypergeometric2F1
[1/3, 1/2, 3/2, (b*x^2)/a])/(4*a*(a - b*x^2)^(4/3))
```

Maple [F]

$$\int \frac{b\,x^{2}+3a}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}}dx$$

```
[In] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

```
[Out] int((b*x^2+3*a)/(-b*x^2+a)^(7/3),x)
```

Fricas [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \int \frac{bx^2+3\,a}{\left(-bx^2+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2 + 3*a)*(-b*x^2 + a)^(2/3)/(b^3*x^6 - 3*a*b^2*x^4 + 3*a^2*b
*x^2 - a^3), x)
```

Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.10

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx = \frac{3x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{4}{3}}} + \frac{bx^3\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{3}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{3}}}$$

```
[In] integrate((b*x**2+3*a)/(-b*x**2+a)**(7/3),x)
```

```
[Out] 3*x*hyper((1/2, 7/3), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(4/3) + b*x**3
*hyper((3/2, 7/3), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/3))
```

Maxima [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx=\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="maxima")
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Giac [F]

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx=\int \frac{bx^2+3\,a}{(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate((b*x^2+3*a)/(-b*x^2+a)^(7/3),x, algorithm="giac")
[Out] integrate((b*x^2 + 3*a)/(-b*x^2 + a)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3a+bx^2}{(a-bx^2)^{7/3}}\,dx=\int \frac{b\,x^2+3\,a}{(a-b\,x^2)^{7/3}}\,dx$$

```
[In] int((3*a + b*x^2)/(a - b*x^2)^(7/3),x)
[Out] int((3*a + b*x^2)/(a - b*x^2)^(7/3), x)
```

3.139 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 796

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx = \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$
$$+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$
$$+\frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+}{128a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$-\frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+}{32\sqrt{2}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 3/32*x/a^2/(-b*x^2+a)^(4/3)+21/64*x/a^3/(-b*x^2+a)^(1/3)+21/64*x/a^3/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))+1/64*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2
^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a^(17/6)/b^(1/2)-1/192*arctanh(x*b^(1/2)/
a^(1/2))*2^(1/3)/a^(17/6)/b^(1/2)+1/192*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b
*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)+1/192*ar
ctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(17/6)*3^(1/2)/b^(1/2)-7/64*(a^(1
/3)-(-b*x^2+a)^(1/3))*EllipticF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(
-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x
^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1
/2)*3^(3/4)/a^(8/3)/b/x*2^(1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*
x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)+21/128*(a^(1/3)-(-b*x^2+a)^(1/3)
)*EllipticE((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1
/3)*(1-3^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+
a)^(2/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2
*2^(1/2))*3^(1/4)/a^(8/3)/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^
2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)}\,dx = \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{3x}{32a^2(a-bx^2)^{4/3}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} + \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+\right.}{128a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x} - \frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+\right.}{32\sqrt{2}a^{8/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (3*x)/(32*a^2*(a - b*x^2)^(4/3)) + (21*x)/(64*a^3*(a - b*x^2)^(1/3)) + (21*
x)/(64*a^3*((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))) + ArcTan[(Sqrt[3]*S
qrt[a])/(Sqrt[b]*x)]/(32*2^(2/3)*Sqrt[3]*a^(17/6)*Sqrt[b]) + ArcTan[(Sqrt[3
]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)]/(32*2^(2/3)*S
qrt[3]*a^(17/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(96*2^(2/3)*a^(17/6
)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1
/3)))]/(32*2^(2/3)*a^(17/6)*Sqrt[b]) + (21*3^(1/4)*Sqrt[2 + Sqrt[3]]*(a^(1/
3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin
[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a -
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(128*a^(8/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3)
- (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (7
*3^(3/4)*(a^(1/3) - (a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^
(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(32*Sqrt[2]*a^(8/3)*b*x*Sqr
```

```
t[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b
*x^2)^(1/3))^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{3\int \frac{\frac{23ab}{3}+\frac{5b^2x^2}{3}}{(a-bx^2)^{4/3}(3a+bx^2)}\,dx}{32a^2b}\\
&= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{9\int \frac{-\frac{68}{9}a^2b^2-\frac{28}{9}ab^3x^2}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{256a^4b^2}\\
&= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{21x}{64a^3\sqrt[3]{a-bx^2}}-\frac{7\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{64a^3}+\frac{\int \frac{1}{\sqrt[3]{a-bx^2}(3a+bx^2)}\,dx}{16a^2}
\end{aligned}
$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$+ \frac{\left(21\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int \frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{128a^3bx}$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}} + \frac{21x}{64a^3\sqrt[3]{a-bx^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$+ \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$- \frac{\left(21\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{128a^3bx}$$

$$+ \frac{\left(21\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{128a^{8/3}bx}$$

$$= \frac{3x}{32a^2\left(a-bx^2\right)^{4/3}}+\frac{21x}{64a^3\sqrt[3]{a-bx^2}}+\frac{21x}{64a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{32\ 2^{2/3}\sqrt{3}a^{17/6}\sqrt{b}}$$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{96\ 2^{2/3}a^{17/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{32\ 2^{2/3}a^{17/6}\sqrt{b}}$$

$$+\frac{21\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx}}\right.\right.}{128a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$-\frac{7\ 3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+\left(a-bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+\right.}{32\sqrt{2}a^{8/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.32 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.31

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\frac{x\left(-7bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+27a\left(\frac{9a-7bx^2}{a-bx^2}-\frac{\quad}{\left(3a+bx^2\right)}\right.\right.}{576}$$

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x]
```

```
[Out] (x*(-7*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -1
/3*(b*x^2)/a] + 27*a*((9*a - 7*b*x^2)/(a - b*x^2) - (51*a^2*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((3*a + b*x^2)*(9*a*AppellF1[1/2, 1
/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/
2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*
(b*x^2)/a]))))))/(576*a^4*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}} \left(b\,x^{2}+3a\right)}dx$$

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^{2}\right)^{7/3}\left(3a+bx^{2}\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a-bx^{2}\right)^{7/3}\left(3a+bx^{2}\right)}\,dx = \int \frac{1}{\left(a-bx^{2}\right)^{\frac{7}{3}}\cdot\left(3a+bx^{2}\right)}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a-bx^{2}\right)^{7/3}\left(3a+bx^{2}\right)}\,dx = \int \frac{1}{\left(bx^{2}+3\,a\right)\left(-bx^{2}+a\right)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Giac [F]

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(7/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx=\int \frac{1}{\left(a-b\,x^2\right)^{7/3}\,\left(b\,x^2+3\,a\right)}\,dx$$

```
[In] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)),x)
[Out] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)), x)
```

3.140 $\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 827

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}}$$

$$+ \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)} + \frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}$$

$$+ \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$

$$+ \frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4}{512\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

$$- \frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4\sqrt{}}{384\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

```
[Out] 5/384*x/a^3/(-b*x^2+a)^(4/3)+79/768*x/a^4/(-b*x^2+a)^(1/3)+1/24*x/a^2/(-b*x
^2+a)^(4/3)/(b*x^2+3*a)+79/768*x/a^4/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))
)+3/256*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/
3)/a^(23/6)/b^(1/2)-1/256*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(23/6)/b^(1/
2)+1/256*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2
))*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)+1/256*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))
*2^(1/3)/a^(23/6)*3^(1/2)/b^(1/2)-79/2304*(a^(1/3)-(-b*x^2+a)^(1/3))*Ellipt
icF((-(-b*x^2+a)^(1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3
^(1/2))),2*I-I*3^(1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3)
)/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*3^(3/4)/a^(11/3)/b/x*2^(
1/2)/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)+79/1536*(a^(1/3)-(-b*x^2+a)^(1/3))*EllipticE((-(-b*x^2+a)^(
1/3)+a^(1/3)*(1+3^(1/2)))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(1/2))),2*I-I*3^(
1/2))*((a^(2/3)+a^(1/3)*(-b*x^2+a)^(1/3)+(-b*x^2+a)^(2/3))/(-(-b*x^2+a)^(1/
3)+a^(1/3)*(1-3^(1/2)))^2)^(1/2)*(1/2*6^(1/2)+1/2*2^(1/2))*3^(1/4)/a^(11/3)
/b/x/(-a^(1/3)*(a^(1/3)-(-b*x^2+a)^(1/3))/(-(-b*x^2+a)^(1/3)+a^(1/3)*(1-3^(
1/2)))^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 827, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{425, 541, 544, 241, 310, 225, 1893, 402\}$

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(bx^2+3a)} + \frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)|-7+4}{512\ 3^{3/4}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x} - \frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a-bx^2}\sqrt[3]{a}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right),-7+4}{384\sqrt{2}\sqrt[4]{3}a^{11/3}b\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}x}$$

```
[In] Int[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (5*x)/(384*a^3*(a - b*x^2)^(4/3)) + (79*x)/(768*a^4*(a - b*x^2)^(1/3)) + x/
(24*a^2*(a - b*x^2)^(4/3)*(3*a + b*x^2)) + (79*x)/(768*a^4*((1 - Sqrt[3])*a
^(1/3) - (a - b*x^2)^(1/3))) + (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x
)])/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (Sqrt[3]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1
/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)])/(128*2^(2/3)*a^(23/6)*Sqrt[
b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(128*2^(2/3)*a^(23/6)*Sqrt[b]) + (3*ArcT
anh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a - b*x^2)^(1/3)))])/(128*2^(2
/3)*a^(23/6)*Sqrt[b]) + (79*Sqrt[2 + Sqrt[3]]*(a^(1/3) - (a - b*x^2)^(1/3))
*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))], -7 + 4*
Sqrt[3]])/(512*3^(3/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a - b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2)]) - (79*(a^(1/3) - (
a - b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a - b*x^2)^(1/3) + (a - b*x^2)^(
2/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
```

```
 Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(384*Sqrt[2]*3^(1/4)*a^(11/3)*b*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a - b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a - b*x^2)^(1/3)
)^2)])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 241

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[3*(Sqrt[b*x^2]/(2*b*x))
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 310

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(-(1 + Sqrt[3]))*(s/r), Int[1/Sqrt[a + b*x^
3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x]]
 /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1893

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{\int \frac{-7ab+\frac{11b^2x^2}{3}}{\left(a-bx^2\right)^{7/3}\left(3a+bx^2\right)}\,dx}{24a^2b} \\
&= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{\int \frac{-\frac{194}{3}a^2b^2-\frac{50}{9}ab^3x^2}{\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)}\,dx}{256a^4b^2} \\
&= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} \\
&\quad + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} - \frac{3\int \frac{\frac{344a^3b^3}{9}+\frac{632}{27}a^2b^4x^2}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{2048a^6b^3} \\
&= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} \\
&\quad - \frac{79\int \frac{1}{\sqrt[3]{a-bx^2}}\,dx}{2304a^4} + \frac{3\int \frac{1}{\sqrt[3]{a-bx^2}\left(3a+bx^2\right)}\,dx}{64a^3}
\end{aligned}
$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\left(79\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int\frac{x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{1536a^4bx}$$

$$= \frac{5x}{384a^3\left(a-bx^2\right)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2\left(a-bx^2\right)^{4/3}\left(3a+bx^2\right)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} - \frac{\left(79\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-x}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{1536a^4bx} + \frac{\left(79\left(1+\sqrt{3}\right)\sqrt{-bx^2}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-a+x^3}}\,dx, x, \sqrt[3]{a-bx^2}\right)}{1536a^{11/3}bx}$$

$$= \frac{5x}{384a^3(a-bx^2)^{4/3}} + \frac{79x}{768a^4\sqrt[3]{a-bx^2}} + \frac{x}{24a^2(a-bx^2)^{4/3}(3a+bx^2)}$$
$$+ \frac{79x}{768a^4\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$+ \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$- \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}} + \frac{3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{128\ 2^{2/3}a^{23/6}\sqrt{b}}$$
$$+ \frac{79\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}E\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right.\right.}{512\ 3^{3/4}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$
$$- \frac{79\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a-bx^2}+(a-bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}F\left(\sin^{-1}\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}}\right)\middle|-7+4\right.}{384\sqrt{2}\sqrt[4]{3}a^{11/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \frac{x\left(-79bx^2\sqrt[3]{1-\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},1,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\frac{27a\left(\frac{299a^2-148abx^2-79b^2}{a-bx^2}\right.}{}\right.}{2073}$$

```
[In] Integrate[1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x]
```

```
[Out] (x*(-79*b*x^2*(1 - (b*x^2)/a)^(1/3)*AppellF1[3/2, 1/3, 1, 5/2, (b*x^2)/a, -
1/3*(b*x^2)/a] + (27*a*((299*a^2 - 148*a*b*x^2 - 79*b^2*x^4)/(a - b*x^2) -
(387*a^2*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/(9*a*Appell
F1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a] + 2*b*x^2*(-AppellF1[3/2, 1
/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)
/a, -1/3*(b*x^2)/a]))))/(3*a + b*x^2)))/(20736*a^5*(a - b*x^2)^(1/3))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{3}}\left(b\,x^{2}+3a\right)^{2}}dx$$

```
[In] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
[Out] int(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{7/3}\,(3a+bx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a-bx^2)^{7/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(a-bx^2)^{\frac{7}{3}}\,(3a+bx^2)^2}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(7/3)/(b*x**2+3*a)**2,x)
[Out] Integral(1/((a - b*x**2)**(7/3)*(3*a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{7/3}\,(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(bx^2+3\,a)^2(-bx^2+a)^{\frac{7}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(7/3)/(b*x^2+3*a)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)^2*(-b*x^2 + a)^(7/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-bx^2)^{7/3}(3a+bx^2)^2}\,dx = \int \frac{1}{(a-b\,x^2)^{7/3}\,(b\,x^2+3\,a)^2}\,dx$$

```
[In] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2),x)
[Out] int(1/((a - b*x^2)^(7/3)*(3*a + b*x^2)^2), x)
```

3.141 $\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{-a}+\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}\right)}{2\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

```
[Out] 1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/4*arct
anh((-a)^(1/3)*x*b^(1/2)/((-a)^(1/3)+2^(1/3)*(b*x^2-a)^(1/3))/a^(1/2))*2^(1
/3)/(-a)^(1/3)/a^(1/2)/b^(1/2)-1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/
3)/(-a)^(1/3)*3^(1/2)/a^(1/2)/b^(1/2)-1/12*arctan(((-a)^(1/3)-2^(1/3)*(b*x^
2-a)^(1/3))*3^(1/2)*a^(1/2)/(-a)^(1/3)/x/b^(1/2))*2^(1/3)/(-a)^(1/3)*3^(1/2
)/a^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {402}

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{bx^2-a}\right)}{\sqrt[3]{-a}\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[3]{-a}\sqrt{b}x}{\sqrt{a}\left(\sqrt[3]{2}\sqrt[3]{bx^2-a}+\sqrt[3]{-a}\right)}\right)}{2\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

```
[In] Int[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt
[a]*Sqrt[b]) - ArcTan[(Sqrt[3]*Sqrt[a]*((-a)^(1/3) - 2^(1/3)*(-a + b*x^2)^(
1/3)))/((-a)^(1/3)*Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*(-a)^(1/3)*Sqrt[a]*Sqrt[b
]) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b]) -
ArcTanh[((-a)^(1/3)*Sqrt[b]*x)/(Sqrt[a]*((-a)^(1/3) + 2^(1/3)*(-a + b*x^2)^
(1/3)))]/(2*2^(2/3)*(-a)^(1/3)*Sqrt[a]*Sqrt[b])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}\left(\sqrt[3]{-a}-\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}{\sqrt[3]{-a}\sqrt{bx}}\right)}{2\ 2^{2/3}\sqrt{3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{6\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt[3]{-a}\sqrt{bx}}{\sqrt{a}\left(\sqrt[3]{-a}+\sqrt[3]{2}\sqrt[3]{-a+bx^2}\right)}\right)}{2\ 2^{2/3}\sqrt[3]{-a}\sqrt{a}\sqrt{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.65

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx =$$

$$-\frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{-a+bx^2}\,(3a+bx^2)\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\text{Ap}\right.\right.}$$

```
[In] Integrate[1/((-3*a - b*x^2)*(-a + b*x^2)^(1/3)),x]
```

```
[Out] (-9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((-a + b*x^2
)^(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^
2)/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + A
ppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{1}{(-b\,x^2-3a)\,(b\,x^2-a)^{\frac{1}{3}}}\,dx$$

```
[In] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = -\int \frac{1}{3a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2-3*a)/(b*x**2-a)**(1/3),x)
[Out] -Integral(1/(3*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \int -\frac{1}{(bx^2+3\,a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{(-3a-bx^2)\sqrt[3]{-a+bx^2}}\,dx = \int -\frac{1}{(bx^2+3\,a)(bx^2-a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2-3*a)/(b*x^2-a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 3*a)*(b*x^2 - a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-3a - bx^2)\sqrt[3]{-a + bx^2}}\,dx = -\int \frac{1}{(b\,x^2 - a)^{1/3}\,(b\,x^2 + 3\,a)}\,dx$$

```
[In] int(-1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)),x)
[Out] -int(1/((b*x^2 - a)^(1/3)*(3*a + b*x^2)), x)
```

3.142 $\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 202

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

```
[Out] 1/4*arctan(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(b*x^2+a)^(1/3)))*2^(1/3)/a^(
5/6)/b^(1/2)-1/12*arctan(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)-1/12*ar
ctanh(a^(1/6)*(a^(1/3)-2^(1/3)*(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/3)/
a^(5/6)*3^(1/2)/b^(1/2)-1/12*arctanh(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a^(
5/6)*3^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used

$= \{401\}$

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a+bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

```
[In] Int[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2^(2/3)*a^(5/6)*Sqrt[b]) + ArcTan[(Sqrt[b
]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a + b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sq
rt[b]) - ArcTanh[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*
Sqrt[b]) - ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3)))/
(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.82

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx$$
$$= \frac{9ax\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)}{(3a-bx^2)\sqrt[3]{a+bx^2}\left(9a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},\frac{bx^2}{3a}\right)-\text{Appell}\right.\right.}$$

```
[In] Integrate[1/((3*a - b*x^2)*(a + b*x^2)^(1/3)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)/(3*a)])/((3*a - b*x
^2)*(a + b*x^2)^(1/3)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), (b*x^2)
/(3*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)]
- AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), (b*x^2)/(3*a)])))
```

Maple [F]

$$\int \frac{1}{(-b\,x^2+3a)\,(b\,x^2+a)^{\frac{1}{3}}}dx$$

```
[In] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

```
[Out] int(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(3a-bx^2)\sqrt[3]{a+bx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(3a - bx^2)\sqrt[3]{a + bx^2}}\, dx = -\int \frac{1}{-3a\sqrt[3]{a + bx^2} + bx^2\sqrt[3]{a + bx^2}}\, dx$$

```
[In] integrate(1/(-b*x**2+3*a)/(b*x**2+a)**(1/3),x)
[Out] -Integral(1/(-3*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3a - bx^2)\sqrt[3]{a + bx^2}}\, dx = \int -\frac{1}{(bx^2 + a)^{\frac{1}{3}}(bx^2 - 3\,a)}\, dx$$

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Giac [F]

$$\int \frac{1}{(3a - bx^2)\sqrt[3]{a + bx^2}}\, dx = \int -\frac{1}{(bx^2 + a)^{\frac{1}{3}}(bx^2 - 3\,a)}\, dx$$

```
[In] integrate(1/(-b*x^2+3*a)/(b*x^2+a)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + a)^(1/3)*(b*x^2 - 3*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3a - bx^2)\sqrt[3]{a + bx^2}}\, dx = \int \frac{1}{\left(b\,x^2 + a\right)^{1/3}\,\left(3\,a - b\,x^2\right)}\, dx$$

```
[In] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)),x)
[Out] int(1/((a + b*x^2)^(1/3)*(3*a - b*x^2)), x)
```

3.143 $\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\,2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\,2^{2/3}c^{5/6}\sqrt{d}}$$

```
[Out] -1/4*arctanh(c^(1/6)*(c^(1/3)-2^(1/3)*(3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3)
/c^(5/6)/d^(1/2)-1/4*arctanh(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)-1
/12*arctan(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/4*a
rctan(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(3*d*x^2+c)^(1/3)))*3^(1/2
)*2^(1/3)/c^(5/6)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used

$= \{401\}$

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c+3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\arctan\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

```
[In] Int[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2^(2/3)*Sqrt[3]*c^(5/6)*Sqrt[d])
+ (Sqrt[3]*ArcTan[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c + 3*d*
x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[Sqrt[c]/(Sqrt[d]*x)]/(
2*2^(2/3)*c^(5/6)*Sqrt[d]) - ArcTanh[(c^(1/6)*(c^(1/3) - 2^(1/3)*(c + 3*d*x
^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c+3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.94 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.75

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx$$
$$= \frac{3cx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)}{(c-dx^2)\sqrt[3]{c+3dx^2}\left(3c\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)+2dx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3dx^2}{c},\frac{dx^2}{c}\right)-\text{Appe}\right.\right.}$$

```
[In] Integrate[1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c])/((c - d*x^2)*(c
 + 3*d*x^2)^(1/3)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (-3*d*x^2)/c, (d*x^2)/c]
+ 2*d*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (-3*d*x^2)/c, (d*x^2)/c] - AppellF1[3
/2, 4/3, 1, 5/2, (-3*d*x^2)/c, (d*x^2)/c])))
```

Maple [F]

$$\int \frac{1}{(-d\,x^2+c)\,(3d\,x^2+c)^{\frac{1}{3}}}dx$$

```
[In] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

```
[Out] int(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = -\int \frac{1}{-c\sqrt[3]{c+3dx^2}+dx^2\sqrt[3]{c+3dx^2}}\,dx$$

```
[In] integrate(1/(-d*x**2+c)/(3*d*x**2+c)**(1/3),x)
[Out] -Integral(1/(-c*(c + 3*d*x**2)**(1/3) + d*x**2*(c + 3*d*x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int -\frac{1}{(3\,dx^2+c)^{\frac{1}{3}}(dx^2-c)}\,dx$$

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Giac [F]

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int -\frac{1}{(3\,dx^2+c)^{\frac{1}{3}}(dx^2-c)}\,dx$$

```
[In] integrate(1/(-d*x^2+c)/(3*d*x^2+c)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((3*d*x^2 + c)^(1/3)*(d*x^2 - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c-dx^2)\sqrt[3]{c+3dx^2}}\,dx = \int \frac{1}{(c-d\,x^2)\,(3\,d\,x^2+c)^{1/3}}\,dx$$

```
[In] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)),x)
[Out] int(1/((c - d*x^2)*(c + 3*d*x^2)^(1/3)), x)
```

3.144 $\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 204

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[Out] 1/4*arctanh(x*b^(1/2)/a^(1/6)/(a^(1/3)+2^(1/3)*(-b*x^2+a)^(1/3)))*2^(1/3)/a
^(5/6)/b^(1/2)-1/12*arctanh(x*b^(1/2)/a^(1/2))*2^(1/3)/a^(5/6)/b^(1/2)+1/12
*arctan(a^(1/6)*(a^(1/3)-2^(1/3)*(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*2^(1/
3)/a^(5/6)*3^(1/2)/b^(1/2)+1/12*arctan(3^(1/2)*a^(1/2)/x/b^(1/2))*2^(1/3)/a
^(5/6)*3^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {402}

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{2}\sqrt[3]{a-bx^2}+\sqrt[3]{a}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}$$

```
[In] Int[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*Sqrt[a])/(Sqrt[b]*x)]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) +
 ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*(a - b*x^2)^(1/3)))/(Sqrt[b]*x)
]/(2*2^(2/3)*Sqrt[3]*a^(5/6)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(6*2^(
2/3)*a^(5/6)*Sqrt[b]) + ArcTanh[(Sqrt[b]*x)/(a^(1/6)*(a^(1/3) + 2^(1/3)*(a
- b*x^2)^(1/3)))]/(2*2^(2/3)*a^(5/6)*Sqrt[b])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt{a}}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{2\ 2^{2/3}\sqrt{3}a^{5/6}\sqrt{b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6\ 2^{2/3}a^{5/6}\sqrt{b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{2}\sqrt[3]{a-bx^2}\right)}\right)}{2\ 2^{2/3}a^{5/6}\sqrt{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx$$
$$= \frac{9ax \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)}{\sqrt[3]{a-bx^2}\,(3a+bx^2)\left(9a \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+2bx^2\left(-\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},-\frac{bx^2}{3a}\right)+\text{Appe}\right.\right.}$$

```
[In] Integrate[1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x]
```

```
[Out] (9*a*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)/a])/((a - b*x^2)^
(1/3)*(3*a + b*x^2)*(9*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, -1/3*(b*x^2)
/a] + 2*b*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a] + App
ellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, -1/3*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}\left(b\,x^{2}+3a\right)}dx$$

```
[In] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{\sqrt[3]{a-bx^2}\cdot(3a+bx^2)}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/3)/(b*x**2+3*a),x)
[Out] Integral(1/((a - b*x**2)**(1/3)*(3*a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(bx^2+3\,a)(-bx^2+a)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(b*x^2+3*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 3*a)*(-b*x^2 + a)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\,(3a+bx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{1/3}\,(b\,x^2+3\,a)}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)),x)
[Out] int(1/((a - b*x^2)^(1/3)*(3*a + b*x^2)), x)
```

3.145 $\int \frac{1}{\sqrt[3]{c-3dx^2}(c+dx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 204

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}+\frac{\arctan\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}+\frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

```
[Out] 1/4*arctan(c^(1/6)*(c^(1/3)-2^(1/3)*(-3*d*x^2+c)^(1/3))/x/d^(1/2))*2^(1/3)/
c^(5/6)/d^(1/2)+1/4*arctan(1/x/d^(1/2)*c^(1/2))*2^(1/3)/c^(5/6)/d^(1/2)-1/1
2*arctanh(x*3^(1/2)*d^(1/2)/c^(1/2))*2^(1/3)/c^(5/6)*3^(1/2)/d^(1/2)+1/4*ar
ctanh(x*3^(1/2)*d^(1/2)/c^(1/6)/(c^(1/3)+2^(1/3)*(-3*d*x^2+c)^(1/3)))*3^(1/
2)*2^(1/3)/c^(5/6)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used

$= \{402\}$

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} + \frac{\arctan\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{2}\sqrt[3]{c-3dx^2}+\sqrt[3]{c}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}}$$

```
[In] Int[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] ArcTan[Sqrt[c]/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqrt[d]) + ArcTan[(c^(1/6)*(
c^(1/3) - 2^(1/3)*(c - 3*d*x^2)^(1/3)))/(Sqrt[d]*x)]/(2*2^(2/3)*c^(5/6)*Sqr
t[d]) - ArcTanh[(Sqrt[3]*Sqrt[d]*x)/Sqrt[c]]/(2*2^(2/3)*Sqrt[3]*c^(5/6)*Sqr
t[d]) + (Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[d]*x)/(c^(1/6)*(c^(1/3) + 2^(1/3)*(c
 - 3*d*x^2)^(1/3)))])/(2*2^(2/3)*c^(5/6)*Sqrt[d])
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{c}}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} + \frac{\tan^{-1}\left(\frac{\sqrt[6]{c}\left(\sqrt[3]{c}-\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}{\sqrt{d}x}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt{c}}\right)}{2\ 2^{2/3}\sqrt{3}c^{5/6}\sqrt{d}} + \frac{\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt{d}x}{\sqrt[6]{c}\left(\sqrt[3]{c}+\sqrt[3]{2}\sqrt[3]{c-3dx^2}\right)}\right)}{2\ 2^{2/3}c^{5/6}\sqrt{d}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$
$$= \frac{3cx \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)}{\sqrt[3]{c-3dx^2}\,(c+dx^2)\left(3c \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+2dx^2\left(-\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3dx^2}{c},-\frac{dx^2}{c}\right)+\operatorname{Ap}\right.\right.}$$

```
[In] Integrate[1/((c - 3*d*x^2)^(1/3)*(c + d*x^2)),x]
```

```
[Out] (3*c*x*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/c)])/((c - 3*d*x^2
)^(1/3)*(c + d*x^2)*(3*c*AppellF1[1/2, 1/3, 1, 3/2, (3*d*x^2)/c, -((d*x^2)/
c)] + 2*d*x^2*(-AppellF1[3/2, 1/3, 2, 5/2, (3*d*x^2)/c, -((d*x^2)/c)] + App
ellF1[3/2, 4/3, 1, 5/2, (3*d*x^2)/c, -((d*x^2)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(-3d\,x^{2}+c\right)^{\frac{1}{3}}\left(d\,x^{2}+c\right)}dx$$

```
[In] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

```
[Out] int(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx$$

```
[In] integrate(1/(-3*d*x**2+c)**(1/3)/(d*x**2+c),x)
[Out] Integral(1/((c - 3*d*x**2)**(1/3)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(dx^2+c)(-3\,dx^2+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(dx^2+c)(-3\,dx^2+c)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*d*x^2+c)^(1/3)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + c)*(-3*d*x^2 + c)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{c-3dx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(d\,x^2+c)\,(c-3\,d\,x^2)^{1/3}}\,dx$$

```
[In] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)),x)
[Out] int(1/((c + d*x^2)*(c - 3*d*x^2)^(1/3)), x)
```

3.146 $\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} \, dx = \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}} + \frac{\operatorname{arctanh}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/12*arctanh(x)*2^(1/3)+1/4*arctanh(x/(1+2^(1/3)*(-x^2+1)^(1/3)))*2^(1/3)+
1/12*arctan(3^(1/2)/x)*2^(1/3)*3^(1/2)+1/12*arctan((1-2^(1/3)*(-x^2+1)^(1/3
))*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {402}

$$\int \frac{1}{\sqrt[3]{1-x^2}(3+x^2)} \, dx = \frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\arctan\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{1-x^2}+1}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}(x)}{6\ 2^{2/3}}$$

```
[In] Int[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] ArcTan[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) + ArcTan[(Sqrt[3]*(1 - 2^(1/3)*(1 - x
^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[x]/(6*2^(2/3)) + ArcTanh[x/(1
+ 2^(1/3)*(1 - x^2)^(1/3))]/(2*2^(2/3))
```

Rule 402

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[q*(ArcTan[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (Simp[q*(ArcTanh[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] - Simp[q*(ArcTanh[q*x]/(6*2^(2/3)*a^(1/3
)*d)), x] + Simp[q*(ArcTan[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\tan^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1-x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}\\ &\quad-\frac{\tanh^{-1}(x)}{6\ 2^{2/3}}+\frac{\tanh^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1-x^2}}\right)}{2\ 2^{2/3}}\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)}{\sqrt[3]{1-x^2}\,(3+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},x^2,-\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},x^2,-\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

```
[In] Integrate[1/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2])/((1 - x^2)^(1/3)*(3 + x^2)
*(-9*AppellF1[1/2, 1/3, 1, 3/2, x^2, -1/3*x^2] + 2*x^2*(AppellF1[3/2, 1/3,
2, 5/2, x^2, -1/3*x^2] - AppellF1[3/2, 4/3, 1, 5/2, x^2, -1/3*x^2])))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 64.53 (sec) , antiderivative size = 704, normalized size of antiderivative = 6.23

method	result	size
trager	Expression too large to display	704

```
[In] int(1/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/144*ln((2*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^2-RootOf(_Z^6+108)^4*x^3-3
*RootOf(_Z^6+108)^4*x-36*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+216*(-x^2+1)^(
2/3)*x+126*RootOf(_Z^6+108)*x^2-54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x+
18)^2/(RootOf(_Z^6+108)^3*x-18))*RootOf(_Z^6+108)^4-1/24*ln((2*(-x^2+1)^(1/
3)*RootOf(_Z^6+108)^5*x^2-RootOf(_Z^6+108)^4*x^3-3*RootOf(_Z^6+108)^4*x-36*
(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+216*(-x^2+1)^(2/3)*x+126*RootOf(_Z^6+10
8)*x^2-54*RootOf(_Z^6+108))/(RootOf(_Z^6+108)^3*x+18)^2/(RootOf(_Z^6+108)^3
*x-18))*RootOf(_Z^6+108)+1/216*RootOf(_Z^6+108)^4*ln((72*RootOf(_Z^6+108)^4
*x^3-1296*RootOf(_Z^6+108)*x^3-RootOf(_Z^6+108)^4*x^6-225*RootOf(_Z^6+108)^
4*x^4+4050*RootOf(_Z^6+108)*x^4-72*x^5*RootOf(_Z^6+108)^4+1296*x^5*RootOf(_
Z^6+108)+486*RootOf(_Z^6+108)-27*RootOf(_Z^6+108)^4+189*RootOf(_Z^6+108)^4*
x^2-3402*RootOf(_Z^6+108)*x^2+18*RootOf(_Z^6+108)*x^6-108*(-x^2+1)^(1/3)*Ro
otOf(_Z^6+108)^5*x^2+324*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x+6*(-x^2+1)^(1/
3)*RootOf(_Z^6+108)^5*x^5+108*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^4-3888*(-
x^2+1)^(2/3)*x+1296*(-x^2+1)^(2/3)*x^4+9072*(-x^2+1)^(2/3)*x^3+3888*(-x^2+1
)^(2/3)*x^2+144*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x^3-36*(-x^2+1)^(1/3)*Roo
tOf(_Z^6+108)^2*x^5-54*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^5*x-648*(-x^2+1)^(1/
3)*RootOf(_Z^6+108)^2*x^4-864*(-x^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^3+648*(-x
^2+1)^(1/3)*RootOf(_Z^6+108)^2*x^2)/(x^2+3)^3)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 1232, normalized size of antiderivative = 10.90

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*log((432^(5/6)*(-1)^(1/6)*(x^6
- 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(1/
3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) + 17
28*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3
)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*432^(
```

```
1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(
x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) + 1)*l
og(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 - 69*x^4 +
 63*x^2 - 27) - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + sqrt(-3)*(5*
x^5 - 30*x^3 + 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x
)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 - sqrt(-3)*(x^5
- 18*x^3 + 9*x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 - sqrt(-3)*(x^
4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/10368*432^(
5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log((432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63
*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27) - 27) + 432*2^(1/3)*(-1)^(2/3)
*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3 + 9*x) + 9*x) + 1728*(9*x^3 - s
qrt(3)*(I*x^4 - 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3)*(-1)^(1/3)*
(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(-1)^(5/
6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 +
 27*x^2 + 27)) + 1/10368*432^(5/6)*(-1)^(1/6)*(sqrt(-3) - 1)*log(-(432^(5/6
)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 - 69*x^4 + 63*x^2 - 27)
 - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 - sqrt(-3)*(5*x^5 - 30*x^3
+ 9*x) + 9*x) - 1728*(9*x^3 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x)*(-x^2 + 1)^
(2/3) + 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + sqrt(-3)*(x^5 - 18*x^3 + 9*
x) + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2 + sqrt(-3)*(x^4 - 3*x^2)))*
(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27)) - 1/5184*432^(5/6)*(-1)^(1/6
)*log(-(432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4 + 63*x^2 - 27) + 432*2^(1/3)*(-1
)^(2/3)*(5*x^5 - 30*x^3 + 9*x) - 864*(9*x^3 - sqrt(3)*(I*x^4 - 9*I*x^2) - 9
*x)*(-x^2 + 1)^(2/3) - 432*(2^(2/3)*(-1)^(1/3)*(x^5 - 18*x^3 + 9*x) + 4*432
^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2))*(-x^2 + 1)^(1/3))/(x^6 + 9*x^4 + 27*x^2 +
27)) + 1/5184*432^(5/6)*(-1)^(1/6)*log((432^(5/6)*(-1)^(1/6)*(x^6 - 69*x^4
+ 63*x^2 - 27) - 432*2^(1/3)*(-1)^(2/3)*(5*x^5 - 30*x^3 + 9*x) + 864*(9*x^3
 - sqrt(3)*(-I*x^4 + 9*I*x^2) - 9*x)*(-x^2 + 1)^(2/3) + 432*(2^(2/3)*(-1)^(
1/3)*(x^5 - 18*x^3 + 9*x) - 4*432^(1/6)*(-1)^(5/6)*(x^4 - 3*x^2))*(-x^2 + 1
)^(1/3))/(x^6 + 9*x^4 + 27*x^2 + 27))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{\sqrt[3]{-(x-1)(x+1)}\,(x^2+3)}\,dx$$

```
[In] integrate(1/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral(1/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(1/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{1}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

```
[In] int(1/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int(1/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.147 $\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{\arctan(x)}{6\ 2^{2/3}} + \frac{\arctan\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[Out] -1/12*arctan(x)*2^(1/3)+1/4*arctan(x/(1+2^(1/3)*(x^2+1)^(1/3)))*2^(1/3)-1/1
2*arctanh(3^(1/2)/x)*2^(1/3)*3^(1/2)-1/12*arctanh((1-2^(1/3)*(x^2+1)^(1/3))
*3^(1/2)/x)*2^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {401}

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt[3]{2}\sqrt[3]{x^2+1}+1}\right)}{2\ 2^{2/3}} - \frac{\arctan(x)}{6\ 2^{2/3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{x^2+1}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

```
[In] Int[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] -1/6*ArcTan[x]/2^(2/3) + ArcTan[x/(1 + 2^(1/3)*(1 + x^2)^(1/3))]/(2*2^(2/3)
) - ArcTanh[Sqrt[3]/x]/(2*2^(2/3)*Sqrt[3]) - ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*
(1 + x^2)^(1/3)))/x]/(2*2^(2/3)*Sqrt[3])
```

Rule 401

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTanh[Sqrt[3]/(q*x)]/(2*2^(2/3)*Sqrt[3]*a^(1/
3)*d)), x] + (-Simp[q*(ArcTan[(a^(1/3)*q*x)/(a^(1/3) + 2^(1/3)*(a + b*x^2)^
(1/3))]/(2*2^(2/3)*a^(1/3)*d)), x] + Simp[q*(ArcTan[q*x]/(6*2^(2/3)*a^(1/3)
*d)), x] + Simp[q*(ArcTanh[Sqrt[3]*((a^(1/3) - 2^(1/3)*(a + b*x^2)^(1/3))/(
a^(1/3)*q*x))]/(2*2^(2/3)*Sqrt[3]*a^(1/3)*d)), x])] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && EqQ[b*c + 3*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}(x)}{6\ 2^{2/3}} + \frac{\tan^{-1}\left(\frac{x}{1+\sqrt[3]{2}\sqrt[3]{1+x^2}}\right)}{2\ 2^{2/3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}}{x}\right)}{2\ 2^{2/3}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{2}\sqrt[3]{1+x^2}\right)}{x}\right)}{2\ 2^{2/3}\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)}{(-3+x^2)\sqrt[3]{1+x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,\frac{x^2}{3}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,\frac{x^2}{3}\right)-\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

```
[In] Integrate[1/((3 - x^2)*(1 + x^2)^(1/3)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3])/((-3 + x^2)*(1 + x^2)^(1/3)*
(9*AppellF1[1/2, 1/3, 1, 3/2, -x^2, x^2/3] + 2*x^2*(AppellF1[3/2, 1/3, 2, 5
/2, -x^2, x^2/3] - AppellF1[3/2, 4/3, 1, 5/2, -x^2, x^2/3])))
```

Maple [F]

$$\int \frac{1}{\left(-x^{2}+3\right) \left(x^{2}+1\right)^{\frac{1}{3}}}dx$$

```
[In] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

```
[Out] int(1/(-x^2+3)/(x^2+1)^(1/3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(77) = 154$.

Time $= 0.70$ (sec) , antiderivative size $= 1103$, normalized size of antiderivative $= 10.12$

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/10368*432^(5/6)*(sqrt(-3) + 1)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + s
qrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sqrt(3)*(x^4 + 9
*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^
5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqr
t(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^
4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) +
1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + sqrt(-3)*(x^6 + 69*x^4 + 63*x^2
 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) -
 432*2^(1/3)*(5*x^5 + 30*x^3 + sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) - 432
*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 - sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9
*x) - 4*432^(1/6)*(x^4 + 3*x^2 - sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 2
7*x^2 - 27)) + 1/10368*432^(5/6)*(sqrt(-3) - 1)*log((432^(5/6)*(x^6 + 69*x^
4 + 63*x^2 - sqrt(-3)*(x^6 + 69*x^4 + 63*x^2 + 27) + 27) - 1728*(9*x^3 + sq
rt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) + 432*2^(1/3)*(5*x^5 + 30*x^3 -
sqrt(-3)*(5*x^5 + 30*x^3 + 9*x) + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5
+ 18*x^3 + sqrt(-3)*(x^5 + 18*x^3 + 9*x) + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2
+ sqrt(-3)*(x^4 + 3*x^2))))/(x^6 - 9*x^4 + 27*x^2 - 27)) - 1/10368*432^(5/6
)*(sqrt(-3) - 1)*log(-(432^(5/6)*(x^6 + 69*x^4 + 63*x^2 - sqrt(-3)*(x^6 + 6
9*x^4 + 63*x^2 + 27) + 27) + 1728*(9*x^3 - sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^
2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^3 - sqrt(-3)*(5*x^5 + 30*x^3 + 9*x
) + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + sqrt(-3)*(x^5 + 18*
x^3 + 9*x) + 9*x) - 4*432^(1/6)*(x^4 + 3*x^2 + sqrt(-3)*(x^4 + 3*x^2))))/(x
^6 - 9*x^4 + 27*x^2 - 27)) + 1/5184*432^(5/6)*log(-(432^(5/6)*(x^6 + 69*x^4
 + 63*x^2 + 27) + 864*(9*x^3 + sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3)
 + 432*2^(1/3)*(5*x^5 + 30*x^3 + 9*x) + 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 +
 18*x^3 + 9*x) + 4*432^(1/6)*(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27)) -
```

```
1/5184*432^(5/6)*log((432^(5/6)*(x^6 + 69*x^4 + 63*x^2 + 27) - 864*(9*x^3
- sqrt(3)*(x^4 + 9*x^2) + 9*x)*(x^2 + 1)^(2/3) - 432*2^(1/3)*(5*x^5 + 30*x^
3 + 9*x) - 432*(x^2 + 1)^(1/3)*(2^(2/3)*(x^5 + 18*x^3 + 9*x) - 4*432^(1/6)*
(x^4 + 3*x^2)))/(x^6 - 9*x^4 + 27*x^2 - 27))
```

Sympy [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[3]{x^2+1}-3\sqrt[3]{x^2+1}}\,dx$$

```
[In] integrate(1/(-x**2+3)/(x**2+1)**(1/3),x)
[Out] -Integral(1/(x**2*(x**2 + 1)**(1/3) - 3*(x**2 + 1)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="maxima")
[Out] -integrate(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Giac [F]

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = \int -\frac{1}{(x^2+1)^{\frac{1}{3}}(x^2-3)}\,dx$$

```
[In] integrate(1/(-x^2+3)/(x^2+1)^(1/3),x, algorithm="giac")
[Out] integrate(-1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(3-x^2)\sqrt[3]{1+x^2}}\,dx = -\int \frac{1}{(x^2+1)^{1/3}\,(x^2-3)}\,dx$$

```
[In] int(-1/((x^2 + 1)^(1/3)*(x^2 - 3)),x)
[Out] -int(1/((x^2 + 1)^(1/3)*(x^2 - 3)), x)
```

3.148 $\int \frac{3-x}{\sqrt[3]{1-x^2}(3+x^2)} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(3+x^2\right)}{2\ 2^{2/3}}+\frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x}+(1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

```
[Out] -1/4*ln(x^2+3)*2^(1/3)+3/4*ln(2^(1/3)*(1-x)^(1/3)+(1+x)^(2/3))*2^(1/3)+1/2*
arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1+x)^(2/3)/(1-x)^(1/3)*3^(1/2))*3^(1/2)*2^
(1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1022}

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(x+1)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}}-\frac{\log\left(x^2+3\right)}{2\ 2^{2/3}}+\frac{3\log\left((x+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{1-x}\right)}{2\ 2^{2/3}}$$

```
[In] Int[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 + x)^(2/3))/(Sqrt[3]*(1 - x)^(1/3
))])/2^(2/3)) - Log[3 + x^2]/(2*2^(2/3)) + (3*Log[2^(1/3)*(1 - x)^(1/3) + (
1 + x)^(2/3)])/(2*2^(2/3))
```

Rule 1022

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}(1+x)^{2/3}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{2^{2/3}} - \frac{\log\left(3+x^2\right)}{2\ 2^{2/3}} + \frac{3\log\left(\sqrt[3]{2}\sqrt[3]{1-x} + (1+x)^{2/3}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.61

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx$$
$$= \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}-2^{2/3}x+\sqrt[3]{1-x^2}}\right) + 2\log\left(2^{2/3}+2^{2/3}x+2\sqrt[3]{1-x^2}\right) - \log\left(-\sqrt[3]{2} - 2\sqrt[3]{2}x - \sqrt[3]{2}x^2 + 2\right.}{2\ 2^{2/3}}$$

```
[In] Integrate[(3 - x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) - 2^(2/3)*x + (1 - x^
2)^(1/3))] + 2*Log[2^(2/3) + 2^(2/3)*x + 2*(1 - x^2)^(1/3)] - Log[-2^(1/3)
- 2*2^(1/3)*x - 2^(1/3)*x^2 + 2^(2/3)*(1 + x)*(1 - x^2)^(1/3) - 2*(1 - x^2)
^(2/3)]])/(2*2^(2/3))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.23 (sec) , antiderivative size = 1033, normalized size of antiderivative = 10.76

method	result	size
trager	Expression too large to display	1033

```
[In] int((3-x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*RootOf(_Z^3-2)*ln((12*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^
2)^2*RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)^3*x^2+36*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)^2*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*
_Z^2)*RootOf(_Z^3-2)^3*x-18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-12*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2
)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-9*(-x^2+1)^(1/3)*RootOf(_Z
^3-2)^2*x-12*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_
Z^2)*RootOf(_Z^3-2)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x
^2-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)*x^2+36*RootOf(RootOf(_Z
^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x+6*RootOf(_Z^3-2)*x-6*(-x^2+1)^(2/3)-1
8*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)-3*RootOf(_Z^3-2))/(2*
RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x+3)
/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+
x-3))+RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*ln(-(4*RootOf(Roo
tOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x^2+6*RootOf(R
ootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x^2+12*RootOf(
RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^2*x+18*RootOf
(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^3*x-18*(-x^2+1
)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^
2-6*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*Root
Of(_Z^3-2)*x-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2*x-6*(-x^2+1)^(1/3)*RootOf(Ro
otOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*RootOf(RootOf(_
Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*x^2-9*(-x^2+1)^(1/3)*RootOf(_Z^3-2)^2+
3*RootOf(_Z^3-2)*x^2-12*(-x^2+1)^(2/3)+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootO
f(_Z^3-2)+4*_Z^2)+9*RootOf(_Z^3-2))/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(
_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x+3)/(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*Roo
tOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2*x+x-3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(70) = 140$.

Time = 3.21 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.97

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{6}$$
$$\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\Big(12\cdot 4^{\frac{2}{3}}(x^4+3x^3+3x^2+9x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^6-18x^5-117x^4-36x^3+207}{6\,(x^6+54x^5+171x^4+108x^3-8}\right.$$
$$-\frac{1}{24}$$
$$\cdot 4^{\frac{2}{3}}\log\left(\frac{6\cdot 4^{\frac{2}{3}}(x^2+3x)(-x^2+1)^{\frac{2}{3}}+4^{\frac{1}{3}}(x^4+18x^3+24x^2-18x-9)-6\,(x^3+7x^2+3x-3)(-x^2+}{x^4+6x^2+9}\right.$$
$$+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(\frac{4^{\frac{2}{3}}(x^2+3)+6\cdot 4^{\frac{1}{3}}(-x^2+1)^{\frac{1}{3}}(x+1)+12\,(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(x^4 + 3*x^3 +
3*x^2 + 9*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^6 - 18*x^5 - 117*x^4 - 36*x^3 +
207*x^2 + 54*x - 27) + 12*(x^5 + 19*x^4 + 42*x^3 + 6*x^2 - 27*x - 9)*(-x^2
+ 1)^(1/3))/(x^6 + 54*x^5 + 171*x^4 + 108*x^3 - 81*x^2 - 162*x - 27)) - 1/2
4*4^(2/3)*log((6*4^(2/3)*(x^2 + 3*x)*(-x^2 + 1)^(2/3) + 4^(1/3)*(x^4 + 18*x
^3 + 24*x^2 - 18*x - 9) - 6*(x^3 + 7*x^2 + 3*x - 3)*(-x^2 + 1)^(1/3))/(x^4
+ 6*x^2 + 9)) + 1/12*4^(2/3)*log((4^(2/3)*(x^2 + 3) + 6*4^(1/3)*(-x^2 + 1)^
(1/3)*(x + 1) + 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\int \frac{x}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\,dx - \int \left(-\frac{3}{x^2\sqrt[3]{1-x^2}+3\sqrt[3]{1-x^2}}\right)dx$$

```
[In] integrate((3-x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] -Integral(x/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x) - Integral(-
3/(x**2*(1 - x**2)**(1/3) + 3*(1 - x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] -integrate((x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int -\frac{x-3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((3-x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate(-(x - 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3-x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\int \frac{x-3}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

```
[In] int(-(x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] -int((x - 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.149 $\int \frac{3+x}{\sqrt[3]{1-x^2}(3+x^2)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{2^{2/3}}+\frac{\log(3+x^2)}{2\ 2^{2/3}} - \frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+x}\right)}{2\ 2^{2/3}}$$

```
[Out] 1/4*ln(x^2+3)*2^(1/3)-3/4*ln((1-x)^(2/3)+2^(1/3)*(1+x)^(1/3))*2^(1/3)-1/2*a
rctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-x)^(2/3)/(1+x)^(1/3)*3^(1/2))*3^(1/2)*2^(
1/3)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1022}

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{\sqrt{3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{x+1}}\right)}{2^{2/3}}+\frac{\log(x^2+3)}{2\ 2^{2/3}} - \frac{3\log\left((1-x)^{2/3}+\sqrt[3]{2}\sqrt[3]{x+1}\right)}{2\ 2^{2/3}}$$

```
[In] Int[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - x)^(2/3))/(Sqrt[3]*(1 + x)^(1/3))
])/2^(2/3) + Log[3 + x^2]/(2*2^(2/3)) - (3*Log[(1 - x)^(2/3) + 2^(1/3)*(1 +
 x)^(1/3)])/(2*2^(2/3))
```

Rule 1022

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2^{2/3}(1-x)^{2/3}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{2^{2/3}} + \frac{\log(3+x^2)}{2\ 2^{2/3}} - \frac{3\log\left((1-x)^{2/3} + \sqrt[3]{2}\sqrt[3]{1+x}\right)}{2\ 2^{2/3}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.59

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \frac{-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{1-x^2}}{-2^{2/3}+2^{2/3}x+\sqrt[3]{1-x^2}}\right) - 2\log\left(-2^{2/3}+2^{2/3}x-2\sqrt[3]{1-x^2}\right) + \log\left(\sqrt[3]{2}-2\sqrt[3]{2}x+\sqrt[3]{2}x^2+2\right.}{2\ 2^{2/3}}$$

```
[In] Integrate[(3 + x)/((1 - x^2)^(1/3)*(3 + x^2)),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^2)^(1/3))/(-2^(2/3) + 2^(2/3)*x + (1 - x
^2)^(1/3))] - 2*Log[-2^(2/3) + 2^(2/3)*x - 2*(1 - x^2)^(1/3)] + Log[2^(1/3)
 - 2*2^(1/3)*x + 2^(1/3)*x^2 + 2^(2/3)*(-1 + x)*(1 - x^2)^(1/3) + 2*(1 - x^
2)^(2/3)]])/(2*2^(2/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 8.11 (sec) , antiderivative size = 1552, normalized size of antiderivative = 16.34

method	result	size
trager	Expression too large to display	1552

```
[In] int((3+x)/(-x^2+1)^(1/3)/(x^2+3),x,method=_RETURNVERBOSE)
```

[Out] RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*ln((-8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2+24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x+18*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+3*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2+4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+3*RootOf(_Z^3+2)*x^2-6*(-x^2+1)^(2/3)-24*x*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-18*RootOf(_Z^3+2)*x-12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)-9*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))-1/2*ln(-(8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+RootOf(_Z^3+2)*x^2-12*(-x^2+1)^(2/3)-12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+3*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))*RootOf(_Z^3+2)-ln(-(8*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x^2-2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x^2+18*(-x^2+1)^(2/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2-24*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)^2*RootOf(_Z^3+2)^2*x+6*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^3*x-18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)*x-6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2*x+18*(-x^2+1)^(1/3)*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)+6*(-x^2+1)^(1/3)*RootOf(_Z^3+2)^2-4*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*x^2+RootOf(_Z^3+2)*x^2-12*(-x^2+1)^(2/3)-12*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)+3*RootOf(_Z^3+2))/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x-3)/(2*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)*RootOf(_Z^3+2)^2*x-x+3))*RootOf(RootOf(_Z^3+2)^2+2*_Z*RootOf(_Z^3+2)+4*_Z^2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(70) = 140$.

Time = 3.18 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.32

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = -\frac{1}{6} \cdot 4^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}} \arctan\left(\frac{4^{\frac{1}{6}}\sqrt{3}\left(12\cdot 4^{\frac{2}{3}}(-1)^{\frac{2}{3}}\,(x^4-3\,x^3+3\,x^2-9\,x)(-x^2+1)^{\frac{2}{3}}+12\,(-1)^{\frac{1}{3}}\,(x^5-19\,x^4+42\right.}{6\,(x^6-54\,x^5+171\,x^4-}\right. - \frac{1}{24} \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(-\frac{6\cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2-3\,x)(-x^2+1)^{\frac{2}{3}}-4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(x^4-18\,x^3+24\,x^2+18\,x-9)-6\,(x^3-7}{x^4+6\,x^2+9}\right. + \frac{1}{12} \cdot 4^{\frac{2}{3}}(-1)^{\frac{1}{3}} \log\left(-\frac{6\cdot 4^{\frac{1}{3}}(-1)^{\frac{2}{3}}\,(-x^2+1)^{\frac{1}{3}}(x-1)+4^{\frac{2}{3}}(-1)^{\frac{1}{3}}\,(x^2+3)-12\,(-x^2+1)^{\frac{2}{3}}}{x^2+3}\right)$$

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(1/6*4^(1/6)*sqrt(3)*(12*4^(2/3)*(-1)
^(2/3)*(x^4 - 3*x^3 + 3*x^2 - 9*x)*(-x^2 + 1)^(2/3) + 12*(-1)^(1/3)*(x^5 -
19*x^4 + 42*x^3 - 6*x^2 - 27*x + 9)*(-x^2 + 1)^(1/3) + 4^(1/3)*(x^6 + 18*x^
5 - 117*x^4 + 36*x^3 + 207*x^2 - 54*x - 27))/(x^6 - 54*x^5 + 171*x^4 - 108*
x^3 - 81*x^2 + 162*x - 27)) - 1/24*4^(2/3)*(-1)^(1/3)*log(-(6*4^(2/3)*(-1)^
(1/3)*(x^2 - 3*x)*(-x^2 + 1)^(2/3) - 4^(1/3)*(-1)^(2/3)*(x^4 - 18*x^3 + 24*
x^2 + 18*x - 9) - 6*(x^3 - 7*x^2 + 3*x + 3)*(-x^2 + 1)^(1/3))/(x^4 + 6*x^2
+ 9)) + 1/12*4^(2/3)*(-1)^(1/3)*log(-(6*4^(1/3)*(-1)^(2/3)*(-x^2 + 1)^(1/3)
*(x - 1) + 4^(2/3)*(-1)^(1/3)*(x^2 + 3) - 12*(-x^2 + 1)^(2/3))/(x^2 + 3))
```

Sympy [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{\sqrt[3]{-(x-1)\,(x+1)}\,(x^2+3)}\,dx$$

```
[In] integrate((3+x)/(-x**2+1)**(1/3)/(x**2+3),x)
```

```
[Out] Integral((x + 3)/((-(x - 1)*(x + 1))**(1/3)*(x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="maxima")
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(x^2+3)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((3+x)/(-x^2+1)^(1/3)/(x^2+3),x, algorithm="giac")
[Out] integrate((x + 3)/((x^2 + 3)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3+x}{\sqrt[3]{1-x^2}\,(3+x^2)}\,dx = \int \frac{x+3}{(1-x^2)^{1/3}\,(x^2+3)}\,dx$$

```
[In] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)),x)
[Out] int((x + 3)/((1 - x^2)^(1/3)*(x^2 + 3)), x)
```

3.150 $\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

```
[Out] 1/12*arctan(1/3*(a^(1/3)-(b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^(5
/6)/d+1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctanh(a^(
1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

```
[In] Int[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*a^(5/6)*d) + (Sqrt[b]*ArcTan[
(a^(1/3) - (a + b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d) - (S
qrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.64 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$

$$= \frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{a+bx^2}\,(9a+bx^2)\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3\right.\right.}$$

```
[In] Integrate[1/((a + b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(a +
 b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
-1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(b
*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{3}} \left(\frac{9 a d}{b}+d\,x^{2}\right)}d x$$

[In] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)

[Out] int(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\text{Timed out}$$

[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\frac{b\int \frac{1}{9a\sqrt[3]{a+bx^2}+bx^2\sqrt[3]{a+bx^2}}\,dx}{d}$$

[In] integrate(1/(b*x**2+a)**(1/3)/(9*a*d/b+d*x**2),x)

[Out] b*Integral(1/(9*a*(a + b*x**2)**(1/3) + b*x**2*(a + b*x**2)**(1/3)), x)/d

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{3}}\left(dx^{2}+\frac{9\,ad}{b}\right)}\,dx$$

[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")

[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/3)*(d*x^2 + 9*a*d/b)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a+bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(b\,x^2+a)^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)
[Out] int(1/((a + b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)
```

3.151 $\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

```
[Out] 1/12*arctanh(1/3*(a^(1/3)-(-b*x^2+a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d-1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d-1/12*arctan(a
^(1/6)*(a^(1/3)-(-b*x^2+a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[In] Int[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] -1/4*(Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - (a - b*x^2)^(1/3)))/(Sqrt[
b]*x)])/(Sqrt[3]*a^(5/6)*d) - (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(1
2*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(a^(1/3) - (a - b*x^2)^(1/3))^2/(3*a^(1/6)*
Sqrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}-\sqrt[3]{a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.77 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\sqrt[3]{a-bx^2}\,(9a-bx^2)\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3\,\text{Appel}\right.\right.}$$

```
[In] Integrate[1/((a - b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(a - b*
x^2)^(1/3)*(9*a - b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2
)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{3}}\left(-\frac{9ad}{b}+d\,x^{2}\right)}dx$$

```
[In] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{-9a\sqrt[3]{a-bx^2}+bx^2\sqrt[3]{a-bx^2}}\,dx}{d}$$

[In] integrate(1/(-b*x**2+a)**(1/3)/(-9*a*d/b+d*x**2),x)

[Out] b*Integral(1/(-9*a*(a - b*x**2)**(1/3) + b*x**2*(a - b*x**2)**(1/3)), x)/d

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)

Giac [F]

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

[In] integrate(1/(-b*x^2+a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")

[Out] integrate(1/((-b*x^2 + a)^(1/3)*(d*x^2 - 9*a*d/b)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{a-bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{{\left(a-b\,x^2\right)}^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

[In] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)),x)

[Out] int(1/((a - b*x^2)^(1/3)*(d*x^2 - (9*a*d)/b)), x)

3.152 $\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx=\frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

```
[Out] -1/12*arctanh(1/3*(a^(1/3)+(b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d+1/12*arctanh(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctan(a
^(1/6)*(a^(1/3)+(b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{bx^2-a}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d}$$

```
[In] Int[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*Sqrt[3]*a^(5/6)*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[a])])/(12*
a^(5/6)*d) - (Sqrt[b]*ArcTanh[(a^(1/3) + (-a + b*x^2)^(1/3))^2/(3*a^(1/6)*S
qrt[b]*x)])/(12*a^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a+bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.86 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx =$$

$$-\frac{27abx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)}{d\left(9a-bx^2\right)\sqrt[3]{-a+bx^2}\left(27a\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+2bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{a},\frac{bx^2}{9a}\right)+3\,\text{App}\right.\right.}$$

```
[In] Integrate[1/((-a + b*x^2)^(1/3)*((-9*a*d)/b + d*x^2)),x]
```

```
[Out] (-27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^2)/(9*a)])/(d*(9*a -
b*x^2)*(-a + b*x^2)^(1/3)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/a, (b*x^
2)/(9*a)] + 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/a, (b*x^2)/(9*a)] +
 3*AppellF1[3/2, 4/3, 1, 5/2, (b*x^2)/a, (b*x^2)/(9*a)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^2-a\right)^{\frac{1}{3}}\left(-\frac{9ad}{b}+d\,x^2\right)}dx$$

```
[In] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{-9a\sqrt[3]{-a+bx^2}+bx^2\sqrt[3]{-a+bx^2}}\,dx}{d}$$

```
[In] integrate(1/(b*x**2-a)**(1/3)/(-9*a*d/b+d*x**2),x)
[Out] b*Integral(1/(-9*a*(-a + b*x**2)**(1/3) + b*x**2*(-a + b*x**2)**(1/3)), x)/
d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{3}}\left(dx^2-\frac{9\,ad}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2-a)^(1/3)/(-9*a*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - a)^(1/3)*(d*x^2 - 9*a*d/b)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a+bx^2}\left(-\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{\left(b\,x^2-a\right)^{1/3}\,\left(d\,x^2-\frac{9\,a\,d}{b}\right)}\,dx$$

```
[In] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)),x)
[Out] int(1/((b*x^2 - a)^(1/3)*(d*x^2 - (9*a*d)/b)), x)
```

3.153 $\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 153

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

```
[Out] -1/12*arctan(1/3*(a^(1/3)+(-b*x^2-a)^(1/3))^2/a^(1/6)/x/b^(1/2))*b^(1/2)/a^
(5/6)/d-1/12*arctan(1/3*x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/6)/d+1/12*arctanh(a
^(1/6)*(a^(1/3)+(-b*x^2-a)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)/a^(5/6)/d*3^(1
/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{-a-bx^2}+\sqrt[3]{a}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

```
[In] Int[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] -1/12*(Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[a])])/(a^(5/6)*d) - (Sqrt[b]*ArcT
an[(a^(1/3) + (-a - b*x^2)^(1/3))^2/(3*a^(1/6)*Sqrt[b]*x)])/(12*a^(5/6)*d)
+ (Sqrt[b]*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + (-a - b*x^2)^(1/3)))/(Sqrt[b
]*x)])/(4*Sqrt[3]*a^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{a}}\right)}{12a^{5/6}d} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)^2}{3\sqrt[6]{a}\sqrt{b}x}\right)}{12a^{5/6}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{3}\sqrt[6]{a}\left(\sqrt[3]{a}+\sqrt[3]{-a-bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}a^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.80 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx$$
$$=\frac{27abx\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)}{d\sqrt[3]{-a-bx^2}\,(9a+bx^2)\left(27a\,\mathrm{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)-2bx^2\left(\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{bx^2}{9a}\right)+3\right.\right.}$$

```
[In] Integrate[1/((-a - b*x^2)^(1/3)*((9*a*d)/b + d*x^2)),x]
```

```
[Out] (27*a*b*x*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a), -1/9*(b*x^2)/a])/(d*(-a
- b*x^2)^(1/3)*(9*a + b*x^2)*(27*a*AppellF1[1/2, 1/3, 1, 3/2, -((b*x^2)/a),
 -1/9*(b*x^2)/a] - 2*b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -((b*x^2)/a), -1/9*(
b*x^2)/a] + 3*AppellF1[3/2, 4/3, 1, 5/2, -((b*x^2)/a), -1/9*(b*x^2)/a])))
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^2-a\right)^{\frac{1}{3}}\left(\frac{9ad}{b}+d\,x^2\right)}dx$$

```
[In] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

```
[Out] int(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{9a\sqrt[3]{-a-bx^2}+bx^2\sqrt[3]{-a-bx^2}}\,dx}{d}$$

[In] integrate(1/(-b*x**2-a)**(1/3)/(9*a*d/b+d*x**2),x)

[Out] b*Integral(1/(9*a*(-a - b*x**2)**(1/3) + b*x**2*(-a - b*x**2)**(1/3)), x)/d

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="maxima")

[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)

Giac [F]

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(-bx^2-a)^{\frac{1}{3}}\left(dx^2+\frac{9\,ad}{b}\right)}\,dx$$

[In] integrate(1/(-b*x^2-a)^(1/3)/(9*a*d/b+d*x^2),x, algorithm="giac")

[Out] integrate(1/((-b*x^2 - a)^(1/3)*(d*x^2 + 9*a*d/b)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-a-bx^2}\left(\frac{9ad}{b}+dx^2\right)}\,dx = \int \frac{1}{(-b\,x^2-a)^{1/3}\,\left(d\,x^2+\frac{9\,a\,d}{b}\right)}\,dx$$

[In] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)),x)

[Out] int(1/((- a - b*x^2)^(1/3)*(d*x^2 + (9*a*d)/b)), x)

3.154 $\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 151

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)} \, dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] 1/24*arctan(1/6*(2^(1/3)-(b*x^2+2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2^(1
/6)/d+1/24*arctan(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d-1/24*arctanh(2^(
1/6)*(2^(1/3)-(b*x^2+2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)} \, dx = \frac{\sqrt{b}\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{bx^2+2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[In] Int[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*2^(5/6)*d) + (Sqrt[b]*ArcTan[
(2^(1/3) - (2 + b*x^2)^(1/3))^2/(3*2^(1/6)*Sqrt[b]*x)])/(12*2^(5/6)*d) - (S
qrt[b]*ArcTanh[(2^(1/6)*Sqrt[3]*(2^(1/3) - (2 + b*x^2)^(1/3)))/(Sqrt[b]*x)]
)/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}-\sqrt[3]{2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 5.40 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = -\frac{27bx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)}{d\sqrt[3]{2+bx^2}\,(18+bx^2)\left(-27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{bx^2}{2},-\frac{bx^2}{18}\right)+\right.\right.}$$

```
[In] Integrate[1/((2 + b*x^2)^(1/3)*((18*d)/b + d*x^2)),x]
```

```
[Out] (-27*b*x*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/18*(b*x^2)])/(d*(2 + b
*x^2)^(1/3)*(18 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -1/2*(b*x^2), -1/1
8*(b*x^2)] + b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)]
 + 3*AppellF1[3/2, 4/3, 1, 5/2, -1/2*(b*x^2), -1/18*(b*x^2)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+2\right)^{\frac{1}{3}} \left(\frac{18 d}{b}+d\,x^{2}\right)}d x$$

```
[In] int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
[Out] int(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2+2}+18\sqrt[3]{bx^2+2}}\,dx}{d}$$

```
[In] integrate(1/(b*x**2+2)**(1/3)/(18*d/b+d*x**2),x)
[Out] b*Integral(1/(b*x**2*(b*x**2 + 2)**(1/3) + 18*(b*x**2 + 2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx=\int \frac{1}{\left(bx^2+2\right)^{\frac{1}{3}}\left(dx^2+\frac{18\,d}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/3)/(18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 2)^(1/3)*(d*x^2 + 18*d/b)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+bx^2}\left(\frac{18d}{b}+dx^2\right)}\,dx=\int \frac{1}{\left(\frac{18\,d}{b}+d\,x^2\right)\,\left(b\,x^2+2\right)^{1/3}}\,dx$$

```
[In] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)),x)
[Out] int(1/(((18*d)/b + d*x^2)*(b*x^2 + 2)^(1/3)), x)
```

3.155 $\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 147

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}$$

```
[Out] -1/24*arctanh(1/6*(2^(1/3)+(b*x^2-2)^(1/3))^2*2^(5/6)/x/b^(1/2))*b^(1/2)*2^
(1/6)/d+1/24*arctanh(1/6*x*b^(1/2)*2^(1/2))*b^(1/2)*2^(1/6)/d+1/24*arctan(2
^(1/6)*(2^(1/3)+(b*x^2-2)^(1/3))*3^(1/2)/x/b^(1/2))*b^(1/2)*2^(1/6)/d*3^(1/
2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{\sqrt{b}\arctan\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\sqrt{b}\text{arctanh}\left(\frac{\left(\sqrt[3]{bx^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d}$$

```
[In] Int[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (Sqrt[b]*ArcTan[(2^(1/6)*Sqrt[3]*(2^(1/3) + (-2 + b*x^2)^(1/3)))/(Sqrt[b]*x
)])/(4*2^(5/6)*Sqrt[3]*d) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(3*Sqrt[2])])/(12*
2^(5/6)*d) - (Sqrt[b]*ArcTanh[(2^(1/3) + (-2 + b*x^2)^(1/3))^2/(3*2^(1/6)*S
qrt[b]*x)])/(12*2^(5/6)*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt[6]{2}\sqrt{3}\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)}{\sqrt{b}x}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{3\sqrt{2}}\right)}{12\ 2^{5/6}d} - \frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+bx^2}\right)^2}{3\sqrt[6]{2}\sqrt{b}x}\right)}{12\ 2^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.66 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx$$
$$= \frac{27bx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)}{d\left(-18+bx^2\right)\sqrt[3]{-2+bx^2}\left(27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)+bx^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{bx^2}{2},\frac{bx^2}{18}\right)+3\,\text{Appell}\right.\right.}$$

```
[In] Integrate[1/((-2 + b*x^2)^(1/3)*((-18*d)/b + d*x^2)),x]
```

```
[Out] (27*b*x*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18])/(d*(-18 + b*x^2)
*(-2 + b*x^2)^(1/3)*(27*AppellF1[1/2, 1/3, 1, 3/2, (b*x^2)/2, (b*x^2)/18] +
 b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, (b*x^2)/2, (b*x^2)/18] + 3*AppellF1[3/2,
 4/3, 1, 5/2, (b*x^2)/2, (b*x^2)/18])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^2-2\right)^{\frac{1}{3}}\left(-\frac{18d}{b}+d\,x^2\right)}dx$$

```
[In] int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

```
[Out] int(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \frac{b\int \frac{1}{bx^2\sqrt[3]{bx^2-2}-18\sqrt[3]{bx^2-2}}\,dx}{d}$$

```
[In] integrate(1/(b*x**2-2)**(1/3)/(-18*d/b+d*x**2),x)
[Out] b*Integral(1/(b*x**2*(b*x**2 - 2)**(1/3) - 18*(b*x**2 - 2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-2)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \int \frac{1}{(bx^2-2)^{\frac{1}{3}}\left(dx^2-\frac{18\,d}{b}\right)}\,dx$$

```
[In] integrate(1/(b*x^2-2)^(1/3)/(-18*d/b+d*x^2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - 2)^(1/3)*(d*x^2 - 18*d/b)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+bx^2}\left(-\frac{18d}{b}+dx^2\right)}\,dx = \int -\frac{1}{\left(\frac{18\,d}{b}-d\,x^2\right)\,\left(b\,x^2-2\right)^{1/3}}\,dx$$

```
[In] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)),x)
[Out] int(-1/(((18*d)/b - d*x^2)*(b*x^2 - 2)^(1/3)), x)
```

3.156 $\int \frac{1}{\sqrt[3]{2+3x^2}(6d+dx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx=\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

```
[Out] -1/8*arctanh(2^(1/6)*(2^(1/3)-(3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctan(1/1
8*(2^(1/3)-(3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*arc
tan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx=\frac{\arctan\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}+\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{3x^2+2}\right)}{x}\right)}{4\,2^{5/6}d}$$

```
[In] Int[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] ArcTan[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTan[(2^(1/3) - (2 + 3*x^2)^(1/
3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/6)*(2^(1
/3) - (2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\,(6+x^2)\sqrt[3]{2+3x^2}\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3\,\text{Ap}\right.\right.}$$

```
[In] Integrate[1/((2 + 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(6 + x^2)*(2 + 3
*x^2)^(1/3)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(App
ellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2
, (-3*x^2)/2, -1/6*x^2])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 76.62 (sec) , antiderivative size = 1066, normalized size of antiderivative = 8.67

method	result	size
trager	Expression too large to display	1066

```
[In] int(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(ln(-(16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*Roo
tOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^
2)^2*RootOf(_Z^6+54)^5*x-(3*x^2+2)^(1/3)*RootOf(_Z^6+54)^5*x+72*RootOf(_Z^6
+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*(3*x^2+2)^(
1/3)*x-1152*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54
)+576*_Z^2)^2*(3*x^2+2)^(1/3)*x-36*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)
^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2+72*RootOf(RootOf(_Z^6+54)^2-24*_Z*Ro
otOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*RootOf(_Z^6+54)^2*(3*x^2+2)^(1
/3)-432*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*
_Z^2)*(3*x^2+2)^(1/3)+54*(3*x^2+2)^(2/3))/(x^2+6))*RootOf(_Z^6+54)-24*RootO
f(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln(-(4*RootOf(_Z^6+54)^
7*x-288*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^
6+54)^6*x+4608*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*R
ootOf(_Z^6+54)^5*x-144*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*Roo
tOf(_Z^6+54)+576*_Z^2)*(3*x^2+2)^(1/3)*x+6912*RootOf(_Z^6+54)^3*RootOf(Root
Of(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*(3*x^2+2)^(1/3)*x-9*x^2*Roo
tOf(_Z^6+54)^4+216*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(
_Z^6+54)+576*_Z^2)*x^2+18*RootOf(_Z^6+54)^4-432*RootOf(RootOf(_Z^6+54)^2-24
*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+2592*RootOf(_Z^6+54)*RootOf
(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*(3*x^2+2)^(1/3)+324*(3*x
^2+2)^(2/3))/(x^2+6))-24*ln(-(16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6
+54)+576*_Z^2)*RootOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootO
f(_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x-(3*x^2+2)^(1/3)*RootOf(_Z^6+54)^
5*x+72*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576
*_Z^2)*(3*x^2+2)^(1/3)*x-1152*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24
*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*(3*x^2+2)^(1/3)*x-36*RootOf(_Z^6+54)^3*Root
Of(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2+72*RootOf(RootOf(_
Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*RootOf(_Z^6+
54)^2*(3*x^2+2)^(1/3)-432*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*Ro
otOf(_Z^6+54)+576*_Z^2)*(3*x^2+2)^(1/3)+54*(3*x^2+2)^(2/3))/(x^2+6))*RootOf
(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1843 vs. $2(90) = 180$.

Time = 192.78 (sec) , antiderivative size = 1843, normalized size of antiderivative = 14.98

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7
*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^
4*x))*(-1/d^6)^(2/3) + 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d
^3)*sqrt(-1/d^6) - 36*x)*(3*x^2 + 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^
4 - 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-1/d
^6)^(5/6) - (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^
5 - 52*d^2*x^3 + 36*d^2*x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) - (1/864)^(1/
6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2
 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/48*(1/
864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5
 - 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-
1/d^6)^(2/3) + 4*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*sqrt
(-1/d^6) - 36*x)*(3*x^2 + 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^4 - 36*d
^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-1/d^6)^(5/6
) + (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^5 - 52*d
^2*x^3 + 36*d^2*x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) + (1/864)^(1/6)*(d*x^
6 - 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d)
 + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/48*(1/864)^(1/
6)*(sqrt(-3) - 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^
4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-1/d^6)^(
2/3) + 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*sqrt(-1/d^6)
 - 36*x)*(3*x^2 + 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 -
 12*d^5 + sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-1/d^6)^(5/6) - (1/4
)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 - 52*d^2*x^3 +
 36*d^2*x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) - (1/864)^(1/6)*(d*x^6 - 210*
d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)
*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) + 1/48*(1/864)^(1/6)*(sqrt
(-3) - 1)*(-1/d^6)^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 -
36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-1/d^6)^(2/3) + 4
*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*sqrt(-1/d^6) - 36*x)
*(3*x^2 + 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5
+ sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-1/d^6)^(5/6) + (1/4)^(1/3)*
(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*
x))*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) + (1/864)^(1/6)*(d*x^6 - 210*d*x^4 +
252*d*x^2 - sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)*(-1/d^6
```

```
)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/24*(1/864)^(1/6)*(-1/d^6)^(1/6
)*log(1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x)*(-1/d^6)^(2/3)
 - 2*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*sqrt(-1/d^6) - 3
6*x)*(3*x^2 + 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*
d^5)*(-1/d^6)^(5/6) - (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x)*(-1/d^6
)^(1/3))*(3*x^2 + 2)^(1/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 +
 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) + 1/24*(1/864)^(1/6)
*(-1/d^6)^(1/6)*log(1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x)*
(-1/d^6)^(2/3) - 2*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*sq
rt(-1/d^6) - 36*x)*(3*x^2 + 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^4 - 36
*d^5*x^2 - 12*d^5)*(-1/d^6)^(5/6) + (1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*
d^2*x)*(-1/d^6)^(1/3))*(3*x^2 + 2)^(1/3) + (1/864)^(1/6)*(d*x^6 - 210*d*x^4
 + 252*d*x^2 + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{3x^2+2}+6\sqrt[3]{3x^2+2}}\,dx}{d}$$

```
[In] integrate(1/(3*x**2+2)**(1/3)/(d*x**2+6*d),x)
[Out] Integral(1/(x**2*(3*x**2 + 2)**(1/3) + 6*(3*x**2 + 2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(3\,x^2+2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(3*x^2+2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + 6*d)*(3*x^2 + 2)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2+3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(3\,x^2+2)^{1/3}\,(d\,x^2+6\,d)}\,dx$$

```
[In] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)),x)
[Out] int(1/((3*x^2 + 2)^(1/3)*(6*d + d*x^2)), x)
```

3.157 $\int \frac{1}{\sqrt[3]{2-3x^2}(-6d+dx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 123

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d}$$

```
[Out] -1/8*arctan(2^(1/6)*(2^(1/3)-(-3*x^2+2)^(1/3))/x)*2^(1/6)/d+1/24*arctanh(1/
18*(2^(1/3)-(-3*x^2+2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24*a
rctanh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = -\frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d} + \frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d} - \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d}$$

```
[In] Int[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] -1/4*ArcTan[(2^(1/6)*(2^(1/3) - (2 - 3*x^2)^(1/3)))/x]/(2^(5/6)*d) - ArcTan
h[x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/3) - (2 - 3*x^2)^(1/3))^
2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}-\sqrt[3]{2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx$$
$$= \frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\sqrt[3]{2-3x^2}\,(-6+x^2)\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+3\,\text{AppellF1}\left(\frac{3}{2},\right.\right.\right.}$$

```
[In] Integrate[1/((2 - 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(2 - 3*x^2)^(1/3)*(-6
 + x^2)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/2
, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
x^2/6])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 81.70 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.45

method	result	size
trager	Expression too large to display	547

```
[In] int(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(24*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*ln((-192
*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6
*x+4*RootOf(_Z^6-54)^7*x-288*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6-54)^2-24*
_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x+6*RootOf(_Z^6-54)^5*(-3*x^2
+2)^(1/3)*x-9*x^2*RootOf(_Z^6-54)^4-18*RootOf(_Z^6-54)^4+108*(-3*x^2+2)^(1/
3)*RootOf(_Z^6-54)^2+324*(-3*x^2+2)^(2/3))/(x^2-6))+RootOf(_Z^6-54)*ln((768
*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)
^5*x-16*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^
6-54)^6*x-1152*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*R
ootOf(_Z^6-54)^3*(-3*x^2+2)^(1/3)*x+72*RootOf(_Z^6-54)^4*RootOf(RootOf(_Z^6
-54)^2-24*_Z*RootOf(_Z^6-54)+576*_Z^2)*(-3*x^2+2)^(1/3)*x-RootOf(_Z^6-54)^5
*(-3*x^2+2)^(1/3)*x+36*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54)+576*_
Z^2)*RootOf(_Z^6-54)^3*x^2+72*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^6-54
)+576*_Z^2)*RootOf(_Z^6-54)^3+432*RootOf(RootOf(_Z^6-54)^2-24*_Z*RootOf(_Z^
6-54)+576*_Z^2)*RootOf(_Z^6-54)*(-3*x^2+2)^(1/3)-18*(-3*x^2+2)^(1/3)*RootOf
(_Z^6-54)^2+54*(-3*x^2+2)^(2/3))/(x^2-6)))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1867 vs. $2(90) = 180$.

Time = 193.66 (sec) , antiderivative size = 1867, normalized size of antiderivative = 15.18

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] -1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(d^(-6))^(1/6)*log(-1/4*(864*(1/864)^(5/
6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5 + sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12
*d^5))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(5/6) + 12*sqrt(1/6)*(d^3*x^4 + 24*d^3*x
^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*sqrt(d^(-6)) + 4*(1/4)^(2/3)*(7*d^4*x^5 + 9
2*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d^(-6
))^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x
^5 + 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/3) - 8*(5*x^3 +
 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 -
```

```
sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^
6 - 18*x^4 + 108*x^2 - 216)) + 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(d^(-6))^(
1/6)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5 + sqrt(-3)
*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(5/6) + 12*
sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*sqrt(d^(-6)) -
 4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 9
2*d^4*x^3 - 36*d^4*x))*(d^(-6))^(2/3) + 2*(1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3
 + 36*d^2*x + sqrt(-3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 + 2)^(1/3
)*(d^(-6))^(1/3) + 8*(5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^(1/6)*(d*x
^6 + 210*d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d
) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) + 1/48*(1/864)^(1
/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 +
36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(-3*x^2 +
 2)^(1/3)*(d^(-6))^(5/6) + 12*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*(-3
*x^2 + 2)^(2/3)*sqrt(d^(-6)) + 4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d
^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d^(-6))^(2/3) - 2*(1/
4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^5 + 52*d^2*x^3
+ 36*d^2*x))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/3) - 8*(5*x^3 + 18*x)*(-3*x^2 +
 2)^(2/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6
+ 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108
*x^2 - 216)) - 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(1/4*(86
4*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36
*d^5*x^2 - 12*d^5))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(5/6) + 12*sqrt(1/6)*(d^3*x
^4 + 24*d^3*x^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*sqrt(d^(-6)) - 4*(1/4)^(2/3)*(
7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d
^4*x))*(d^(-6))^(2/3) + 2*(1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sq
rt(-3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1/3)
 + 8*(5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4 +
 252*d*x^2 + sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)*(d^(-6
))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/24*(1/864)^(1/6)*(d^(-6))^(1/
6)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5)*(-3*x^2 + 2
)^(1/3)*(d^(-6))^(5/6) - 6*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*(-3*x^
2 + 2)^(2/3)*sqrt(d^(-6)) + 4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*
x)*(d^(-6))^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x)*(-3*x^2
 + 2)^(1/3)*(d^(-6))^(1/3) + 4*(5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^
(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4
+ 108*x^2 - 216)) + 1/24*(1/864)^(1/6)*(d^(-6))^(1/6)*log(-1/4*(864*(1/864)
^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5)*(-3*x^2 + 2)^(1/3)*(d^(-6))^(5/6)
- 6*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*(-3*x^2 + 2)^(2/3)*sqrt(d^(-6
)) - 4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x)*(d^(-6))^(2/3) + 2*(
1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x)*(-3*x^2 + 2)^(1/3)*(d^(-6))^(1
/3) - 4*(5*x^3 + 18*x)*(-3*x^2 + 2)^(2/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^
4 + 252*d*x^2 - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{2-3x^2}-6\sqrt[3]{2-3x^2}}\,dx}{d}$$

```
[In] integrate(1/(-3*x**2+2)**(1/3)/(d*x**2-6*d),x)
[Out] Integral(1/(x**2*(2 - 3*x**2)**(1/3) - 6*(2 - 3*x**2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(-3\,x^2+2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 - 6*d)*(-3*x^2 + 2)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{2-3x^2}\,(-6d+dx^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{1/3}\,(6\,d-d\,x^2)}\,dx$$

```
[In] int(-1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)),x)
[Out] -int(1/((2 - 3*x^2)^(1/3)*(6*d - d*x^2)), x)
```

3.158 $\int \frac{1}{\sqrt[3]{-2+3x^2}(-6d+dx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[Out] 1/8*arctan(2^(1/6)*(2^(1/3)+(3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctanh(1/18
*(2^(1/3)+(3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)+1/24*arct
anh(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\ 2^{5/6}d}-\frac{\operatorname{arctanh}\left(\frac{\left(\sqrt[3]{3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

```
[In] Int[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] ArcTan[(2^(1/6)*(2^(1/3) + (-2 + 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d) + ArcTanh[
x/Sqrt[6]]/(4*2^(5/6)*Sqrt[3]*d) - ArcTanh[(2^(1/3) + (-2 + 3*x^2)^(1/3))^2
/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d)
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)}{x}\right)}{4\ 2^{5/6}d} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2+3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)}{d\,(-6+x^2)\sqrt[3]{-2+3x^2}\left(9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},\frac{3x^2}{2},\frac{x^2}{6}\right)+3\,\text{AppellF1}\left(\frac{3}{2}\right.\right.\right.}$$

```
[In] Integrate[1/((-2 + 3*x^2)^(1/3)*(-6*d + d*x^2)),x]
```

```
[Out] (9*x*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6])/(d*(-6 + x^2)*(-2 + 3*x^
2)^(1/3)*(9*AppellF1[1/2, 1/3, 1, 3/2, (3*x^2)/2, x^2/6] + x^2*(AppellF1[3/
2, 1/3, 2, 5/2, (3*x^2)/2, x^2/6] + 3*AppellF1[3/2, 4/3, 1, 5/2, (3*x^2)/2,
 x^2/6])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 74.92 (sec) , antiderivative size = 1063, normalized size of antiderivative = 8.93

method	result	size
trager	Expression too large to display	1063

```
[In] int(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(24*ln(-(768*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)
^2*RootOf(_Z^6-54)^5*x+16*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+57
6*_Z^2)*RootOf(_Z^6-54)^6*x+1152*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+2
4*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^3*x+72*(3*x^2-2)^(1/3)*Roo
tOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^4*x+(
3*x^2-2)^(1/3)*RootOf(_Z^6-54)^5*x+36*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf
(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3*x^2+72*RootOf(RootOf(_Z^6-54)^2+24*_Z
*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3-432*(3*x^2-2)^(1/3)*RootOf(Roo
tOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)-18*RootOf(_Z
^6-54)^2*(3*x^2-2)^(1/3)-54*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(RootOf(_Z^6-54
)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)+24*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootO
f(_Z^6-54)+576*_Z^2)*ln((4608*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54
)+576*_Z^2)^2*RootOf(_Z^6-54)^5*x+288*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf
(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^6*x+4*RootOf(_Z^6-54)^7*x+6912*(3*x^2-2
)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_
Z^6-54)^3*x+144*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-
54)+576*_Z^2)*RootOf(_Z^6-54)^4*x+216*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf
(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3*x^2+9*x^2*RootOf(_Z^6-54)^4+432*RootO
f(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^3+18*Ro
otOf(_Z^6-54)^4-2592*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(
_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)+324*(3*x^2-2)^(2/3))/(x^2-6))+ln(-(768*R
ootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)^2*RootOf(_Z^6-54)^5
*x+16*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-
54)^6*x+1152*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)
+576*_Z^2)^2*RootOf(_Z^6-54)^3*x+72*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^
2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)^4*x+(3*x^2-2)^(1/3)*RootO
f(_Z^6-54)^5*x+36*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576*_Z^2)*
RootOf(_Z^6-54)^3*x^2+72*RootOf(RootOf(_Z^6-54)^2+24*_Z*RootOf(_Z^6-54)+576
*_Z^2)*RootOf(_Z^6-54)^3-432*(3*x^2-2)^(1/3)*RootOf(RootOf(_Z^6-54)^2+24*_Z
*RootOf(_Z^6-54)+576*_Z^2)*RootOf(_Z^6-54)-18*RootOf(_Z^6-54)^2*(3*x^2-2)^(
1/3)-54*(3*x^2-2)^(2/3))/(x^2-6))*RootOf(_Z^6-54))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1771 vs. $2(86) = 172$.

Time = 190.80 (sec) , antiderivative size = 1771, normalized size of antiderivative = 14.88

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="fricas")
```

```
[Out] 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7
*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^
4*x))*(d^(-6))^(2/3) - 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d
^3)*sqrt(d^(-6)) + 36*x)*(3*x^2 - 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^
4 + 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(d^(-
6))^(5/6) + (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^
5 + 52*d^2*x^3 + 36*d^2*x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) + (1/864)^(1/
6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2
 - 72*d) - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/48*(1/
864)^(1/6)*(sqrt(-3) + 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5
 + 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d
^(-6))^(2/3) - 4*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*sqrt
(d^(-6)) + 36*x)*(3*x^2 - 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^4 + 36*d
^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(d^(-6))^(5/6
) - (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^5 + 52*d
^2*x^3 + 36*d^2*x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) - (1/864)^(1/6)*(d*x^
6 + 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d)
 - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/48*(1/864)^(1/
6)*(sqrt(-3) - 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^
4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d^(-6))^(
2/3) - 4*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*sqrt(d^(-6))
 + 36*x)*(3*x^2 - 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 -
 12*d^5 + sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(d^(-6))^(5/6) + (1/4
)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 + 52*d^2*x^3 +
 36*d^2*x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) + (1/864)^(1/6)*(d*x^6 + 210*
d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)
*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) + 1/48*(1/864)^(1/6)*(sqrt
(-3) - 1)*(d^(-6))^(1/6)*log(-1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 -
36*d^4*x - sqrt(-3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x))*(d^(-6))^(2/3) - 4
*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*sqrt(d^(-6)) + 36*x)
*(3*x^2 - 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5
+ sqrt(-3)*(5*d^5*x^4 + 36*d^5*x^2 - 12*d^5))*(d^(-6))^(5/6) - (1/4)^(1/3)*
(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*
x))*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4 +
252*d*x^2 - sqrt(-3)*(d*x^6 + 210*d*x^4 + 252*d*x^2 - 72*d) - 72*d)*(d^(-6)
```

```
)^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) - 1/24*(1/864)^(1/6)*(d^(-6))^(1/6
)*log(1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x)*(d^(-6))^(2/3)
 + 2*(10*x^3 + 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*sqrt(d^(-6)) + 3
6*x)*(3*x^2 - 2)^(2/3) + 2*(432*(1/864)^(5/6)*(5*d^5*x^4 + 36*d^5*x^2 - 12*
d^5)*(d^(-6))^(5/6) + (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*d^2*x)*(d^(-6)
)^(1/3))*(3*x^2 - 2)^(1/3) + (1/864)^(1/6)*(d*x^6 + 210*d*x^4 + 252*d*x^2 -
 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216)) + 1/24*(1/864)^(1/6)
*(d^(-6))^(1/6)*log(1/4*(4*(1/4)^(2/3)*(7*d^4*x^5 + 92*d^4*x^3 - 36*d^4*x)*
(d^(-6))^(2/3) + 2*(10*x^3 - 3*sqrt(1/6)*(d^3*x^4 + 24*d^3*x^2 + 12*d^3)*sq
rt(d^(-6)) + 36*x)*(3*x^2 - 2)^(2/3) - 2*(432*(1/864)^(5/6)*(5*d^5*x^4 + 36
*d^5*x^2 - 12*d^5)*(d^(-6))^(5/6) - (1/4)^(1/3)*(d^2*x^5 + 52*d^2*x^3 + 36*
d^2*x)*(d^(-6))^(1/3))*(3*x^2 - 2)^(1/3) - (1/864)^(1/6)*(d*x^6 + 210*d*x^4
 + 252*d*x^2 - 72*d)*(d^(-6))^(1/6))/(x^6 - 18*x^4 + 108*x^2 - 216))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{3x^2-2}-6\sqrt[3]{3x^2-2}}\,dx}{d}$$

```
[In] integrate(1/(3*x**2-2)**(1/3)/(d*x**2-6*d),x)
[Out] Integral(1/(x**2*(3*x**2 - 2)**(1/3) - 6*(3*x**2 - 2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = \int \frac{1}{(dx^2-6\,d)(3\,x^2-2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(3*x^2-2)^(1/3)/(d*x^2-6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 - 6*d)*(3*x^2 - 2)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2+3x^2}\,(-6d+dx^2)}\,dx = -\int \frac{1}{(3\,x^2-2)^{1/3}\,(6\,d-d\,x^2)}\,dx$$

```
[In] int(-1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)),x)
[Out] -int(1/((3*x^2 - 2)^(1/3)*(6*d - d*x^2)), x)
```

3.159 $\int \frac{1}{\sqrt[3]{-2-3x^2}(6d+dx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = -\frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d} - \frac{\arctan\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\,2^{5/6}d}$$

```
[Out] 1/8*arctanh(2^(1/6)*(2^(1/3)+(-3*x^2-2)^(1/3))/x)*2^(1/6)/d-1/24*arctan(1/1
8*(2^(1/3)+(-3*x^2-2)^(1/3))^2*2^(5/6)/x*3^(1/2))*2^(1/6)/d*3^(1/2)-1/24*ar
ctan(1/6*x*6^(1/2))*2^(1/6)/d*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = -\frac{\arctan\left(\frac{\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\,2^{5/6}\sqrt{3}d} - \frac{\arctan\left(\frac{x}{\sqrt{6}}\right)}{4\,2^{5/6}\sqrt{3}d} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{-3x^2-2}+\sqrt[3]{2}\right)}{x}\right)}{4\,2^{5/6}d}$$

```
[In] Int[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] -1/4*ArcTan[x/Sqrt[6]]/(2^(5/6)*Sqrt[3]*d) - ArcTan[(2^(1/3) + (-2 - 3*x^2)
^(1/3))^2/(3*2^(1/6)*Sqrt[3]*x)]/(4*2^(5/6)*Sqrt[3]*d) + ArcTanh[(2^(1/6)*(
2^(1/3) + (-2 - 3*x^2)^(1/3)))/x]/(4*2^(5/6)*d)
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

integral

$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt{6}}\right)}{4\ 2^{5/6}\sqrt{3}d} - \frac{\tan^{-1}\left(\frac{\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)^2}{3\sqrt[6]{2}\sqrt{3}x}\right)}{4\ 2^{5/6}\sqrt{3}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[6]{2}\left(\sqrt[3]{2}+\sqrt[3]{-2-3x^2}\right)}{x}\right)}{4\ 2^{5/6}d}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.57 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = -\frac{9x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)}{d\sqrt[3]{-2-3x^2}\,(6+x^2)\left(-9\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-\frac{3x^2}{2},-\frac{x^2}{6}\right)+3\,\text{Ap}\right.\right.}$$

```
[In] Integrate[1/((-2 - 3*x^2)^(1/3)*(6*d + d*x^2)),x]
```

```
[Out] (-9*x*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2])/(d*(-2 - 3*x^2)^(1/
3)*(6 + x^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, (-3*x^2)/2, -1/6*x^2] + x^2*(Ap
pellF1[3/2, 1/3, 2, 5/2, (-3*x^2)/2, -1/6*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/
2, (-3*x^2)/2, -1/6*x^2])))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 73.39 (sec) , antiderivative size = 1061, normalized size of antiderivative = 8.92

method	result	size
trager	Expression too large to display	1061

```
[In] int(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(ln((16*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*Root
Of(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2
)^2*RootOf(_Z^6+54)^5*x+(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^5*x-72*(-3*x^2-2)^
(1/3)*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*
_Z^2)*x+1152*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24
*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*x+36*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+5
4)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2-72*RootOf(RootOf(_Z^6+54)^2-24*_Z*
RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*(-3*x^2-2)^(1/3)*RootOf(_Z^6
+54)^2-432*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*
RootOf(_Z^6+54)+576*_Z^2)-54*(-3*x^2-2)^(2/3))/(x^2+6))*RootOf(_Z^6+54)-24*
RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*ln((4*RootOf(_Z^6+
54)^7*x-288*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf
(_Z^6+54)^6*x+4608*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)
^2*RootOf(_Z^6+54)^5*x+144*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^4*RootOf(RootOf
(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x-6912*(-3*x^2-2)^(1/3)*RootOf(
_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*x+9*x
^2*RootOf(_Z^6+54)^4-216*RootOf(_Z^6+54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*R
ootOf(_Z^6+54)+576*_Z^2)*x^2-18*RootOf(_Z^6+54)^4+432*RootOf(RootOf(_Z^6+54
)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+2592*(-3*x^2-2)^(1/3)
*RootOf(_Z^6+54)*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)-3
24*(-3*x^2-2)^(2/3))/(x^2+6))-24*ln((16*RootOf(RootOf(_Z^6+54)^2-24*_Z*Root
Of(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^6*x-768*RootOf(RootOf(_Z^6+54)^2-24*_
Z*RootOf(_Z^6+54)+576*_Z^2)^2*RootOf(_Z^6+54)^5*x+(-3*x^2-2)^(1/3)*RootOf(_
Z^6+54)^5*x-72*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^4*RootOf(RootOf(_Z^6+54)^2-
24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x+1152*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)^3*R
ootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)^2*x+36*RootOf(_Z^6+
54)^3*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*x^2-72*RootO
f(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)*RootOf(_Z^6+54)^3+18*(-
3*x^2-2)^(1/3)*RootOf(_Z^6+54)^2-432*(-3*x^2-2)^(1/3)*RootOf(_Z^6+54)*RootO
f(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2)-54*(-3*x^2-2)^(2/3))/(x
^2+6))*RootOf(RootOf(_Z^6+54)^2-24*_Z*RootOf(_Z^6+54)+576*_Z^2))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1939 vs. $2(86) = 172$.

Time = 191.30 (sec) , antiderivative size = 1939, normalized size of antiderivative = 16.29

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="fricas")
```

```
[Out] 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(-1/d^6)^(1/6)*log(-1/4*(864*(1/864)^(5/6
)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5 + sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*
d^5))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(5/6) + 12*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^
2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sqrt(-1/d^6) + 4*(1/4)^(2/3)*(7*d^4*x^5 - 92
*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-1/d^6
)^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x + sqrt(-3)*(d^2*x^
5 - 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(1/3) + 8*(5*x^3 -
18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 - s
qrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6
 + 18*x^4 + 108*x^2 + 216)) - 1/48*(1/864)^(1/6)*(sqrt(-3) - 1)*(-1/d^6)^(1
/6)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5 + sqrt(-3)*
(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(5/6) + 12*s
qrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sqrt(-1/d^6) -
4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x - sqrt(-3)*(7*d^4*x^5 - 92
*d^4*x^3 - 36*d^4*x))*(-1/d^6)^(2/3) + 2*(1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3
+ 36*d^2*x + sqrt(-3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 - 2)^(1/3)
*(-1/d^6)^(1/3) - 8*(5*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(1/6)*(d*x^
6 - 210*d*x^4 + 252*d*x^2 - sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d)
 + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) - 1/48*(1/864)^(1/
6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(-1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 - 3
6*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5))*(-3*x^2 -
2)^(1/3)*(-1/d^6)^(5/6) + 12*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*(-3*
x^2 - 2)^(2/3)*sqrt(-1/d^6) + 4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^
4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x))*(-1/d^6)^(2/3) - 2*(1/4
)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqrt(-3)*(d^2*x^5 - 52*d^2*x^3 +
 36*d^2*x))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(1/3) + 8*(5*x^3 - 18*x)*(-3*x^2 -
2)^(2/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + sqrt(-3)*(d*x^6 -
 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*
x^2 + 216)) + 1/48*(1/864)^(1/6)*(sqrt(-3) + 1)*(-1/d^6)^(1/6)*log(1/4*(864
*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5 - sqrt(-3)*(5*d^5*x^4 - 36*
d^5*x^2 - 12*d^5))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(5/6) + 12*sqrt(1/6)*(d^3*x^
4 - 24*d^3*x^2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sqrt(-1/d^6) - 4*(1/4)^(2/3)*(7
*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x + sqrt(-3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^
4*x))*(-1/d^6)^(2/3) + 2*(1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x - sqr
t(-3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x))*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(1/3)
```

```
- 8*(5*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4 +
252*d*x^2 + sqrt(-3)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d) + 72*d)*(-1/d^6
)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216)) + 1/24*(1/864)^(1/6)*(-1/d^6)^(1/6
)*log(1/4*(864*(1/864)^(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5)*(-3*x^2 - 2)
^(1/3)*(-1/d^6)^(5/6) - 6*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*(-3*x^2
 - 2)^(2/3)*sqrt(-1/d^6) + 4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x
)*(-1/d^6)^(2/3) - 2*(1/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x)*(-3*x^2
- 2)^(1/3)*(-1/d^6)^(1/3) - 4*(5*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(
1/6)*(d*x^6 - 210*d*x^4 + 252*d*x^2 + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 +
 108*x^2 + 216)) - 1/24*(1/864)^(1/6)*(-1/d^6)^(1/6)*log(-1/4*(864*(1/864)^
(5/6)*(5*d^5*x^4 - 36*d^5*x^2 - 12*d^5)*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(5/6) -
 6*sqrt(1/6)*(d^3*x^4 - 24*d^3*x^2 + 12*d^3)*(-3*x^2 - 2)^(2/3)*sqrt(-1/d^6
) - 4*(1/4)^(2/3)*(7*d^4*x^5 - 92*d^4*x^3 - 36*d^4*x)*(-1/d^6)^(2/3) + 2*(1
/4)^(1/3)*(d^2*x^5 - 52*d^2*x^3 + 36*d^2*x)*(-3*x^2 - 2)^(1/3)*(-1/d^6)^(1/
3) + 4*(5*x^3 - 18*x)*(-3*x^2 - 2)^(2/3) - (1/864)^(1/6)*(d*x^6 - 210*d*x^4
 + 252*d*x^2 + 72*d)*(-1/d^6)^(1/6))/(x^6 + 18*x^4 + 108*x^2 + 216))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \frac{\int \frac{1}{x^2\sqrt[3]{-3x^2-2}+6\sqrt[3]{-3x^2-2}}\,dx}{d}$$

```
[In] integrate(1/(-3*x**2-2)**(1/3)/(d*x**2+6*d),x)
[Out] Integral(1/(x**2*(-3*x**2 - 2)**(1/3) + 6*(-3*x**2 - 2)**(1/3)), x)/d
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="maxima")
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(dx^2+6\,d)(-3\,x^2-2)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-3*x^2-2)^(1/3)/(d*x^2+6*d),x, algorithm="giac")
[Out] integrate(1/((d*x^2 + 6*d)*(-3*x^2 - 2)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{-2-3x^2}\,(6d+dx^2)}\,dx = \int \frac{1}{(-3\,x^2-2)^{1/3}\,(d\,x^2+6\,d)}\,dx$$

```
[In] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)),x)
[Out] int(1/((- 3*x^2 - 2)^(1/3)*(6*d + d*x^2)), x)
```

3.160 $\int \frac{1}{\sqrt[3]{1+x^2}(9+x^2)} dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \frac{1}{12}\arctan\left(\frac{x}{3}\right)+\frac{1}{12}\arctan\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

```
[Out] 1/12*arctan(1/3*x)+1/12*arctan(1/3*(1-(x^2+1)^(1/3))^2/x)-1/12*arctanh((1-(
x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \frac{1}{12}\arctan\left(\frac{\left(1-\sqrt[3]{x^2+1}\right)^2}{3x}\right)+\frac{1}{12}\arctan\left(\frac{x}{3}\right)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{x^2+1}\right)}{x}\right)}{4\sqrt{3}}$$

```
[In] Int[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] ArcTan[x/3]/12 + ArcTan[(1 - (1 + x^2)^(1/3))^2/(3*x)]/12 - ArcTanh[(Sqrt[3
]*(1 - (1 + x^2)^(1/3)))/x]/(4*Sqrt[3])
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{1}{12}\tan^{-1}\left(\frac{x}{3}\right) + \frac{1}{12}\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+x^2}\right)^2}{3x}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+x^2}\right)}{x}\right)}{4\sqrt{3}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = -\frac{27x\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right)}{\sqrt[3]{1+x^2}\,(9+x^2)\left(-27\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-x^2,-\frac{x^2}{9}\right)+2x^2\left(\text{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-x^2,-\frac{x^2}{9}\right)+3\,\text{Appell}\right.\right.}$$

```
[In] Integrate[1/((1 + x^2)^(1/3)*(9 + x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2])/((1 + x^2)^(1/3)*(9 + x^
2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -x^2, -1/9*x^2] + 2*x^2*(AppellF1[3/2, 1
/3, 2, 5/2, -x^2, -1/9*x^2] + 3*AppellF1[3/2, 4/3, 1, 5/2, -x^2, -1/9*x^2])
))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.47 (sec) , antiderivative size = 623, normalized size of antiderivative = 8.90

method	result
trager	$144\,\mathrm{RootOf}\left(20736\_Z^4-144\_Z^2+1\right)^3 \ln\left(-\frac{497664\,\mathrm{RootOf}\left(20736\_Z^4-144\_Z^2+1\right)^5 (x^2+1)^{\frac{1}{3}} x-995328\,\mathrm{Ro}}{}\right.$

```
[In] int(1/(x^2+1)^(1/3)/(x^2+9),x,method=_RETURNVERBOSE)
```

```
[Out] 144*RootOf(20736*_Z^4-144*_Z^2+1)^3*ln(-(497664*RootOf(20736*_Z^4-144*_Z^2+
1)^5*(x^2+1)^(1/3)*x-995328*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf(2
0736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2+1)
^3*x+144*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2-864*RootOf(20736*_Z^4-144*_Z^2
+1)^2*(x^2+1)^(1/3)-432*RootOf(20736*_Z^4-144*_Z^2+1)^2-96*RootOf(20736*_Z^
4-144*_Z^2+1)*x+6*(x^2+1)^(2/3)-x^2+3)/(x^2+9))-RootOf(20736*_Z^4-144*_Z^2+
1)*ln(-(497664*RootOf(20736*_Z^4-144*_Z^2+1)^5*(x^2+1)^(1/3)*x-995328*RootO
f(20736*_Z^4-144*_Z^2+1)^5*x-6912*RootOf(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(
1/3)*x+20736*RootOf(20736*_Z^4-144*_Z^2+1)^3*x+144*RootOf(20736*_Z^4-144*_Z
^2+1)^2*x^2-864*RootOf(20736*_Z^4-144*_Z^2+1)^2*(x^2+1)^(1/3)-432*RootOf(20
736*_Z^4-144*_Z^2+1)^2-96*RootOf(20736*_Z^4-144*_Z^2+1)*x+6*(x^2+1)^(2/3)-x
^2+3)/(x^2+9))+RootOf(20736*_Z^4-144*_Z^2+1)*ln(-(82944*RootOf(20736*_Z^4-1
44*_Z^2+1)^5*(x^2+1)^(1/3)*x-165888*RootOf(20736*_Z^4-144*_Z^2+1)^5*x-1728*
RootOf(20736*_Z^4-144*_Z^2+1)^3*(x^2+1)^(1/3)*x+2304*RootOf(20736*_Z^4-144*
_Z^2+1)^3*x-24*RootOf(20736*_Z^4-144*_Z^2+1)^2*x^2+144*RootOf(20736*_Z^4-14
4*_Z^2+1)^2*(x^2+1)^(1/3)+8*RootOf(20736*_Z^4-144*_Z^2+1)*(x^2+1)^(1/3)*x+7
2*RootOf(20736*_Z^4-144*_Z^2+1)^2+(x^2+1)^(2/3)-(x^2+1)^(1/3))/(x^2+9))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 1059, normalized size of antiderivative = 15.13

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="fricas")
```

```
[Out] -1/144*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 + sqrt(2)*(x^6 -
 315*x^4 + 567*x^2 + 243)*sqrt(-I*sqrt(3) + 1) + 9*(40*x^3 + (sqrt(3)*sqrt(
2)*(-I*x^4 + 36*I*x^2 - 27*I) + sqrt(2)*(x^4 - 36*x^2 + 27))*sqrt(-I*sqrt(3
) + 1) - 216*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 - 138*I*x^3 - 81*I*x)
- 3*(2*x^5 - 156*x^3 - 2*sqrt(3)*(I*x^5 - 78*I*x^3 + 81*I*x) - 3*(sqrt(3)*s
```

```
qrt(2)*(-5*I*x^4 + 54*I*x^2 + 27*I) - sqrt(2)*(5*x^4 - 54*x^2 - 27))*sqrt(-
I*sqrt(3) + 1) + 162*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 + 27*x^4 + 243*x^2 +
729)) + 1/144*sqrt(2)*sqrt(-I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 - sqrt(2)
*(x^6 - 315*x^4 + 567*x^2 + 243)*sqrt(-I*sqrt(3) + 1) + 9*(40*x^3 + (sqrt(3
)*sqrt(2)*(I*x^4 - 36*I*x^2 + 27*I) - sqrt(2)*(x^4 - 36*x^2 + 27))*sqrt(-I*
sqrt(3) + 1) - 216*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(7*I*x^5 - 138*I*x^3 - 81
*I*x) - 3*(2*x^5 - 156*x^3 - 2*sqrt(3)*(I*x^5 - 78*I*x^3 + 81*I*x) - 3*(sqr
t(3)*sqrt(2)*(5*I*x^4 - 54*I*x^2 - 27*I) + sqrt(2)*(5*x^4 - 54*x^2 - 27))*s
qrt(-I*sqrt(3) + 1) + 162*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 + 27*x^4 + 243*x
^2 + 729)) - 1/144*sqrt(2)*sqrt(I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 + 72*
(5*x^3 - 27*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(-7*I*x^5 + 138*I*x^3 + 81*I*x)
+ (9*(sqrt(3)*sqrt(2)*(I*x^4 - 36*I*x^2 + 27*I) + sqrt(2)*(x^4 - 36*x^2 + 2
7))*(x^2 + 1)^(2/3) + sqrt(2)*(x^6 - 315*x^4 + 567*x^2 + 243) + 9*(sqrt(3)*
sqrt(2)*(5*I*x^4 - 54*I*x^2 - 27*I) - sqrt(2)*(5*x^4 - 54*x^2 - 27))*(x^2 +
 1)^(1/3))*sqrt(I*sqrt(3) + 1) - 6*(x^5 - 78*x^3 - sqrt(3)*(-I*x^5 + 78*I*x
^3 - 81*I*x) + 81*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 + 27*x^4 + 243*x^2 + 729
)) + 1/144*sqrt(2)*sqrt(I*sqrt(3) + 1)*log(-(42*x^5 - 828*x^3 + 72*(5*x^3 -
 27*x)*(x^2 + 1)^(2/3) + 6*sqrt(3)*(-7*I*x^5 + 138*I*x^3 + 81*I*x) + (9*(sq
rt(3)*sqrt(2)*(-I*x^4 + 36*I*x^2 - 27*I) - sqrt(2)*(x^4 - 36*x^2 + 27))*(x^
2 + 1)^(2/3) - sqrt(2)*(x^6 - 315*x^4 + 567*x^2 + 243) + 9*(sqrt(3)*sqrt(2)
*(-5*I*x^4 + 54*I*x^2 + 27*I) + sqrt(2)*(5*x^4 - 54*x^2 - 27))*(x^2 + 1)^(1
/3))*sqrt(I*sqrt(3) + 1) - 6*(x^5 - 78*x^3 - sqrt(3)*(-I*x^5 + 78*I*x^3 - 8
1*I*x) + 81*x)*(x^2 + 1)^(1/3) - 486*x)/(x^6 + 27*x^4 + 243*x^2 + 729)) - 1
/36*arctan(6*(11*x^5 + 30*x^3 + 6*(23*x^3 + 27*x)*(x^2 + 1)^(2/3) + (x^5 -
240*x^3 - 81*x)*(x^2 + 1)^(1/3) - 81*x)/(x^6 - 1971*x^4 - 1701*x^2 - 729))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{\sqrt[3]{x^2+1}\,(x^2+9)}\,dx$$

```
[In] integrate(1/(x**2+1)**(1/3)/(x**2+9),x)
[Out] Integral(1/((x**2 + 1)**(1/3)*(x**2 + 9)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="maxima")
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+9)(x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/3)/(x^2+9),x, algorithm="giac")
[Out] integrate(1/((x^2 + 9)*(x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+x^2}\,(9+x^2)}\,dx = \int \frac{1}{(x^2+1)^{1/3}\,(x^2+9)}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)),x)
[Out] int(1/((x^2 + 1)^(1/3)*(x^2 + 9)), x)
```

3.161 $\int \frac{1}{\sqrt[3]{1+bx^2}(9+bx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}+\frac{\arctan\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

```
[Out] 1/12*arctan(1/3*(1-(b*x^2+1)^(1/3))^2/x/b^(1/2))/b^(1/2)+1/12*arctan(1/3*x*
b^(1/2))/b^(1/2)-1/12*arctanh((1-(b*x^2+1)^(1/3))*3^(1/2)/x/b^(1/2))*3^(1/2
)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {403}

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx=\frac{\arctan\left(\frac{\left(1-\sqrt[3]{bx^2+1}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}}+\frac{\arctan\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{bx^2+1}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

```
[In] Int[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/3]/(12*Sqrt[b]) + ArcTan[(1 - (1 + b*x^2)^(1/3))^2/(3*Sq
rt[b]*x)]/(12*Sqrt[b]) - ArcTanh[(Sqrt[3]*(1 - (1 + b*x^2)^(1/3)))/(Sqrt[b]
*x)]/(4*Sqrt[3]*Sqrt[b])
```

Rule 403

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b/a, 2]}, Simp[q*(ArcTan[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Simp[q*
(ArcTan[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a, 3]*d
)), x] - Simp[q*(ArcTanh[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[a, 3]
*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && EqQ[b*c - 9*a*d, 0] && PosQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{3}\right)}{12\sqrt{b}} + \frac{\tan^{-1}\left(\frac{\left(1-\sqrt[3]{1+bx^2}\right)^2}{3\sqrt{b}x}\right)}{12\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1+bx^2}\right)}{\sqrt{b}x}\right)}{4\sqrt{3}\sqrt{b}}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 4.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.32

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx =$$

$$-\frac{27x \operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-bx^2,-\frac{bx^2}{9}\right)}{\sqrt[3]{1+bx^2}\,(9+bx^2)\left(-27\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{3},1,\frac{3}{2},-bx^2,-\frac{bx^2}{9}\right)+2bx^2\left(\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{3},2,\frac{5}{2},-bx^2,-\frac{bx^2}{9}\right)+\right.\right.}$$

```
[In] Integrate[1/((1 + b*x^2)^(1/3)*(9 + b*x^2)),x]
```

```
[Out] (-27*x*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)])/((1 + b*x^2)^(1/
3)*(9 + b*x^2)*(-27*AppellF1[1/2, 1/3, 1, 3/2, -(b*x^2), -1/9*(b*x^2)] + 2*
b*x^2*(AppellF1[3/2, 1/3, 2, 5/2, -(b*x^2), -1/9*(b*x^2)] + 3*AppellF1[3/2,
 4/3, 1, 5/2, -(b*x^2), -1/9*(b*x^2)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+1\right)^{\frac{1}{3}} \left(b\,x^{2}+9\right)}dx$$

```
[In] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
[Out] int(1/(b*x^2+1)^(1/3)/(b*x^2+9),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{\sqrt[3]{bx^2+1}\,(bx^2+9)}\,dx$$

```
[In] integrate(1/(b*x**2+1)**(1/3)/(b*x**2+9),x)
[Out] Integral(1/((b*x**2 + 1)**(1/3)*(b*x**2 + 9)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(bx^2+9)(bx^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(b*x^2+1)^(1/3)/(b*x^2+9),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 9)*(b*x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1+bx^2}\,(9+bx^2)}\,dx = \int \frac{1}{(b\,x^2+1)^{1/3}\,(b\,x^2+9)}\,dx$$

```
[In] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)),x)
[Out] int(1/((b*x^2 + 1)^(1/3)*(b*x^2 + 9)), x)
```

3.162 $\int \frac{1}{\sqrt[3]{1-x^2}(9-x^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}+\frac{1}{12}\operatorname{arctanh}\left(\frac{x}{3}\right)-\frac{1}{12}\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

```
[Out] 1/12*arctanh(1/3*x)-1/12*arctanh(1/3*(1-(-x^2+1)^(1/3))^2/x)+1/12*arctan((1
-(-x^2+1)^(1/3))*3^(1/2)/x)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {404}

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx=\frac{\arctan\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}}-\frac{1}{12}\operatorname{arctanh}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)+\frac{1}{12}\operatorname{arctanh}\left(\frac{x}{3}\right)$$

```
[In] Int[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] ArcTan[(Sqrt[3]*(1 - (1 - x^2)^(1/3)))/x]/(4*Sqrt[3]) + ArcTanh[x/3]/12 - A
rcTanh[(1 - (1 - x^2)^(1/3))^2/(3*x)]/12
```

Rule 404

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/3)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[(-q)*(ArcTanh[q*(x/3)]/(12*Rt[a, 3]*d)), x] + (Si
mp[q*(ArcTanh[(Rt[a, 3] - (a + b*x^2)^(1/3))^2/(3*Rt[a, 3]^2*q*x)]/(12*Rt[a
, 3]*d)), x] - Simp[q*(ArcTan[(Sqrt[3]*(Rt[a, 3] - (a + b*x^2)^(1/3)))/(Rt[
a, 3]*q*x)]/(4*Sqrt[3]*Rt[a, 3]*d)), x])] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && EqQ[b*c - 9*a*d, 0] && NegQ[b/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{\sqrt{3}\left(1-\sqrt[3]{1-x^2}\right)}{x}\right)}{4\sqrt{3}} + \frac{1}{12}\tanh^{-1}\left(\frac{x}{3}\right) - \frac{1}{12}\tanh^{-1}\left(\frac{\left(1-\sqrt[3]{1-x^2}\right)^2}{3x}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 3 in optimal.

Time = 8.55 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx$$
$$= \frac{\sqrt[3]{\frac{-1+x}{-3+x}}\sqrt[3]{\frac{1+x}{-3+x}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},-\frac{4}{-3+x},-\frac{2}{-3+x}\right) - \sqrt[3]{\frac{-1+x}{3+x}}\sqrt[3]{\frac{1+x}{3+x}}\,\text{AppellF1}\left(\frac{2}{3},\frac{1}{3},\frac{1}{3},\frac{5}{3},\frac{2}{3+x},\right.}{4\sqrt[3]{1-x^2}}$$

```
[In] Integrate[1/((1 - x^2)^(1/3)*(9 - x^2)),x]
```

```
[Out] (((-1 + x)/(-3 + x))^(1/3)*((1 + x)/(-3 + x))^(1/3)*AppellF1[2/3, 1/3, 1/3,
 5/3, -4/(-3 + x), -2/(-3 + x)] - ((-1 + x)/(3 + x))^(1/3)*((1 + x)/(3 + x)
)^(1/3)*AppellF1[2/3, 1/3, 1/3, 5/3, 2/(3 + x), 4/(3 + x)])/(4*(1 - x^2)^(1
/3))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.63 (sec) , antiderivative size = 539, normalized size of antiderivative = 7.28

method	result
trager	$-\frac{\ln\left(\frac{288\left(-x^2+1\right)^{\frac{1}{3}}\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x+36\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)\left(-x^2+1\right)^{\frac{1}{3}}x-576\operatorname{RootOf}\left(144\_Z^2+12\_Z+1\right)^2 x-}{}\right.}{}$

[In] int(1/(-x^2+1)^(1/3)/(-x^2+9),x,method=_RETURNVERBOSE)

[Out] -1/12*ln((288*(-x^2+1)^(1/3)*RootOf(144*_Z^2+12*_Z+1)^2*x+36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-576*RootOf(144*_Z^2+12*_Z+1)^2*x-6*RootOf(144*_Z^2+12*_Z+1)*x^2+3*(-x^2+1)^(2/3)-36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)+x*(-x^2+1)^(1/3)-24*RootOf(144*_Z^2+12*_Z+1)*x-3*(-x^2+1)^(1/3)-18*RootOf(144*_Z^2+12*_Z+1))/(-3+x)/(3+x))-ln((288*(-x^2+1)^(1/3)*RootOf(144*_Z^2+12*_Z+1)^2*x+36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-576*RootOf(144*_Z^2+12*_Z+1)^2*x-6*RootOf(144*_Z^2+12*_Z+1)*x^2+3*(-x^2+1)^(2/3)-36*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)+x*(-x^2+1)^(1/3)-24*RootOf(144*_Z^2+12*_Z+1)*x-3*(-x^2+1)^(1/3)-18*RootOf(144*_Z^2+12*_Z+1))/(-3+x)/(3+x))*RootOf(144*_Z^2+12*_Z+1)+RootOf(144*_Z^2+12*_Z+1)*ln((576*(-x^2+1)^(1/3)*RootOf(144*_Z^2+12*_Z+1)^2*x+24*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)*x-1152*RootOf(144*_Z^2+12*_Z+1)^2*x+12*RootOf(144*_Z^2+12*_Z+1)*x^2+6*(-x^2+1)^(2/3)+72*RootOf(144*_Z^2+12*_Z+1)*(-x^2+1)^(1/3)-144*RootOf(144*_Z^2+12*_Z+1)*x+x^2+36*RootOf(144*_Z^2+12*_Z+1)-4*x+3)/(-3+x)/(3+x))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(53) = 106$.

Time = 0.54 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.64

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx =$$

$$-\frac{1}{36}\sqrt{3}\arctan\left(\frac{36\sqrt{3}(x^4-32x^3-42x^2+9)(-x^2+1)^{\frac{2}{3}}+12\sqrt{3}(x^5+27x^4-210x^3-54x^2+81x+}{3\,(x^6+108x^5-1647x^4-1080x^3}\right.$$

$$-\frac{1}{72}\log\left(\frac{x^3+33x^2+18(-x^2+1)^{\frac{2}{3}}(x+1)-6(x^2+6x-3)(-x^2+1)^{\frac{1}{3}}-9x-9}{x^3+9x^2+27x+27}\right)$$

$$+\frac{1}{36}\log\left(-\frac{x^3-33x^2+18(-x^2+1)^{\frac{2}{3}}(x-1)+6(x^2-6x-3)(-x^2+1)^{\frac{1}{3}}-9x+9}{x^3+9x^2+27x+27}\right)$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(3)*arctan(1/3*(36*sqrt(3)*(x^4 - 32*x^3 - 42*x^2 + 9)*(-x^2 + 1)
^(2/3) + 12*sqrt(3)*(x^5 + 27*x^4 - 210*x^3 - 54*x^2 + 81*x + 27)*(-x^2 + 1
)^(1/3) + sqrt(3)*(x^6 - 108*x^5 - 567*x^4 + 1080*x^3 + 459*x^2 - 972*x - 4
05))/(x^6 + 108*x^5 - 1647*x^4 - 1080*x^3 + 891*x^2 + 972*x + 243)) - 1/72*
log((x^3 + 33*x^2 + 18*(-x^2 + 1)^(2/3)*(x + 1) - 6*(x^2 + 6*x - 3)*(-x^2 +
 1)^(1/3) - 9*x - 9)/(x^3 + 9*x^2 + 27*x + 27)) + 1/36*log(-(x^3 - 33*x^2 +
 18*(-x^2 + 1)^(2/3)*(x - 1) + 6*(x^2 - 6*x - 3)*(-x^2 + 1)^(1/3) - 9*x + 9
)/(x^3 + 9*x^2 + 27*x + 27))
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = -\int \frac{1}{x^2\sqrt[3]{1-x^2}-9\sqrt[3]{1-x^2}}\,dx$$

```
[In] integrate(1/(-x**2+1)**(1/3)/(-x**2+9),x)
```

```
[Out] -Integral(1/(x**2*(1 - x**2)**(1/3) - 9*(1 - x**2)**(1/3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = \int -\frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = \int -\frac{1}{(x^2-9)(-x^2+1)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/3)/(-x^2+9),x, algorithm="giac")
```

```
[Out] integrate(-1/((x^2 - 9)*(-x^2 + 1)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{1-x^2}\,(9-x^2)}\,dx = -\int \frac{1}{(1-x^2)^{1/3}\,(x^2-9)}\,dx$$

```
[In] int(-1/((1 - x^2)^(1/3)*(x^2 - 9)),x)
[Out] -int(1/((1 - x^2)^(1/3)*(x^2 - 9)), x)
```

3.163 $\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 79

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx=\frac{x\sqrt{-1+c^2x^2}}{2d\,(d-c^2dx^2)^{3/2}}+\frac{\sqrt{-1+c^2x^2}\operatorname{arctanh}(cx)}{2cd^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*x*(c^2*x^2-1)^(1/2)/d/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1)
^(1/2)/c/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 205, 214}

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx=\frac{\sqrt{c^2x^2-1}\operatorname{arctanh}(cx)}{2cd^2\sqrt{d-c^2dx^2}}+\frac{x\sqrt{c^2x^2-1}}{2d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}}$$

```
[In] Int[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[-1 + c^2*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
 + c^2*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-1+c^2x^2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{\sqrt{d-c^2dx^2}} \\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+c^2x^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{d-c^2dx^2}} \\
&= \frac{x\sqrt{-1+c^2x^2}}{2d^2\,(1-c^2x^2)\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+c^2x^2}\tanh^{-1}(cx)}{2cd^2\sqrt{d-c^2dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$
\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \frac{-cx+(-1+c^2x^2)\operatorname{arctanh}(cx)}{2cd^2\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}
$$

```
[In] Integrate[Sqrt[-1 + c^2*x^2]/(d - c^2*d*x^2)^(5/2),x]
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d -
 c^2*d*x^2])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\sqrt{-(c^2x^2-1)d}\,(\ln(cx-1)c^2x^2-\ln(cx+1)c^2x^2+2cx-\ln(cx-1)+\ln(cx+1))}{4\sqrt{c^2x^2-1}\,d^3(cx-1)c(cx+1)}$	94
risch	$-\frac{x}{2d^2\sqrt{c^2x^2-1}\,\sqrt{-(c^2x^2-1)d}}+\frac{\sqrt{c^2x^2-1}\,\ln(-cx-1)}{4d^2\sqrt{-(c^2x^2-1)d}\,c}-\frac{\sqrt{c^2x^2-1}\,\ln(cx-1)}{4d^2\sqrt{-(c^2x^2-1)d}\,c}$	112

```
[In] int((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*c
^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^3/(c*x-1)/c/(c*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.99

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+5c^4dx^4-5c^2dx^2-4}{c^6x^6}\right.}{8(c^5d^3x^4-2c^3d^3x^2+cd^3)}\right.$$

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3), 1/4*(2*sqrt(-c^2
*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arcta
n(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c
^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3)]
```

Sympy [F]

$$\int \frac{\sqrt{-1+c^2x^2}}{(d-c^2dx^2)^{5/2}}\,dx = \int \frac{\sqrt{(cx-1)(cx+1)}}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral(sqrt((c*x - 1)*(c*x + 1))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx = -\frac{x}{2\left(c^2\sqrt{-d}d^2x^2-\sqrt{-d}d^2\right)} - \frac{\sqrt{-d}\log\left(cx+1\right)}{4\,cd^3} + \frac{\sqrt{-d}\log\left(cx-1\right)}{4\,cd^3}$$

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
[Out] -1/2*x/(c^2*sqrt(-d)*d^2*x^2 - sqrt(-d)*d^2) - 1/4*sqrt(-d)*log(c*x + 1)/(c
*d^3) + 1/4*sqrt(-d)*log(c*x - 1)/(c*d^3)
```

Giac [F]

$$\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{c^2x^2-1}}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(c^2*x^2 - 1)/(-c^2*d*x^2 + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+c^2x^2}}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{c^2\,x^2-1}}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

```
[In] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2),x)
[Out] int((c^2*x^2 - 1)^(1/2)/(d - c^2*d*x^2)^(5/2), x)
```

3.164 $\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 74

$$\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx=\frac{dx\sqrt{-1+c^2x^2}}{2\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{-1+c^2x^2}\operatorname{arctanh}(cx)}{2c\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*d*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)+1/2*arctanh(c*x)*(c^2*x^2-1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 205, 214}

$$\int \frac{1}{\left(-1+c^2x^2\right)^{3/2}\sqrt{d-c^2dx^2}}\,dx=\frac{\operatorname{arctanh}(cx)\left(d-c^2dx^2\right)^{3/2}}{2cd^2\left(c^2x^2-1\right)^{3/2}}+\frac{x\left(d-c^2dx^2\right)^{3/2}}{2d^2\left(1-c^2x^2\right)\left(c^2x^2-1\right)^{3/2}}$$

```
[In] Int[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (x*(d - c^2*d*x^2)^(3/2))/(2*d^2*(1 - c^2*x^2)*(-1 + c^2*x^2)^(3/2)) + ((d
- c^2*d*x^2)^(3/2)*ArcTanh[c*x])/(2*c*d^2*(-1 + c^2*x^2)^(3/2))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(d-c^2dx^2)^{3/2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{(-1+c^2x^2)^{3/2}} \\
&= \frac{x(d-c^2dx^2)^{3/2}}{2d^2(1-c^2x^2)(-1+c^2x^2)^{3/2}} + \frac{(d-c^2dx^2)^{3/2}\int \frac{1}{d-c^2dx^2}\,dx}{2d(-1+c^2x^2)^{3/2}} \\
&= \frac{x(d-c^2dx^2)^{3/2}}{2d^2(1-c^2x^2)(-1+c^2x^2)^{3/2}} + \frac{(d-c^2dx^2)^{3/2}\tanh^{-1}(cx)}{2cd^2(-1+c^2x^2)^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$
\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \frac{-cx+(-1+c^2x^2)\operatorname{arctanh}(cx)}{2c\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}
$$

```
[In] Integrate[1/((-1 + c^2*x^2)^(3/2)*Sqrt[d - c^2*d*x^2]),x]
[Out] (-(c*x) + (-1 + c^2*x^2)*ArcTanh[c*x])/(2*c*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2
*d*x^2])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

method	result	size
default	$\frac{\sqrt{-(c^2x^2-1)d}\,(\ln(cx-1)c^2x^2-\ln(cx+1)c^2x^2+2cx-\ln(cx-1)+\ln(cx+1))}{4\sqrt{c^2x^2-1}\,d(cx-1)c(cx+1)}$	94
risch	$-\frac{x}{2\sqrt{c^2x^2-1}\,\sqrt{-(c^2x^2-1)d}}+\frac{\sqrt{c^2x^2-1}\,\ln(-cx-1)}{4\sqrt{-(c^2x^2-1)d}\,c}-\frac{\sqrt{c^2x^2-1}\,\ln(cx-1)}{4\sqrt{-(c^2x^2-1)d}\,c}$	103

```
[In] int(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*c
^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d/(c*x-1)/c/(c*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.09

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \left[\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c}{}\right.}{8\,(c^5dx^4-2\,c^3dx^2+cd)}\right.$$

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 +
 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)))/(c^5*d*x^4 - 2*c^3*d*x^2 + c*d), 1/4*(2*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arctan(2*sq
rt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/(c^5*d*x
^4 - 2*c^3*d*x^2 + c*d)]
```

Sympy [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \int \frac{1}{((cx-1)(cx+1))^{\frac{3}{2}}\sqrt{-d(cx-1)(cx+1)}}\,dx$$

```
[In] integrate(1/(c**2*x**2-1)**(3/2)/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(((c*x - 1)*(c*x + 1))**(3/2)*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

Maxima [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \int \frac{1}{\sqrt{-c^2dx^2+d}(c^2x^2-1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \int \frac{1}{\sqrt{-c^2dx^2+d}(c^2x^2-1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c^2*x^2-1)^(3/2)/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-c^2*d*x^2 + d)*(c^2*x^2 - 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-1+c^2x^2)^{3/2}\sqrt{d-c^2dx^2}}\,dx = \int \frac{1}{\sqrt{d-c^2\,d\,x^2}\,(c^2\,x^2-1)^{3/2}}\,dx$$

```
[In] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)),x)
[Out] int(1/((d - c^2*d*x^2)^(1/2)*(c^2*x^2 - 1)^(3/2)), x)
```

3.165 $\int \frac{1}{\sqrt{-1+c^2x^2}(d-c^2dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 76

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx=-\frac{x\sqrt{-1+c^2x^2}}{2\,(d-c^2dx^2)^{3/2}}-\frac{\sqrt{-1+c^2x^2}\,\mathrm{arctanh}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] -1/2*x*(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2)-1/2*arctanh(c*x)*(c^2*x^2-1)^
(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {23, 205, 214}

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx=\frac{\mathrm{arctanh}(cx)\sqrt{d-c^2dx^2}}{2cd^2\sqrt{c^2x^2-1}}+\frac{x\sqrt{d-c^2dx^2}}{2d^2\,(1-c^2x^2)\,\sqrt{c^2x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[d - c^2*d*x^2])/(2*d^2*(1 - c^2*x^2)*Sqrt[-1 + c^2*x^2]) + (Sqrt[d
- c^2*d*x^2]*ArcTanh[c*x])/(2*c*d^2*Sqrt[-1 + c^2*x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{d-c^2dx^2}\int \frac{1}{(d-c^2dx^2)^2}\,dx}{\sqrt{-1+c^2x^2}} \\
&= \frac{x\sqrt{d-c^2dx^2}}{2d^2(1-c^2x^2)\sqrt{-1+c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\int \frac{1}{d-c^2dx^2}\,dx}{2d\sqrt{-1+c^2x^2}} \\
&= \frac{x\sqrt{d-c^2dx^2}}{2d^2(1-c^2x^2)\sqrt{-1+c^2x^2}} + \frac{\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{2cd^2\sqrt{-1+c^2x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.75

$$
\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \frac{cx+(1-c^2x^2)\operatorname{arctanh}(cx)}{2cd\sqrt{-1+c^2x^2}\sqrt{d-c^2dx^2}}
$$

```
[In] Integrate[1/(Sqrt[-1 + c^2*x^2]*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (c*x + (1 - c^2*x^2)*ArcTanh[c*x])/(2*c*d*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d
*x^2])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.24

method	result	size
default	$-\frac{\sqrt{-\left(c^{2} x^{2}-1\right) d}\,\left(\ln \left(c x -1\right) c^{2} x^{2}-\ln \left(c x +1\right) c^{2} x^{2}+2 c x -\ln \left(c x -1\right)+\ln \left(c x +1\right)\right)}{4 \sqrt{c^{2} x^{2}-1}\, d^{2} \left(c x -1\right) c \left(c x +1\right)}$	94
risch	$\frac{x}{2 d \sqrt{c^{2} x^{2}-1}\, \sqrt{-\left(c^{2} x^{2}-1\right) d}}-\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(-c x -1\right)}{4 d \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}+\frac{\sqrt{c^{2} x^{2}-1}\, \ln \left(c x -1\right)}{4 d \sqrt{-\left(c^{2} x^{2}-1\right) d}\, c}$	112

```
[In] int(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(c^2*x^2-1)^(1/2)*(-(c^2*x^2-1)*d)^(1/2)*(ln(c*x-1)*c^2*x^2-ln(c*x+1)*
c^2*x^2+2*c*x-ln(c*x-1)+ln(c*x+1))/d^2/(c*x-1)/c/(c*x+1)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.13

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \left[-\frac{4\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx+(c^4x^4-2\,c^2x^2+1)\sqrt{-d}\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2+4\,(c^3x^3+cx)\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\sqrt{-d}-d}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right)}{8\,(c^5d^2x^4-2\,c^3d^2x^2+cd^2)}, -\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}cx-(c^4x^4-2\,c^2x^2+1)\sqrt{d}\arctan\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}c\sqrt{d}x}{c^4dx^4-d}\right)}{4\,(c^5d^2x^4-2\,c^3d^2x^2+cd^2)}\right]$$

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x + (c^4*x^4 - 2*c^2*x^2
+ 1)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^
4 + 3*c^2*x^2 - 1)))/(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2), -1/4*(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(d)*arc
tan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)))/
(c^5*d^2*x^4 - 2*c^3*d^2*x^2 + c*d^2)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{(cx-1)\,(cx+1)}\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(c**2*x**2-1)**(1/2)/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(1/(sqrt((c*x - 1)*(c*x + 1))*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{(-c^2dx^2+d)^{\frac{3}{2}}\sqrt{c^2x^2-1}}\,dx$$

```
[In] integrate(1/(c^2*x^2-1)^(1/2)/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-c^2*d*x^2 + d)^(3/2)*sqrt(c^2*x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+c^2x^2}\,(d-c^2dx^2)^{3/2}}\,dx = \int \frac{1}{(d-c^2\,d\,x^2)^{3/2}\,\sqrt{c^2\,x^2-1}}\,dx$$

```
[In] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)),x)
[Out] int(1/((d - c^2*d*x^2)^(3/2)*(c^2*x^2 - 1)^(1/2)), x)
```

3.166 $\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 328

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx=\frac{\left(7ac-\frac{2bc^2}{d}+\frac{3a^2d}{b}\right)x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}}-\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d}+\frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d}+\frac{\sqrt{c}(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{c^{3/2}(bc-9ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] 1/5*b*x*(d*x^2+c)^(3/2)*(b*x^2+a)^(1/2)/d+1/15*(7*a*c-2*b*c^2/d+3*a^2*d/b)*
x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-1/15*c^(3/2)*(-9*a*d+b*c)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+1/15*(-3*a^2*d^2-7*a*b*c*d+2*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*c^(1/2)*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)-2/15*(-3*a*d+b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}(-3a^2d^2-7abcd+2b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{a+bx^2}\left(\frac{3a^2d}{b}+7ac-\frac{2bc^2}{d}\right)}{15\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{a+bx^2}(bc-9ad)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} - \frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-3ad)}{15d}$$

```
[In] Int[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] ((7*a*c - (2*b*c^2)/d + (3*a^2*d)/b)*x*Sqrt[a + b*x^2])/(15*Sqrt[c + d*x^2]
) - (2*(b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d) + (b*x*Sqrt[
a + b*x^2]*(c + d*x^2)^(3/2))/(5*d) + (Sqrt[c]*(2*b^2*c^2 - 7*a*b*c*d - 3*a
^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(15*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
- (c^(3/2)*(b*c - 9*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(15*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} + \frac{\int \frac{\sqrt{c+dx^2}(-a(bc-5ad)-2b(bc-3ad)x^2)}{\sqrt{a+bx^2}}\,dx}{5d} \\
&= -\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} \\
&\quad + \frac{\int \frac{-abc(bc-9ad)-b(2b^2c^2-7abcd-3a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd} \\
&= -\frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} \\
&\quad - \frac{(ac(bc-9ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d} - \frac{(2b^2c^2-7abcd-3a^2d^2)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d}
\end{aligned}
$$

$$= \frac{\left(7ac - \frac{2bc^2}{d} + \frac{3a^2d}{b}\right) x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}} - \frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} - \frac{c^{3/2}(bc-9ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(c(2b^2c^2-7abcd-3a^2d^2))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15bd}$$

$$= \frac{\left(7ac - \frac{2bc^2}{d} + \frac{3a^2d}{b}\right) x\sqrt{a+bx^2}}{15\sqrt{c+dx^2}} - \frac{2(bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d} + \frac{bx\sqrt{a+bx^2}(c+dx^2)^{3/2}}{5d} + \frac{\sqrt{c}(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}(bc-9ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.74

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(6ad+b(c+3dx^2)) - ic(-2b^2c^2+7abcd+3a^2d^2)\sqrt{1+\frac{bx^2}{a}}\cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^2)^(3/2)*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*a*d + b*(c + 3*d*x^2)) - I*c*(-2*
b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(b^2*c^2 - 4*a*b*c*d
 + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2
])
```

Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.26

method	result
risch	$\frac{x(3bdx^2+6ad+bc)\sqrt{bx^2+a}\,\sqrt{dx^2+c}}{15d}+\frac{\left(\frac{9a^2cd\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bdx^4+adx^2+cbx^2+ac}}-\frac{bc^2a\sqrt{1+\frac{bx^2}{a}}\,\sqrt{1+\frac{dx^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\cdots}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bdx^4+adx^2+cbx^2+ac}}\right.}{}$
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{bx^3\sqrt{bdx^4+adx^2+cbx^2+ac}}{5}+\frac{\left(2abd+b^2c-\frac{b(4ad+4bc)}{5}\right)x\sqrt{bdx^4+adx^2+cbx^2+ac}}{3bd}+\frac{\left(a^2c-\frac{\left(2abd+b^2c-\frac{b(4ad+4bc)}{5}\right)}{3bd}\right.}{\sqrt{-\frac{b}{a}}\cdots}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\,\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+9\sqrt{-\frac{b}{a}}\,abd^3x^5+4\sqrt{-\frac{b}{a}}\,b^2cd^2x^5+6\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abcd^2x^3+\sqrt{-\frac{b}{a}}\,b^2c^2dx^3+6\sqrt{\frac{bx^2+\cdots}{a}}\right.}{}$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(3*b*d*x^2+6*a*d+b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d+1/15/d*(9*a^
2*c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-b*c^2*a/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(3*a^2*d^2+7*a
*b*c*d-2*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d
*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

$$\int (a+bx^2)^{3/2}\sqrt{c+dx^2}\,dx=\frac{(2\,b^2c^3-7\,abc^2d-3\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^2c^3-7\,abc^2d-\cdots}{}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*((2*b^2*c^3 - 7*a*b*c^2*d - 3*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellipt
ic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 - 7*a*b*c^2*d - 9*a^2*d^
3 - (3*a^2 - a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d
```

```
)/x), a*d/(b*c)) + (3*b^2*d^3*x^4 - 2*b^2*c^2*d + 7*a*b*c*d^2 + 3*a^2*d^3 +
 (b^2*c*d^2 + 6*a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*d^3*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx=\int \left(a+bx^2\right)^{\frac{3}{2}}\sqrt{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\,dx=\int \left(b\,x^2+a\right)^{3/2}\,\sqrt{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2), x)
```

3.167 $\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 249

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} - \frac{\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] 1/3*(a*d+b*c)*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+2/3*c^(3/2)*(1/(1+d*x^2/c
))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1
-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d
*x^2+c)^(1/2)-1/3*(a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipti
cE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)
^(1/2)/b/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*x*(b*x
^2+a)^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used

= {428, 545, 429, 506, 422}

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \frac{2c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{x\sqrt{a+bx^2}(ad+bc)}{3b\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] ((b*c + a*d)*x*Sqrt[a + b*x^2])/(3*b*Sqrt[c + d*x^2]) + (x*Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/3 - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(
a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (2*c^(3/2)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
```

```
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2}{3}\int \frac{ac+\frac{1}{2}(bc+ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{1}{3}(2ac)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&\quad + \frac{1}{3}(bc+ad)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} \\
&\quad + \frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(c(bc+ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3b} \\
&= \frac{(bc+ad)x\sqrt{a+bx^2}}{3b\sqrt{c+dx^2}} + \frac{1}{3}x\sqrt{a+bx^2}\sqrt{c+dx^2} \\
&\quad - \frac{\sqrt{c}(bc+ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{2c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2) - ic(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{\frac{b}{a}}x\Big)\,|\frac{ad}{bc}\Big) - ic(-bc+ad)\sqrt{1+\frac{b}{}} \cdots}{3\sqrt{\frac{b}{a}}d\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - I*c*(b*c + a*d)*Sqrt[1 + (b*x^2)/a
]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*
(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3}+\frac{\Big(\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\Big(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\Big)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(ad+bc)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\Big(F\Big(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\Big)-E\Big(x\sqrt{-} \cdots}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d}\cdots}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\Big(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3}+\frac{2ac\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\Big(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\Big)}{3\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\big(\frac{ad}{3}+\frac{bc}{3}\big)c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\Big(F\Big(x\sqrt{-} \cdots}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4} \cdots}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\Big(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\Big(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\Big)d-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\Big(x\sqrt{-} \cdots}{3(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)+(2/3*a*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/
2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-1/3*(a*d+b*c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx =$$

$$-\frac{(bc^2+acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2+acd+2\,ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) -}{3\,bd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((b*c^2 + a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x
), a*d/(b*c)) - (b*c^2 + a*c*d + 2*a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f
(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^2 + b*c*d + a*d^2)*sqrt(b*x^2
+ a)*sqrt(d*x^2 + c))/(b*d^2*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\sqrt{c+dx^2}\,dx = \int \sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.168 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= c\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\ &= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}\end{aligned}$$

$$= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - \sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - \sqrt{bx^2+a}\sqrt{dx^2+c}d}{bdx}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.169 $\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[Out] (1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(1/2)/b^(1/2)/(b*x^2+a)^(1/
2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {422}

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(
Sqrt[a]*Sqrt[b]*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{b}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \frac{x(c+dx^2)+\frac{ic\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{\sqrt{\frac{b}{a}}}}{a\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(3/2),x]
```

```
[Out] (x*(c + d*x^2) + (I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(EllipticE[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)]))/Sqrt[b/a])/(a*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.15

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\left(\sqrt{-\frac{b}{a}}\,d\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)c+\sqrt{-\frac{b}{a}}\,cx\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)a\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{\left(bd\,x^2+bc\right)x}{ba\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{d}{b}-\frac{ad-bc}{ba}-\frac{c}{a}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\right.\right.}{a\sqrt{-}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-b/a)^(1/2)*d*x^3+((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c-((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*c+(-b/a
)^(1/2)*c*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}ax - (bx^2+a)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}) + (bx^2+a)\sqrt{ac}\sqrt{-}}{a^2bx^2+a^3}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*x - (b*x^2 + a)*sqrt(a*c)*sqrt(-b/a)*ell
iptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (b*x^2 + a)*sqrt(a*c)*sqrt(-b/a)
*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a^2*b*x^2 + a^3)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(3/2), x)
```

3.170 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{x\sqrt{c+dx^2}}{3a\,(a+bx^2)^{3/2}} + \frac{(2bc-ad)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a*
d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*x*(d*x^2+c)^(1/2
)/a/(b*x^2+a)^(3/2)+1/3*(-a*d+2*b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2
)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c
)^(1/2)/a^(3/2)/(-a*d+b*c)/b^(1/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used

$= \{423, 539, 429, 422\}$

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c+dx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}}$$

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(3*a*(a + b*x^2)^(3/2)) + ((2*b*c - a*d)*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3*a^(3/2)*Sqrt[
b]*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (c^
(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b
*c)/(a*d)])/(3*a^2*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2c-dx^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{3a}$$

$$= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} - \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)} + \frac{(2bc-ad)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx}{3a(bc-ad)}$$

$$= \frac{x\sqrt{c+dx^2}}{3a\left(a+bx^2\right)^{3/2}} + \frac{(2bc-ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{b}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.68 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(2a^2d-2b^2cx^2+ab(-3c+dx^2)\right)+ic(-2bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1}}{3a^2\sqrt{\frac{b}{a}}(-}$$

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(2*a^2*d - 2*b^2*c*x^2 + a*b*(-3*c + d*x^2)) + I*c
*(-2*b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d)*(a + b*x^2
)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
 (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(-(b*c) + a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d
*x^2])
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.76

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3b^2a\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)x(ad-2bc)}{3b\,a^2(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{d}{3ab}-\frac{ad-2bc}{3b\,a^2}-\frac{c(ad-2bc)}{3a^2(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F(}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+a}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
default	$\frac{\sqrt{-\frac{b}{a}}\,ab\,d^2x^5-2\sqrt{-\frac{b}{a}}\,b^2cd\,x^5+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd\,x^2-2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2x^2-\sqrt{b}}{}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/b^2/a*x*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)/b/a^2/(a*d-
b*c)*x*(a*d-2*b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(1/3/a/b*d-1/3*(a*d-2*b*
c)/b/a^2-1/3*c/a^2/(a*d-b*c)*(a*d-2*b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))+1/3*(a*d-2*b*c)/(a*d-b*c)/a^2*c/(-b/a)^(1/2)*(1
+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(Elli
pticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1
+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{5/2}}\,dx =$$

$$-\frac{\left(2\,a^2b^2c-a^3bd+\left(2\,b^4c-ab^3d\right)x^4+2\left(2\,ab^3c-a^2b^2d\right)x^2\right)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-\left(2\,a^2b^2c+\right.}{}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*a^2*b^2*c - a^3*b*d + (2*b^4*c - a*b^3*d)*x^4 + 2*(2*a*b^3*c - a^2
*b^2*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c
)) - (2*a^2*b^2*c + (2*b^4*c + (a^2*b^2 - a*b^3)*d)*x^4 + 2*(2*a*b^3*c + (a
^3*b - a^2*b^2)*d)*x^2 + (a^4 - a^3*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(a
rcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b^3*c - a^2*b^2*d)*x^3 + (3*a^2*b^2
*c - 2*a^3*b*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^2*c - a^6*b*d +
(a^3*b^4*c - a^4*b^3*d)*x^4 + 2*(a^4*b^3*c - a^5*b^2*d)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(5/2),x)
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{5/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(5/2), x)
```

3.171 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 309

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \frac{x\sqrt{c+dx^2}}{5a\,(a+bx^2)^{5/2}} + \frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\,(a+bx^2)^{3/2}} + \frac{(8b^2c^2-13abcd+3a^2d^2)\,\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{2c^{3/2}\sqrt{d}(2bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -2/15*c^(3/2)*(-3*a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Ellipt
icF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a
)^(1/2)/a^3/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/
5*x*(d*x^2+c)^(1/2)/a/(b*x^2+a)^(5/2)+1/15*(-3*a*d+4*b*c)*x*(d*x^2+c)^(1/2)
/a^2/(-a*d+b*c)/(b*x^2+a)^(3/2)+1/15*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)*(1/(1
+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^
(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/(-a*d+b*c)^2/b^(1/2)/(b*x^
2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {423, 541, 539, 429, 422}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = -\frac{2c^{3/2}\sqrt{d}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{c+dx^2}(4bc-3ad)}{15a^2(a+bx^2)^{3/2}(bc-ad)} + \frac{\sqrt{c+dx^2}(3a^2d^2-13abcd+8b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} + \frac{x\sqrt{c+dx^2}}{5a(a+bx^2)^{5/2}}$$

```
[In] Int[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(5*a*(a + b*x^2)^(5/2)) + ((4*b*c - 3*a*d)*x*Sqrt[c + d
*x^2])/(15*a^2*(b*c - a*d)*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 13*a*b*c*d +
3*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)
/(b*c)])/(15*a^(5/2)*Sqrt[b]*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x
^2))/(c*(a + b*x^2))]) - (2*c^(3/2)*Sqrt[d]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]
*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*
d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{-4c-3dx^2}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx}{5a}\\
&= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}+\frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{c(8bc-9ad)+d(4bc-3ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)}\\
&= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}+\frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}}\\
&\quad-\frac{(2cd(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)^2}+\frac{\left(8b^2c^2-13abcd+3a^2d^2\right)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx}{15a^2(bc-ad)^2}\\
&= \frac{x\sqrt{c+dx^2}}{5a\left(a+bx^2\right)^{5/2}}+\frac{(4bc-3ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)\left(a+bx^2\right)^{3/2}}\\
&\quad+\frac{\left(8b^2c^2-13abcd+3a^2d^2\right)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{b}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a\left(c+dx^2\right)}{c\left(a+bx^2\right)}}}\\
&\quad-\frac{2c^{3/2}\sqrt{d}(2bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^2\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.87 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+a(-bc+ad)(-4bc+3ad)\left(a+bx^2\right)+(8b^2c^2-13abcd\right.}{}$$

```
[In] Integrate[Sqrt[c + d*x^2]/(a + b*x^2)^(7/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-4*b*c +
3*a*d)*(a + b*x^2) + (8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)^2) +
I*c*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((8*b^2*c^2 - 13*
a*b*c*d + 3*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8*b
^2*c^2 + 17*a*b*c*d - 9*a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^2*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5b^3a\left(x^2+\frac{a}{b}\right)^3}+\frac{(3ad-4bc)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{15(ad-bc)a^2b^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd\,x^2+bc\right)x\left(3a^2d^2-13abcd+8b^2c^2\right)}{15b\,a^3(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{d(3ad-4bc}{15b(ad-bc)c}\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5/b^3/a*x*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3+1/15*(3*a*d-4*b*c)/(a*d-b*c)/
a^2/b^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/15*(b*d*x^2+b*c
)/b/a^3/(a*d-b*c)^2*x*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)/((x^2+a/b)*(b*d*x^2+
b*c))^(1/2)+(1/15*d*(3*a*d-4*b*c)/b/(a*d-b*c)/a^2-1/15/(a*d-b*c)/b*(3*a^2*d
^2-13*a*b*c*d+8*b^2*c^2)/a^3-1/15*c/a^3/(a*d-b*c)^2*(3*a^2*d^2-13*a*b*c*d+8
*b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/1
5*(3*a^2*d^2-13*a*b*c*d+8*b^2*c^2)/(a*d-b*c)^2/a^3*c/(-b/a)^(1/2)*(1+b*x^2/
a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x
*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b
*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(291) = 582$.

Time = 0.11 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx =$$

$$-\frac{(8\,a^3b^3c^2 - 13\,a^4b^2cd + 3\,a^5bd^2 + (8\,b^6c^2 - 13\,ab^5cd + 3\,a^2b^4d^2)x^6 + 3\,(8\,ab^5c^2 - 13\,a^2b^4cd + 3\,a^3b^3d^2)x^4 +}{}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*((8*a^3*b^3*c^2 - 13*a^4*b^2*c*d + 3*a^5*b*d^2 + (8*b^6*c^2 - 13*a*b^
5*c*d + 3*a^2*b^4*d^2)*x^6 + 3*(8*a*b^5*c^2 - 13*a^2*b^4*c*d + 3*a^3*b^3*d^
2)*x^4 + 3*(8*a^2*b^4*c^2 - 13*a^3*b^3*c*d + 3*a^4*b^2*d^2)*x^2)*sqrt(a*c)*
sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (8*a^3*b^3*c^2 + (
8*b^6*c^2 + (4*a^2*b^4 - 13*a*b^5)*c*d - 3*(2*a^3*b^3 - a^2*b^4)*d^2)*x^6 +
 3*(8*a*b^5*c^2 + (4*a^3*b^3 - 13*a^2*b^4)*c*d - 3*(2*a^4*b^2 - a^3*b^3)*d^
2)*x^4 + (4*a^5*b - 13*a^4*b^2)*c*d - 3*(2*a^6 - a^5*b)*d^2 + 3*(8*a^2*b^4*
c^2 + (4*a^4*b^2 - 13*a^3*b^3)*c*d - 3*(2*a^5*b - a^4*b^2)*d^2)*x^2)*sqrt(a
*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((8*a*b^5*c^2
- 13*a^2*b^4*c*d + 3*a^3*b^3*d^2)*x^5 + (20*a^2*b^4*c^2 - 33*a^3*b^3*c*d +
9*a^4*b^2*d^2)*x^3 + (15*a^3*b^3*c^2 - 26*a^4*b^2*c*d + 9*a^5*b*d^2)*x)*sqr
t(b*x^2 + a)*sqrt(d*x^2 + c))/(a^7*b^3*c^2 - 2*a^8*b^2*c*d + a^9*b*d^2 + (a
^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2)*x^6 + 3*(a^5*b^5*c^2 - 2*a^6*b^4*
c*d + a^7*b^3*d^2)*x^4 + 3*(a^6*b^4*c^2 - 2*a^7*b^3*c*d + a^8*b^2*d^2)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/(a + b*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/(b*x^2 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{7/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(7/2), x)
```

3.172 $\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} dx$

Optimal result

Integrand size = 23, antiderivative size = 410

$$\int (a+bx^2)^{3/2} (c+dx^2)^{3/2} dx = -\frac{2(bc+ad)(b^2c^2-6abcd+a^2d^2)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2} + \frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{2\sqrt{c}(bc+ad)(b^2c^2-6abcd+a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}(b^2c^2-18abcd+a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/d/(d*x^2+
c)^(1/2)-1/35*c^(3/2)*(a^2*d^2-18*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1
+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))*(b*x^2+a)^(1/2)/b/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(
1/2)+2/35*(a*d+b*c)*(a^2*d^2-6*a*b*c*d+b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*c^(1/2)*(b*x^2+a)^(1/2)/b^2/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x
^2+c)^(1/2)+2/35*(-a*d+4*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/b+1/7*d*x*(
b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/b+1/35*(9*a*c+b*c^2/d-2*a^2*d/b)*x*(b*x^2+a)
^(1/2)*(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} dx = \frac{2\sqrt{c}\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{c^{3/2}\sqrt{a+bx^2}(a^2d^2-18abcd+b^2c^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{2x\sqrt{a+bx^2}(ad+bc)\left(a^2d^2-6abcd+b^2c^2\right)}{35b^2d\sqrt{c+dx^2}} + \frac{1}{35}x\sqrt{a+bx^2}\sqrt{c+dx^2}\left(-\frac{2a^2d}{b}+9ac+\frac{bc^2}{d}\right)+\frac{dx\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}{7b}+\frac{2x\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}(4bc-ad}{35b}$$

```
[In] Int[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (-2*(b*c + a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*Sqrt[a + b*x^2])/(35*b^2*
d*Sqrt[c + d*x^2]) + ((9*a*c + (b*c^2)/d - (2*a^2*d)/b)*x*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/35 + (2*(4*b*c - a*d)*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/
(35*b) + (d*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(7*b) + (2*Sqrt[c]*(b*c +
a*d)*(b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt
[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(35*b^2*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*
(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b^2*c^2 - 18*a*b*c*d + a^2*d^2)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
5*b*d^(3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} + \frac{\int \frac{(a+bx^2)^{3/2}(c(7bc-ad)+2d(4bc-ad)x^2)}{\sqrt{c+dx^2}}\,dx}{7b} \\
&= \frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b} + \frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b} \\
&\quad + \frac{\int \frac{\sqrt{a+bx^2}(3acd(9bc-ad)+3d(b^2c^2+9abcd-2a^2d^2)x^2)}{\sqrt{c+dx^2}}\,dx}{35bd}
\end{aligned}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b}$$
$$+\frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b}+\frac{\int\frac{-3acd(b^2c^2-18abcd+a^2d^2)-6d(bc+ad)(b^2c^2-6abcd+a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{105bd^2}$$

$$= \frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}+\frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b}$$
$$+\frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b}-\frac{(ac(b^2c^2-18abcd+a^2d^2))\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{35bd}$$
$$-\frac{(2(bc+ad)(b^2c^2-6abcd+a^2d^2))\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{35bd}$$

$$= -\frac{2(bc+ad)(b^2c^2-6abcd+a^2d^2)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}}$$
$$+\frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$+\frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b}+\frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b}$$
$$-\frac{c^{3/2}(b^2c^2-18abcd+a^2d^2)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+\frac{(2c(bc+ad)(b^2c^2-6abcd+a^2d^2))\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{35b^2d}$$

$$= -\frac{2(bc+ad)(b^2c^2-6abcd+a^2d^2)x\sqrt{a+bx^2}}{35b^2d\sqrt{c+dx^2}}$$
$$+\frac{1}{35}\left(9ac+\frac{bc^2}{d}-\frac{2a^2d}{b}\right)x\sqrt{a+bx^2}\sqrt{c+dx^2}$$
$$+\frac{2(4bc-ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35b}+\frac{dx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7b}$$
$$+\frac{2\sqrt{c}(bc+ad)(b^2c^2-6abcd+a^2d^2)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35b^2d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$-\frac{c^{3/2}(b^2c^2-18abcd+a^2d^2)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{35bd^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.18 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.74

$$\int (a+bx^2)^{3/2}(c+dx^2)^{3/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(a^2d^2+abd(17c+8dx^2)+b^2(c^2+8cdx^2+5d^2x^4))+2ic(b^3c^3-5c}{}$$

```
[In] Integrate[(a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(a^2*d^2 + a*b*d*(17*c + 8*d*x^2) +
b^2*(c^2 + 8*c*d*x^2 + 5*d^2*x^4)) + (2*I)*c*(b^3*c^3 - 5*a*b^2*c^2*d - 5*a
^2*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b^3*c^3 - 11*a*b^2*c^2*d + 8*a^2
*b*c*d^2 + a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*Arc
Sinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(35*b*Sqrt[b/a]*d^2*Sqrt[a + b*x^2]*Sqrt[c
 + d*x^2])
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.38

method	result
risch	$\frac{x(5b^2d^2x^4+8x^2abd^2+8x^2b^2cd+a^2d^2+17abcd+b^2c^2)\sqrt{bx^2+a}\sqrt{dx^2+c}}{35bd} - \frac{\left(\frac{a^3cd^2\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cbx^2+ac}}+\frac{b^2c}{}\right.}{}$
elliptic	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{bdx^5\sqrt{bdx^4+adx^2+cbx^2+ac}}{7}+\frac{\left(2abd^2+2b^2cd-\frac{bd(6ad+6bc)}{7}\right)x^3\sqrt{bdx^4+adx^2+cbx^2+ac}}{5bd}+\frac{\left(a^2d\right.}{}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(5\sqrt{-\frac{b}{a}}b^3d^4x^9+13\sqrt{-\frac{b}{a}}ab^2d^4x^7+13\sqrt{-\frac{b}{a}}b^3cd^3x^7+9\sqrt{-\frac{b}{a}}a^2bd^4x^5+38\sqrt{-\frac{b}{a}}ab^2cd^3x^5+9\sqrt{-\frac{b}{a}}b^3c^2d^2x^5+\right.}{}$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/35/b/d*x*(5*b^2*d^2*x^4+8*a*b*d^2*x^2+8*b^2*c*d*x^2+a^2*d^2+17*a*b*c*d+b^
2*c^2)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)-1/35/b/d*(a^3*c*d^2/(-b/a)^(1/2)*(1+
b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*Ellipt
icF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b^2*c^3*a/(-b/a)^(1/2)*(1+b*x^
2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-18*a^2*b*c^2*d/(-b/a)^(1/2)*(1+b*x
^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF
(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^3*d^3-10*a^2*b*c*d^2-10*a*b^
2*c^2*d+2*b^3*c^3)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*
x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/
b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(
d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.77

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \, dx = \frac{2\left(b^3c^4 - 5\,ab^2c^3d - 5\,a^2bc^2d^2 + a^3cd^3\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \left(2\,b^3c^4 - 10\,ab^2c^3d\right.}{}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*(b^3*c^4 - 5*a*b^2*c^3*d - 5*a^2*b*c^2*d^2 + a^3*c*d^3)*sqrt(b*d)*x
*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^3*c^4 - 10*a
*b^2*c^3*d + a^3*d^4 - (10*a^2*b - a*b^2)*c^2*d^2 + 2*(a^3 - 9*a^2*b)*c*d^3
)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (5*b
^3*d^4*x^6 - 2*b^3*c^3*d + 10*a*b^2*c^2*d^2 + 10*a^2*b*c*d^3 - 2*a^3*d^4 +
8*(b^3*c*d^3 + a*b^2*d^4)*x^4 + (b^3*c^2*d^2 + 17*a*b^2*c*d^3 + a^2*b*d^4)*
x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^3*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)^{3/2} \left(c+dx^2\right)^{3/2} \, dx = \int \left(a+bx^2\right)^{\frac{3}{2}} \left(c+dx^2\right)^{\frac{3}{2}} \, dx$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}\,dx=\int \left(b\,x^2+a\right)^{3/2}\left(d\,x^2+c\right)^{3/2}\,dx$$

```
[In] int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2), x)
```

3.173 $\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 336

$$\int \sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx = \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}} + \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} - \frac{\sqrt{c}(3b^2c^2+7abcd-2a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(9bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] 1/15*(-2*a^2*d^2+7*a*b*c*d+3*b^2*c^2)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2)
+1/15*c^(3/2)*(-a*d+9*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/b/
d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/15*(-2*a^2*d^2+7*
a*b*c*d+3*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/b^
2/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*d*x*(b*x^2+a)
^(3/2)*(d*x^2+c)^(1/2)/b+2/15*(-a*d+3*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2
)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {427, 542, 545, 429, 506, 422}

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx = -\frac{\sqrt{c}\sqrt{a+bx^2}(-2a^2d^2+7abcd+3b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{a+bx^2}(-2a^2d^2+7abcd+3b^2c^2)}{15b^2\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}(9bc-ad)\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} + \frac{2x\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-ad)}{15b}$$

```
[In] Int[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] ((3*b^2*c^2 + 7*a*b*c*d - 2*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*b^2*Sqrt[c + d*
x^2]) + (2*(3*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*b) + (d*x*(
a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*b) - (Sqrt[c]*(3*b^2*c^2 + 7*a*b*c*d -
 2*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(15*b^2*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]) + (c^(3/2)*(9*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b}+\frac{\int \frac{\sqrt{a+bx^2}\left(c(5bc-ad)+2d(3bc-ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5b}\\
&= \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b}\\
&\quad+\frac{\int \frac{acd(9bc-ad)+d\left(3b^2c^2+7abcd-2a^2d^2\right)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15bd}\\
&= \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b}+\frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b}\\
&\quad+\frac{(ac(9bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b}+\frac{\left(3b^2c^2+7abcd-2a^2d^2\right)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15b}
\end{aligned}$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}} + \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} + \frac{c^{3/2}(9bc-ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(c(3b^2c^2+7abcd-2a^2d^2))\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15b^2}$$

$$= \frac{(3b^2c^2+7abcd-2a^2d^2)\,x\sqrt{a+bx^2}}{15b^2\sqrt{c+dx^2}} + \frac{2(3bc-ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15b} + \frac{dx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5b} - \frac{\sqrt{c}(3b^2c^2+7abcd-2a^2d^2)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b^2\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(9bc-ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15b\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.73

$$\int\sqrt{a+bx^2}(c+dx^2)^{3/2}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(6bc+ad+3bdx^2)+ic(-3b^2c^2-7abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\ldots}}{15b\sqrt{\ldots}}$$

```
[In] Integrate[Sqrt[a + b*x^2]*(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(6*b*c + a*d + 3*b*d*x^2) + I*c*(-3*
b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*El
lipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(-3*b^2*c^2 + 2*a*b*c*d
+ a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)])/(15*b*Sqrt[b/a]*d*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.22

method	result
risch	$\frac{x\left(3bd\,x^2+ad+6bc\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15b}-\frac{\left(\frac{a^2cd\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{9b\,c^2a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}}{}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{d\,x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5}+\frac{\left(a\,d^2+2bcd-\frac{d(4ad+4bc)}{5}\right)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3bd}+\frac{\left(c^2a-\frac{\left(a\,d^2+2bcd-\frac{d(4ad+4bc)}{5}\right)}{3bd}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^2d^3x^7+4\sqrt{-\frac{b}{a}}\,ab\,d^3x^5+9\sqrt{-\frac{b}{a}}\,b^2c\,d^2x^5+\sqrt{-\frac{b}{a}}\,a^2d^3x^3+10\sqrt{-\frac{b}{a}}\,abc\,d^2x^3+6\sqrt{-\frac{b}{a}}\,b^2c^2d\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\right.}{}$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*(3*b*d*x^2+a*d+6*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b-1/15/b*(a^2*
c*d/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-9*b*c^2*a/
(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a^2*d^2-7*a
*b*c*d-3*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))*((b*x^2+a)*(d
*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.69

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx =$$

$$-\frac{(3\,b^2c^3+7\,abc^2d-2\,a^2cd^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(3\,b^2c^3+7\,abc^2d-a^2d^3-(2\,a^2-9\,ab)cd^2)}{}$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*((3*b^2*c^3 + 7*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*ellip
tic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^2*c^3 + 7*a*b*c^2*d - a^2*d^3
 - (2*a^2 - 9*a*b)*c*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
```

```
d)/x), a*d/(b*c)) - (3*b^2*d^3*x^4 + 3*b^2*c^2*d + 7*a*b*c*d^2 - 2*a^2*d^3
+ (6*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d^2*x)
```

Sympy [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{a+bx^2}\left(c+dx^2\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(3/2),x)
[Out] Integral(sqrt(a + b*x**2)*(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{bx^2+a}\left(dx^2+c\right)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}\,dx=\int \sqrt{b\,x^2+a}\,{\left(d\,x^2+c\right)}^{3/2}\,dx$$

```
[In] int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2), x)
```

3.174 $\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b} - \frac{2\sqrt{c}\sqrt{d}(2bc-ad)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] 2/3*d*(-a*d+2*b*c)*x*(b*x^2+a)^(1/2)/b^2/(d*x^2+c)^(1/2)+1/3*c^(3/2)*(-a*d+
3*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/
(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/a/b/d^(1/2)/(c*(b*x^2+
a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2
)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)/(d*x^2+c)^(1/2)+1/3*d*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {427, 545, 429, 506, 422}

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}\sqrt{a+bx^2}(3bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{2dx\sqrt{a+bx^2}(2bc-ad)}{3b^2\sqrt{c+dx^2}} + \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}$$

```
[In] Int[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*Sqrt[a + b*x^2])/(3*b^2*Sqrt[c + d*x^2]) + (d*x*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (2*Sqrt[c]*Sqrt[d]*(2*b*c - a*d)*Sqrt[a
+ b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*b^2*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*(3*b*c - a*
d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(3*a*b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}+\frac{\int \frac{c(3bc-ad)+2d(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b}\\
&= \frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}+\frac{(2d(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b}+\frac{(c(3bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3b}\\
&= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\\
&\quad+\frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\
&\quad-\frac{(2cd(2bc-ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3b^2}\\
&= \frac{2d(2bc-ad)x\sqrt{a+bx^2}}{3b^2\sqrt{c+dx^2}}+\frac{dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}\\
&\quad-\frac{2\sqrt{c}\sqrt{d}(2bc-ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3b^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\
&\quad+\frac{c^{3/2}(3bc-ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ab\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)+2ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)\cdots}{3b\sqrt{\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+\cdots}}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + (2*I)*c*(-2*b*c + a*d)*Sqrt[1 + (b
*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*Sqrt[a + b*x^2]*Sqrt[c +
d*x^2])
```

Maple [A] (verified)

Time = 4.42 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.14

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(\frac{dx\sqrt{bd x^4+ad x^2+cb x^2+ac}}{3b}+\frac{\left(c^2-\frac{dac}{3b}\right)\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{\left(2cd-\frac{d(2ad+2bc)}{3b}\right)c\sqrt{1+\frac{b x^2}{a}}\cdots}{\cdots}\right.}{\sqrt{b x^2+a}\sqrt{d x^2+c}}$
default	$\frac{\sqrt{d x^2+c}\sqrt{b x^2+a}\left(\sqrt{-\frac{b}{a}}\,b d^2x^5+\sqrt{-\frac{b}{a}}\,a d^2x^3+\sqrt{-\frac{b}{a}}\,bcd x^3+ac\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{b x^2+a}{a}}\sqrt{\frac{d x^2+c}{c}}F\left(x\cdots\right.\right.}{3(bd x^4+ad x^2+cb x^2+\cdots}$
risch	$\frac{dx\sqrt{b x^2+a}\sqrt{d x^2+c}}{3b}-\frac{\left(\frac{acd\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{3b c^2\sqrt{1+\frac{b x^2}{a}}\sqrt{1+\frac{d x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd x^4+ad x^2+cb x^2+ac}}-\frac{\left(2a d^2-\cdots\right.}{\cdots}\right.}{3b\sqrt{b x^2+a}\sqrt{\cdots}}$

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*d/b*x*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(c^2-1/3*d/b*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*c*d-1/3*d/b*(2*a*d+2*b*c))*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx =$$
$$-\frac{2\,(2\,bc^2-acd)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(4\,bc^2-(2\,a-3\,b)cd-ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{3\,b^2dx}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(2*b*c^2 - a*c*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (4*b*c^2 - (2*a - 3*b)*c*d - a*d^2)*sqrt(b*d)*x*sqrt(-c
/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^2 + 4*b*c*d - 2*
a*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b^2*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(1/2), x)
```

3.175 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 267

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}}+\frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}}+\frac{\sqrt{c}\sqrt{d}(bc-2ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -d*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/a/b^2/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/b/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)/(d*x^2+c)^(1/2)+(-2*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1
/2)*(b*x^2+a)^(1/2)/a/b^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(
-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {424, 545, 429, 506, 422}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{ab\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{dx\sqrt{a+bx^2}(bc-2ad)}{ab^2\sqrt{c+dx^2}} + \frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}$$

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((d*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(a*b^2*Sqrt[c + d*x^2])) + ((b*c - a*
d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*(b*c - 2*a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*b^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sq
rt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d
)])/(a*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{\int \frac{acd-d(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ab} \\
&= \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} + \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{b} - \frac{(d(bc-2ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{ab} \\
&= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(cd(bc-2ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{ab^2} \\
&= -\frac{d(bc-2ad)x\sqrt{a+bx^2}}{ab^2\sqrt{c+dx^2}} + \frac{(bc-ad)x\sqrt{c+dx^2}}{ab\sqrt{a+bx^2}} \\
&\quad + \frac{\sqrt{c}\sqrt{d}(bc-2ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{ab\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(bc-ad)\left(\sqrt{\frac{b}{a}}x(c+dx\right.}{a^2\left(\frac{b}{a}\right)^{3/2}\sqrt{a+bx^2}\sqrt{c+dx}}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x]
```

```
[Out] ((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[
I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2)
 - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a
]*x], (a*d)/(b*c)]))/(a^2*(b/a)^(3/2)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.24

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\,c^2+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\right.}{b(bd\,x^4+ad\,x^2+cb\,x^2+c}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{\left(bd\,x^2+bc\right)(ad-bc)x}{a\,b^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(-\frac{d(ad-2bc)}{b^2}+\frac{(ad-bc)^2}{b^2a}+\frac{c(ad-bc)}{ba}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*a*d^2*x^3+(-b/a)^(1/2)*b*c*d*x^3-a*c*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2+2
*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)
^(1/2))*a*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b*c^2-(-b/a)^(1/2)*a*c*d*x+(-b/a)^(1/2)*b*c^2*x)*(d*x^
2+c)^(1/2)*(b*x^2+a)^(1/2)/b/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/a/(-b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{((b^2c^2-2\,abcd)x^3+(abc^2-2\,a^2cd)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((b^2c^2-2\,abcd}{}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (((b^2*c^2 - 2*a*b*c*d)*x^3 + (a*b*c^2 - 2*a^2*c*d)*x)*sqrt(b*d)*sqrt(-c/d)
*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c^2 - 2*a*b*c*d - a*b*
d^2)*x^3 + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic
_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d^2*x^2 - a*b*c*d + 2*a^2*d^2)*s
qrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a*b^3*d*x^3 + a^2*b^2*d*x)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(3/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2),x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(3/2), x)
```

3.176 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 229

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx &= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab(a+bx^2)^{3/2}} \\ &+ \frac{2(bc+ad)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ &- \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] -1/3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^
(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/b/(c
*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*(-a*d+b*c)*x*(d*x^2+c)^(1
/2)/a/b/(b*x^2+a)^(3/2)+2/3*(a*d+b*c)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/
2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*(d*x^2+
c)^(1/2)/a^(3/2)/b^(3/2)/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {424, 539, 429, 422}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{2\sqrt{c+dx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{c+dx^2}(bc-ad)}{3ab\,(a+bx^2)^{3/2}}$$

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(3*a*b*(a + b*x^2)^(3/2)) + (2*(b*c + a*d)*
Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(3
*a^(3/2)*b^(3/2)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (
c^(3/2)*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(3*a^2*b*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2
])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}} + \frac{\int \frac{c(2bc+ad)+d(bc+2ad)x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{3ab}\\
&= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}} - \frac{(cd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3ab} + \frac{(2(bc+ad))\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx}{3ab}\\
&= \frac{(bc-ad)x\sqrt{c+dx^2}}{3ab\,(a+bx^2)^{3/2}} + \frac{2(bc+ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}b^{3/2}\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\\
&\quad - \frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.38 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(a^2d+2b^2cx^2+ab(3c+2dx^2)\right)+2ic(bc+ad)\left(a+bx^2\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+}}{3a^3\left(\frac{b}{a}\right)^{3}}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b/a]*x*(c + d*x^2)*(a^2*d + 2*b^2*c*x^2 + a*b*(3*c + 2*d*x^2)) + (2*I
)*c*(b*c + a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*c*(2*b*c + a*d)*(a + b*x^2)*Sqr
t[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)])/(3*a^3*(b/a)^(3/2)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.84

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{(ad-bc)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3a\,b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{2\left(bd\,x^2+bc\right)x(ad+bc)}{3b^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{d^2}{b^2}-\frac{d(ad-bc)}{3b^2a}-\frac{2(ad+bc)(ad-bc)}{3b^2a^2}-\frac{2c(ad+bc)}{3b\,a^2}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d}}$
default	$\frac{2\sqrt{-\frac{b}{a}}\,ab\,d^2x^5+2\sqrt{-\frac{b}{a}}\,b^2cd\,x^5+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd\,x^2+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2x^2-2\sqrt{}}{}$

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3*(a*d-b*c)
/a/b^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+2/3*(b*d*x^2+b*c)/
b^2/a^2*x*(a*d+b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(d^2/b^2-1/3/b^2*d*(a*d
-b*c)/a-2/3*(a*d+b*c)/b^2*(a*d-b*c)/a^2-2/3/b*c/a^2*(a*d+b*c))/(-b/a)^(1/2)
*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*El
lipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+2/3*(a*d+b*c)/b/a^2*c/(-b/
a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^
(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a
)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx =$$

$$-\frac{2\,(a^2b^2c+a^3bd+(b^4c+ab^3d)x^4+2\,(ab^3c+a^2b^2d)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(2\,a^2b^2c+(2\,b}{}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(a^2*b^2*c + a^3*b*d + (b^4*c + a*b^3*d)*x^4 + 2*(a*b^3*c + a^2*b^2
*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) -
 (2*a^2*b^2*c + (2*b^4*c + (a^2*b^2 + 2*a*b^3)*d)*x^4 + 2*(2*a*b^3*c + (a^3
*b + 2*a^2*b^2)*d)*x^2 + (a^4 + 2*a^3*b)*d)*sqrt(a*c)*sqrt(-b/a)*elliptic_f
(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*(a*b^3*c + a^2*b^2*d)*x^3 + (3*a^2*b
^2*c + a^3*b*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^3*b^4*x^4 + 2*a^4*b^
3*x^2 + a^5*b^2)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(5/2),x)
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2),x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(5/2), x)
```

3.177 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 315

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}}$$
$$+ \frac{(8b^2c^2-3abcd-2a^2d^2)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$- \frac{c^{3/2}\sqrt{d}(4bc-ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3b(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -1/15*c^(3/2)*(-a*d+4*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*Elliptic
F(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^
(1/2)/a^3/b/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/5*
(-a*d+b*c)*x*(d*x^2+c)^(1/2)/a/b/(b*x^2+a)^(5/2)+2/15*(a*d+2*b*c)*x*(d*x^2+
c)^(1/2)/a^2/b/(b*x^2+a)^(3/2)+1/15*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(1/(1+
b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(
1/2),(1-a*d/b/c)^(1/2))*(d*x^2+c)^(1/2)/a^(5/2)/b^(3/2)/(-a*d+b*c)/(b*x^2+a
)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {424, 541, 539, 429, 422}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = -\frac{c^{3/2}\sqrt{d}\sqrt{a+bx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3b\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{2x\sqrt{c+dx^2}(ad+2bc)}{15a^2b\,(a+bx^2)^{3/2}}$$
$$+\frac{\sqrt{c+dx^2}(-2a^2d^2-3abcd+8b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}\sqrt{a+bx^2}(bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}+\frac{x\sqrt{c+dx^2}(bc-ad)}{5ab\,(a+bx^2)^{5/2}}$$

```
[In] Int[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]
```

```
[Out] ((b*c - a*d)*x*Sqrt[c + d*x^2])/(5*a*b*(a + b*x^2)^(5/2)) + (2*(2*b*c + a*d
)*x*Sqrt[c + d*x^2])/(15*a^2*b*(a + b*x^2)^(3/2)) + ((8*b^2*c^2 - 3*a*b*c*d
 - 2*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a
*d)/(b*c)])/(15*a^(5/2)*b^(3/2)*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*
x^2))/(c*(a + b*x^2))]) - (c^(3/2)*Sqrt[d]*(4*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*a^3*b*(b*c - a*d
)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{\int \frac{c(4bc+ad)+d(3bc+2ad)x^2}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx}{5ab}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} - \frac{\int \frac{-c(bc-ad)(8bc+ad)-2d(bc-ad)(2bc+ad)x^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2b(bc-ad)}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} - \frac{(cd(4bc-ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2b(bc-ad)} + \frac{(8b^2c^2-3abcd-2a^2d^2)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx}{15a^2b(bc-ad)}$$

$$= \frac{(bc-ad)x\sqrt{c+dx^2}}{5ab\,(a+bx^2)^{5/2}} + \frac{2(2bc+ad)x\sqrt{c+dx^2}}{15a^2b\,(a+bx^2)^{3/2}} + \frac{(8b^2c^2-3abcd-2a^2d^2)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}b^{3/2}(bc-ad)\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{c^{3/2}\sqrt{d}(4bc-ad)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3b(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.59 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+2a(bc-ad)(2bc+ad)(a+bx^2)+(8b^2c^2-3abcd-2a^2\right.}{}$$

[In] Integrate[(c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x]

[Out] (Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 2*a*(b*c - a*d)*(2*b*c + a*
d)*(a + b*x^2) + (8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*(a + b*x^2)^2) - I*c*(
a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*((-8*b^2*c^2 + 3*a*b*c
*d + 2*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*b^2*c^2
 - 7*a*b*c*d - a^2*d^2)*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(1
5*a^4*(b/a)^(3/2)*(b*c - a*d)*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b x^2+a)(d x^2+c)}\left(-\frac{(ad-bc)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{5a b^4\left(x^2+\frac{a}{b}\right)^3}+\frac{2(ad+2bc)x\sqrt{bd x^4+ad x^2+cb x^2+ac}}{15a^2b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bd x^2+bc\right)x\left(2a^2d^2+3abcd-8b^2c^2\right)}{15b^2a^3(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd x^2+bc\right)}}+\frac{\left(2d\right.}{}\right.}{}$
default	Expression too large to display

[In] int((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5*(a*d-b*c)
/a/b^4*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3+2/15*(a*d+2*b*c)/a
^2/b^3*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/15*(b*d*x^2+b*c)
/b^2/a^3/(a*d-b*c)*x*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2)/((x^2+a/b)*(b*d*x^2+b*
c))^(1/2)+(2/15*d*(a*d+2*b*c)/a^2/b^2-1/15/b^2*(2*a^2*d^2+3*a*b*c*d-8*b^2*c
^2)/a^3-1/15/b*c/a^3/(a*d-b*c)*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2))/(-b/a)^(1/2
)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+1/15/b*(2*a^2*d^2+3*a*b*c
*d-8*b^2*c^2)/(a*d-b*c)/a^3*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1
/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+
b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(297) = 594$.

Time = 0.10 (sec) , antiderivative size = 635, normalized size of antiderivative = 2.02

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx =$$

$$-\frac{(8\,a^3b^3c^2-3\,a^4b^2cd-2\,a^5bd^2+(8\,b^6c^2-3\,ab^5cd-2\,a^2b^4d^2)x^6+3\,(8\,ab^5c^2-3\,a^2b^4cd-2\,a^3b^3d^2)x^4+3\,(}{}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*((8*a^3*b^3*c^2 - 3*a^4*b^2*c*d - 2*a^5*b*d^2 + (8*b^6*c^2 - 3*a*b^5*
c*d - 2*a^2*b^4*d^2)*x^6 + 3*(8*a*b^5*c^2 - 3*a^2*b^4*c*d - 2*a^3*b^3*d^2)*
x^4 + 3*(8*a^2*b^4*c^2 - 3*a^3*b^3*c*d - 2*a^4*b^2*d^2)*x^2)*sqrt(a*c)*sqrt
(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (8*a^3*b^3*c^2 + (8*b^
6*c^2 + (4*a^2*b^4 - 3*a*b^5)*c*d - (a^3*b^3 + 2*a^2*b^4)*d^2)*x^6 + 3*(8*a
*b^5*c^2 + (4*a^3*b^3 - 3*a^2*b^4)*c*d - (a^4*b^2 + 2*a^3*b^3)*d^2)*x^4 + (
4*a^5*b - 3*a^4*b^2)*c*d - (a^6 + 2*a^5*b)*d^2 + 3*(8*a^2*b^4*c^2 + (4*a^4*
b^2 - 3*a^3*b^3)*c*d - (a^5*b + 2*a^4*b^2)*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*e
lliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((8*a*b^5*c^2 - 3*a^2*b^4*c*d
- 2*a^3*b^3*d^2)*x^5 + 2*(10*a^2*b^4*c^2 - 4*a^3*b^3*c*d - 3*a^4*b^2*d^2)*x
^3 + (15*a^3*b^3*c^2 - 11*a^4*b^2*c*d - a^5*b*d^2)*x)*sqrt(b*x^2 + a)*sqrt(
d*x^2 + c))/(a^7*b^3*c - a^8*b^2*d + (a^4*b^6*c - a^5*b^5*d)*x^6 + 3*(a^5*b
^5*c - a^6*b^4*d)*x^4 + 3*(a^6*b^4*c - a^7*b^3*d)*x^2)
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)**(7/2),x)
```

```
[Out] Integral((c + d*x**2)**(3/2)/(a + b*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)^(7/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/(b*x^2 + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)^{7/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)^{7/2}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2),x)
[Out] int((c + d*x^2)^(3/2)/(a + b*x^2)^(7/2), x)
```

3.178 $\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 235

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} - \frac{\sqrt{2}(3b+2d)\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{2\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] 1/3*(3*b+2*d)*x*(b*x^2+2)^(1/2)/b/(d*x^2+3)^(1/2)-1/3*(3*b+2*d)*(1/(3*d*x^2
+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),
1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/b/d^(1/2)/((b*x^2+2)/(d*x^2+3)
)^(1/2)/(d*x^2+3)^(1/2)+2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF
(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)
^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+1/3*x*(b*x^2+2)^
(1/2)*(d*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used

$= \{428, 545, 429, 506, 422\}$

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \frac{2\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} - \frac{\sqrt{2}(3b+2d)\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} + \frac{1}{3}x\sqrt{bx^2+2}\sqrt{dx^2+3} + \frac{x(3b+2d)\sqrt{bx^2+2}}{3b\sqrt{dx^2+3}}$$

```
[In] Int[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] ((3*b + 2*d)*x*Sqrt[2 + b*x^2])/(3*b*Sqrt[3 + d*x^2]) + (x*Sqrt[2 + b*x^2]*
Sqrt[3 + d*x^2])/3 - (Sqrt[2]*(3*b + 2*d)*Sqrt[2 + b*x^2]*EllipticE[ArcTan[
(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(3*b*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
d*x^2)]*Sqrt[3 + d*x^2]) + (2*Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqr
t[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*
Sqrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 428

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^p*((c + d*x^n)^q/(n*(p + q) + 1)), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
```

```
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2}+\frac{2}{3}\int\frac{6+\frac{1}{2}(3b+2d)x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx \\
&= \frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2}+4\int\frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx+\frac{1}{3}(3b+2d)\int\frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx \\
&= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} \\
&\quad+\frac{2\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}+\frac{(-3b-2d)\int\frac{\sqrt{2+bx^2}}{(3+dx^2)^{3/2}}\,dx}{b} \\
&= \frac{(3b+2d)x\sqrt{2+bx^2}}{3b\sqrt{3+dx^2}}+\frac{1}{3}x\sqrt{2+bx^2}\sqrt{3+dx^2} \\
&\quad-\frac{\sqrt{2}(3b+2d)\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{3b\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} \\
&\quad+\frac{2\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.54

$$\int\sqrt{2+bx^2}\sqrt{3+dx^2}\,dx$$

$$=\frac{\sqrt{b}dx\sqrt{2+bx^2}\sqrt{3+dx^2}-i\sqrt{3}(3b+2d)E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right)|\frac{2d}{3b}\right)+i\sqrt{3}(3b-2d)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{b}x}{\sqrt{2}}\right.\right.}{3\sqrt{b}d}$$

```
[In] Integrate[Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2],x]
```

```
[Out] (Sqrt[b]*d*x*Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2] - I*Sqrt[3]*(3*b + 2*d)*Ellipt
icE[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)] + I*Sqrt[3]*(3*b - 2*d)*El
lipticF[I*ArcSinh[(Sqrt[b]*x)/Sqrt[2]], (2*d)/(3*b)])/(3*Sqrt[b]*d)
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07

method	result
risch	$\frac{x\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}{3}+\frac{\left(\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{(3b+2d)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\right.\right.}{3\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}\,b}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{x\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}{3}+\frac{2\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\left(b+\frac{2d}{3}\right)\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3}\right.\right.}{\sqrt{-3d}\,\sqrt{bd\,x}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$
default	$\frac{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\left(b^2d\,x^5\sqrt{-d}+3b^2x^3\sqrt{-d}+2bd\,x^3\sqrt{-d}+3\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)b\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}-2\sqrt{2}\,F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\right.\right.}{3(bd\,x^4+3b}$

```
[In] int((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(b*x^2+2)^(1/2)*(d*x^2+3)^(1/2)+(2/(-3*d)^(1/2)*(3*d*x^2+9)^(1/2)*(2*
b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*d)^(1/
2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-1/3*(3*b+2*d)/(-3*d)^(1/2)*(3*d*x^2+9)^(1/
2)*(2*b*x^2+4)^(1/2)/(b*d*x^4+3*b*x^2+2*d*x^2+6)^(1/2)/b*(EllipticF(1/3*x*(
-3*d)^(1/2),1/2*(-4+2*(3*b+2*d)/d)^(1/2))-EllipticE(1/3*x*(-3*d)^(1/2),1/2*
(-4+2*(3*b+2*d)/d)^(1/2))))*((b*x^2+2)*(d*x^2+3))^(1/2)/(b*x^2+2)^(1/2)/(d*
x^2+3)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.63

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx =$$

$$-\frac{3\sqrt{3}\sqrt{bd}(3\,b+2\,d)x\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)\mid\frac{2\,d}{3\,b})-\sqrt{3}\sqrt{bd}(4\,d^2+9\,b+6\,d)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right.}{3\,bd^2x}$$

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*sqrt(3)*sqrt(b*d)*(3*b + 2*d)*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3
)*sqrt(-1/d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(4*d^2 + 9*b + 6*d)*x*sqrt(-1
/d)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - (b*d^2*x^2 + 3*b*d
+ 2*d^2)*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3))/(b*d^2*x)
```

Sympy [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

```
[In] integrate((b*x**2+2)**(1/2)*(d*x**2+3)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3), x)
```

Maxima [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Giac [F]

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{bx^2+2}\sqrt{dx^2+3}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)*(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+bx^2}\sqrt{3+dx^2}\,dx = \int \sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,dx$$

```
[In] int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2),x)
[Out] int((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2), x)
```

3.179 $\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 38

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \sqrt{\frac{2}{3}}x\sqrt{1-4x^4} + \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-1\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticF(x*2^(1/2),I)*3^(1/2)+1/3*x*6^(1/2)*(-4*x^4+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {254, 201, 227}

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-1\right)}{\sqrt{3}} + \sqrt{\frac{2}{3}}\sqrt{1-4x^4}x$$

```
[In] Int[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
```

```
[Out] Sqrt[2/3]*x*Sqrt[1 - 4*x^4] + (2*EllipticF[ArcSin[Sqrt[2]*x], -1])/Sqrt[3]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \sqrt{6-24x^4}\,dx \\ &= \sqrt{\frac{2}{3}}x\sqrt{1-4x^4}+4\int \frac{1}{\sqrt{6-24x^4}}\,dx \\ &= \sqrt{\frac{2}{3}}x\sqrt{1-4x^4}+\frac{2F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-1\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx=\sqrt{6}x\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},4x^4\right)$$

```
[In] Integrate[Sqrt[3 - 6*x^2]*Sqrt[2 + 4*x^2],x]
[Out] Sqrt[6]*x*Hypergeometric2F1[-1/2, 1/4, 5/4, 4*x^4]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(29) = 58.

Time = 2.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.97

method	result	size
default	$-\frac{\sqrt{-6x^2+3}\,\sqrt{2}\,\sqrt{2x^2+1}\,\left(\sqrt{2}\,\sqrt{3}\,\sqrt{-6x^2+3}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)-12x^5+3x\right)}{9(4x^4-1)}$	75
elliptic	$-\frac{\sqrt{-6x^2+3}\,\sqrt{4x^2+2}\,\sqrt{-24x^4+6}\,\left(\frac{x\sqrt{-24x^4+6}}{3}+\frac{2\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)}{\sqrt{-24x^4+6}}\right)}{6(4x^4-1)}$	92
risch	$-\frac{x(2x^2-1)\,(2x^2+1)\,\sqrt{(-6x^2+3)(4x^2+2)}\,\sqrt{6}}{3\sqrt{-(2x^2-1)(2x^2+1)}\,\sqrt{-6x^2+3}\,\sqrt{4x^2+2}}+\frac{\sqrt{2}\,\sqrt{-2x^2+1}\,\sqrt{2x^2+1}\,F\left(\sqrt{2}\,x,i\right)\sqrt{(-6x^2+3)(4x^2+2)}\,\sqrt{6}}{3\sqrt{-4x^4+1}\,\sqrt{-6x^2+3}\,\sqrt{4x^2+2}}$	153

```
[In] int((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*(-6*x^2+3)^(1/2)*2^(1/2)*(2*x^2+1)^(1/2)*(2^(1/2)*3^(1/2)*(-6*x^2+3)^(
1/2)*(2*x^2+1)^(1/2)*EllipticF(2^(1/2)*x,I)-12*x^5+3*x)/(4*x^4-1)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \frac{1}{3}\,\sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}x+\frac{1}{3}\,\sqrt{2}\sqrt{-6}F(\arcsin\left(\frac{\sqrt{2}}{2\,x}\right)\,|-1)$$

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3)*x + 1/3*sqrt(2)*sqrt(-6)*elliptic_f(ar
csin(1/2*sqrt(2)/x), -1)
```

Sympy [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \sqrt{6}\int \sqrt{1-2x^2}\sqrt{2x^2+1}\,dx$$

```
[In] integrate((-6*x**2+3)**(1/2)*(4*x**2+2)**(1/2),x)
```

```
[Out] sqrt(6)*Integral(sqrt(1 - 2*x**2)*sqrt(2*x**2 + 1), x)
```

Maxima [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Giac [F]

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\sqrt{-6\,x^2+3}\,dx$$

```
[In] integrate((-6*x^2+3)^(1/2)*(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x^2 + 2)*sqrt(-6*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{3-6x^2}\sqrt{2+4x^2}\,dx = \int \sqrt{4\,x^2+2}\,\sqrt{3-6\,x^2}\,dx$$

```
[In] int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2),x)
[Out] int((4*x^2 + 2)^(1/2)*(3 - 6*x^2)^(1/2), x)
```

3.180 $\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx=\sqrt{6}x+2\sqrt{\frac{2}{3}}x^3$$

```
[Out] 2/3*x^3*6^(1/2)+x*6^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {22}

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx=2\sqrt{\frac{2}{3}}x^3+\sqrt{6}x$$

```
[In] Int[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
```

```
[Out] Sqrt[6]*x + 2*Sqrt[2/3]*x^3
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] && !(IntegerQ[m] || IntegerQ[n])
```

Rubi steps

$$\begin{aligned}\text{integral}&=\sqrt{\frac{2}{3}}\int \left(3+6x^2\right)\,dx\\&=\sqrt{6}x+2\sqrt{\frac{2}{3}}x^3\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \sqrt{6}\left(x+\frac{2x^3}{3}\right)$$

```
[In] Integrate[Sqrt[2 + 4*x^2]*Sqrt[3 + 6*x^2],x]
[Out] Sqrt[6]*(x + (2*x^3)/3)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 2 vs. order 1.

Time = 2.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

method	result	size
gosper	$\frac{x(2x^2+3)\sqrt{4x^2+2}}{\sqrt{6x^2+3}}$	38

```
[In] int((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*x*(2*x^2+3)*(4*x^2+2)^(1/2)*(6*x^2+3)^(1/2)/(2*x^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(14) = 28$.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{(2\,x^3+3\,x)\sqrt{6\,x^2+3}\sqrt{4\,x^2+2}}{3\,(2\,x^2+1)}$$

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*x^3 + 3*x)*sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2)/(2*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{2\sqrt{6}x^3}{3} + \sqrt{6}x$$

```
[In] integrate((4*x**2+2)**(1/2)*(6*x**2+3)**(1/2),x)
[Out] 2*sqrt(6)*x**3/3 + sqrt(6)*x
```

Maxima [F]

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \int \sqrt{6\,x^2+3}\sqrt{4\,x^2+2}\,dx$$

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(6*x^2 + 3)*sqrt(4*x^2 + 2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \frac{1}{3}\,\sqrt{3}\sqrt{2}\left(2\,x^3+3\,x\right)$$

```
[In] integrate((4*x^2+2)^(1/2)*(6*x^2+3)^(1/2),x, algorithm="giac")
[Out] 1/3*sqrt(3)*sqrt(2)*(2*x^3 + 3*x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+4x^2}\sqrt{3+6x^2}\,dx = \int \sqrt{4\,x^2+2}\,\sqrt{6\,x^2+3}\,dx$$

```
[In] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2),x)
[Out] int((4*x^2 + 2)^(1/2)*(6*x^2 + 3)^(1/2), x)
```

3.181 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*E
llipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(
b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3*d*
x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/
2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3
))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} - \frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} + \frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}$$

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx\\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{\left(3+dx^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{\sqrt{2}E\left(\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]

[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result
default	$\frac{E\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\frac{x\sqrt{-3d}}{3},\sqrt{-\right.\right.}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$

[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2)
/(-d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = -\frac{9\sqrt{3}\sqrt{bd}bx\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)|\frac{2d}{3b})-\sqrt{3}\sqrt{bd}(2d^2+9b)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)|\frac{2d}{3b})-3\sqrt{}}{3bd^2x}$$

[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")

[Out] -1/3*(9*sqrt(3)*sqrt(b*d)*b*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/
d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(2*d^2 + 9*b)*x*sqrt(-1/d)*elliptic_f(a
rcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - 3*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)*b
*d)/(b*d^2*x)

Sympy [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

```
[In] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
[Out] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.182 $\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx=-\frac{\sqrt{c+dx^2}E\left(\arcsin\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}}+\frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}$$

```
[Out] -EllipticE(1/2*x,2*(-d/c)^(1/2))*(d*x^2+c)^(1/2)/d/(1+d*x^2/c)^(1/2)+(c+4*d
)*EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/d/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {434, 437, 435, 432, 430}

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$$
$$=\frac{(c+4d)\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}-\frac{\sqrt{c+dx^2}E\left(\arcsin\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[c + d*x^2]*EllipticE[ArcSin[x/2], (-4*d)/c])/(d*Sqrt[1 + (d*x^2)/c]
)) + ((c + 4*d)*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/(d*Sq
rt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{\sqrt{c+dx^2}}{\sqrt{4-x^2}}\,dx}{d} - \frac{(-c-4d)\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx}{d} \\
&= -\frac{\sqrt{c+dx^2}\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{4-x^2}}\,dx}{d\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((-c-4d)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}} \\
&= -\frac{\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{1+\frac{dx^2}{c}}} + \frac{(c+4d)\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)|-\frac{4d}{c}\right)}{d\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx=\frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -1/4*c/d])/(Sqrt[-
(d/c)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result
default	$\frac{\left(cF\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)+4F\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)d-cE\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)\right)\sqrt{\frac{d\,x^{2}+c}{c}}}{\sqrt{d\,x^{2}+c}\,d}$
elliptic	$\frac{\sqrt{-\left(d\,x^{2}+c\right)\left(x^{2}-4\right)}\left(\frac{4\sqrt{-x^{2}+4}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}}+\frac{c\sqrt{-x^{2}+4}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)-E\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)\right)}{\sqrt{-d\,x^{4}-c\,x^{2}+4d\,x^{2}+4c}\,d}\right)}{\sqrt{-x^{2}+4}\,\sqrt{d\,x^{2}+c}}$

```
[In] int((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*EllipticF(1/2*x,2*(-d/c)^(1/2))+4*EllipticF(1/2*x,2*(-d/c)^(1/2))*d-c*El
lipticE(1/2*x,2*(-d/c)^(1/2)))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx$$
$$=\frac{2\left(4\,xE(\arcsin\left(\frac{2}{x}\right)\,|-\frac{c}{4\,d})-3\,xF(\arcsin\left(\frac{2}{x}\right)\,|-\frac{c}{4\,d})\right)\sqrt{-d}+\sqrt{dx^2+c}\sqrt{-x^2+4}}{dx}$$

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*(4*x*elliptic_e(arcsin(2/x), -1/4*c/d) - 3*x*elliptic_f(arcsin(2/x), -1/
4*c/d))*sqrt(-d) + sqrt(d*x^2 + c)*sqrt(-x^2 + 4))/(d*x)
```

Sympy [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{4-x^2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((4 - x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.183 $\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}} - \frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}} + \frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

```
[Out] x*(d*x^2+c)^(1/2)/d/(x^2+4)^(1/2)-(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+4)^(1/
2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/d/((d*x^2+c)/c/(x^2+4))^(1/2)+4*(1/(x^2
+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2)/c/((d
*x^2+c)/c/(x^2+4))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \frac{4\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}} - \frac{\sqrt{c+dx^2}E\left(\arctan\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}} + \frac{x\sqrt{c+dx^2}}{d\sqrt{x^2+4}}$$

```
[In] Int[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2])/(d*Sqrt[4 + x^2]) - (Sqrt[c + d*x^2]*EllipticE[ArcTan[x
/2], 1 - (4*d)/c])/(d*Sqrt[4 + x^2]*Sqrt[(c + d*x^2)/(c*(4 + x^2))]) + (4*S
qrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt[(
c + d*x^2)/(c*(4 + x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 4\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx+\int \frac{x^2}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}-\frac{4\int \frac{\sqrt{c+dx^2}}{(4+x^2)^{3/2}}\,dx}{d}\\
&= \frac{x\sqrt{c+dx^2}}{d\sqrt{4+x^2}}-\frac{\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{d\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}+\frac{4\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{c}{4d}\right)}{\sqrt{-\frac{d}{c}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (2*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], c/(4*d)])/(Sqrt[-(
d/c)]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

method	result
default	$\frac{2E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)\sqrt{\frac{d\,x^2+c}{c}}}{\sqrt{d\,x^2+c}\,\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(x^2+4)}\left(\frac{2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{x^2+4}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^4+c\,x^2+4d\,x^2+4c}}-\frac{2\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{x^2+4}\,\left(F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)-E\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^4+c\,x^2+4d\,x^2+4c}}\right)}{\sqrt{d\,x^2+c}\,\sqrt{x^2+4}}$

```
[In] int((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*EllipticE(x*(-d/c)^(1/2),1/2*(c/d)^(1/2))*((d*x^2+c)/c)^(1/2)/(d*x^2+c)^(
1/2)/(-d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = -\frac{c^2\sqrt{d}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)|\frac{4d}{c}) - (c^2+4d^2)\sqrt{d}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)|\frac{4d}{c}) - \sqrt{dx^2+c}\sqrt{x^2+4}}{cd^2x}$$

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(c^2*sqrt(d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), 4*d/c) - (c^2 +
 4*d^2)*sqrt(d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), 4*d/c) - sqrt
(d*x^2 + c)*sqrt(x^2 + 4)*c*d)/(c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(x**2 + 4)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 4)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((x^2 + 4)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.184 $\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {435}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 - 3*x^2],x]

[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 2/3]/Sqrt[3]

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{\sqrt{2}\left(F\left(x,\frac{\sqrt{6}}{2}\right)+2E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{6}$	23
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4-5x^2+2}}-\frac{\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,\left(F\left(x,\frac{\sqrt{6}}{2}\right)-E\left(x,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4-5x^2+2}}\right)}{\sqrt{-x^2+1}\,\sqrt{-3x^2+2}}$	128

[In] int((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6*2^(1/2)*(EllipticF(x,1/2*6^(1/2))+2*EllipticE(x,1/2*6^(1/2)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{\sqrt{3}xE\left(\arcsin\left(\frac{1}{x}\right)\mid\frac{2}{3}\right)+\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}{3\,x}$$

[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] -1/3*(sqrt(3)*x*elliptic_e(arcsin(1/x), 2/3) + sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2))/x

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((-x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.185 $\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 21

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}} \, dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {435}

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}} \, dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx=\frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 - 3*x^2],x]
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{2E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\sqrt{3}}{3}$	18
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-4)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-x^2+4}\,F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)}{3\sqrt{3x^4-14x^2+8}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{\sqrt{6}}{6}\right)\right)}{3\sqrt{3x^4-14x^2+8}}\right)}{\sqrt{-x^2+4}\,\sqrt{-3x^2+2}}$	149

```
[In] int((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*6^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.52

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{8\sqrt{3}xE\left(\arcsin\left(\frac{2}{x}\right)|\frac{1}{6}\right)-6\sqrt{3}xF\left(\arcsin\left(\frac{2}{x}\right)|\frac{1}{6}\right)+\sqrt{-x^2+4}\sqrt{-3x^2+2}}{3x}$$

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(8*sqrt(3)*x*elliptic_e(arcsin(2/x), 1/6) - 6*sqrt(3)*x*elliptic_f(arc
sin(2/x), 1/6) + sqrt(-x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{1}{6}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((-x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 1/6)/3, (x > -sqrt(6)/3)
 & (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4-x^2}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((4 - x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.186 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {435}

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], 8/3]/Sqrt[3]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{\sqrt{2}\left(5F\left(2x,\frac{\sqrt{6}}{4}\right)-8E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{12}$	29
elliptic	$\frac{\sqrt{(3x^2-2)(4x^2-1)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\,F\left(2x,\frac{\sqrt{6}}{4}\right)}{4\sqrt{12x^4-11x^2+2}}-\frac{2\sqrt{-4x^2+1}\,\sqrt{-6x^2+4}\left(F\left(2x,\frac{\sqrt{6}}{4}\right)-E\left(2x,\frac{\sqrt{6}}{4}\right)\right)}{3\sqrt{12x^4-11x^2+2}}\right)}{\sqrt{-4x^2+1}\,\sqrt{-3x^2+2}}$	136

```
[In] int((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/12*2^(1/2)*(5*EllipticF(2*x,1/4*6^(1/2))-8*EllipticE(2*x,1/4*6^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{16\,\sqrt{2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|\frac{3}{8})-7\,\sqrt{2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|\frac{3}{8})+12\,\sqrt{-3\,x^2+2}\sqrt{-4\,x^2+1}}{36\,x}$$

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/36*(16*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) - 7*sqrt
(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 3/8) + 12*sqrt(-3*x^2 + 2)*
sqrt(-4*x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{8}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((-4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), 8/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((-4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{1-4\,x^2}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((1 - 4*x^2)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.187 $\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 4

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

```
[Out] EllipticE(x,I)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {435}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

```
[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
[Out] EllipticE[ArcSin[x], -1]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = E\left(\sin^{-1}(x)\middle| -1\right)$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = E(\arcsin(x)|-1)$$

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^2],x]
[Out] EllipticE[ArcSin[x], -1]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$E(x,i)$	5
elliptic	$\frac{\sqrt{-x^4+1}\left(\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,F(x,i)}{\sqrt{-x^4+1}}-\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{\sqrt{-x^4+1}}\right)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	96

```
[In] int((x^2+1)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticE(x,I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 10.25

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \frac{-i\,xE\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)+2i\,xF\left(\arcsin\left(\frac{1}{x}\right)\,|-1\right)-\sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] (-I*x*elliptic_e(arcsin(1/x), -1) + 2*I*x*elliptic_f(arcsin(1/x), -1) - sqr
t(x^2 + 1)*sqrt(-x^2 + 1))/x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(2) = 4$.

Time = 1.37 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \begin{cases} E(\operatorname{asin}(x)|-1) & \text{for } x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate((x**2+1)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Piecewise((elliptic_e(asin(x), -1), (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{1-x^2}}\,dx$$

```
[In] int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

3.188 $\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}} \, dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right) \middle| -\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {435}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}} \, dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right) \middle| -\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) \middle| -\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+1)}\left(\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^4-x^2+2}}-\frac{\sqrt{6}\sqrt{-6x^2+4}\sqrt{x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\right)}{6\sqrt{-3x^4-x^2+2}}\right)}{\sqrt{x^2+1}\sqrt{-3x^2+2}}$	147

```
[In] int((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticE(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.80

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{4\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{2}) - 13\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{2}) + 18\sqrt{x^2+1}\sqrt{-3x^2+}}{54x}$$

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/54*(4*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x
), -3/2) - 13*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt
(2)/x), -3/2) + 18*sqrt(x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

Sympy [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.189 $\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}} dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}} dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {435}

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}} dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{2E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \frac{2E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 - 3*x^2],x]
[Out] (2*EllipticE[ArcSin[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{2E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+4)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+4}\,F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)}{3\sqrt{-3x^4-10x^2+8}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{x^2+4}\left(F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{6}\right)\right)}{3\sqrt{-3x^4-10x^2+8}}\right)}{\sqrt{x^2+4}\,\sqrt{-3x^2+2}}$	147

```
[In] int((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/3*EllipticE(1/2*x*6^(1/2),1/6*I*6^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.62

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{2\,\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|-6) - 20\,\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right)|-6) + 9\,\sqrt{x^2+4}\sqrt{-3\,x^2+}}{27\,x}$$

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/27*(2*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x
), -6) - 20*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2
)/x), -6) + 9*sqrt(x^2 + 4)*sqrt(-3*x^2 + 2))/x
```

Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{2\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{1}{6}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((x**2+4)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((2*sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -1/6)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 4)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((x^2 + 4)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.190 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 20

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx=\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {435}

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx=\frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral}=\frac{E\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 - 3*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[3/2]*x], -8/3]/Sqrt[3]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(4x^2+1)}\left(\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)}{6\sqrt{-12x^4+5x^2+2}}-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{4x^2+1}\left(F\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)-E\left(\frac{x\sqrt{6}}{2},\frac{2i\sqrt{6}}{3}\right)\right)}{6\sqrt{-12x^4+5x^2+2}}\right)}{\sqrt{4x^2+1}\,\sqrt{-3x^2+2}}$	155

```
[In] int((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticE(1/2*x*6^(1/2),2/3*I*6^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.90

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{16\sqrt{3}\sqrt{2}\sqrt{-3}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{8}) - 25\sqrt{3}\sqrt{2}\sqrt{-3}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3x}\right)|-\frac{3}{8}) + 36\sqrt{4x^2+1}\sqrt{-3x^2}}{108x}$$

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/108*(16*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)
/x), -3/8) - 25*sqrt(3)*sqrt(2)*sqrt(-3)*x*elliptic_f(arcsin(1/3*sqrt(3)*sq
rt(2)/x), -3/8) + 36*sqrt(4*x^2 + 1)*sqrt(-3*x^2 + 2))/x
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \begin{cases} \frac{\sqrt{3}E\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{8}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate((4*x**2+1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_e(asin(sqrt(6)*x/2), -8/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((4*x^2+1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((4*x^2 + 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.191 $\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 13

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -E(\arcsin(x)|-1) + 2\,\mathrm{EllipticF}(\arcsin(x), -1)$$

```
[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {434, 435, 254, 227}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = 2\,\mathrm{EllipticF}(\arcsin(x), -1) - E(\arcsin(x)|-1)$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
[Out] -EllipticE[ArcSin[x], -1] + 2*EllipticF[ArcSin[x], -1]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
```

```
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{1-x^2}\sqrt{1+x^2}}\,dx - \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx \\
&= -E\left(\sin^{-1}(x)\middle|-1\right) + 2\int \frac{1}{\sqrt{1-x^4}}\,dx \\
&= -E\left(\sin^{-1}(x)\middle|-1\right) + 2F\left(\sin^{-1}(x)\middle|-1\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$
\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = -iE(i\text{arcsinh}(x)|-1)
$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x^2],x]
[Out] (-I)*EllipticE[I*ArcSinh[x], -1]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
default	$-E(x,i)+2F(x,i)$	14
elliptic	$\frac{\sqrt{-x^4+1}\left(\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,F(x,i)}{\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(F(x,i)-E(x,i))}{\sqrt{-x^4+1}}\right)}{\sqrt{x^2+1}\,\sqrt{-x^2+1}}$	95

```
[In] int((-x^2+1)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -EllipticE(x,I)+2*EllipticF(x,I)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \frac{i\,xE(\arcsin\left(\frac{1}{x}\right)\,|-1)+\sqrt{x^2+1}\sqrt{-x^2+1}}{x}$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*x*elliptic_e(arcsin(1/x), -1) + sqrt(x^2 + 1)*sqrt(-x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{x^2+1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{x^2+1}}\,dx$$

```
[In] int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(x^2 + 1)^(1/2), x)
```

3.192 $\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)+\frac{5\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{3\sqrt{2}}$$

```
[Out] 5/6*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)-1/3*EllipticE(x,1/2*I*6^(1/2))*2^(1/
2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {434, 435, 430}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \frac{5\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{3\sqrt{2}}-\frac{1}{3}\sqrt{2}E\left(\arcsin(x)\left|-\frac{3}{2}\right.\right)$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x], -3/2]) + (5*EllipticF[ArcSin[x], -3/2])/
(3*Sqrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-x^2}}\,dx\right) + \frac{5}{3}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx \\
&= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}(x)|-\frac{3}{2}\right) + \frac{5F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{3\sqrt{2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$
\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}}
$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -2/3])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\left(5F\left(x,\frac{i\sqrt{6}}{2}\right)-2E\left(x,\frac{i\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$	27
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,F\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^4+x^2+2}}+\frac{\sqrt{-x^2+1}\,\sqrt{6x^2+4}\,\left(F\left(x,\frac{i\sqrt{6}}{2}\right)-E\left(x,\frac{i\sqrt{6}}{2}\right)\right)}{3\sqrt{-3x^4+x^2+2}}\right)}{\sqrt{-x^2+1}\,\sqrt{3x^2+2}}$	128

```
[In] int((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*(5*EllipticF(x,1/2*I*6^(1/2))-2*EllipticE(x,1/2*I*6^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{-3}xE(\arcsin\left(\frac{1}{x}\right)\,|-\frac{2}{3}) + \sqrt{3\,x^2+2}\sqrt{-x^2+1}}{3\,x}$$

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(sqrt(-3)*x*elliptic_e(arcsin(1/x), -2/3) + sqrt(3*x^2 + 2)*sqrt(-x^2 +
 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.193 $\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)+\frac{7}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)$$

```
[Out] -1/3*EllipticE(1/2*x,I*6^(1/2))*2^(1/2)+7/3*EllipticF(1/2*x,I*6^(1/2))*2^(1
/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {434, 435, 430}

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \frac{7}{3}\sqrt{2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-6\right)-\frac{1}{3}\sqrt{2}E\left(\arcsin\left(\frac{x}{2}\right)\middle|-6\right)$$

```
[In] Int[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] -1/3*(Sqrt[2]*EllipticE[ArcSin[x/2], -6]) + (7*Sqrt[2]*EllipticF[ArcSin[x/2
], -6])/3
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{1}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{4-x^2}}\,dx\right)+\frac{14}{3}\int \frac{1}{\sqrt{4-x^2}\sqrt{2+3x^2}}\,dx\\ &= -\frac{1}{3}\sqrt{2}E\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)+\frac{7}{3}\sqrt{2}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-6\right)\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[4 - x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -1/6])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(7F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)\sqrt{2}}{3}$	31
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2-4)}\left(\frac{2\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,F\left(\frac{x}{2},i\sqrt{6}\right)}{\sqrt{-3x^4+10x^2+8}}+\frac{\sqrt{-x^2+4}\,\sqrt{6x^2+4}\,\left(F\left(\frac{x}{2},i\sqrt{6}\right)-E\left(\frac{x}{2},i\sqrt{6}\right)\right)}{3\sqrt{-3x^4+10x^2+8}}\right)}{\sqrt{-x^2+4}\,\sqrt{3x^2+2}}$	138

```
[In] int((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] 1/3*(7*EllipticF(1/2*x,I*6^(1/2))-EllipticE(1/2*x,I*6^(1/2)))*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx$$
$$= \frac{8\sqrt{-3}xE\left(\arcsin\left(\frac{2}{x}\right)\middle|-\frac{1}{6}\right)-6\sqrt{-3}xF\left(\arcsin\left(\frac{2}{x}\right)\middle|-\frac{1}{6}\right)+\sqrt{3\,x^2+2}\sqrt{-x^2+4}}{3\,x}$$

[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/3*(8*sqrt(-3)*x*elliptic_e(arcsin(2/x), -1/6) - 6*sqrt(-3)*x*elliptic_f(a
rcsin(2/x), -1/6) + sqrt(3*x^2 + 2)*sqrt(-x^2 + 4))/x

Sympy [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(x-2)(x+2)}}{\sqrt{3x^2+2}}\,dx$$

[In] integrate((-x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)

[Out] Integral(sqrt(-(x - 2)*(x + 2))/sqrt(3*x**2 + 2), x)

Maxima [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)

Giac [F]

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((-x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4-x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4-x^2}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((4 - x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.194 $\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2}{3}\sqrt{2}E\left(\arcsin(2x)\left|-\frac{3}{8}\right.\right)+\frac{11\,\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{6\sqrt{2}}$$

```
[Out] 11/12*EllipticF(2*x,1/4*I*6^(1/2))*2^(1/2)-2/3*EllipticE(2*x,1/4*I*6^(1/2))
*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {434, 435, 430}

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \frac{11\,\text{EllipticF}\left(\arcsin(2x),-\frac{3}{8}\right)}{6\sqrt{2}}-\frac{2}{3}\sqrt{2}E\left(\arcsin(2x)\left|-\frac{3}{8}\right.\right)$$

```
[In] Int[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (-2*Sqrt[2]*EllipticE[ArcSin[2*x], -3/8])/3 + (11*EllipticF[ArcSin[2*x], -3
/8])/(6*Sqrt[2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\sqrt{1-4x^2}}\,dx\right)+\frac{11}{3}\int \frac{1}{\sqrt{1-4x^2}\sqrt{2+3x^2}}\,dx\\ &= -\frac{2}{3}\sqrt{2}E\left(\sin^{-1}(2x)|-\frac{3}{8}\right)+\frac{11F\left(\sin^{-1}(2x)|-\frac{3}{8}\right)}{6\sqrt{2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{8}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[1 - 4*x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], -8/3])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\left(11F\left(2x,\frac{i\sqrt{6}}{4}\right)-8E\left(2x,\frac{i\sqrt{6}}{4}\right)\right)\sqrt{2}}{12}$	31
elliptic	$\frac{\sqrt{-(3x^2+2)(4x^2-1)}\left(\frac{\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\,F\left(2x,\frac{i\sqrt{6}}{4}\right)}{4\sqrt{-12x^4-5x^2+2}}+\frac{2\sqrt{-4x^2+1}\,\sqrt{6x^2+4}\,\left(F\left(2x,\frac{i\sqrt{6}}{4}\right)-E\left(2x,\frac{i\sqrt{6}}{4}\right)\right)}{3\sqrt{-12x^4-5x^2+2}}\right)}{\sqrt{-4x^2+1}\,\sqrt{3x^2+2}}$	140

```
[In] int((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/12*(11*EllipticF(2*x,1/4*I*6^(1/2))-8*EllipticE(2*x,1/4*I*6^(1/2)))*2^(1/
2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{-3}xE\left(\arcsin\left(\frac{1}{2\,x}\right)\mid -\frac{8}{3}\right) + 3\sqrt{-3}xF\left(\arcsin\left(\frac{1}{2\,x}\right)\mid -\frac{8}{3}\right) + 4\sqrt{3\,x^2+2}\sqrt{-4\,x^2+1}}{12\,x}$$

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(sqrt(-3)*x*elliptic_e(arcsin(1/2/x), -8/3) + 3*sqrt(-3)*x*elliptic_f(
arcsin(1/2/x), -8/3) + 4*sqrt(3*x^2 + 2)*sqrt(-4*x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-(2x-1)(2x+1)}}{\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((-4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(-(2*x - 1)*(2*x + 1))/sqrt(3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{-4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((-4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{1-4\,x^2}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2),x)
```

```
[Out] int((1 - 4*x^2)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.195 $\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 131

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} + \frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+
1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)-1
/3*(1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*2^(1/2)*(3*x^
2+2)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}} - \frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan(x)\left|-\frac{1}{2}\right.\right)}{3\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}} + \frac{\sqrt{3x^2+2}\,x}{3\sqrt{x^2+1}}$$

```
[In] Int[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x], -1/2])/(3*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]) + (Sqrt[2 +
 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 + 3*x^2)
/(1 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx + \int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx\\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} - \frac{1}{3}\int \frac{\sqrt{2+3x^2}}{(1+x^2)^{3/2}}\,dx\\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{3\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}} + \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\text{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\mid\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[1 + x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 2/3])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.23

method	result	size
default	$-\frac{i\left(F\left(ix,\frac{\sqrt{6}}{2}\right)+2E\left(ix,\frac{\sqrt{6}}{2}\right)\right)\sqrt{2}}{6}$	30
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+1)}\left(-\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,F\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^4+5x^2+2}}+\frac{i\sqrt{x^2+1}\,\sqrt{6x^2+4}\,\left(F\left(ix,\frac{\sqrt{6}}{2}\right)-E\left(ix,\frac{\sqrt{6}}{2}\right)\right)}{3\sqrt{3x^4+5x^2+2}}\right)}{\sqrt{3x^2+2}\,\sqrt{x^2+1}}$	133

```
[In] int((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*I*(EllipticF(I*x,1/2*6^(1/2))+2*EllipticE(I*x,1/2*6^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{4\sqrt{-2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\mid\frac{3}{2}) - 13\sqrt{-2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\mid\frac{3}{2}) - 6\sqrt{3x^2+2}\sqrt{x^2+1}}{18x}$$

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/18*(4*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 13*sq
rt(-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 3/2) - 6*sqrt(3*x^2 + 2
)*sqrt(x^2 + 1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}} \, dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3x^2+2}} \, dx$$

```
[In] integrate((x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}} \, dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}} \, dx$$

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}} \, dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}} \, dx$$

```
[In] integrate((x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}}{\sqrt{2+3x^2}} \, dx = \int \frac{\sqrt{x^2+1}}{\sqrt{3\,x^2+2}} \, dx$$

```
[In] int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.196 $\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}} - \frac{\sqrt{2}\sqrt{2+3x^2}E\left(\arctan\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}} + \frac{2\sqrt{2}\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}$$

```
[Out] 1/3*x*(3*x^2+2)^(1/2)/(x^2+4)^(1/2)-1/3*(1/(x^2+4))^(1/2)*EllipticE(x/(x^2+
4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(x^2+4))^(1/2)+2*(1/
(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),I*5^(1/2))*2^(1/2)*(3*x^2+2)^(1/2)
/((3*x^2+2)/(x^2+4))^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \frac{2\sqrt{2}\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),-5\right)}{\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}} - \frac{\sqrt{2}\sqrt{3x^2+2}E\left(\arctan\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{x^2+4}\sqrt{\frac{3x^2+2}{x^2+4}}} + \frac{\sqrt{3x^2+2}x}{3\sqrt{x^2+4}}$$

```
[In] Int[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (x*Sqrt[2 + 3*x^2])/(3*Sqrt[4 + x^2]) - (Sqrt[2]*Sqrt[2 + 3*x^2]*EllipticE[
ArcTan[x/2], -5])/(3*Sqrt[4 + x^2]*Sqrt[(2 + 3*x^2)/(4 + x^2)]) + (2*Sqrt[2
]*Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x/2], -5])/(Sqrt[4 + x^2]*Sqrt[(2 + 3*x^
2)/(4 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 4\int \frac{1}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx+\int \frac{x^2}{\sqrt{4+x^2}\sqrt{2+3x^2}}\,dx\\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}+\frac{2\sqrt{2}\sqrt{2+3x^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}-\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\left(4+x^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{2+3x^2}}{3\sqrt{4+x^2}}-\frac{\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{3\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}+\frac{2\sqrt{2}\sqrt{2+3x^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)\middle|-5\right)}{\sqrt{4+x^2}\sqrt{\frac{2+3x^2}{4+x^2}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)\,|\,\frac{1}{6}\right)}{\sqrt{3}}$$

```
[In] Integrate[Sqrt[4 + x^2]/Sqrt[2 + 3*x^2],x]
[Out] ((-2*I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 1/6])/Sqrt[3]
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.19

method	result	size
default	$-\frac{i\left(5F\left(\frac{ix}{2},\sqrt{6}\right)+E\left(\frac{ix}{2},\sqrt{6}\right)\right)\sqrt{2}}{3}$	26
elliptic	$\frac{\sqrt{(3x^2+2)(x^2+4)}\left(-\frac{2i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,F\left(\frac{ix}{2},\sqrt{6}\right)}{\sqrt{3x^4+14x^2+8}}+\frac{i\sqrt{x^2+4}\,\sqrt{6x^2+4}\,\left(F\left(\frac{ix}{2},\sqrt{6}\right)-E\left(\frac{ix}{2},\sqrt{6}\right)\right)}{3\sqrt{3x^4+14x^2+8}}\right)}{\sqrt{3x^2+2}\,\sqrt{x^2+4}}$	127

```
[In] int((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*I*(5*EllipticF(1/2*I*x,6^(1/2))+EllipticE(1/2*I*x,6^(1/2)))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{2\sqrt{-2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\,|\,6)-20\sqrt{-2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{-2}}{3x}\right)\,|\,6)-3\sqrt{3x^2+2}\sqrt{x^2+4}}{9x}$$

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/9*(2*sqrt(-2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 20*sqrt(
-2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(-2)/x), 6) - 3*sqrt(3*x^2 + 2)*sqr
t(x^2 + 4))/x
```

Sympy [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((x**2+4)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(x**2 + 4)/sqrt(3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((x^2+4)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 4)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{4+x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{x^2+4}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((x^2 + 4)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.197 $\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 148

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} \, dx = \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}} - \frac{2\sqrt{2}\sqrt{2+3x^2}E\left(\arctan(2x)\,\middle|\,\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}} + \frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}$$

```
[Out] 4/3*x*(3*x^2+2)^(1/2)/(4*x^2+1)^(1/2)+1/4*(1/(4*x^2+1))^(1/2)*EllipticF(2*x
/(4*x^2+1)^(1/2),1/4*10^(1/2))*(3*x^2+2)^(1/2)*2^(1/2)/((3*x^2+2)/(4*x^2+1)
)^(1/2)-2/3*(1/(4*x^2+1))^(1/2)*EllipticE(2*x/(4*x^2+1)^(1/2),1/4*10^(1/2))
*2^(1/2)*(3*x^2+2)^(1/2)/((3*x^2+2)/(4*x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}} \, dx = \frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(2x),\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}} - \frac{2\sqrt{2}\sqrt{3x^2+2}E\left(\arctan(2x)\,\middle|\,\frac{5}{8}\right)}{3\sqrt{\frac{3x^2+2}{4x^2+1}}\sqrt{4x^2+1}} + \frac{4\sqrt{3x^2+2}x}{3\sqrt{4x^2+1}}$$

```
[In] Int[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]
```

```
[Out] (4*x*Sqrt[2 + 3*x^2])/(3*Sqrt[1 + 4*x^2]) - (2*Sqrt[2]*Sqrt[2 + 3*x^2]*Elli
pticE[ArcTan[2*x], 5/8])/(3*Sqrt[(2 + 3*x^2)/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
+ (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[2*x], 5/8])/(2*Sqrt[2]*Sqrt[(2 + 3*x^2)
/(1 + 4*x^2)]*Sqrt[1 + 4*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 4\int \frac{x^2}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx+\int \frac{1}{\sqrt{2+3x^2}\sqrt{1+4x^2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}+\frac{\sqrt{2+3x^2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}-\frac{4}{3}\int \frac{\sqrt{2+3x^2}}{\left(1+4x^2\right)^{3/2}}\,dx \\
&= \frac{4x\sqrt{2+3x^2}}{3\sqrt{1+4x^2}}-\frac{2\sqrt{2}\sqrt{2+3x^2}E\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{3\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}+\frac{\sqrt{2+3x^2}F\left(\tan^{-1}(2x)|\frac{5}{8}\right)}{2\sqrt{2}\sqrt{\frac{2+3x^2}{1+4x^2}}\sqrt{1+4x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = -\frac{iE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{3}{2}}x\right)|\frac{8}{3}\right)}{\sqrt{3}}$$

[In] Integrate[Sqrt[1 + 4*x^2]/Sqrt[2 + 3*x^2],x]

[Out] ((-I)*EllipticE[I*ArcSinh[Sqrt[3/2]*x], 8/3])/Sqrt[3]

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.14

method	result	size
default	$-\frac{iE\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\sqrt{3}}{3}$	20
elliptic	$\frac{\sqrt{(3x^2+2)(4x^2+1)}\left(-\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)}{6\sqrt{12x^4+11x^2+2}}+\frac{i\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{4x^2+1}\,\left(F\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)-E\left(\frac{ix\sqrt{6}}{2},\frac{2\sqrt{6}}{3}\right)\right)}{6\sqrt{12x^4+11x^2+2}}\right)}{\sqrt{3x^2+2}\,\sqrt{4x^2+1}}$	156

[In] int((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/3*I*EllipticE(1/2*I*x*6^(1/2),2/3*6^(1/2))*3^(1/2)

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \frac{-i\sqrt{3}xE(\arcsin\left(\frac{i}{2x}\right)|\frac{8}{3}) + 5i\sqrt{3}xF(\arcsin\left(\frac{i}{2x}\right)|\frac{8}{3}) + 4\sqrt{4x^2+1}\sqrt{3x^2+2}}{12x}$$

[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")

[Out] 1/12*(-I*sqrt(3)*x*elliptic_e(arcsin(1/2*I/x), 8/3) + 5*I*sqrt(3)*x*ellipti
c_f(arcsin(1/2*I/x), 8/3) + 4*sqrt(4*x^2 + 1)*sqrt(3*x^2 + 2))/x

Sympy [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4x^2+1}}{\sqrt{3x^2+2}}\,dx$$

```
[In] integrate((4*x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(4*x**2 + 1)/sqrt(3*x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] integrate((4*x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(4*x^2 + 1)/sqrt(3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+4x^2}}{\sqrt{2+3x^2}}\,dx = \int \frac{\sqrt{4\,x^2+1}}{\sqrt{3\,x^2+2}}\,dx$$

```
[In] int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2),x)
[Out] int((4*x^2 + 1)^(1/2)/(3*x^2 + 2)^(1/2), x)
```

3.198 $\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}} \, dx = \frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+2x^2}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2),1/2*2^(1/2))*(-2*x^2+1)^(1/2)*2^(1/2)/(2*x^2-1)^(1/
2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {438, 435}

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}} \, dx = \frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{2x^2-1}}$$

```
[In] Int[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/(Sqrt[2]*Sqrt[-1 + 2*x^
2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-2x^2}\int \frac{\sqrt{1-x^2}}{\sqrt{1-2x^2}}\,dx}{\sqrt{-1+2x^2}}\\ &= \frac{\sqrt{1-2x^2}E\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+2x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}E\left(\arcsin\left(\sqrt{2}x\right)|\frac{1}{2}\right)}{\sqrt{-2+4x^2}}$$

```
[In] Integrate[Sqrt[1 - x^2]/Sqrt[-1 + 2*x^2],x]
[Out] (Sqrt[1 - 2*x^2]*EllipticE[ArcSin[Sqrt[2]*x], 1/2])/Sqrt[-2 + 4*x^2]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\left(F\left(x,\sqrt{2}\right)+E\left(x,\sqrt{2}\right)\right)\sqrt{-2x^2+1}}{2\sqrt{2x^2-1}}$	32
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\left(\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{-2x^4+3x^2-1}}-\frac{\sqrt{-x^2+1}\,\sqrt{-2x^2+1}\,\left(F\left(x,\sqrt{2}\right)-E\left(x,\sqrt{2}\right)\right)}{2\sqrt{-2x^4+3x^2-1}}\right)}{\sqrt{-x^2+1}\,\sqrt{2x^2-1}}$	122

```
[In] int((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(EllipticF(x,2^(1/2))+EllipticE(x,2^(1/2)))*(-2*x^2+1)^(1/2)/(2*x^2-1)^
(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{-2}xE(\arcsin\left(\frac{1}{x}\right)\mid\frac{1}{2}) + \sqrt{2\,x^2-1}\sqrt{-x^2+1}}{2\,x}$$

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/2*(sqrt(-2)*x*elliptic_e(arcsin(1/x), 1/2) + sqrt(2*x^2 - 1)*sqrt(-x^2 +
1))/x
```

Sympy [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{2x^2-1}}\,dx$$

```
[In] integrate((-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{2\,x^2-1}}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/sqrt(2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{\sqrt{-1+2x^2}}\,dx = \int \frac{\sqrt{1-x^2}}{\sqrt{2\,x^2-1}}\,dx$$

```
[In] int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2),x)
[Out] int((1 - x^2)^(1/2)/(2*x^2 - 1)^(1/2), x)
```

3.199 $\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 423

$$
\begin{aligned}
\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx &= -\frac{8(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} \\
&+ \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\
&- \frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2} + \frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\
&+ \frac{8\sqrt{c}(bc-2ad)\left(6b^2c^2-11abcd+11a^2d^2\right)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&- \frac{\sqrt{c}(3bc-7ad)\left(8b^2c^2-11abcd+15a^2d^2\right)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

```
[Out] -8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^3
/(d*x^2+c)^(1/2)+8/105*(-2*a*d+b*c)*(11*a^2*d^2-11*a*b*c*d+6*b^2*c^2)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/105*(-7*a*d+3*b*c)*(15*a^2*d^2-11*a*b*c*d+8
*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*
(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-6/35*b*(-2*a*d+b*c)*x*(b*x^2+a
)^(3/2)*(d*x^2+c)^(1/2)/d^2+1/7*b*x*(b*x^2+a)^(5/2)*(d*x^2+c)^(1/2)/d+1/105
*b*(71*a^2*d^2-71*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {427, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c}\sqrt{a+bx^2}(3bc-7ad)\left(15a^2d^2-11abcd+8b^2c^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{8\sqrt{c}\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}\left(71a^2d^2-71abcd+24b^2c^2\right)}{105d^3}$$
$$-\frac{8x\sqrt{a+bx^2}(bc-2ad)\left(11a^2d^2-11abcd+6b^2c^2\right)}{105d^3\sqrt{c+dx^2}}$$
$$-\frac{6bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-2ad)}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}$$

```
[In] Int[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-8*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*x*Sqrt[a + b*x^2])/
(105*d^3*Sqrt[c + d*x^2]) + (b*(24*b^2*c^2 - 71*a*b*c*d + 71*a^2*d^2)*x*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])/(105*d^3) - (6*b*(b*c - 2*a*d)*x*(a + b*x^2)^
(3/2)*Sqrt[c + d*x^2])/(35*d^2) + (b*x*(a + b*x^2)^(5/2)*Sqrt[c + d*x^2])/(
7*d) + (8*Sqrt[c]*(b*c - 2*a*d)*(6*b^2*c^2 - 11*a*b*c*d + 11*a^2*d^2)*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^
(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(3*
b*c - 7*a*d)*(8*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(105*d^(7/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}+\frac{\int \frac{(a+bx^2)^{3/2}(-a(bc-7ad)-6b(bc-2ad)x^2)}{\sqrt{c+dx^2}}\,dx}{7d}\\
&= -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d}\\
&\quad+\frac{\int \frac{\sqrt{a+bx^2}\,(a(6b^2c^2-17abcd+35a^2d^2)+b(24b^2c^2-71abcd+71a^2d^2)x^2)}{\sqrt{c+dx^2}}\,dx}{35d^2}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\
&\quad -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\
&\quad +\frac{\int\frac{-a(3bc-7ad)(8b^2c^2-11abcd+15a^2d^2)-8b(bc-2ad)(6b^2c^2-11abcd+11a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{105d^3} \\
&= \frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\
&\quad -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\
&\quad -\frac{(8b(bc-2ad)(6b^2c^2-11abcd+11a^2d^2))\int\frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{105d^3} \\
&\quad -\frac{(a(3bc-7ad)(8b^2c^2-11abcd+15a^2d^2))\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{105d^3} \\
&= -\frac{8(bc-2ad)(6b^2c^2-11abcd+11a^2d^2)\,x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} \\
&\quad +\frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\
&\quad -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\
&\quad -\frac{\sqrt{c}(3bc-7ad)(8b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad +\frac{(8c(bc-2ad)(6b^2c^2-11abcd+11a^2d^2))\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{105d^3} \\
&= -\frac{8(bc-2ad)(6b^2c^2-11abcd+11a^2d^2)\,x\sqrt{a+bx^2}}{105d^3\sqrt{c+dx^2}} \\
&\quad +\frac{b(24b^2c^2-71abcd+71a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{105d^3} \\
&\quad -\frac{6b(bc-2ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{35d^2}+\frac{bx(a+bx^2)^{5/2}\sqrt{c+dx^2}}{7d} \\
&\quad +\frac{8\sqrt{c}(bc-2ad)(6b^2c^2-11abcd+11a^2d^2)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad -\frac{\sqrt{c}(3bc-7ad)(8b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{105d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)\,(c+dx^2)\,(122a^2d^2+abd(-89c+66dx^2)+3b^2(8c^2-6cdx^2+5d^2x^4)) \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^2)^(7/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(122*a^2*d^2 + a*b*d*(-89*c + 66*d
*x^2) + 3*b^2*(8*c^2 - 6*c*d*x^2 + 5*d^2*x^4)) - (8*I)*b*c*(-6*b^3*c^3 + 23
*a*b^2*c^2*d - 33*a^2*b*c*d^2 + 22*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(48*b^4*c^4 - 2
08*a*b^3*c^3*d + 353*a^2*b^2*c^2*d^2 - 298*a^3*b*c*d^3 + 105*a^4*d^4)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)])/(105*Sqrt[b/a]*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 7.29 (sec) , antiderivative size = 626, normalized size of antiderivative = 1.48

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b^3x^5\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{7d}+\frac{\left(4a\,b^3-\frac{b^3(6ad+6bc)}{7d}\right)x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5bd}+\frac{\left(6a^2b^2-\frac{5a\,b^3c}{7d}-\frac{\left(4a\,b^3-\frac{b^3(6ad+6\cdots}{7d}\right)\cdots}{5bd\cdots}\cdots\right)\cdots}{\cdots}\cdots\right)}{\cdots}$
risch	$\frac{bx\left(15b^2d^2x^4+66x^2ab\,d^2-18x^2b^2cd+122a^2d^2-89abcd+24b^2c^2\right)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{105d^3}+\frac{\left(\frac{105a^4d^3\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad\cdots}{c\cdots}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\cdots\right)}{\cdots}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(15\sqrt{-\frac{b}{a}}\,b^4d^4x^9+81\sqrt{-\frac{b}{a}}\,a\,b^3d^4x^7-3\sqrt{-\frac{b}{a}}\,b^4c\,d^3x^7+188\sqrt{-\frac{b}{a}}\,a^2b^2d^4x^5-26\sqrt{-\frac{b}{a}}\,a\,b^3c\,d^3x^5+6\sqrt{-\frac{b}{a}}\,b^4c^2d^2x^5\cdots\right)}{\cdots}$

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/7*b^3/d*x^5*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/5*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*c))/b
/d*x^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(6*a^2*b^2-5/7*a*b^3*c/d-1/5
*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*x*(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)+(a^4-1/3*(6*a^2*b^2-5/7*a*b^3*c/d-1/5*(4*a*b^3-1/7*b^3
/d*(6*a*d+6*b*c))/b/d*(4*a*d+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2
)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(4*a^3*b-3/5*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*
c))/b/d*a*c-1/3*(6*a^2*b^2-5/7*a*b^3*c/d-1/5*(4*a*b^3-1/7*b^3/d*(6*a*d+6*b*
c))/b/d*(4*a*d+4*b*c))/b/d*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*
(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b
)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \frac{8\,(6\,b^4c^5 - 23\,ab^3c^4d + 33\,a^2b^2c^3d^2 - 22\,a^3bc^2d^3)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (4}{}$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*b^4*c^5 - 23*a*b^3*c^4*d + 33*a^2*b^2*c^3*d^2 - 22*a^3*b*c^2*d^
3)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (48
*b^4*c^5 - 184*a*b^3*c^4*d + 122*a^3*b*c*d^4 - 105*a^4*d^5 + 24*(11*a^2*b^2
 + a*b^3)*c^3*d^2 - (176*a^3*b + 89*a^2*b^2)*c^2*d^3)*sqrt(b*d)*x*sqrt(-c/d
)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (15*b^4*c*d^4*x^6 - 48*b^4*
c^4*d + 184*a*b^3*c^3*d^2 - 264*a^2*b^2*c^2*d^3 + 176*a^3*b*c*d^4 - 6*(3*b^
4*c^2*d^3 - 11*a*b^3*c*d^4)*x^4 + (24*b^4*c^3*d^2 - 89*a*b^3*c^2*d^3 + 122*
a^2*b^2*c*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{7}{2}}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(7/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(7/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(7/2)/(c + d*x^2)^(1/2), x)
```

3.200 $\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 344

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx &= \frac{(8b^2c^2-23abcd+23a^2d^2)\,x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} \\ &- \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} \\ &- \frac{\sqrt{c}(8b^2c^2-23abcd+23a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{c}(4b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] 1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/d^2/(d*x^2+c)^(1/2
)-1/15*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^
(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1
/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
+1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+
1/5*b*x*(b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/d-4/15*b*(-2*a*d+b*c)*x*(b*x^2+a)^(
1/2)*(d*x^2+c)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {427, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}(15a^2d^2-11abcd+4b^2c^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$-\frac{\sqrt{c}\sqrt{a+bx^2}(23a^2d^2-23abcd+8b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{x\sqrt{a+bx^2}(23a^2d^2-23abcd+8b^2c^2)}{15d^2\sqrt{c+dx^2}}$$
$$-\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{15d^2}+\frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d}$$

```
[In] Int[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] ((8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*x*Sqrt[a + b*x^2])/(15*d^2*Sqrt[c +
d*x^2]) - (4*b*(b*c - 2*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(15*d^2) +
(b*x*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(5*d) - (Sqrt[c]*(8*b^2*c^2 - 23*a*
b*c*d + 23*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2]) + (Sqrt[c]*(4*b^2*c^2 - 11*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(5/2)*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\int \frac{\sqrt{a+bx^2}\left(-a(bc-5ad)-4b(bc-2ad)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5d} \\
&= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} \\
&\quad + \frac{\int \frac{a(4b^2c^2-11abcd+15a^2d^2)+b(8b^2c^2-23abcd+23a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d^2} \\
&= -\frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} \\
&\quad + \frac{(a(4b^2c^2-11abcd+15a^2d^2))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d^2} \\
&\quad + \frac{(b(8b^2c^2-23abcd+23a^2d^2))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d^2}
\end{aligned}
$$

$$= \frac{(8b^2c^2 - 23abcd + 23a^2d^2)\, x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} + \frac{\sqrt{c}(4b^2c^2 - 11abcd + 15a^2d^2)\sqrt{a+bx^2}\, F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(c(8b^2c^2 - 23abcd + 23a^2d^2))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15d^2}$$

$$= \frac{(8b^2c^2 - 23abcd + 23a^2d^2)\, x\sqrt{a+bx^2}}{15d^2\sqrt{c+dx^2}} - \frac{4b(bc-2ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15d^2} + \frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5d} - \frac{\sqrt{c}(8b^2c^2 - 23abcd + 23a^2d^2)\sqrt{a+bx^2}\, E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(4b^2c^2 - 11abcd + 15a^2d^2)\sqrt{a+bx^2}\, F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.69 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4bc+11ad+3bdx^2) - ibc(8b^2c^2 - 23abcd + 23a^2d^2)\sqrt{1+}}{}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(-4*b*c + 11*a*d + 3*b*d*x^2) - I*
b*c*(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x
^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(-8*b^3*c^3 + 27*
a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*
x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*Sqrt[b/a]*d^3*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 6.43 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{b^2x^3\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{5d}+\frac{\left(3a\,b^2-\frac{b^2(4ad+4bc)}{5d}\right)x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3bd}+\frac{\left(a^3-\frac{\left(3a\,b^2-\frac{b^2(4ad+4bc)}{5d}\right)ac}{3bd}\right)\sqrt{}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd}}\right.}{}$
risch	$\frac{bx(3bd\,x^2+11ad-4bc)\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{15d^2}+\frac{\left(\frac{15a^3d^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{4a\,b^2c^2\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2}}\right.}{}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(3\sqrt{-\frac{b}{a}}\,b^3d^3x^7+14\sqrt{-\frac{b}{a}}\,a\,b^2d^3x^5-\sqrt{-\frac{b}{a}}\,b^3c\,d^2x^5+11\sqrt{-\frac{b}{a}}\,a^2b\,d^3x^3+10\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^2x^3-4\sqrt{-\frac{b}{a}}\,b^3c^2d\,x^3+15\right.}{}$

```
[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/5*b^2/d*x^3*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+1/3*(3*a*b^2-1/5*b^2/d*(4*a*d+4*b*c))/b
/d*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^3-1/3*(3*a*b^2-1/5*b^2/d*(4*a*d
+4*b*c))/b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/
2))-(3*a^2*b-3/5*a*b^2*c/d-1/3*(3*a*b^2-1/5*b^2/d*(4*a*d+4*b*c))/b/d*(2*a*d
+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^
2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-
EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^3c^4-23\,ab^2c^3d+23\,a^2bc^2d^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^3c^4-23\,ab^2c^3d-11\,a^2bcd^3+15}{}$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((8*b^3*c^4 - 23*a*b^2*c^3*d + 23*a^2*b*c^2*d^2)*sqrt(b*d)*x*sqrt(-c/
d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^3*c^4 - 23*a*b^2*c^3*
```

```
d - 11*a^2*b*c*d^3 + 15*a^3*d^4 + (23*a^2*b + 4*a*b^2)*c^2*d^2)*sqrt(b*d)*x
*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^3*c*d^3*x^4
+ 8*b^3*c^3*d - 23*a*b^2*c^2*d^2 + 23*a^2*b*c*d^3 - (4*b^3*c^2*d^2 - 11*a*b
^2*c*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c*d^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{5}{2}}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
[Out] Integral((a + b*x**2)**(5/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{5}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^(1/2), x)
```

3.201 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 260

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}}+\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{2\sqrt{c}(bc-2ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}(bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -2/3*(-2*a*d+b*c)*x*(b*x^2+a)^(1/2)/d/(d*x^2+c)^(1/2)+2/3*(-2*a*d+b*c)*(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d
*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)+1/3*b*x*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {427, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{2\sqrt{c}\sqrt{a+bx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{2x\sqrt{a+bx^2}(bc-2ad)}{3d\sqrt{c+dx^2}}$$

```
[In] Int[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (-2*(b*c - 2*a*d)*x*Sqrt[a + b*x^2])/(3*d*Sqrt[c + d*x^2]) + (b*x*Sqrt[a +
b*x^2]*Sqrt[c + d*x^2])/(3*d) + (2*Sqrt[c]*(b*c - 2*a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(3/2)*Sqrt[(c*(
a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*(b*c - 3*a*d)*Sqrt
[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(
3/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} + \frac{\int \frac{-a(bc-3ad)-2b(bc-2ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} \\
&= \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} - \frac{(a(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} - \frac{(2b(bc-2ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d} \\
&= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} \\
&\quad - \frac{\sqrt{c}(bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(2c(bc-2ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3d} \\
&= -\frac{2(bc-2ad)x\sqrt{a+bx^2}}{3d\sqrt{c+dx^2}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d} \\
&\quad + \frac{2\sqrt{c}(bc-2ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{\sqrt{c}(bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)-2ibc(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{a}{b}\right.}{3\sqrt{\frac{b}{a}}d^2\sqrt{a+bx}}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) - (2*I)*b*c*(-(b*c) + 2*a*d)*Sqrt[
1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] - I*(2*b^2*c^2 - 5*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*d^
2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.19

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{bx\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3d}+\frac{\left(a^2-\frac{bac}{3d}\right)\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\left(2ab-\frac{b(2ad+2bc)}{3d}\right)c\sqrt{1+\frac{b\,x^2}{a}}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$
risch	$\frac{bx\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{3d}+\frac{\left(\frac{3a^2d\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{abc\sqrt{1+\frac{b\,x^2}{a}}\sqrt{1+\frac{d\,x^2}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(4abd-2}{}\right.}{3d\sqrt{b\,x^2+a}\,\sqrt{d}}$
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{-\frac{b}{a}}\,ab\,d^2x^3+\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+3\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2d^2-5\sqrt{\frac{b\,x^2+a}{a}}\sqrt{\frac{d\,x^2+c}{c}}\right.}{}$

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3*b/d*x*(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(a^2-1/3*b/d*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^
(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b
/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(2*a*b-1/3*b/d*(2*a*d+2*b*c))*c/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)
^(1/2),(-1+(a*d+b*c)/c/b)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{2\left(b^2c^3-2\,abc^2d\right)\sqrt{bd}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \left(2\,b^2c^3-4\,abc^2d+abcd^2-3\,a^2d^3\right.}{3\,bcd}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*(b^2*c^3 - 2*a*b*c^2*d)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 - 4*a*b*c^2*d + a*b*c*d^2 - 3*a^2*d^3)*
sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b^2*c
*d^2*x^2 - 2*b^2*c^2*d + 4*a*b*c*d^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(b*c
*d^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^(1/2), x)
```

3.202 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*
x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\left(bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\,\left(\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}}-\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad}{c}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}+cb\,x^{2}+ac}\,d}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-\sqrt{bx^2+a}\sqrt{dx^2+c}bcd-(bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{bcd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt(
-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.203 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {429}

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}$	122

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)}{bc}$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.204 $\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 273

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -d*x*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(d*x^2+c)^(1/2)+(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))
^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*
d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2
+a)^(1/2)/a/(-a*d+b*c)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+b*x*
(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {425, 21, 433, 429, 506, 422}

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{c+dx^2}}{a\sqrt{a+bx^2}(bc-ad)} - \frac{dx\sqrt{a+bx^2}}{a\sqrt{c+dx^2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(a*(b*c - a*d)*Sqrt[c + d*x^2])) + (b*x*Sqrt[c + d*
x^2])/(a*(b*c - a*d)*Sqrt[a + b*x^2]) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a
 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*(b*c
- a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
```

```
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}}-\frac{\int \frac{ad+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx}{a(bc-ad)}\\
&= \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}}-\frac{d\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad}-\frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{a(bc-ad)}\\
&= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}}+\frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}}\\
&\quad -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{a(bc-ad)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{dx\sqrt{a+bx^2}}{a(bc-ad)\sqrt{c+dx^2}} + \frac{bx\sqrt{c+dx^2}}{a(bc-ad)\sqrt{a+bx^2}} \\
&\quad + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.41

$$
\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{-bx(c+dx^2) + \frac{ad\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}}}{a(-bc+ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}
$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (-(b*x*(c + d*x^2)) + (a*d*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(d/c)]*x], (b*c)/(a*d)])/Sqrt[-(d/c)])/(a*(-(b*c) + a*d)*Sqr
t[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.39 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bd\,x^3+a\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\right.\right.}{a\sqrt{-\frac{b}{a}}\,(ad-bc)(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{\left(bd\,x^2+bc\right)x}{a(ad-bc)\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(\frac{1}{a}+\frac{bc}{a(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+}}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*b*d*x^3+a*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c+((b*x^2+a)/a)^(1/2)*((d*x^2+c)
/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c-(-b/a)^(1/2)*b*c*x)
*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/a/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+
b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}ab^2cx - (b^3cx^2+ab^2c)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) + \cdots}{a^3b^2c^2 - a^4bcd + (a^2b^3c^2 \cdots}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*b^2*c*x - (b^3*c*x^2 + a*b^2*c)*sqrt(a*c
)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a*b^2*c + a^3*d
 + (b^3*c + a^2*b*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/
a)), a*d/(b*c)))/(a^3*b^2*c^2 - a^4*b*c*d + (a^2*b^3*c^2 - a^3*b^2*c*d)*x^2
)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)), x)
```

3.205 $\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 255

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2\sqrt{b}(bc-2ad)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}(bc-3ad)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -1/3*(-3*a*d+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/
2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(
1/2)/a^2/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b
*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(3/2)+2/3*(-2*a*d+b*c)*(1/(1+b*x^
2/a))^(1/2)*(1+b*x^2/a)^(1/2)*EllipticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2)
,(1-a*d/b/c)^(1/2))*b^(1/2)*(d*x^2+c)^(1/2)/a^(3/2)/(-a*d+b*c)^2/(b*x^2+a)^
(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {425, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{2\sqrt{b}\sqrt{c+dx^2}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}\sqrt{a+bx^2}(bc-ad)^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{c+dx^2}}{3a\,(a+bx^2)^{3/2}\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)) + (2*Sqrt[b]*(b*c
 - 2*a*d)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]], 1 - (a*d)/
(b*c)])/(3*a^(3/2)*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*(b*c - a*d)^2*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-2bc+3ad-bdx^2}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)} \\
&= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} - \frac{(d(bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)^2} + \frac{(2b(bc-2ad))\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}}\,dx}{3a(bc-ad)^2} \\
&= \frac{bx\sqrt{c+dx^2}}{3a(bc-ad)\left(a+bx^2\right)^{3/2}} + \frac{2\sqrt{b}(bc-2ad)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3a^{3/2}(bc-ad)^2\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}(bc-3ad)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}x(c+dx^2)\left(-5a^2d+2b^2cx^2+ab(3c-4dx^2)\right)-2ibc(-bc+2ad)\left(a+bx^2\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(-5*a^2*d + 2*b^2*c*x^2 + a*b*(3*c - 4*d*x^2)) -
 (2*I)*b*c*(-(b*c) + 2*a*d)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2
)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*(2*b^2*c^2 - 5*a*b*
c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*a^2*Sqrt[b/a]*(b*c - a*d)^2*(a
 + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{x\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}{3ba(ad-bc)\left(x^2+\frac{a}{b}\right)^2}-\frac{2\left(bd\,x^2+bc\right)x(2ad-bc)}{3a^2(ad-bc)^2\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2+bc\right)}}+\frac{\left(-\frac{d}{3(ad-bc)a}+\frac{\frac{4ad}{3}-\frac{2bc}{3}}{a^2(ad-bc)}+\frac{2bc(2ad-bc)}{3a^2(ad-bc)^2}\right)\sqrt{1+\frac{b\,x^2}{a}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x}}$
default	$\frac{-4\sqrt{-\frac{b}{a}}\,a\,b^2d^2x^5+2\sqrt{-\frac{b}{a}}\,b^3cd\,x^5+3\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2b\,d^2x^2-5\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a\,b^2cd\,x}{}$

[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/3/b/a/(a*d-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-2/3*(b*d*x^2+b*c)/a^2/(a*d-b*c)^2*x*(2*a*d-b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(-1/3*d/(a*d-b*c)/a+2/3*(2*a*d-b*c)/(a*d-b*c)/a^2+2/3*b*c/a^2/(a*d-b*c)^2*(2*a*d-b*c))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-2/3*b*(2*a*d-b*c)/(a*d-b*c)^2/a^2*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)^{5/2}\sqrt{c+dx^2}}\,dx =$$
$$-\frac{2\left(a^2b^3c^2-2\,a^3b^2cd+(b^5c^2-2\,ab^4cd)x^4+2\,(ab^4c^2-2\,a^2b^3cd)x^2\right)\sqrt{ac}\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc}\right)-(2}{}$$

[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] -1/3*(2*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + (b^5*c^2 - 2*a*b^4*c*d)*x^4 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*a^2*b^3*c^2 - 3*a^5*d^2 + (2*b^5*c^2 - 3*a^3*b^2*d^2 + (a^2*b^3 - 4*a*b^4)*c*d)*x^4 + (a^4*b - 4*a^3*b^2)*c*d + 2*(2*a*b^4*c^2 - 3*a^4*b*d^2 + (a^3*b^2 - 4*a^2*b^3)*c*d)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*(a*b^4*c^2 - 2*a^2*b^3*c*d)*x^3 + (3*a^2*b^3*c^2 - 5*a^3*b^2*c*d)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^3*c^3 - 2*a^6*b^2*c^2*d + a^7*b*c*d^2 + (a^3*b^5*c^3 - 2*a^4*b^4*c^2*d +

```
a^5*b^3*c*d^2)*x^4 + 2*(a^4*b^4*c^3 - 2*a^5*b^3*c^2*d + a^6*b^2*c*d^2)*x^2
)
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{2}}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**(5/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx=\int \frac{1}{{\left(b\,x^2+a\right)}^{5/2}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(1/2)), x)
```

3.206 $\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 334

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)(a+bx^2)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2(a+bx^2)^{3/2}} + \frac{\sqrt{b}(8b^2c^2-23abcd+23a^2d^2)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}(bc-ad)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}(4b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -1/15*(15*a^2*d^2-11*a*b*c*d+4*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(
1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/
2)*d^(1/2)*(b*x^2+a)^(1/2)/a^3/(-a*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
/(d*x^2+c)^(1/2)+1/5*b*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(b*x^2+a)^(5/2)+4/15*
b*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)/a^2/(-a*d+b*c)^2/(b*x^2+a)^(3/2)+1/15*(23*
a^2*d^2-23*a*b*c*d+8*b^2*c^2)*(1/(1+b*x^2/a))^(1/2)*(1+b*x^2/a)^(1/2)*Ellip
ticE(x*b^(1/2)/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))*b^(1/2)*(d*x^2+
c)^(1/2)/a^(5/2)/(-a*d+b*c)^3/(b*x^2+a)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/
2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {425, 541, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{4bx\sqrt{c+dx^2}(bc-2ad)}{15a^2(a+bx^2)^{3/2}(bc-ad)^2} + \frac{\sqrt{b}\sqrt{c+dx^2}(23a^2d^2-23abcd+8b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}\sqrt{a+bx^2}(bc-ad)^3\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(15a^2d^2-11abcd+4b^2c^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^3\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{c+dx^2}}{5a(a+bx^2)^{5/2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(5*a*(b*c - a*d)*(a + b*x^2)^(5/2)) + (4*b*(b*c - 2*a
*d)*x*Sqrt[c + d*x^2])/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(3/2)) + (Sqrt[b]*
(8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqr
t[b]*x)/Sqrt[a]], 1 - (a*d)/(b*c)])/(15*a^(5/2)*(b*c - a*d)^3*Sqrt[a + b*x^
2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]) - (Sqrt[c]*Sqrt[d]*(4*b^2*c^2 - 1
1*a*b*c*d + 15*a^2*d^2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(15*a^3*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} - \frac{\int \frac{-4bc+5ad-3bdx^2}{\left(a+bx^2\right)^{5/2}\sqrt{c+dx^2}}\,dx}{5a(bc-ad)} \\
&= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\int \frac{8b^2c^2-19abcd+15a^2d^2+4bd(bc-2ad)x^2}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)^2} \\
&= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} \\
&\quad - \frac{\left(d\left(4b^2c^2-11abcd+15a^2d^2\right)\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15a^2(bc-ad)^3} \\
&\quad + \frac{\left(b\left(8b^2c^2-23abcd+23a^2d^2\right)\right)\int \frac{\sqrt{c+dx^2}}{\left(a+bx^2\right)^{3/2}}\,dx}{15a^2(bc-ad)^3}
\end{aligned}$$

$$= \frac{bx\sqrt{c+dx^2}}{5a(bc-ad)\left(a+bx^2\right)^{5/2}} + \frac{4b(bc-2ad)x\sqrt{c+dx^2}}{15a^2(bc-ad)^2\left(a+bx^2\right)^{3/2}} + \frac{\sqrt{b}(8b^2c^2-23abcd+23a^2d^2)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15a^{5/2}(bc-ad)^3\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{d}(4b^2c^2-11abcd+15a^2d^2)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^3(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.28 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a+bx^2\right)^{7/2}\sqrt{c+dx^2}}\,dx = \frac{b\sqrt{\frac{b}{a}}x(c+dx^2)\left(3a^2(bc-ad)^2+4a(bc-2ad)(bc-ad)\left(a+bx^2\right)+(8b^2c^2-2\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)^(7/2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (b*Sqrt[b/a]*x*(c + d*x^2)*(3*a^2*(b*c - a*d)^2 + 4*a*(b*c - 2*a*d)*(b*c -
a*d)*(a + b*x^2) + (8*b^2*c^2 - 23*a*b*c*d + 23*a^2*d^2)*(a + b*x^2)^2) + I
*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*(b*c*(8*b^2*c^2 - 23
*a*b*c*d + 23*a^2*d^2)*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-8
*b^3*c^3 + 27*a*b^2*c^2*d - 34*a^2*b*c*d^2 + 15*a^3*d^3)*EllipticF[I*ArcSin
h[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*a^3*Sqrt[b/a]*(b*c - a*d)^3*(a + b*x^2)^
(5/2)*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 6.83 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(-\frac{x\sqrt{bdx^4+adx^2+cbx^2+ac}}{5b^2a(ad-bc)\left(x^2+\frac{a}{b}\right)^3}-\frac{4(2ad-bc)x\sqrt{bdx^4+adx^2+cbx^2+ac}}{15b(ad-bc)^2a^2\left(x^2+\frac{a}{b}\right)^2}-\frac{\left(bdx^2+bc\right)x\left(23a^2d^2-23abcd+8b^2c^2\right)}{15a^3(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bdx^2+bc\right)}}+\left(-\frac{4d(2ad}{15a^2(ad}\right.\right.}{}$
default	Expression too large to display

```
[In] int(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(-1/5/b^2/a/(a*
d-b*c)*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^3-4/15*(2*a*d-b*c)/b
```

```
/(a*d-b*c)^2/a^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2-1/15*(b*
d*x^2+b*c)/a^3/(a*d-b*c)^3*x*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/((x^2+a/b)*(
b*d*x^2+b*c))^(1/2)+(-4/15*d*(2*a*d-b*c)/a^2/(a*d-b*c)^2+1/15/(a*d-b*c)^2*(
23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/a^3+1/15*b*c/a^3/(a*d-b*c)^3*(23*a^2*d^2-2
3*a*b*c*d+8*b^2*c^2))/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d
*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)
^(1/2))-1/15*b*(23*a^2*d^2-23*a*b*c*d+8*b^2*c^2)/(a*d-b*c)^3/a^3*c/(-b/a)^(
1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2
)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1
/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(316) = 632$.

Time = 0.11 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.53

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(8\,a^3b^4c^3 - 23\,a^4b^3c^2d + 23\,a^5b^2cd^2 + (8\,b^7c^3 - 23\,ab^6c^2d + 23\,a^2b^5cd^2)x^6 + 3\,(8\,ab^6c^3 - 23\,a^2b^5c^2d + 23\,a^3}{}$$

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((8*a^3*b^4*c^3 - 23*a^4*b^3*c^2*d + 23*a^5*b^2*c*d^2 + (8*b^7*c^3 -
23*a*b^6*c^2*d + 23*a^2*b^5*c*d^2)*x^6 + 3*(8*a*b^6*c^3 - 23*a^2*b^5*c^2*d
+ 23*a^3*b^4*c*d^2)*x^4 + 3*(8*a^2*b^5*c^3 - 23*a^3*b^4*c^2*d + 23*a^4*b^3*
c*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)
) - (8*a^3*b^4*c^3 + 15*a^7*d^3 + (8*b^7*c^3 + 15*a^4*b^3*d^3 + (4*a^2*b^5
- 23*a*b^6)*c^2*d - (11*a^3*b^4 - 23*a^2*b^5)*c*d^2)*x^6 + 3*(8*a*b^6*c^3 +
 15*a^5*b^2*d^3 + (4*a^3*b^4 - 23*a^2*b^5)*c^2*d - (11*a^4*b^3 - 23*a^3*b^4
)*c*d^2)*x^4 + (4*a^5*b^2 - 23*a^4*b^3)*c^2*d - (11*a^6*b - 23*a^5*b^2)*c*d
^2 + 3*(8*a^2*b^5*c^3 + 15*a^6*b*d^3 + (4*a^4*b^3 - 23*a^3*b^4)*c^2*d - (11
*a^5*b^2 - 23*a^4*b^3)*c*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x
*sqrt(-b/a)), a*d/(b*c)) - ((8*a*b^6*c^3 - 23*a^2*b^5*c^2*d + 23*a^3*b^4*c*
d^2)*x^5 + 2*(10*a^2*b^5*c^3 - 29*a^3*b^4*c^2*d + 27*a^4*b^3*c*d^2)*x^3 + (
15*a^3*b^4*c^3 - 41*a^4*b^3*c^2*d + 34*a^5*b^2*c*d^2)*x)*sqrt(b*x^2 + a)*sq
rt(d*x^2 + c))/(a^7*b^4*c^4 - 3*a^8*b^3*c^3*d + 3*a^9*b^2*c^2*d^2 - a^10*b*
c*d^3 + (a^4*b^7*c^4 - 3*a^5*b^6*c^3*d + 3*a^6*b^5*c^2*d^2 - a^7*b^4*c*d^3)
*x^6 + 3*(a^5*b^6*c^4 - 3*a^6*b^5*c^3*d + 3*a^7*b^4*c^2*d^2 - a^8*b^3*c*d^3
)*x^4 + 3*(a^6*b^5*c^4 - 3*a^7*b^4*c^3*d + 3*a^8*b^3*c^2*d^2 - a^9*b^2*c*d^
3)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{7}{2}}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(7/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**(7/2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{2}}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/2)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{7/2}\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(7/2)*(c + d*x^2)^(1/2)), x)
```

3.207 $\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 445

$$
\begin{aligned}
\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx &= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} \\
&- \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\
&+ \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\
&- \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&+ \frac{b\sqrt{c}(24b^2c^2-61abcd+45a^2d^2)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(5/2)/c/d/(d*x^2+c)^(1/2)+1/15*(-15*a^3*d^3+103*a^2
*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*x*(b*x^2+a)^(1/2)/c/d^3/(d*x^2+c)^(1/2
)-1/15*(-15*a^3*d^3+103*a^2*b*c*d^2-128*a*b^2*c^2*d+48*b^3*c^3)*(1/(1+d*x^2
/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),
(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(7/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c)
)^(1/2)/(d*x^2+c)^(1/2)+1/15*b*(45*a^2*d^2-61*a*b*c*d+24*b^2*c^2)*(1/(1+d*x
^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2
),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(7/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)/(d*x^2+c)^(1/2)+1/5*b*(-5*a*d+6*b*c)*x*(b*x^2+a)^(3/2)*(d*x^2+c)^
(1/2)/c/d^2-1/15*b*(15*a^2*d^2-43*a*b*c*d+24*b^2*c^2)*x*(b*x^2+a)^(1/2)*(d*
x^2+c)^(1/2)/c/d^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {424, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{c}\sqrt{a+bx^2}(45a^2d^2-61abcd+24b^2c^2)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$-\frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(15a^2d^2-43abcd+24b^2c^2)}{15cd^3}$$
$$-\frac{\sqrt{a+bx^2}(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{x\sqrt{a+bx^2}(-15a^3d^3+103a^2bcd^2-128ab^2c^2d+48b^3c^3)}{15cd^3\sqrt{c+dx^2}}$$
$$+\frac{bx(a+bx^2)^{3/2}\sqrt{c+dx^2}(6bc-5ad)}{5cd^2}-\frac{x(a+bx^2)^{5/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[In] Int[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2 - 15*a^3*d^3)*x*Sqrt[a + b
*x^2])/(15*c*d^3*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(5/2))/(c*d*
Sqrt[c + d*x^2]) - (b*(24*b^2*c^2 - 43*a*b*c*d + 15*a^2*d^2)*x*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2])/(15*c*d^3) + (b*(6*b*c - 5*a*d)*x*(a + b*x^2)^(3/2)*Sq
rt[c + d*x^2])/(5*c*d^2) - ((48*b^3*c^3 - 128*a*b^2*c^2*d + 103*a^2*b*c*d^2
 - 15*a^3*d^3)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(15*Sqrt[c]*d^(7/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt
[c + d*x^2]) + (b*Sqrt[c]*(24*b^2*c^2 - 61*a*b*c*d + 45*a^2*d^2)*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(15*d^(7/2)*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

```
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}}+\frac{\int \frac{(a+bx^2)^{3/2}(abc+b(6bc-5ad)x^2)}{\sqrt{c+dx^2}}\,dx}{cd}\\ &= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}}+\frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2}\\ &\quad+\frac{\int \frac{\sqrt{a+bx^2}\left(-2abc(3bc-5ad)-b\left(24b^2c^2-43abcd+15a^2d^2\right)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5cd^2}\end{aligned}$$

$$
\begin{aligned}
&= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\
&\quad + \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\
&\quad + \frac{\int \frac{abc(24b^2c^2-61abcd+45a^2d^2)+b(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15cd^3} \\
&= -\frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\
&\quad + \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\
&\quad + \frac{(ab(24b^2c^2-61abcd+45a^2d^2))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15d^3} \\
&\quad + \frac{(b(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{15cd^3} \\
&= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} \\
&\quad - \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\
&\quad + \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\
&\quad + \frac{b\sqrt{c}(24b^2c^2-61abcd+45a^2d^2)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{15d^3} \\
&= \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\,x\sqrt{a+bx^2}}{15cd^3\sqrt{c+dx^2}} \\
&\quad - \frac{(bc-ad)x(a+bx^2)^{5/2}}{cd\sqrt{c+dx^2}} - \frac{b(24b^2c^2-43abcd+15a^2d^2)\,x\sqrt{a+bx^2}\sqrt{c+dx^2}}{15cd^3} \\
&\quad + \frac{b(6bc-5ad)x(a+bx^2)^{3/2}\sqrt{c+dx^2}}{5cd^2} \\
&\quad - \frac{(48b^3c^3-128ab^2c^2d+103a^2bcd^2-15a^3d^3)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15\sqrt{c}d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{b\sqrt{c}(24b^2c^2-61abcd+45a^2d^2)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15d^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\,(-45a^2bcd^2+15a^3d^3+ab^2cd(61c+16dx^2)-3b^3c(8c^2+2cdx^2-d^2x^4}{}$$

```
[In] Integrate[(a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-45*a^2*b*c*d^2 + 15*a^3*d^3 + a*b^2*c*d*(61*c
+ 16*d*x^2) - 3*b^3*c*(8*c^2 + 2*c*d*x^2 - d^2*x^4)) + I*b*c*(-48*b^3*c^3 +
 128*a*b^2*c^2*d - 103*a^2*b*c*d^2 + 15*a^3*d^3)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (4*I)*b*c*(1
2*b^3*c^3 - 38*a*b^2*c^2*d + 41*a^2*b*c*d^2 - 15*a^3*d^3)*Sqrt[1 + (b*x^2)/
a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*
Sqrt[b/a]*c*d^4*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 9.53 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.58

method	result
risch	$\frac{b^2x(3bdx^2+16ad-9bc)\sqrt{bx^2+a}\sqrt{dx^2+c}}{15d^3}+\frac{\left(-\frac{b^2(58a^2d^2-83abcd+33b^2c^2)c\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cbx^2+ac}\,d}\right.}{}$
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{(bdx^2+ad)(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3)x}{cd^4\sqrt{\left(x^2+\frac{c}{d}\right)(bdx^2+ad)}}+\frac{b^3x^3\sqrt{bdx^4+adx^2+cbx^2+ac}}{5d^2}+\frac{\left(\frac{b^3(4ad-bc)}{d^2}-\frac{b^3(4ad+4bc)}{5d^2}\right)x\sqrt{bdx^4}}{3bd}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(3\sqrt{-\frac{b}{a}}b^4cd^3x^7+19\sqrt{-\frac{b}{a}}ab^3cd^3x^5-6\sqrt{-\frac{b}{a}}b^4c^2d^2x^5+15\sqrt{-\frac{b}{a}}a^3bd^4x^3-29\sqrt{-\frac{b}{a}}a^2b^2cd^3x^3+55\sqrt{-\frac{b}{a}}ab^3c^2\right.}{}$

```
[In] int((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b^2*x*(3*b*d*x^2+16*a*d-9*b*c)*(b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/d^3+1/1
5/d^3*(-b^2*(58*a^2*d^2-83*a*b*c*d+33*b^2*c^2)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(
1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(
```

```
-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c
)/c/b)^(1/2)))+b*(60*a^3*d^3-106*a^2*b*c*d^2+69*a*b^2*c^2*d-15*b^3*c^3)/d/(
-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+(15*a^4*d^4-60*
a^3*b*c*d^3+90*a^2*b^2*c^2*d^2-60*a*b^3*c^3*d+15*b^4*c^4)/d*((b*d*x^2+a*d)/
c/(a*d-b*c)*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/c-1/c/(a*d-b*c)*a*d)/(-b/a
)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))+b/(a*d-b*c)/(-b/a)^
(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(
1/2),(-1+(a*d+b*c)/c/b)^(1/2)))))*((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/
2)/(d*x^2+c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((48\,b^3c^4d-128\,ab^2c^3d^2+103\,a^2bc^2d^3-15\,a^3cd^4)x^3+(48\,b^3c^5-128\,ab^2c^4d+103\,a^2bc^3d^2-15\,a^3c^2d^3)x)\sqrt{}}{}$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*(((48*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 103*a^2*b*c^2*d^3 - 15*a^3*c*d^
4)*x^3 + (48*b^3*c^5 - 128*a*b^2*c^4*d + 103*a^2*b*c^3*d^2 - 15*a^3*c^2*d^3
)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((4
8*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 45*a^3*d^5 + (103*a^2*b + 24*a*b^2)*c^2*d
^3 - (15*a^3 + 61*a^2*b)*c*d^4)*x^3 + (48*b^3*c^5 - 128*a*b^2*c^4*d + 45*a^
3*c*d^4 + (103*a^2*b + 24*a*b^2)*c^3*d^2 - (15*a^3 + 61*a^2*b)*c^2*d^3)*x)*
sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (3*b^3*c
*d^4*x^6 + 48*b^3*c^4*d - 128*a*b^2*c^3*d^2 + 103*a^2*b*c^2*d^3 - 15*a^3*c*
d^4 - 2*(3*b^3*c^2*d^3 - 8*a*b^2*c*d^4)*x^4 + (24*b^3*c^3*d^2 - 67*a*b^2*c^
2*d^3 + 58*a^2*b*c*d^4)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d^6*x^3 +
c^2*d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{7}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(7/2)/(d*x**2+c)**(3/2),x)
[Out] Integral((a + b*x**2)**(7/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{7}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(7/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(7/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{7/2}}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(7/2)/(c + d*x^2)^(3/2), x)
```

3.208 $\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 346

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = &-\frac{(8b^2c^2-13abcd+3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} \\
&-\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}}+\frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} \\
&+\frac{(8b^2c^2-13abcd+3a^2d^2)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&-\frac{2b\sqrt{c}(2bc-3ad)\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(3/2)/c/d/(d*x^2+c)^(1/2)-1/3*(3*a^2*d^2-13*a*b*c*d
+8*b^2*c^2)*x*(b*x^2+a)^(1/2)/c/d^2/(d*x^2+c)^(1/2)+1/3*(3*a^2*d^2-13*a*b*c
*d+8*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/d^(5/2)/c^(1/2)
/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2/3*b*(-3*a*d+2*b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(5/2)/(c*(b*x^2+a)/a/(d*
x^2+c))^(1/2)/(d*x^2+c)^(1/2)+1/3*b*(-3*a*d+4*b*c)*x*(b*x^2+a)^(1/2)*(d*x^2
+c)^(1/2)/c/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {424, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{a+bx^2}(3a^2d^2-13abcd+8b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{x\sqrt{a+bx^2}(3a^2d^2-13abcd+8b^2c^2)}{3cd^2\sqrt{c+dx^2}} - \frac{2b\sqrt{c}\sqrt{a+bx^2}(2bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3cd^2} - \frac{x(a+bx^2)^{3/2}\,(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[In] Int[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -1/3*((8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*x*Sqrt[a + b*x^2])/(c*d^2*Sqrt[c
 + d*x^2]) - ((b*c - a*d)*x*(a + b*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) + (b*(
4*b*c - 3*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*c*d^2) + ((8*b^2*c^2 -
 13*a*b*c*d + 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(3*Sqrt[c]*d^(5/2)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2]) - (2*b*Sqrt[c]*(2*b*c - 3*a*d)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*d^(5/2)*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}}+\frac{\int \frac{\sqrt{a+bx^2}(abc+b(4bc-3ad)x^2)}{\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}}+\frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2}\\
&\quad+\frac{\int \frac{-2abc(2bc-3ad)-b(8b^2c^2-13abcd+3a^2d^2)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3cd^2}\\
&= -\frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}}+\frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2}\\
&\quad-\frac{(2ab(2bc-3ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3d^2}\\
&\quad-\frac{(b(8b^2c^2-13abcd+3a^2d^2))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3cd^2}
\end{aligned}$$

$$= -\frac{(8b^2c^2 - 13abcd + 3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} - \frac{2b\sqrt{c}(2bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(8b^2c^2 - 13abcd + 3a^2d^2)\int\frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{(8b^2c^2 - 13abcd + 3a^2d^2)\,x\sqrt{a+bx^2}}{3cd^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{b(4bc-3ad)x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cd^2} + \frac{(8b^2c^2 - 13abcd + 3a^2d^2)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3\sqrt{c}d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{2b\sqrt{c}(2bc-3ad)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3d^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)\left(-6abcd+3a^2d^2+b^2c(4c+dx^2)\right)+ibc(8b^2c^2-13abcd+3a^2d^2)\sqrt{1-}}{}$$

```
[In] Integrate[(a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b/a]*d*x*(a + b*x^2)*(-6*a*b*c*d + 3*a^2*d^2 + b^2*c*(4*c + d*x^2)) +
 I*b*c*(8*b^2*c^2 - 13*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d
*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(8*b^2*c^2
- 17*a*b*c*d + 9*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*Sqrt[b/a]*c*d^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])
```

Maple [A] (verified)

Time = 9.22 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{(bdx^2+ad)(a^2d^2-2abcd+b^2c^2)x}{cd^3\sqrt{\left(x^2+\frac{c}{d}\right)(bdx^2+ad)}}+\frac{b^2x\sqrt{bdx^4+adx^2+cbx^2+ac}}{3d^2}+\frac{\left(\frac{b(3a^2d^2-3abcd+b^2c^2)}{d^3}+\frac{(a^2d^2-2abcd+b^2c^2)(ad-\cdots}{d^3c}\right.}{\sqrt{\cdots}}\right.}{}$
default	$\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}\left(\sqrt{-\frac{b}{a}}b^3cd^2x^5+3\sqrt{-\frac{b}{a}}a^2bd^3x^3-5\sqrt{-\frac{b}{a}}ab^2cd^2x^3+4\sqrt{-\frac{b}{a}}b^3c^2dx^3+9\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2\cdots\right.}{}$
risch	$\frac{b^2x\sqrt{bx^2+a}\sqrt{dx^2+c}}{3d^2}+\frac{\left(\frac{(3a^3d^3-9a^2bcd^2+9ab^2c^2d-3b^3c^3)\left(\frac{(bdx^2+ad)x}{c(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)(bdx^2+ad)}}+\frac{\left(\frac{1}{c}-\frac{ad}{c(ad-bc)}\right)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(x\sqrt{\cdots}\right.}{\sqrt{-\frac{b}{a}}\sqrt{bdx^4+adx^2+cbx^2\cdots}}\right.}{d}\right.}{}$

[In] int((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*((b*d*x^2+a*d)*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+1/3*b^2/d^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+(b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(a*d-b*c)/c-a/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c-1/3*b^2/d^2*a*c)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-(b^2/d^2*(3*a*d-b*c)-(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^2*b/c-1/3*b^2/d^2*(2*a*d+2*b*c))*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \frac{((8\,b^2c^3d-13\,abc^2d^2+3\,a^2cd^3)x^3+(8\,b^2c^4-13\,abc^3d+3\,a^2c^2d^2)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\cdots}{\cdots}$$

[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")

[Out] 1/3*(((8*b^2*c^3*d - 13*a*b*c^2*d^2 + 3*a^2*c*d^3)*x^3 + (8*b^2*c^4 - 13*a*b*c^3*d + 3*a^2*c^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/

```
d)/x), a*d/(b*c)) - ((8*b^2*c^3*d - 13*a*b*c^2*d^2 - 6*a^2*d^4 + (3*a^2 + 4
*a*b)*c*d^3)*x^3 + (8*b^2*c^4 - 13*a*b*c^3*d - 6*a^2*c*d^3 + (3*a^2 + 4*a*b
)*c^2*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) + (b^2*c*d^3*x^4 - 8*b^2*c^3*d + 13*a*b*c^2*d^2 - 3*a^2*c*d^3 - (4*b^2*
c^2*d^2 - 7*a*b*c*d^3)*x^2)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(c*d^5*x^3 + c
^2*d^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(a+bx^2)^{\frac{5}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
[Out] Integral((a + b*x**2)**(5/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx=\int \frac{(bx^2+a)^{\frac{5}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{5/2}}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(5/2)/(c + d*x^2)^(3/2), x)
```

3.209 $\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 258

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}} dx &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} + \frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}} \\ &- \frac{(2bc-ad)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &+ \frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] -(-a*d+b*c)*x*(b*x^2+a)^(1/2)/c/d/(d*x^2+c)^(1/2)+(-a*d+2*b*c)*x*(b*x^2+a)^
(1/2)/c/d/(d*x^2+c)^(1/2)-(-a*d+2*b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1
/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2
+a)^(1/2)/d^(3/2)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+b
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x
^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(3/2)/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {424, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{a+bx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}} + \frac{x\sqrt{a+bx^2}(2bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[In] Int[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] -(((b*c - a*d)*x*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + ((2*b*c - a*d)*x
*Sqrt[a + b*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[c]*d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*Sqrt[c]*Sqrt[a + b
*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(d^(3/2)*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}+\frac{\int \frac{abc+b(2bc-ad)x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}+\frac{(ab)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{d}+\frac{(b(2bc-ad))\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{cd}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}+\frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}\\
&\quad+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{(2bc-ad)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{d}\\
&= -\frac{(bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}+\frac{(2bc-ad)x\sqrt{a+bx^2}}{cd\sqrt{c+dx^2}}\\
&\quad-\frac{(2bc-ad)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\
&\quad+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.73 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \frac{ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc+ad)\left(\sqrt{\frac{b}{a}}dx(a+bx\right.}{\sqrt{\frac{b}{a}}cd^2\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x]
```

```
[Out] (I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^
2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[
Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c*d^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.34

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,ab\,d^2x^3-\sqrt{-\frac{b}{a}}\,b^2cd\,x^3+2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd-2\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.}{(bd\,x^4+ad\,x^2+cb\,x^2+}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{\left(bd\,x^2+ad\right)(ad-bc)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{b(2ad-bc)}{d^2}+\frac{(ad-bc)^2}{d^2c}-\frac{a(ad-bc)}{dc}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{\left(\frac{b^2}{d}-\right.}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*a*b*d^2*x^3-(-b/a)^(1/2)*b^2*
c*d*x^3+2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*a*b*c*d-2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Elliptic
F(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)
^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+2*((b*x^2+a)/a)^(1
/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(
-b/a)^(1/2)*a^2*d^2*x-(-b/a)^(1/2)*a*b*c*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
/d^2/c/(-b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,bc^2d-acd^2)x^3+(2\,bc^3-ac^2d)x)\sqrt{bd}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((2\,bc^2d-acd^2+ad^3)x^3+(2\,bc^3}{cd^4x^3+c^2d}$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((((2*b*c^2*d - a*c*d^2)*x^3 + (2*b*c^3 - a*c^2*d)*x)*sqrt(b*d)*sqrt(-c/d)*
elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((2*b*c^2*d - a*c*d^2 + a*d^3
)*x^3 + (2*b*c^3 - a*c^2*d + a*c*d^2)*x)*sqrt(b*d)*sqrt(-c/d)*elliptic_f(ar
csin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d^2*x^2 + 2*b*c^2*d - a*c*d^2)*sqrt(b
*x^2 + a)*sqrt(d*x^2 + c))/(c*d^4*x^3 + c^2*d^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(3/2)/(c + d*x^2)^(3/2), x)
```

3.210 $\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 84

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}} \, dx = \frac{\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/c^(1/2)/d^(1/2)/(c*(b*x^2+a)/
a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {422}

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}} \, dx = \frac{\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(
Sqrt[c]*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{\frac{x(a+bx^2)}{c}+\frac{ia\sqrt{\frac{b}{a}}\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\left(E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)-\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)\right)}{d}}{\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2),x]
```

```
[Out] ((x*(a + b*x^2))/c + (I*a*Sqrt[b/a]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]
*(EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/d)/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{b}{a}}\,bd\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)bc+\sqrt{-\frac{b}{a}}\,adx\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)dc\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,(\frac{(bd\,x^2+ad)x}{dc\sqrt{(x^2+\frac{c}{d})(bd\,x^2+ad)}}+\frac{(\frac{b}{d}+\frac{ad-bc}{dc}-\frac{a}{c})\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{b\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,(F(x\sqrt{\cdots}}{\sqrt{-\frac{b}{a}}\cdots}}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b/a)^(1/2)*b*d*x^3+((b*x^2+a)/a)^(1/2)*(
(d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c-((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c
+(-b/a)^(1/2)*a*d*x)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/d/c/(-b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}adx - (bdx^2+bc)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) + (bdx^2+bc)}{acd^2x^2+ac^2d}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*d*x - (b*d*x^2 + b*c)*sqrt(a*c)*sqrt(-b/
a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (b*d*x^2 + b*c)*sqrt(a*c)*
sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*c*d^2*x^2 + a*c^
2*d)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\left(d\,x^2+c\right)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2),x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(3/2), x)
```

3.211 $\int \frac{1}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1/2
)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+
d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))*d^(1/2)*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {425, 21, 433, 429, 506, 422}

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{b\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(Sqrt[c]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c
+ d*x^2])) + (b*Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c
]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d
*x^2))]*Sqrt[c + d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}}+\frac{\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c(bc-ad)} \\
&= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx}{c(bc-ad)} \\
&= -\frac{dx\sqrt{a+bx^2}}{c(bc-ad)\sqrt{c+dx^2}}+\frac{b\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad}+\frac{(bd)\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{c(bc-ad)} \\
&= \frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-\frac{d\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad} \\
&= -\frac{\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{b\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.58

$$
\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\frac{-dx(a+bx^2)+\frac{bc\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}}}{c(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}
$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
[Out] (-(d*x*(a + b*x^2)) + (b*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
E[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/Sqrt[-(b/a)])/(c*(b*c - a*d)*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.74

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\,b d\,x^{3}-\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right) b c+\sqrt{-\frac{b}{a}}\,a d x\right) \sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}}{c\sqrt{-\frac{b}{a}}\,(a d-b c)(b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c)}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(d\,x^{2}+c)}\left(\frac{\left(b d\,x^{2}+a d\right) x}{c(a d-b c)\sqrt{\left(x^{2}+\frac{c}{d}\right)\left(b d\,x^{2}+a d\right)}}+\frac{\left(\frac{1}{c}-\frac{a d}{c(a d-b c)}\right)\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}+c b\,x^{2}+a c}}+\frac{b\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(\right.}{(ad}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}+c}}$

[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((-b/a)^(1/2)*b*d*x^3-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c+(-b/a)^(1/2)*a*d*x)*(d*x^2+c)^(1/2)*(b*x^2
+a)^(1/2)/c/(-b/a)^(1/2)/(a*d-b*c)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{\sqrt{bx^2+a}\sqrt{dx^2+c}adx-(bdx^2+bc)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})+((a+b)dx^2+(a+b)c)\sqrt{ac}\sqrt{-\frac{b}{a}}}{abc^3-a^2c^2d+(abc^2d-a^2cd^2)x^2}$$

[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")

[Out] -(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*a*d*x - (b*d*x^2 + b*c)*sqrt(a*c)*sqrt(-b
/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + ((a + b)*d*x^2 + (a + b)*
c)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*b*c
^3 - a^2*c^2*d + (a*b*c^2*d - a^2*c*d^2)*x^2)

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(3/2),x)

[Out] Integral(1/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.212 $\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 242

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx &= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}}\\ &\quad+\frac{\sqrt{d}(bc+ad)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &\quad-\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] b*x/a/(-a*d+b*c)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+(a*d+b*c)*(1/(1+d*x^2/c))^
(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*
c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/c^(1/2)/(c*(b*x^2+a)/a
/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-2*b*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/
2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)
*d^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)^2/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*
x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {425, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = -\frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{d}\sqrt{a+bx^2}(ad+bc)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{bx}{a\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(b*c + a*d
)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[c]*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x
^2]) - (2*b*Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sq
rt[c]], 1 - (b*c)/(a*d)])/(a*(b*c - a*d)^2*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{\int \frac{ad-bdx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx}{a(bc-ad)} \\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} - \frac{(2bd)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} + \frac{(d(bc+ad))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{a(bc-ad)^2} \\
&= \frac{bx}{a(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\sqrt{d}(bc+ad)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad - \frac{2b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a(bc-ad)^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.93

$$
\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}\left(\sqrt{\frac{b}{a}}x(a^2d^2+abd^2x^2+b^2c(c+dx^2))+ibc(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right.}{bc(bc}
$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*(Sqrt[b/a]*x*(a^2*d^2 + a*b*d^2*x^2 + b^2*c*(c + d*x^2)) + I*b*c
*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sq
rt[b/a]*x], (a*d)/(b*c)] + I*b*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1
+ (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c*(b*c - a
*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.46

method	result
default	$\frac{\left(\sqrt{-\frac{b}{a}}\, ab\, d^2x^3+\sqrt{-\frac{b}{a}}\, b^2cd\, x^3-\sqrt{\frac{b\,x^2+a}{a}}\, \sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)abcd+\sqrt{\frac{b\,x^2+a}{a}}\, \sqrt{\frac{d\,x^2+c}{c}}\, F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b^2c^2-\sqrt{\frac{b\,x^2+a}{a}}\, \sqrt{}\right.}{c\sqrt{-\frac{b}{a}}\, a(ad-bc)^2(bd\,x^4+}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{2bd\left(-\frac{(ad+bc)x^3}{2ac\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(a^2d^2+b^2c^2\right)x}{2ac\left(a^2d^2-2abcd+b^2c^2\right)bd}\right)}{\sqrt{\left(x^4+\frac{(ad+bc)x^2}{bd}+\frac{ac}{bd}\right)bd}}+\frac{\left(\frac{1}{ac}-\frac{a^2d^2+b^2c^2}{ac\left(a^2d^2-2abcd+b^2c^2\right)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+}}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-b/a)^(1/2)*a*b*d^2*x^3+(-b/a)^(1/2)*b^2*c*d*x^3-((b*x^2+a)/a)^(1/2)*((d*
x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^
2*c^2-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*b*c*d-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-
b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(-b/a)^(1/2)*a^2*d^2*x+(-b/a)^(1/2)*b^2
*c^2*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/c/(-b/a)^(1/2)/a/(a*d-b*c)^2/(b*d*x
^4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.68

$$\int \frac{1}{(a+bx^2)^{3/2}\,(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(ab^2c^2+a^2bcd+(b^3cd+ab^2d^2)x^4+(b^3c^2+2\,ab^2cd+a^2bd^2)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(ab^2}{a^3b^2c^4-2}$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a*b^2*c^2 + a^2*b*c*d + (b^3*c*d + a*b^2*d^2)*x^4 + (b^3*c^2 + 2*a*b^2*c
*d + a^2*b*d^2)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)),
a*d/(b*c)) - (a*b^2*c^2 + (b^3*c*d + (2*a^2*b + a*b^2)*d^2)*x^4 + (2*a^3 +
a^2*b)*c*d + (b^3*c^2 + 2*(a^2*b + a*b^2)*c*d + (2*a^3 + a^2*b)*d^2)*x^2)*s
qrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b^2*c
*d + a^2*b*d^2)*x^3 + (a*b^2*c^2 + a^3*d^2)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 +
 c))/(a^3*b^2*c^4 - 2*a^4*b*c^3*d + a^5*c^2*d^2 + (a^2*b^3*c^3*d - 2*a^3*b^
2*c^2*d^2 + a^4*b*c*d^3)*x^4 + (a^2*b^3*c^4 - a^3*b^2*c^3*d - a^4*b*c^2*d^2
 + a^5*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(3/2),x)
[Out] Integral(1/((a + b*x**2)**(3/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{3/2}\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(3/2)), x)
```

3.213 $\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx &= \frac{bx}{3a(bc-ad)(a+bx^2)^{3/2}\sqrt{c+dx^2}} \\ &+ \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} \\ &+ \frac{\sqrt{d}(2b^2c^2-7abcd-3a^2d^2)\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &- \frac{b\sqrt{c}\sqrt{d}(bc-9ad)\sqrt{a+bx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] 1/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)+2/3*b*(-3*a*d+b*c)*x/a
^2/(-a*d+b*c)^2/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)+1/3*(-3*a^2*d^2-7*a*b*c*d+2
*b^2*c^2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/
2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a*d+b
*c)^3/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-1/3*b*(-9*a*d
+b*c)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/a^2/(-a
*d+b*c)^3/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {425, 541, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{a+bx^2}(-3a^2d^2-7abcd+2b^2c^2)\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$-\frac{b\sqrt{c}\sqrt{d}\sqrt{a+bx^2}(bc-9ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{2bx(bc-3ad)}{3a^2\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-ad)^2}+\frac{bx}{3a\,(a+bx^2)^{3/2}\sqrt{c+dx^2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2]) + (2*b*(b*c - 3*a
*d)*x)/(3*a^2*(b*c - a*d)^2*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]) + (Sqrt[d]*(2*
b^2*c^2 - 7*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3*a^2*Sqrt[c]*(b*c - a*d)^3*Sqrt[(c*(a + b*
x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (b*Sqrt[c]*Sqrt[d]*(b*c - 9*a*d)*
Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(3
*a^2*(b*c - a*d)^3*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
```

```
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} - \frac{\int \frac{-2bc+3ad-3bdx^2}{\left(a+bx^2\right)^{3/2}\left(c+dx^2\right)^{3/2}}\,dx}{3a(bc-ad)} \\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} \\
&\quad + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} + \frac{\int \frac{ad(bc+3ad)+2bd(bc-3ad)x^2}{\sqrt{a+bx^2}\left(c+dx^2\right)^{3/2}}\,dx}{3a^2(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} \\
&\quad - \frac{(bd(bc-9ad))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{3a(bc-ad)^3} + \frac{\left(d\left(2b^2c^2-7abcd-3a^2d^2\right)\right)\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{3a^2(bc-ad)^3} \\
&= \frac{bx}{3a(bc-ad)\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}} + \frac{2b(bc-3ad)x}{3a^2(bc-ad)^2\sqrt{a+bx^2}\sqrt{c+dx^2}} \\
&\quad + \frac{\sqrt{d}\left(2b^2c^2-7abcd-3a^2d^2\right)\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}(bc-ad)^3\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}} \\
&\quad - \frac{b\sqrt{c}\sqrt{d}(bc-9ad)\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2(bc-ad)^3\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.40 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}x(3a^4d^3+6a^3bd^3x^2-2b^4c^2x^2(c+dx^2)+a^2b^2d(8c^2+8cdx^2+3d^2x^4)+}{}$$

```
[In] Integrate[1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*x*(3*a^4*d^3 + 6*a^3*b*d^3*x^2 - 2*b^4*c^2*x^2*(c + d*x^2) + a^2
*b^2*d*(8*c^2 + 8*c*d*x^2 + 3*d^2*x^4) + a*b^3*c*(-3*c^2 + 4*c*d*x^2 + 7*d^
2*x^4)) + I*b*c*(-2*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)
] + (2*I)*b*c*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2)*(a + b*x^2)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3
*a^2*Sqrt[b/a]*c*(-(b*c) + a*d)^3*(a + b*x^2)^(3/2)*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(bx^2+a)(dx^2+c)}\left(\frac{x\sqrt{bdx^4+adx^2+cbx^2+ac}}{3a(ad-bc)^2\left(x^2+\frac{a}{b}\right)^2}+\frac{\left(bdx^2+bc\right)bx(7ad-2bc)}{3a^2(ad-bc)^3\sqrt{\left(x^2+\frac{a}{b}\right)\left(bdx^2+bc\right)}}+\frac{\left(bdx^2+ad\right)d^2x}{c(ad-bc)^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(bdx^2+ad\right)}}+\frac{\left(\frac{bd}{3(ad-bc)^2a}-\frac{b(7a}{3(ad}\right.}{}\right.}{}$
default	$-\frac{-3\sqrt{-\frac{b}{a}}\,a^2b^2d^3x^5-7\sqrt{-\frac{b}{a}}\,ab^3cd^2x^5+2\sqrt{-\frac{b}{a}}\,b^4c^2dx^5+6\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^2b^2cd^2x^2-8\sqrt{\frac{bx^2+a}{a}}\,\sqrt{\frac{dx^2+c}{c}}\,F}{}$

```
[In] int(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b*x^2+a)*(d*x^2+c))^(1/2)/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)*(1/3/a/(a*d-b*c
)^2*x*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)/(x^2+a/b)^2+1/3*(b*d*x^2+b*c)*b/a
^2/(a*d-b*c)^3*x*(7*a*d-2*b*c)/((x^2+a/b)*(b*d*x^2+b*c))^(1/2)+(b*d*x^2+a*d
)*d^2/c/(a*d-b*c)^3*x/((x^2+c/d)*(b*d*x^2+a*d))^(1/2)+(1/3*b*d/(a*d-b*c)^2/
a-1/3/(a*d-b*c)^2*b*(7*a*d-2*b*c)/a^2-1/3*b^2*c/a^2/(a*d-b*c)^3*(7*a*d-2*b*
c)+d^2/(a*d-b*c)^2/c-a*d^3/c/(a*d-b*c)^3)/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1
+d*x^2/c)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*EllipticF(x*(-b/a)^(1/2
),(-1+(a*d+b*c)/c/b)^(1/2))-(-1/3*b^2*d*(7*a*d-2*b*c)/(a*d-b*c)^3/a^2-b*d^3
/(a*d-b*c)^3/c)*c/(-b/a)^(1/2)*(1+b*x^2/a)^(1/2)*(1+d*x^2/c)^(1/2)/(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)/d*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^
(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d+b*c)/c/b)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(305) = 610$.

Time = 0.13 (sec) , antiderivative size = 856, normalized size of antiderivative = 2.65

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx =$$
$$-\frac{(2a^2b^3c^3 - 7a^3b^2c^2d - 3a^4bcd^2 + (2b^5c^2d - 7ab^4cd^2 - 3a^2b^3d^3)x^6 + (2b^5c^3 - 3ab^4c^2d - 17a^2b^3cd^2 - 6a}{}$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*a^2*b^3*c^3 - 7*a^3*b^2*c^2*d - 3*a^4*b*c*d^2 + (2*b^5*c^2*d - 7*a
*b^4*c*d^2 - 3*a^2*b^3*d^3)*x^6 + (2*b^5*c^3 - 3*a*b^4*c^2*d - 17*a^2*b^3*c
*d^2 - 6*a^3*b^2*d^3)*x^4 + (4*a*b^4*c^3 - 12*a^2*b^3*c^2*d - 13*a^3*b^2*c*
d^2 - 3*a^4*b*d^3)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)
), a*d/(b*c)) - (2*a^2*b^3*c^3 + (2*b^5*c^2*d + (a^2*b^3 - 7*a*b^4)*c*d^2 -
 3*(3*a^3*b^2 + a^2*b^3)*d^3)*x^6 + (2*b^5*c^3 + (a^2*b^3 - 3*a*b^4)*c^2*d
- (7*a^3*b^2 + 17*a^2*b^3)*c*d^2 - 6*(3*a^4*b + a^3*b^2)*d^3)*x^4 + (a^4*b
- 7*a^3*b^2)*c^2*d - 3*(3*a^5 + a^4*b)*c*d^2 + (4*a*b^4*c^3 + 2*(a^3*b^2 -
6*a^2*b^3)*c^2*d - (17*a^4*b + 13*a^3*b^2)*c*d^2 - 3*(3*a^5 + a^4*b)*d^3)*x
^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*
a*b^4*c^2*d - 7*a^2*b^3*c*d^2 - 3*a^3*b^2*d^3)*x^5 + 2*(a*b^4*c^3 - 2*a^2*b
^3*c^2*d - 4*a^3*b^2*c*d^2 - 3*a^4*b*d^3)*x^3 + (3*a^2*b^3*c^3 - 8*a^3*b^2*
c^2*d - 3*a^5*d^3)*x)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c))/(a^5*b^3*c^5 - 3*a^6
*b^2*c^4*d + 3*a^7*b*c^3*d^2 - a^8*c^2*d^3 + (a^3*b^5*c^4*d - 3*a^4*b^4*c^3
*d^2 + 3*a^5*b^3*c^2*d^3 - a^6*b^2*c*d^4)*x^6 + (a^3*b^5*c^5 - a^4*b^4*c^4*
d - 3*a^5*b^3*c^3*d^2 + 5*a^6*b^2*c^2*d^3 - 2*a^7*b*c*d^4)*x^4 + (2*a^4*b^4
*c^5 - 5*a^5*b^3*c^4*d + 3*a^6*b^2*c^3*d^2 + a^7*b*c^2*d^3 - a^8*c*d^4)*x^2
)
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{2}}(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**(5/2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/2)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/2}(c+dx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)^{5/2}\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)^(5/2)*(c + d*x^2)^(3/2)), x)
```

3.214 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] (1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {429}

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(
a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
(a*d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{\sqrt{-\frac{b}{a}}\,(bd\,x^4+ad\,x^2+cb\,x^2+ac)}$	100
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}$	122

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)
^(1/2)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{ac}\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right)}{bc}$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*c)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.215 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] EllipticF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+
d*x^2/c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[b]*
x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{c+dx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{a-bx^2}\sqrt{c+dx^2}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], -((
a*d)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{\frac{b}{a}}\,(-bd\,x^4+ad\,x^2-cb\,x^2+ac)}$	103
elliptic	$\frac{\sqrt{(-b\,x^2+a)(d\,x^2+c)}\,\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}}$	127

```
[In] int(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a
)^(1/2)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/(-b*d*x^4+a*d*x^2-b*c*
x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{b}{a}}F\left(\arcsin\left(x\sqrt{\frac{b}{a}}\right) \mid -\frac{ad}{bc}\right)}{bc}$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*c)*sqrt(b/a)*elliptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c))/(b*c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{dx^2+c}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((a - b*x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.216 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] EllipticF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-
d*x^2/c)^(1/2)/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\
&= \frac{\left(\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int\frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\\
&= \frac{\sqrt{c}\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$
\int\frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right),-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{c-dx^2}}
$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c - d*x^2]),x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[-(b/a)]*x],
-((a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}{\sqrt{\frac{d}{c}}\,(-bd\,x^4-ad\,x^2+cb\,x^2+ac)}$	103
elliptic	$\frac{\sqrt{(b\,x^2+a)(-d\,x^2+c)}\,\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+cb\,x^2+ac}}$	127

```
[In] int(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F(\arcsin\left(x\sqrt{\frac{d}{c}}\right) \mid -\frac{bc}{ad})}{ad}$$

[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d))/(a*d)

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx$$

[In] integrate(1/(b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)

[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c - d*x**2)), x)

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{c-d\,x^2}}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.217 $\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] EllipticF(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(1-b*x^2/a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[In] Int[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
```

```
[Out] (Sqrt[c]*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(Sqrt[d]*
x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{a-bx^2}\sqrt{c-dx^2}} \\
&= \frac{\sqrt{c}\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$
\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{\frac{c-dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right),\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{c-dx^2}}
$$

```
[In] Integrate[1/(Sqrt[a - b*x^2]*Sqrt[c - d*x^2]),x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[(c - d*x^2)/c]*EllipticF[ArcSin[Sqrt[b/a]*x], (a*
d)/(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 4.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{F\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{-d\,x^2+c}\,\sqrt{-b\,x^2+a}}{\sqrt{\frac{d}{c}}\,(bd\,x^4-ad\,x^2-cb\,x^2+ac)}$	104
elliptic	$\frac{\sqrt{(-b\,x^2+a)(-d\,x^2+c)}\,\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-cb\,x^2+ac}}$	131

```
[In] int(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c
)^(1/2)*(-d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)/(d/c)^(1/2)/(b*d*x^4-a*d*x^2-b*c*
x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \frac{\sqrt{ac}\sqrt{\frac{d}{c}}F\left(\arcsin\left(x\sqrt{\frac{d}{c}}\right) \mid \frac{bc}{ad}\right)}{ad}$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*c)*sqrt(d/c)*elliptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d))/(a*d)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(a - b*x**2)*sqrt(c - d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{-bx^2+a}\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx = \int \frac{1}{\sqrt{a-b\,x^2}\,\sqrt{c-d\,x^2}}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)),x)
[Out] int(1/((a - b*x^2)^(1/2)*(c - d*x^2)^(1/2)), x)
```

3.218 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {430}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x), -\frac{5}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 5*x^2]),x]
[Out] EllipticF[ArcSin[x], -5/2]/Sqrt[2]
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x, \frac{i\sqrt{10}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(5x^2+2)}\,\sqrt{10x^2+4}\,F\left(x, \frac{i\sqrt{10}}{2}\right)}{2\sqrt{5x^2+2}\,\sqrt{-5x^4+3x^2+2}}$	59

```
[In] int(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)} \mid -\frac{5}{2})$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -5/2)
```

Sympy [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(x\right)\middle|-\frac{5}{2}\right)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -5/2)/2, (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{5\,x^2+2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.219 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {430}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F(\sin^{-1}(x)|-2)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = -\frac{i\sqrt{1-x^2}\sqrt{1+2x^2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{1+x^2-2x^4}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 4*x^2]),x]
[Out] ((-1/2*I)*Sqrt[1 - x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x], -1/
2])/Sqrt[1 + x^2 - 2*x^4]
```

Maple [A] (verified)

Time = 2.91 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{F\left(x,i\sqrt{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(2x^2+1)}\,F\left(x,i\sqrt{2}\right)}{\sqrt{-4x^4+2x^2+2}}$	40

```
[In] int(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,I*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid -2)$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2x^2+1}}\,dx}{2}$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(2*x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{4\,x^2+2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.220 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx=\frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {430}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx=\frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral}=\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]),x]
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x,\frac{i\sqrt{6}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(3x^2+2)(x^2-1)}\,\sqrt{6x^2+4}\,F\left(x,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{3x^2+2}\,\sqrt{-3x^4+x^2+2}}$	57

```
[In] int(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\mid -\frac{3}{2})$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -3/2)
```

Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{2} & \text{for } x > -1 \land x < 1 \end{cases}$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -3/2)/2, (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{3\,x^2+2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.221 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {254, 227}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
```

))

Rubi steps

$$\begin{aligned} \text{integral} &= \int \frac{1}{\sqrt{2-2x^4}}\,dx \\ &= \frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + 2*x^2]),x]
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{F(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\,F(x,i)}{\sqrt{-2x^4+2}}$	24

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\,|-1)$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(8) = 16.

Time = 11.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 7.30

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = -\frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}} + \frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle|\,\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
```

```
[Out] -sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
(0,)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3/2)) + sqrt(2)*meijerg(((-1/4, 0,
 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi**(
3/2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2+2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.222 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = -i\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 + x^2]),x]
[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[2]], -2]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{F\left(x,\frac{i\sqrt{2}}{2}\right)\sqrt{2}}{2}$	14
elliptic	$\frac{\sqrt{-(x^2-1)(x^2+2)}\,\sqrt{2x^2+4}\,F\left(x,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{x^2+2}\,\sqrt{-x^4-x^2+2}}$	55

```
[In] int(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid -\frac{1}{2})$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -1/2)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\ x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Piecewise((sqrt(2)*elliptic_f(asin(x), -1/2)/2, (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{x^2+2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.223 $\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {430}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[2 - x^2]),x]
[Out] EllipticF[ArcSin[x], 1/2]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{2}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(x^2-1)(x^2-2)}\,\sqrt{-2x^2+4}\,F\left(x,\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{x^4-3x^2+2}}$	53

```
[In] int(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \frac{1}{2}\,\sqrt{2}F(\arcsin{(x)}\mid\frac{1}{2})$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), 1/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{2-x^2}}\,dx$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(-x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-x^2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.224 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 8

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{\operatorname{arctanh}(x)}{\sqrt{2}}$$

```
[Out] 1/2*arctanh(x)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {22, 212}

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{\operatorname{arctanh}(x)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] ArcTanh[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{1}{1-x^2}\,dx}{\sqrt{2}}\\ &= \frac{\tanh^{-1}(x)}{\sqrt{2}}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(8) = 16.

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = -\frac{\frac{1}{2}\log(1-x)-\frac{1}{2}\log(1+x)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 - x^2]),x]
[Out] -((Log[1 - x]/2 - Log[1 + x]/2)/Sqrt[2])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\operatorname{arctanh}(x)\sqrt{2}}{2}$	8

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctanh(x)*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(7) = 14.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 8.50

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{x^6+5\,x^4-2\,\sqrt{2}(x^3+x)\sqrt{-x^2+1}\sqrt{-2\,x^2+2}-5\,x^2-1}{x^6-3\,x^4+3\,x^2-1}\right)$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(-(x^6 + 5*x^4 - 2*sqrt(2)*(x^3 + x)*sqrt(-x^2 + 1)*sqrt(-2*
x^2 + 2) - 5*x^2 - 1)/(x^6 - 3*x^4 + 3*x^2 - 1))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = -\sqrt{2}\left(\frac{\log(x-1)}{4} - \frac{\log(x+1)}{4}\right)$$

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
[Out] -sqrt(2)*(log(x - 1)/4 - log(x + 1)/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-2\,x^2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.225 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {430}

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}}$$

[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]),x]

[Out] EllipticF[ArcSin[x], 3/2]/Sqrt[2]

Maple [A] (verified)

Time = 2.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(3x^2-2)(x^2-1)}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{3x^4-5x^2+2}}$	57

[In] int(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2*EllipticF(x,1/2*6^(1/2))*2^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x) \mid \frac{3}{2})$$

[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), 3/2)

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 1.43 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|\frac{2}{3}\right)}{3} & \text{for}\ x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3) &
 (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-3\,x^2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.226 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {430}

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|2\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 - x^2]),x]
[Out] EllipticF[ArcSin[x], 2]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{2}}{2}$	11
elliptic	$\frac{\sqrt{(2x^2-1)(x^2-1)}\,F\left(x,\sqrt{2}\right)}{\sqrt{4x^4-6x^2+2}}$	36

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\mid 2)$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), 2)
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.90

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\ x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqrt(2
)/2) & (x < sqrt(2)/2)))/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-4\,x^2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.227 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{430\}$

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{\operatorname{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 - x^2]),x]
[Out] EllipticF[ArcSin[x], 5/2]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{2}}{2}$	13
elliptic	$\frac{\sqrt{(5x^2-2)(x^2-1)}\,\sqrt{-10x^2+4}\,F\left(x,\frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{5x^4-7x^2+2}}$	57

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin{(x)}\mid\frac{5}{2})$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), 5/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 1.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \begin{cases} \frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|\frac{2}{5}\right)}{5} & \text{for}\ x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2+1)**(1/2),x)
[Out] Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)/5)
 & (x < sqrt(10)/5)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2-5\,x^2}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.228 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{2+5x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2))*(5*x^2+2)^(1
/2)*2^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 5*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+5x^2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{5}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 5*x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], 5/2])/Sqrt[2]
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.33

method	result	size
default	$-\frac{iF\left(ix,\frac{\sqrt{10}}{2}\right)\sqrt{2}}{2}$	17
elliptic	$-\frac{i\sqrt{(x^2+1)(5x^2+2)}\,\sqrt{10x^2+4}\,F\left(ix,\frac{\sqrt{10}}{2}\right)}{2\sqrt{5x^2+2}\,\sqrt{5x^4+7x^2+2}}$	61

```
[In] int(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x,1/2*10^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\mid\frac{5}{2})$$

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 5/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{5x^2+2}}\,dx$$

```
[In] integrate(1/(x**2+1)**(1/2)/(5*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(5*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{5\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.229 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx=\frac{\sqrt{1+2x^2}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)^(1/2)*2^(1/2)/
((2*x^2+1)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx=\frac{\sqrt{2x^2+1}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(1 +
 2*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+2x^2}F(\tan^{-1}(x)|-1)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{i\,\text{EllipticF}(i\text{arcsinh}(x),2)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 4*x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], 2])/Sqrt[2]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.31

method	result	size
default	$-\frac{iF\left(ix,\sqrt{2}\right)\sqrt{2}}{2}$	15
elliptic	$-\frac{i\sqrt{(x^2+1)(2x^2+1)}\,F\left(ix,\sqrt{2}\right)}{\sqrt{4x^4+6x^2+2}}$	41

```
[In] int(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x,2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\;|2)$$

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{x^2+1}\sqrt{2x^2+1}}\,dx}{2}$$

```
[In] integrate(1/(x**2+1)**(1/2)/(4*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(x**2 + 1)*sqrt(2*x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{4\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.230 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2))*(3*x^2+2)^(1
/2)*2^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2
 + 3*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{3}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], 3/2])/Sqrt[2]
```

Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.33

method	result	size
default	$-\frac{iF\left(ix,\frac{\sqrt{6}}{2}\right)\sqrt{2}}{2}$	17
elliptic	$-\frac{i\sqrt{(3x^2+2)(x^2+1)}\,\sqrt{6x^2+4}\,F\left(ix,\frac{\sqrt{6}}{2}\right)}{2\sqrt{3x^2+2}\,\sqrt{3x^4+5x^2+2}}$	61

```
[In] int(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x,1/2*6^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{1}{2}i\,\sqrt{2}F(\arcsin{(i\,x)}\mid\frac{3}{2})$$

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*I*sqrt(2)*elliptic_f(arcsin(I*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{3x^2+2}}\,dx$$

```
[In] integrate(1/(x**2+1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{3\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.231 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 8

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\arctan(x)}{\sqrt{2}}$$

```
[Out] 1/2*arctan(x)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {22, 209}

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\arctan(x)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] ArcTan[x]/Sqrt[2]
```

Rule 22

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x
] && EqQ[b*c - a*d, 0] && GtQ[b/d, 0] &&  !(IntegerQ[m] || IntegerQ[n])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\text{integral} = \sqrt{2}\int \frac{1}{2+2x^2}\,dx$$
$$= \frac{\tan^{-1}(x)}{\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\arctan(x)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + 2*x^2]),x]
[Out] ArcTan[x]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

method	result	size
meijerg	$\frac{\arctan(x)\sqrt{2}}{2}$	8

```
[In] int(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*arctan(x)*2^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(7) = 14.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 4.25

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{2\,x^2+2}\sqrt{x^2+1}x}{x^4-1}\right)$$

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*sqrt(2*x^2 + 2)*sqrt(x^2 + 1)*x/(x^4 - 1))
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\,(x)}{2}$$

```
[In] integrate(1/(x**2+1)**(1/2)/(2*x**2+2)**(1/2),x)
[Out] sqrt(2)*atan(x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.232 $\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*(x^2+2)^(1/2)*
2^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {429}

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[1 + x^2]*Sqrt[(2 +
x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+x^2}F\left(\tan^{-1}(x)|\frac{1}{2}\right)}{\sqrt{2}\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[1 + x^2]*Sqrt[2 + x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], 1/2])/Sqrt[2]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.32

method	result	size
default	$-iF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)$	15
elliptic	$-\frac{i\sqrt{(x^2+1)(x^2+2)}\,\sqrt{2}\,\sqrt{2x^2+4}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2\sqrt{x^2+2}\,\sqrt{x^4+3x^2+2}}$	59

```
[In] int(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*EllipticF(1/2*I*x*2^(1/2),2^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = -i\,F(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2)$$

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] -I*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{x^2+2}}\,dx$$

```
[In] integrate(1/(x**2+1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(x**2 + 1)*sqrt(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(x^2+1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{x^2+2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.233 $\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)$$

```
[In] Int[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = -\frac{i\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[1 + x^2]),x]
[Out] ((-I)*EllipticF[I*ArcSinh[x], -1/2])/Sqrt[2]
```

Maple [A] (verified)

Time = 2.68 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$F\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)$	14
elliptic	$\frac{\sqrt{-(x^2-2)(x^2+1)}\,\sqrt{2}\,\sqrt{-2x^2+4}\,F\left(\frac{\sqrt{2}\,x}{2},i\sqrt{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{-x^4+x^2+2}}$	63

```
[In] int(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(1/2*2^(1/2)*x,I*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = F(\arcsin\left(\frac{1}{2}\sqrt{2}x\right)\mid -2)$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{2-x^2}\sqrt{x^2+1}}\,dx$$

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-x^2+2}}\,dx$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-x^2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.234 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 10

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {254, 227}

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
```

```
))
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{\sqrt{2-2x^4}}\,dx\\ &= \frac{F(\sin^{-1}(x)|-1)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}(\arcsin(x),-1)}{\sqrt{2}}$$

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[x], -1]/Sqrt[2]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{F(x,i)\sqrt{2}}{2}$	10
elliptic	$\frac{\sqrt{-x^4+1}\,F(x,i)}{\sqrt{-2x^4+2}}$	24

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,I)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{2}F(\arcsin(x)\,|-1)$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*elliptic_f(arcsin(x), -1)
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(8) = 16.

Time = 11.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 7.60

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle|\,\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle|\,\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4, 0, 1/4, 1/2, 3/
4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi
**(3/2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-2\,x^2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.235 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{3}}{3}$	19
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2+1)}\,\sqrt{6}\,\sqrt{-6x^2+4}\,F\left(\frac{x\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{-3x^2+2}\,\sqrt{-3x^4-x^2+2}}$	67

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \frac{1}{3}\,\sqrt{3}F(\arcsin\left(\frac{1}{2}\,\sqrt{3}\sqrt{2}x\right)\,|-\frac{2}{3})$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*elliptic_f(arcsin(1/2*sqrt(3)*sqrt(2)*x), -2/3)
```

Sympy [A] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \begin{cases} \frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3} & \text{for } x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Piecewise((sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3, (x > -sqrt(6)/3)
& (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-3\,x^2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.236 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\operatorname{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{1}{2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)$$

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/2]/2
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{F\left(\sqrt{2}\,x,\frac{i\sqrt{2}}{2}\right)}{2}$	15
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2+1)}\,\sqrt{2}\,F\left(\sqrt{2}\,x,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-4x^4-2x^2+2}}$	48

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(2^(1/2)*x,1/2*I*2^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{1}{2}\,F(\arcsin\left(\sqrt{2}x\right)\mid-\frac{1}{2})$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/2*elliptic_f(arcsin(sqrt(2)*x), -1/2)
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} \frac{\sqrt{2}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for } x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
[Out] sqrt(2)*Piecewise((sqrt(2)*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sqrt(
2)/2) & (x < sqrt(2)/2)))/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-4\,x^2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.237 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*5^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {430}

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rubi steps

$$\text{integral} = \frac{F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}}$$

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[1 + x^2]),x]
[Out] EllipticF[ArcSin[Sqrt[5/2]*x], -2/5]/Sqrt[5]
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{F\left(\frac{\sqrt{10}\,x}{2},\frac{i\sqrt{10}}{5}\right)\sqrt{5}}{5}$	19
elliptic	$\frac{\sqrt{-(5x^2-2)(x^2+1)}\,\sqrt{10}\,\sqrt{-10x^2+4}\,F\left(\frac{\sqrt{10}\,x}{2},\frac{i\sqrt{10}}{5}\right)}{10\sqrt{-5x^2+2}\,\sqrt{-5x^4-3x^2+2}}$	67

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*EllipticF(1/2*10^(1/2)*x,1/5*I*10^(1/2))*5^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \frac{1}{5}\,\sqrt{5}F(\arcsin\left(\frac{1}{2}\,\sqrt{5}\sqrt{2}x\right)\,|-\frac{2}{5})$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
[Out] 1/5*sqrt(5)*elliptic_f(arcsin(1/2*sqrt(5)*sqrt(2)*x), -2/5)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \begin{cases} \frac{\sqrt{5}F\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|-\frac{2}{5}\right)}{5} & \text{for}\: x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2+1)**(1/2),x)
[Out] Piecewise((sqrt(5)*elliptic_f(asin(sqrt(10)*x/2), -2/5)/5, (x > -sqrt(10)/5
) & (x < sqrt(10)/5)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}\,\sqrt{2-5\,x^2}}\,dx$$

```
[In] int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.238 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+5x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 5*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] (verified)

Time = 2.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{iF\left(\frac{ix\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)\sqrt{-x^2+1}\,\sqrt{5}}{5\sqrt{x^2-1}}$	37
elliptic	$-\frac{i\sqrt{(x^2-1)(5x^2+2)}\,\sqrt{10}\,\sqrt{10x^2+4}\,\sqrt{-x^2+1}\,F\left(\frac{ix\sqrt{10}}{2},\frac{i\sqrt{10}}{5}\right)}{10\sqrt{x^2-1}\,\sqrt{5x^2+2}\,\sqrt{5x^4-3x^2-2}}$	84

```
[In] int(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*I*10^(1/2))*(-x^2+1)^(1/2)*5^(1/2)/(x
^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-\frac{5}{2})$$

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -5/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{5x^2+2}}\,dx$$

```
[In] integrate(1/(x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(5*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+5x^2}}\,dx=\int \frac{1}{\sqrt{x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.239 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+4x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F(\sin^{-1}(x)|-2)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),-2)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 4*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

method	result	size
default	$-\frac{iF\left(ix\sqrt{2},\frac{i\sqrt{2}}{2}\right)\sqrt{-x^2+1}}{2\sqrt{x^2-1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-1)(2x^2+1)}\,\sqrt{2}\,\sqrt{-x^2+1}\,F\left(ix\sqrt{2},\frac{i\sqrt{2}}{2}\right)}{2\sqrt{x^2-1}\,\sqrt{4x^4-2x^2-2}}$	66

```
[In] int(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*2^(1/2)*x,1/2*I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-2)$$

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

```
[In] integrate(1/(x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.240 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+3x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{iF\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)\sqrt{-x^2+1}\,\sqrt{3}}{3\sqrt{x^2-1}}$	37
elliptic	$-\frac{i\sqrt{(3x^2+2)(x^2-1)}\,\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{-x^2+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{i\sqrt{6}}{3}\right)}{6\sqrt{3x^2+2}\,\sqrt{x^2-1}\,\sqrt{3x^4-x^2-2}}$	84

```
[In] int(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))*(-x^2+1)^(1/2)*3^(1/2)/(x^2
-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-\frac{3}{2})$$

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{3x^2+2}}\,dx$$

```
[In] integrate(1/(x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+3x^2}}\,dx=\int \frac{1}{\sqrt{x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.241 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right),\frac{1}{2}\right)$$

```
[Out] 1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {259, 228}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
```

```
[Out] EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2]/2
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
```

```
b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{-1+x^2}\sqrt{2+2x^2}} \\
&= \frac{1}{2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$
\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{x\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)}{\sqrt{-1+x^2}\sqrt{2+2x^2}}
$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + 2*x^2]),x]
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[-1 + x^2]*Sqr
t[2 + 2*x^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{iF(ix,i)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{-x^2+1}\,F(ix,i)}{\sqrt{x^2-1}\,\sqrt{2x^4-2}}$	43

```
[In] int(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*EllipticF(I*x,I)*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\mid -1)$$

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.87 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}iG_{6,6}^{5,3}\left(\begin{matrix} \frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4} \\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0 \end{matrix}\middle| \frac{e^{2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}iG_{6,6}^{3,5}\left(\begin{matrix} -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1 \\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4} \end{matrix}\middle| \frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

```
[In] integrate(1/(x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
[Out] sqrt(2)*I*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4),
 (0,)), exp_polar(2*I*pi)/x**4)/(16*pi**(3/2)) - sqrt(2)*I*meijerg(((-1/4,
0, 1/4, 1/2, 3/4), (1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), x**(-4))/(16*pi*
*(3/2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.242 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*I*2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx=\frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{2+x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[2 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -1/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{iF\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)\sqrt{-x^2+1}}{\sqrt{x^2-1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-1)(x^2+2)}\,\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},i\sqrt{2}\right)}{2\sqrt{x^2-1}\,\sqrt{x^2+2}\,\sqrt{x^4+x^2-2}}$	76

```
[In] int(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -I*EllipticF(1/2*I*x*2^(1/2),I*2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x)\,|-\frac{1}{2})$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -1/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2+2}}\,dx$$

```
[In] integrate(1/(x**2-1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2+2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.243 $\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 12

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx=-\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),2\right)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF(1/2*(-2*x^2+4)^(1/2),2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {431}

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx=-\operatorname{EllipticF}\left(\arccos\left(\frac{x}{\sqrt{2}}\right),2\right)$$

```
[In] Int[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] -EllipticF[ArcCos[x/Sqrt[2]], 2]
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\text{integral}=-F\left(\cos^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. $2(12) = 24$.

Time = 10.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.92

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\sqrt{1-\frac{x^2}{2}}\,\text{EllipticF}\left(\arcsin(x),\frac{1}{2}\right)}{\sqrt{-2+3x^2-x^4}}$$

```
[In] Integrate[1/(Sqrt[2 - x^2]*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^2]*Sqrt[1 - x^2/2]*EllipticF[ArcSin[x], 1/2])/Sqrt[-2 + 3*x^2 -
 x^4]
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

method	result	size
default	$\frac{F\left(\frac{\sqrt{2}\,x}{2},\sqrt{2}\right)\sqrt{-x^2+1}}{\sqrt{x^2-1}}$	28
elliptic	$\frac{\sqrt{-(x^2-1)(x^2-2)}\,\sqrt{2}\,\sqrt{-2x^2+4}\,\sqrt{-x^2+1}\,F\left(\frac{\sqrt{2}\,x}{2},\sqrt{2}\right)}{2\sqrt{-x^2+2}\,\sqrt{x^2-1}\,\sqrt{-x^4+3x^2-2}}$	78

```
[In] int(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(1/2*2^(1/2)*x,2^(1/2))*(-x^2+1)^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x) \mid \frac{1}{2})$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), 1/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)\,(x+1)}\sqrt{2-x^2}}\,dx$$

```
[In] integrate(1/(-x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(2 - x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-x^2+2}}\,dx$$

```
[In] integrate(1/(-x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-x^2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.244 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}} \, dx = -\frac{\sqrt{-1+x^2}\operatorname{arctanh}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

```
[Out] -1/2*arctanh(x)*(x^2-1)^(1/2)*2^(1/2)/(-x^2+1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {23, 213}

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}} \, dx = -\frac{\sqrt{x^2-1}\operatorname{arctanh}(x)}{\sqrt{2}\sqrt{1-x^2}}$$

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
[Out] -((Sqrt[-1 + x^2]*ArcTanh[x])/(Sqrt[2]*Sqrt[1 - x^2]))
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{-1+x^2}\int \frac{1}{-1+x^2}\,dx}{\sqrt{2-2x^2}}\\ &= -\frac{\sqrt{-1+x^2}\tanh^{-1}(x)}{\sqrt{2}\sqrt{1-x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = -\frac{(-1+x^2)\operatorname{arctanh}(x)}{\sqrt{2}\sqrt{-(-1+x^2)^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 + x^2]),x]
[Out] -(((-1 + x^2)*ArcTanh[x])/(Sqrt[2]*Sqrt[-(-1 + x^2)^2]))
```

Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{-x^2+1}\,\operatorname{arctanh}(x)}{2\sqrt{x^2-1}}$	24
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}(x^2-1)}\,\operatorname{arctanh}(x)}{2\sqrt{\operatorname{signum}(x^2-1)}}$	26
risch	$\frac{\sqrt{\frac{-2x^2+2}{x^2-1}}\,\sqrt{x^2-1}\left(-\frac{\sqrt{-2}\,\ln(-1+x)}{4}+\frac{\sqrt{-2}\,\ln(1+x)}{4}\right)}{\sqrt{-2x^2+2}}$	54

```
[In] int(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)*arctanh(x)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{x^2-1}\sqrt{-2\,x^2+2}x}{x^4-1}\right)$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)*x/(x^4 - 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{1-x^2}\sqrt{x^2-1}}\,dx}{2}$$

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(1 - x**2)*sqrt(x**2 - 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.245 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 3/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3])/(Sqrt[3]*Sqrt[-1 + x^2]
)
```

Maple [A] (verified)

Time = 3.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{6}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-(3x^2-2)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-6x^2+4}\,F\left(x,\frac{\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{x^2-1}\,\sqrt{-3x^4+5x^2-2}}$	74

```
[In] int(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*6^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)} \mid \frac{3}{2})$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), 3/2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \begin{cases} -\frac{\sqrt{3}iF\left(\operatorname{asin}{\left(\frac{\sqrt{6}x}{2}\right)}\middle| \frac{2}{3}\right)}{3} & \text{for}\: x > -\frac{\sqrt{6}}{3} \wedge x < \frac{\sqrt{6}}{3} \end{cases}$$

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] Piecewise((-sqrt(3)*I*elliptic_f(asin(sqrt(6)*x/2), 2/3)/3, (x > -sqrt(6)/3
) & (x < sqrt(6)/3)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.246 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-4x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}$$

$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\,2\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),\frac{1}{2}\right)}{2\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2]*x], 1/2])/(2*Sqrt[-1 + x^2])
```

Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	27
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\,\sqrt{-x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{x^2-1}\,\sqrt{-4x^4+6x^2-2}}$	53

```
[In] int(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,2^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.30

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x)\,|\,2)$$

[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")

[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), 2)

Sympy [A] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} -\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|\frac{1}{2}\right)}{2} & \text{for}\ x > -\frac{\sqrt{2}}{2} \wedge x < \frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

[In] integrate(1/(-4*x**2+2)**(1/2)/(x**2-1)**(1/2),x)

[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), 1/2)/2, (x > -sqr t(2)/2) & (x < sqrt(2)/2)))/2

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

[In] integrate(1/(-4*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-4*x^2 + 2)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.247 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

```
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], 5/2])/(Sqrt[2]*Sqrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1-x^2}}\,dx}{\sqrt{-1+x^2}}$$
$$= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1+x^2}}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 + x^2]),x]
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], 2/5])/(Sqrt[5]*Sqrt[-1 + x^2]
)
```

Maple [A] (verified)

Time = 3.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{F\left(x,\frac{\sqrt{10}}{2}\right)\sqrt{-x^2+1}\,\sqrt{2}}{2\sqrt{x^2-1}}$	29
elliptic	$\frac{\sqrt{-(5x^2-2)(x^2-1)}\,\sqrt{-x^2+1}\,\sqrt{-10x^2+4}\,F\left(x,\frac{\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{x^2-1}\,\sqrt{-5x^4+7x^2-2}}$	74

```
[In] int(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(x,1/2*10^(1/2))*(-x^2+1)^(1/2)*2^(1/2)/(x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin(x)\,|\,\frac{5}{2})$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), 5/2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \begin{cases} -\frac{\sqrt{5}iF\left(\operatorname{asin}\left(\frac{\sqrt{10}x}{2}\right)\middle|\frac{2}{5}\right)}{5} & \text{for}\ x > -\frac{\sqrt{10}}{5} \wedge x < \frac{\sqrt{10}}{5} \end{cases}$$

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(x**2-1)**(1/2),x)
[Out] Piecewise((-sqrt(5)*I*elliptic_f(asin(sqrt(10)*x/2), 2/5)/5, (x > -sqrt(10)
/5) & (x < sqrt(10)/5)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.248 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{2+5x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*6^(1/2)
)*(5*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((5*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {429}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx=\frac{\sqrt{5x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{5x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
```

```
[Out] (Sqrt[2 + 5*x^2]*EllipticF[ArcTan[x], -3/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 5*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+5x^2}F\left(\tan^{-1}(x)\middle|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+5x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{5}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 5*x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 5/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{iF\left(\frac{ix\sqrt{10}}{2},\frac{\sqrt{10}}{5}\right)\sqrt{5}\,\sqrt{-x^2-1}}{5\sqrt{x^2+1}}$	36
elliptic	$-\frac{i\sqrt{-(x^2+1)(5x^2+2)}\,\sqrt{10}\,\sqrt{10x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{10}}{2},\frac{\sqrt{10}}{5}\right)}{10\sqrt{-x^2-1}\,\sqrt{5x^2+2}\,\sqrt{-5x^4-7x^2-2}}$	84

```
[In] int(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/5*I*EllipticF(1/2*I*x*10^(1/2),1/5*10^(1/2))/(x^2+1)^(1/2)*5^(1/2)*(-x^2-
1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \frac{1}{2}i\,\sqrt{-2}F(\arcsin\left(i\,x\right)\mid\frac{5}{2})$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 5/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{5x^2+2}}\,dx$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(5*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(5*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{5\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(5*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(5*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+5x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{5\,x^2+2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(5*x^2 + 2)^(1/2)), x)
```

3.249 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{1+2x^2}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),I)*(2*x^2+1)^
(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((2*x^2+1)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {429}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2x^2+1}\,\text{EllipticF}(\arctan(x),-1)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{2x^2+1}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
```

```
[Out] (Sqrt[1 + 2*x^2]*EllipticF[ArcTan[x], -1])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(1
+ 2*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+2x^2}F(\tan^{-1}(x)|-1)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{1+2x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}(i\text{arcsinh}(x),2)}{\sqrt{2}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 4*x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{iF\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	33
elliptic	$-\frac{i\sqrt{-(x^2+1)(2x^2+1)}\,\sqrt{2}\,\sqrt{x^2+1}\,F\left(ix\sqrt{2},\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{-4x^4-6x^2-2}}$	66

```
[In] int(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*2^(1/2)*x,1/2*2^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{1}{2}i\,\sqrt{-2}F(\arcsin{(i\,x)}\,|\,2)$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2-1}\sqrt{2x^2+1}}\,dx}{2}$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(4*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(2*x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{4\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(4*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(4*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+4x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{4\,x^2+2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(4*x^2 + 2)^(1/2)), x)
```

3.250 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{2+3x^2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*I*2^(1/2)
)*(3*x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((3*x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {429}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx=\frac{\sqrt{3x^2+2}\,\text{EllipticF}\left(\arctan(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
```

```
[Out] (Sqrt[2 + 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(
2 + 3*x^2)/(1 + x^2)])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+3x^2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+3x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{3}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 3*x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], 3/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{iF\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)\sqrt{3}\,\sqrt{-x^2-1}}{3\sqrt{x^2+1}}$	36
elliptic	$-\frac{i\sqrt{-(3x^2+2)(x^2+1)}\,\sqrt{6}\,\sqrt{6x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{6}}{2},\frac{\sqrt{6}}{3}\right)}{6\sqrt{-x^2-1}\,\sqrt{3x^2+2}\,\sqrt{-3x^4-5x^2-2}}$	84

```
[In] int(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3*I*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))/(x^2+1)^(1/2)*3^(1/2)*(-x^2-1)
^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \frac{1}{2}i\,\sqrt{-2}F(\arcsin{(i\,x)}\mid\frac{3}{2})$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(-2)*elliptic_f(arcsin(I*x), 3/2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{3x^2+2}}\,dx$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(3*x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(3*x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{3\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(3*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+3x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{3\,x^2+2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(3*x^2 + 2)^(1/2)), x)
```

3.251 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 28

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx=\frac{\sqrt{1+x^2}\arctan(x)}{\sqrt{2}\sqrt{-1-x^2}}$$

```
[Out] 1/2*arctan(x)*(x^2+1)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {23, 209}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx=\frac{\sqrt{x^2+1}\arctan(x)}{\sqrt{2}\sqrt{-x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
[Out] (Sqrt[1 + x^2]*ArcTan[x])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{2+2x^2}\int \frac{1}{2+2x^2}\,dx}{\sqrt{-1-x^2}}\\
&= \frac{\sqrt{1+x^2}\tan^{-1}(x)}{\sqrt{2}\sqrt{-1-x^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{(1+x^2)\arctan(x)}{\sqrt{2}\sqrt{-(1+x^2)^2}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + 2*x^2]),x]
[Out] ((1 + x^2)*ArcTan[x])/(Sqrt[2]*Sqrt[-(1 + x^2)^2])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.32

method	result	size
meijerg	$-\frac{i\sqrt{2}\,\arctan(x)}{2}$	9
default	$-\frac{\sqrt{-x^2-1}\,\sqrt{2}\,\arctan(x)}{2\sqrt{x^2+1}}$	24
risch	$\frac{\sqrt{\frac{(-x^2-1)(2x^2+2)}{(x^2+1)^2}}\,(x^2+1)\left(-\frac{i\sqrt{-2}\,\ln(x+i)}{4}+\frac{i\sqrt{-2}\,\ln(x-i)}{4}\right)}{\sqrt{-x^2-1}\,\sqrt{2x^2+2}}$	72

```
[In] int(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*2^(1/2)*arctan(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(23) = 46$.

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.71

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}x+\sqrt{2}(x^4-1)\right)}{x^4+2x^2+1}\right) - \frac{1}{8}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2x^2+2}\sqrt{-x^2-1}x-\sqrt{2}(x^4-1)\right)}{x^4+2x^2+1}\right)$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x + sqrt(2)*(x^4 - 1))/
(x^4 + 2*x^2 + 1)) - 1/8*sqrt(2)*log(2*(2*sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)*x
- sqrt(2)*(x^4 - 1))/(x^4 + 2*x^2 + 1))
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \frac{\sqrt{2}\int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+1}}\,dx}{2}$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(2*x**2+2)**(1/2),x)
[Out] sqrt(2)*Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 1)), x)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(2*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+2x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2\,x^2+2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2*x^2 + 2)^(1/2)), x)
```

3.252 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] 1/2*(1/(x^2+1))^(1/2)*(x^2+1)^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
(x^2+2)^(1/2)*2^(1/2)/(-x^2-1)^(1/2)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-x^2-1}\sqrt{\frac{x^2+2}{x^2+1}}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
```

```
[Out] (Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(Sqrt[2]*Sqrt[-1 - x^2]*Sqrt[(2 +
 x^2)/(1 + x^2)])
```

<u>Rule 429</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+x^2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = -\frac{i\sqrt{1+x^2}\sqrt{2+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),\frac{1}{2}\right)}{\sqrt{2}\sqrt{-\left((1+x^2)\left(2+x^2\right)\right)}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 + x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + x^2]*EllipticF[I*ArcSinh[x], 1/2])/(Sqrt[2]*Sq
rt[-((1 + x^2)*(2 + x^2))])
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{iF\left(ix,\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	33
elliptic	$-\frac{i\sqrt{-(x^2+1)(x^2+2)}\,\sqrt{x^2+1}\,\sqrt{2x^2+4}\,F\left(ix,\frac{\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{x^2+2}\,\sqrt{-x^4-3x^2-2}}$	74

```
[In] int(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \frac{1}{2}i\,\sqrt{2}\sqrt{-2}F\left(\arcsin\left(\frac{1}{2}i\,\sqrt{2}x\right)\,|\,2\right)$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/2*I*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*I*sqrt(2)*x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{x^2+2}}\,dx$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(-x**2 - 1)*sqrt(x**2 + 2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2+x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{x^2+2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(x^2 + 2)^(1/2)), x)
```

3.253 $\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 31

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}} \, dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{\sqrt{-1-x^2}}$$

```
[Out] EllipticF(1/2*x*2^(1/2),I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}} \, dx = \frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{\sqrt{2}}\right),-2\right)}{\sqrt{-x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[x/Sqrt[2]], -2])/Sqrt[-1 - x^2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-2\right)}{\sqrt{-1-x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = -\frac{i\sqrt{1+x^2}\,\text{EllipticF}\left(i\text{arcsinh}(x),-\frac{1}{2}\right)}{\sqrt{2}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 - x^2]*Sqrt[2 - x^2]),x]
[Out] ((-I)*Sqrt[1 + x^2]*EllipticF[I*ArcSinh[x], -1/2])/(Sqrt[2]*Sqrt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{iF\left(ix,\frac{i\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-2x^2+4}\,F\left(ix,\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-x^2-1}\,\sqrt{-x^2+2}\,\sqrt{x^4-x^2-2}}$	74

```
[In] int(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*I*2^(1/2))/(x^2+1)^(1/2)*2^(1/2)*(-x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = -\frac{1}{2}\,\sqrt{2}\sqrt{-2}F(\arcsin\left(\frac{1}{2}\,\sqrt{2}x\right)\,|-2)$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(2)*x), -2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{2-x^2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x**2-1)**(1/2)/(-x**2+2)**(1/2),x)
[Out] Integral(1/(sqrt(2 - x**2)*sqrt(-x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-x^2-1)^(1/2)/(-x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 + 2)*sqrt(-x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1-x^2}\sqrt{2-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-x^2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - x^2)^(1/2)), x)
```

3.254 $\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}} dx$

Optimal result

Integrand size = 23, antiderivative size = 42

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}} dx = -\frac{\sqrt{1-\frac{1}{x^4}}x^2 \operatorname{EllipticF}\left(\csc^{-1}(x),-1\right)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}$$

```
[Out] -x^2*EllipticF(1/x,I)*(1-1/x^4)^(1/2)/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.55, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {259, 228}

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}} dx = \frac{\sqrt{x^2-1}\sqrt{x^2+1} \operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}x}{\sqrt{x^2-1}}\right),\frac{1}{2}\right)}{2\sqrt{-x^2-1}\sqrt{1-x^2}}$$

```
[In] Int[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]],
 1/2])/(2*Sqrt[-1 - x^2]*Sqrt[1 - x^2])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{-2+2x^4}\int \frac{1}{\sqrt{-2+2x^4}}\,dx}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\\
&= \frac{\sqrt{-1+x^2}\sqrt{1+x^2}F\left(\sin^{-1}\left(\frac{\sqrt{2}x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{2\sqrt{-1-x^2}\sqrt{1-x^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$
\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{x\sqrt{1-x^4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^4\right)}{\sqrt{2-2x^2}\sqrt{-1-x^2}}
$$

```
[In] Integrate[1/(Sqrt[2 - 2*x^2]*Sqrt[-1 - x^2]),x]
[Out] (x*Sqrt[1 - x^4]*Hypergeometric2F1[1/4, 1/2, 5/4, x^4])/(Sqrt[2 - 2*x^2]*Sq
rt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{iF(ix,i)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	30
elliptic	$-\frac{i\sqrt{x^4-1}\,\sqrt{x^2+1}\,F(ix,i)}{\sqrt{-x^2-1}\,\sqrt{2x^4-2}}$	43

```
[In] int(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,I)*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{2}\sqrt{-2}F(\arcsin{(x)}\,|-1)$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(-2)*elliptic_f(arcsin(x), -1)
```

Sympy [A] (verification not implemented)

Time = 11.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{2}G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{2},1,1 & \frac{3}{4},\frac{3}{4},\frac{5}{4}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4} & 0\end{matrix}\middle|\frac{1}{x^4}\right)}{16\pi^{\frac{3}{2}}} - \frac{\sqrt{2}G_{6,6}^{3,5}\left(\begin{matrix}-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4} & 1\\ 0,\frac{1}{2},0 & -\frac{1}{4},\frac{1}{4},\frac{1}{4}\end{matrix}\middle|\frac{e^{-2i\pi}}{x^4}\right)}{16\pi^{\frac{3}{2}}}$$

```
[In] integrate(1/(-2*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] sqrt(2)*meijerg(((1/2, 1, 1), (3/4, 3/4, 5/4)), ((1/4, 1/2, 3/4, 1, 5/4), (
0,)), x**(-4))/(16*pi**(3/2)) - sqrt(2)*meijerg(((-1/4, 0, 1/4, 1/2, 3/4),
(1,)), ((0, 1/2, 0), (-1/4, 1/4, 1/4)), exp_polar(-2*I*pi)/x**4)/(16*pi**(3
/2))
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-2\,x^2+2}}\,dx$$

```
[In] integrate(1/(-2*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-2*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-2x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-2\,x^2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 2*x^2)^(1/2)), x)
```

3.255 $\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

```
[Out] 1/3*EllipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))*(x^2+1)^(1/2)*3^(1/2)/(-x^2-1)^(
1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-3x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}\\ &= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right),-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 3*x^2]*Sqrt[-1 - x^2]),x]
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3])/(Sqrt[3]*Sqrt[-1 - x^2
])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{iF\left(ix,\frac{i\sqrt{6}}{2}\right)\sqrt{-x^2-1}\,\sqrt{2}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(3x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-6x^2+4}\,F\left(ix,\frac{i\sqrt{6}}{2}\right)}{2\sqrt{-3x^2+2}\,\sqrt{-x^2-1}\,\sqrt{3x^4+x^2-2}}$	76

```
[In] int(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*I*6^(1/2))/(x^2+1)^(1/2)*(-x^2-1)^(1/2)*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{6}\sqrt{3}\sqrt{2}\sqrt{-2}F(\arcsin\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right) | -\frac{2}{3})$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/6*sqrt(3)*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(3)*sqrt(2)*x), -2/
3)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{2-3x^2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-3*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 3*x**2)*sqrt(-x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-3*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-3x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-3\,x^2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 3*x^2)^(1/2)), x)
```

3.256 $\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

```
[Out] 1/2*EllipticF(x*2^(1/2),1/2*I*2^(1/2))*(x^2+1)^(1/2)/(-x^2-1)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{432, 430\}$

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-4x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}$$
$$= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{2}x\right),-\frac{1}{2}\right)}{2\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 4*x^2]*Sqrt[-1 - x^2]),x]
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/2])/(2*Sqrt[-1 - x^2])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{iF\left(ix,i\sqrt{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(2x^2-1)(x^2+1)}\,\sqrt{x^2+1}\,F\left(ix,i\sqrt{2}\right)}{\sqrt{-x^2-1}\,\sqrt{4x^4+2x^2-2}}$	60

```
[In] int(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,I*2^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{4}\sqrt{2}\sqrt{-2}F(\arcsin\left(\sqrt{2}x\right)|-\frac{1}{2})$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/4*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(sqrt(2)*x), -1/2)
```

Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{2}\left(\begin{cases} -\frac{\sqrt{2}iF\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{2}\right)}{2} & \text{for}\ x>-\frac{\sqrt{2}}{2}\wedge x<\frac{\sqrt{2}}{2} \end{cases}\right)}{2}$$

```
[In] integrate(1/(-4*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] sqrt(2)*Piecewise((-sqrt(2)*I*elliptic_f(asin(sqrt(2)*x), -1/2)/2, (x > -sq
rt(2)/2) & (x < sqrt(2)/2)))/2
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-4\,x^2+2}}\,dx$$

```
[In] integrate(1/(-4*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-4*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-4x^2}\sqrt{-1-x^2}}\,dx=\int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-4\,x^2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 4*x^2)^(1/2)), x)
```

3.257 $\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

```
[Out] 1/5*EllipticF(1/2*x*10^(1/2),1/5*I*10^(1/2))*(x^2+1)^(1/2)*5^(1/2)/(-x^2-1)
^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{x^2+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-x^2-1}}$$

```
[In] Int[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
```

```
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1+x^2}\int \frac{1}{\sqrt{2-5x^2}\sqrt{1+x^2}}\,dx}{\sqrt{-1-x^2}}\\ &= \frac{\sqrt{1+x^2}F\left(\sin^{-1}\left(\sqrt{\frac{5}{2}}x\right)|-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \frac{\sqrt{1+x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{5}{2}}x\right),-\frac{2}{5}\right)}{\sqrt{5}\sqrt{-1-x^2}}$$

```
[In] Integrate[1/(Sqrt[2 - 5*x^2]*Sqrt[-1 - x^2]),x]
[Out] (Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[5/2]*x], -2/5])/(Sqrt[5]*Sqrt[-1 - x^2
])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{iF\left(ix,\frac{i\sqrt{10}}{2}\right)\sqrt{2}\,\sqrt{-x^2-1}}{2\sqrt{x^2+1}}$	34
elliptic	$-\frac{i\sqrt{(5x^2-2)(x^2+1)}\,\sqrt{x^2+1}\,\sqrt{-10x^2+4}\,F\left(ix,\frac{i\sqrt{10}}{2}\right)}{2\sqrt{-5x^2+2}\,\sqrt{-x^2-1}\,\sqrt{5x^4+3x^2-2}}$	78

```
[In] int(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*I*EllipticF(I*x,1/2*I*10^(1/2))*2^(1/2)/(x^2+1)^(1/2)*(-x^2-1)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = -\frac{1}{10}\sqrt{5}\sqrt{2}\sqrt{-2}F(\arcsin\left(\frac{1}{2}\sqrt{5}\sqrt{2}x\right)|-\frac{2}{5})$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="fricas")
[Out] -1/10*sqrt(5)*sqrt(2)*sqrt(-2)*elliptic_f(arcsin(1/2*sqrt(5)*sqrt(2)*x), -2
/5)
```

Sympy [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{2-5x^2}\sqrt{-x^2-1}}\,dx$$

```
[In] integrate(1/(-5*x**2+2)**(1/2)/(-x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(2 - 5*x**2)*sqrt(-x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\sqrt{-5\,x^2+2}}\,dx$$

```
[In] integrate(1/(-5*x^2+2)^(1/2)/(-x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-x^2 - 1)*sqrt(-5*x^2 + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2-5x^2}\sqrt{-1-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2-1}\,\sqrt{2-5\,x^2}}\,dx$$

```
[In] int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)),x)
[Out] int(1/((- x^2 - 1)^(1/2)*(2 - 5*x^2)^(1/2)), x)
```

3.258 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\middle|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\
&= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int\frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\\
&= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)
*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(
d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{-bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid-\frac{ad}{bc})+\sqrt{bx^2+a}\sqrt{-dx^2+c}bcd-(bc^2+ad^2)\sqrt{-bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{}}{}\right.}{bcd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c))
+ sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*sqr
t(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2)/sqrt(c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{\sqrt{c-d\,x^2}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.259 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}}\\
&= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\\
&= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}}$$

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(75) = 150.

Time = 2.48 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2+c}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{-d\,x^2+c}{c}}\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)}{\left(-bd\,x^4-ad\,x^2+cb\,x^2+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-(b\,x^2+a)(-d\,x^2+c)}\left(-\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-cb\,x^2-ac}}-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1-\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2-cb\,x^2-ac}\,d}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2+c}}$

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*
(a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2
),(-a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*d*x
^4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{-dx^2+c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right.}{bcd^2x}$$

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
 sqrt(-b*x^2 - a)*sqrt(-d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt
(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(-a - b*x**2)/sqrt(c - d*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)

Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 + c), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

[In] int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)

[Out] int((- a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)

3.260 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}} \, dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}} \, dx = \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\
&= \frac{\left(\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int\frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&= \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c)
/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(73) = 146.

Time = 2.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

method	result
default	$\frac{\left(-aF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}}+\frac{bc\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac}\,d}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/
2),(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x^2
+a)^(1/2)*(d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(-b*d*x^
4-a*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$
$$=\frac{\sqrt{bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc})+\sqrt{bx^2+a}\sqrt{dx^2-c}bcd-(bc^2+ad^2)\sqrt{bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{a}{b}}{bcd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
sqrt(b*x^2 + a)*sqrt(d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt(c/
d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2-c}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.261 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)*(1-d
*x^2/c)^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[d]*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int\frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\
&= \frac{\left(\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int\frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&= \frac{\sqrt{c}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\frac{\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], -((b*c
)/(a*d))])/(Sqrt[d/c]*Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(bd\,x^{4}+ad\,x^{2}-cb\,x^{2}-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(-d\,x^{2}+c)}\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/(d/c)^(1/2)*(-b*x^2-a)^(1/2)*(d*x^2-c)^(1/2
)*a*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/
a/d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{-bd}bc^2x\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{dx^2-c}bcd - (bc^2+ad^2)\sqrt{-bd}x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)}{bcd^2x}$$

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*b*c^2*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
 sqrt(-b*x^2 - a)*sqrt(d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*sqrt
(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

```
[In] integrate((-b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

```
[In] int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
[Out] int((- a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.262 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}} \\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} \\
&= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}$$

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\cdots}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}\cdots\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2+c)^(1/2)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(
d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2+c}b}{bdx}$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqrt
(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sqrt
(-b*x^2 + a)*sqrt(-d*x^2 + c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx$$

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{c-d\,x^2}}\,dx$$

```
[In] int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.263 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*x
^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{c-dx^2}} \\
&= \frac{\left(\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} \\
&= \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$
\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}}
$$

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c - d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(74) = 148$.

Time = 2.49 (sec), antiderivative size = 161, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\left(aF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}+\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}\,d}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

[In] int((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] (b*x^2-a)^(1/2)*(-d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*(a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec), antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = -\frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})-\sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})+\sqrt{bx^2-a}\sqrt{-dx^2+c}b}{bdx}$$

[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqrt(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sqrt(b*x^2 - a)*sqrt(-d*x^2 + c)*b)/(b*d*x)

Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx$$

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2+c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c-dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{c-d\,x^2}}\,dx$$

```
[In] int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(c - d*x^2)^(1/2), x)
```

3.264 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2+a)^(1/2)*(1-d*
x^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/S
qrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\
&= \frac{\left(\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&= \frac{\sqrt{c}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/(
a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.

Time = 2.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.82

method	result
default	$\frac{\left(-aF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcE\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}}-\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac}\,d}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),
(a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2+a)^(
1/2)*(d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)/(b*d*x^4-a*d
*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$
$$= \frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2-c}b}{bdx}$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqrt
(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sqr
t(-b*x^2 + a)*sqrt(d*x^2 - c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx$$

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2-c)**(1/2),x)
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2-c}}\,dx$$

```
[In] int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.265 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}} dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}} dx = \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*c^(1/2)*(b*x^2-a)^(1/2)*(1-d*x
^2/c)^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}} dx = \frac{\sqrt{c}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/
Sqrt[c]], (b*c)/(a*d)])/(Sqrt[d]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{dx^2}{c}}\int \frac{\sqrt{-a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{-c+dx^2}}\\
&= \frac{\left(\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&= \frac{\sqrt{c}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\mid\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$
\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}\,E\left(\arcsin\left(\sqrt{\frac{d}{c}}x\right)\mid\frac{bc}{ad}\right)}{\sqrt{\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}}
$$

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c + d*x^2],x]
```

```
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c]*EllipticE[ArcSin[Sqrt[d/c]*x], (b*c)/
(a*d)])/(Sqrt[d/c]*Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}\,a\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{\frac{d}{c}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}+\frac{a\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(d/c)^(1/2)*(b*x^2-a)^(1/2)*(d*x^2-c)^(1/2
)*a*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/
a/d)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$
$$=\frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})-\sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid\frac{bc}{ad})+\sqrt{bx^2-a}\sqrt{dx^2-c}b}{bdx}$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) - sqrt(
b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)) + sqrt(
b*x^2 - a)*sqrt(d*x^2 - c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx=\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx$$

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2-c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2-c}}\,dx$$

```
[In] int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(d*x^2 - c)^(1/2), x)
```

3.266 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 194

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*E
llipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*
x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x
^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcT
an[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - c\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\,dx\\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)\mid\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[c + d*x^2],x]
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\,\left(\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] (b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(a*
EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a)^(1/2),(a
*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/(b*d*x^4+a*d*
x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}bcd - (bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right.}{bcd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt(
-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.267 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} dx = \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{-a-bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] x*(-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-
b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[c + d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE[
ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))
/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(a\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx\right)-b\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx\\
&= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}-c\int \frac{\sqrt{-a-bx^2}}{(c+dx^2)^{3/2}}\,dx
\end{aligned}$$

$$= \frac{x\sqrt{-a-bx^2}}{\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.51

method	result
default	$\frac{\sqrt{-b\,x^2-a}\,\sqrt{d\,x^2+c}\,a\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-(b\,x^2+a)(d\,x^2+c)}\left(-\frac{a\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}}+\frac{a\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{d\,x^2+c}}$

```
[In] int((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-d
/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{-bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\Bigg(\frac{\sqrt{-\frac{c}{d}}}{x}\Bigg) \mid \frac{ad}{bc}) - \sqrt{-bx^2-a}\sqrt{dx^2+c}bcd - (bc^2+ad^2)\sqrt{-bd}x\sqrt{-\frac{c}{d}}F(\arcsin\Bigg(}{bcd^2x}$$

[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(-b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(-b*x^2 - a)*sqrt(d*x^2 + c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)

Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

[In] integrate((-b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(sqrt(-a - b*x**2)/sqrt(c + d*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)

Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((-b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-b\,x^2-a}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((- a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.268 $\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 203

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*
EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b
*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1/(
1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)
^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/
(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}$$

```
[In] Int[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqr
t[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*Sqrt
[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx + b\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + c\int \frac{\sqrt{a+bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{a+bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{a+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}$$

```
[In] Integrate[Sqrt[a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*c
)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.53

method	result
default	$\frac{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}-cb\,x^{2}-ac}}}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

```
[In] int((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-
d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$
$$= \frac{\sqrt{-bd}bc^2x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{-dx^2-c}bcd - (bc^2+ad^2)\sqrt{-bd}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-}}{x}\right.}{bcd^2x}$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(-d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(-b*d)*x*sqr
t(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)/sqrt(-c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{-d\,x^2-c}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((a + b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.269 $\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 212

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] x*(-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)
*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(
-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+(1
/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/
c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(-b*x^2-a)^(1/2)/d^(1/2)/(-d*x^2-c)^(1/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{c}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}$$

```
[In] Int[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (x*Sqrt[-a - b*x^2])/Sqrt[-c - d*x^2] - (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticE
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (Sqrt[c]*Sqrt[-a - b*x^2]*EllipticF
[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[-c - d*x^2]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(a\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\right)-b\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\\
&= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}+\frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+c\int \frac{\sqrt{-a-bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx
\end{aligned}
$$

$$
= \frac{x\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.43

$$
\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}}
$$

```
[In] Integrate[Sqrt[-a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], (b*
c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.78

method	result
default	$\frac{\left(-aF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bcE\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)\right)\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2-c}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}}{\left(bd\,x^4+ad\,x^2+cb\,x^2+ac\right)\sqrt{-\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{a\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{bc\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}\,d}\right)}{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2-c}}$

```
[In] int((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(-b/a)^(1/2
),(a*d/b/c)^(1/2))-b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))*(-b*x^2-a
)^(1/2)*(-d*x^2-c)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)/(-b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$
$$= \frac{\sqrt{bd}bc^2x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-\sqrt{-bx^2-a}\sqrt{-dx^2-c}bcd-(bc^2+ad^2)\sqrt{bd}x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\ldots}}{x}\right.\right.}{bcd^2x}$$

[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")

[Out] (sqrt(b*d)*b*c^2*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)*b*c*d - (b*c^2 + a*d^2)*sqrt(b*d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)))/(b*c*d^2*x)

Sympy [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

[In] integrate((-b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)

[Out] Integral(sqrt(-a - b*x**2)/sqrt(-c - d*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)

Giac [F]

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

```
[In] int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((- a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

```
[Out] -EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(d*x^2+c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sq
rt[b]*x)/Sqrt[a]], -((a*d)/(b*c))]])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
) + (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sq
rt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx}{d} + \frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{d}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}$$

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(aF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d+bcF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)-bcE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)}{(-bd\,x^4+ad\,x^2-cb\,x^2+ac)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(d\,x^2+c)}\left(\frac{a\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}}+\frac{bc\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}\,d}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*(
a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d+b*c*EllipticF(x*(b/a)^(1/2),(
-a*d/b/c)^(1/2))-b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))/(-b*d*x^4+a
*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2+c}b}{bdx}$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sqr
t(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + s
qrt(-b*x^2 + a)*sqrt(d*x^2 + c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((-b*x**2+a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.271 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{c+dx^2}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(
1/2)*(d*x^2+c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Elliptic
F(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)
^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}} - \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt
[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
- (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]*Sqr
t[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx}{d}-\frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{c+dx^2}}\,dx}{d}\\
&= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}}-\frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}}\\
&= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}-\frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{c+dx^2}}\\
&= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\\
&\quad-\frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$
\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx=\frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}}
$$

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[c + d*x^2],x]
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}+\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}}$

```
[In] int((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2-a)^(1/2)*(d*x^2+c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*
EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(
-d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{dx^2+c}b}{bdx}$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sqrt
(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + sqr
t(b*x^2 - a)*sqrt(d*x^2 + c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2-a)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-a + b*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(c + d*x^2)^(1/2), x)
```

3.272 $\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(
1/2)*(-d*x^2-c)^(1/2)/d/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}} + \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqr
t[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
+ (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2]*Sqrt
[-c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx}{d}+\frac{(bc+ad)\int \frac{1}{\sqrt{a-bx^2}\sqrt{-c-dx^2}}\,dx}{d}\\
&= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}}+\frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-c-dx^2}}\\
&= \frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}+\frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{a-bx^2}\sqrt{-c-dx^2}}\\
&= \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\
&\quad+\frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{-c-dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.47

$$
\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx=\frac{\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}\,E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}
$$

```
[In] Integrate[Sqrt[a - b*x^2]/Sqrt[-c - d*x^2],x]
[Out] (Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((b
*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}\,a\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)}{(bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{-(-b\,x^{2}+a)(d\,x^{2}+c)}\left(\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}-\frac{a\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

```
[In] int((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x^2+a)^(1/2)*(-d*x^2-c)^(1/2)*a*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/
(-d/c)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx =$$
$$-\frac{\sqrt{bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) - \sqrt{bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2-c}b}{bdx}$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sqr
t(b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + sq
rt(-b*x^2 + a)*sqrt(-d*x^2 - c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx$$

```
[In] integrate((-b*x**2+a)**(1/2)/(-d*x**2-c)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**2)/sqrt(-c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-bx^2+a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((-b*x^2+a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-b*x^2 + a)/sqrt(-d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{a-b\,x^2}}{\sqrt{-d\,x^2-c}}\,dx$$

```
[In] int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((a - b*x^2)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.273 $\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

```
[Out] -EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^
(1/2)*(-d*x^2-c)^(1/2)/d/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/
c)^(1/2)/d/b^(1/2)/(b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}(ad+bc)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{bx^2-a}\sqrt{-c-dx^2}} - \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
```

```
[Out] -((Sqrt[a]*Sqrt[b]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(d*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c
])) - (Sqrt[a]*(b*c + a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[-a + b*x^2]
*Sqrt[-c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\frac{b\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx}{d} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{-c-dx^2}}\,dx}{d}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-c-dx^2}}$$

$$= -\frac{\left(b\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\left((bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

$$= -\frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{d\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} - \frac{\sqrt{a}(bc+ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{-a+bx^2}\sqrt{-c-dx^2}}$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \frac{\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}}$$

```
[In] Integrate[Sqrt[-a + b*x^2]/Sqrt[-c - d*x^2],x]
[Out] (Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c)]*x], -((
b*c)/(a*d))])/(Sqrt[-(d/c)]*Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

method	result
default	$\frac{\left(-aF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)d-bcF\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)+bcE\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)\right)\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{b}{a}}\,d}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{a\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}-\frac{bc\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{a}{}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}\,d}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

```
[In] int((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(b/a)^(1/2)
,(-a*d/b/c)^(1/2))+b*c*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2)))*(b*x^2-a)
^(1/2)*(-d*x^2-c)^(1/2)*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)/(-b*d*x^4+
a*d*x^2-b*c*x^2+a*c)/(b/a)^(1/2)/d
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = -\frac{\sqrt{-bd}ax\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) - \sqrt{-bd}(a-b)x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{-dx^2-}}{bdx}$$

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b*d)*a*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) - sq
rt(-b*d)*(a - b)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
sqrt(b*x^2 - a)*sqrt(-d*x^2 - c)*b)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx$$

```
[In] integrate((b*x**2-a)**(1/2)/(-d*x**2-c)**(1/2),x)
[Out] Integral(sqrt(-a + b*x**2)/sqrt(-c - d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Giac [F]

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{bx^2-a}}{\sqrt{-dx^2-c}}\,dx$$

```
[In] integrate((b*x^2-a)^(1/2)/(-d*x^2-c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 - a)/sqrt(-d*x^2 - c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-a+bx^2}}{\sqrt{-c-dx^2}}\,dx = \int \frac{\sqrt{b\,x^2-a}}{\sqrt{-d\,x^2-c}}\,dx$$

```
[In] int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2),x)
[Out] int((b*x^2 - a)^(1/2)/(- c - d*x^2)^(1/2), x)
```

3.274 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2+a}\,c\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(-bd\,x^4+ad\,x^2-cb\,x^2+ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^2+a\right)\left(d\,x^2+c\right)}\left(\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}}-\frac{c\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticE(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(
b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2+c}abd - (b^2c+a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\right.}{ab^2dx}$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d))
+ sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*a*b*d - (b^2*c + a^2*d)*sqrt(-b*d)*x*sqr
t(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{a-b\,x^2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.275 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}\\ &= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\ &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(75) = 150$.

Time = 2.42 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.87

method	result
default	$\frac{\sqrt{-d\,x^{2}-c}\,\sqrt{-b\,x^{2}+a}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b-adE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(-b\,x^{2}+a)(d\,x^{2}+c)}\,\left(-\frac{c\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}-\frac{da\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}\,b}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}-c}}$

[In] int((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*(a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*d*x^4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx =$$
$$-\frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2-c}abd - (b^2c+a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right.}{ab^2dx}$$

[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) + sqrt(-b*x^2 + a)*sqrt(-d*x^2 - c)*a*b*d - (b^2*c + a^2*d)*sqrt(b*d)*x*sqrt(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)

Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

```
[In] int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((- c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.276 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*
x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\,E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int\frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a + b*x^2],x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d)
/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(73) = 146$.

Time = 2.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.90

method	result
default	$\frac{\left(-adF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cF\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adE\left(x\sqrt{-\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^{2}+c}\,\sqrt{b\,x^{2}-a}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}}{\left(-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(\frac{c\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}}+\frac{da\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac}\,b}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}+c}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2)
,(-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x^2
+c)^(1/2)*(b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$
$$=\frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad})+\sqrt{bx^2-a}\sqrt{dx^2+c}abd-(b^2c+a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -}{ab^2dx}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
sqrt(b*x^2 - a)*sqrt(d*x^2 + c)*a*b*d - (b^2*c + a^2*d)*sqrt(b*d)*x*sqrt(a/
b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a + bx^2}}\,dx = \int \frac{\sqrt{c + dx^2}}{\sqrt{-a + bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a + bx^2}}\,dx = \int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 - a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a + bx^2}}\,dx = \int \frac{\sqrt{dx^2 + c}}{\sqrt{bx^2 - a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c + dx^2}}{\sqrt{-a + bx^2}}\,dx = \int \frac{\sqrt{d\,x^2 + c}}{\sqrt{b\,x^2 - a}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.277 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d
*x^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1+d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{\frac{dx^2}{c}+1}}$$

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 + (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1+\frac{dx^2}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], -((a*d
)/(b*c))])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-d\,x^{2}-c}\,\sqrt{b\,x^{2}-a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)}{\left(bd\,x^{4}-ad\,x^{2}+cb\,x^{2}-ac\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(d\,x^{2}+c\right)}\left(-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}+\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1+\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad}{c}}\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^{4}+ad\,x^{2}-cb\,x^{2}+ac}}\right.}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}-c}}$

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(b*d*x^4-a*d*x^2+b*c*x^2-a*c)/(b/a)^(1/2)*(-d*x^2-c)^(1/2)*(b*x^2-a)^(1/2
)*c*((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*d/
b/c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\mid -\frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{-dx^2-c}abd - (b^2c+a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right.}{ab^2dx}$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), -b*c/(a*d)) +
 sqrt(b*x^2 - a)*sqrt(-d*x^2 - c)*a*b*d - (b^2*c + a^2*d)*sqrt(-b*d)*x*sqrt
(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), -b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

```
[In] int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
[Out] int((- c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.278 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d*
x^2+c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$
\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}
$$

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(73) = 146$.

Time = 2.44 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.82

method	result
default	$\frac{\left(-adF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+cF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+adE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^{2}+c}\,\sqrt{-b\,x^{2}+a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\left(\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}-\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}\,b}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

```
[In] int((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(b
*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2+c)^(
1/2)*(-b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)/(b*d*x^4-a*
d*x^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{-dx^2+c}abd + (b^2c-a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)}{ab^2dx}$$

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*a*b*d + (b^2*c - a^2*d)*sqrt(b*d)*x*sqrt(
a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx$$

```
[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(c - d*x**2)/sqrt(a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{a-b\,x^2}}\,dx$$

```
[In] int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(a - b*x^2)^(1/2), x)
```

3.279 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*x
^2-c)^(1/2)/b^(1/2)/(-b*x^2+a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}}\\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int\frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int\frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a - b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{-b\,x^{2}+a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)}{\left(b d\,x^{4}-a d\,x^{2}-c b\,x^{2}+a c\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}+c b\,x^{2}-a c}}+\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\right.\right.}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}+c b\,x^{2}-a c}}\right.}{\sqrt{-b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

[In] int((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (d*x^2-c)^(1/2)*(-b*x^2+a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4-a*d*x^2-b*c*x^2+a*c)/(b
/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = -\frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{-bx^2+a}\sqrt{dx^2-c}abd + (b^2c-a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right.}{ab^2dx}$$

[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
 sqrt(-b*x^2 + a)*sqrt(d*x^2 - c)*a*b*d + (b^2*c - a^2*d)*sqrt(-b*d)*x*sqrt
(a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)

Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx$$

[In] integrate((d*x**2-c)**(1/2)/(-b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(-c + d*x**2)/sqrt(a - b*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)

Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2+a}}\,dx$$

[In] integrate((d*x^2-c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{a-b\,x^2}}\,dx$$

[In] int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2),x)

[Out] int((d*x^2 - c)^(1/2)/(a - b*x^2)^(1/2), x)

3.280 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 89

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(-d*
x^2+c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/S
qrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c-dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}}\\ &= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\\ &= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a + b*x^2],x]
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/(
b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{b\,x^{2}-a}\,c\sqrt{\frac{-b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)}{\left(-b d\,x^{4}+a d\,x^{2}+c b\,x^{2}-a c\right)\sqrt{\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}+c b\,x^{2}-a c}}-\frac{c\sqrt{1-\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\frac{a d+b c}{c b}}\right)-E\left(x\sqrt{\frac{b}{a}},\sqrt{-1+\cdots}\right)\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-b d\,x^{4}+a d\,x^{2}+c b\,x^{2}-a c}}\right)}{\sqrt{b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2-a)^(1/2)*c*((-b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(b/a)^(1/2),(a*d/b/c)^(1/2))/(-b*d*x^4+a*d*x^2+b*c*x^2-a*c)/(
b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{-bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{-dx^2+c}abd + (b^2c-a^2d)\sqrt{-bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right)\cdots}{ab^2dx}$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) +
sqrt(b*x^2 - a)*sqrt(-d*x^2 + c)*a*b*d + (b^2*c - a^2*d)*sqrt(-b*d)*x*sqrt(
a/b)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx$$

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2-a}}\,dx$$

```
[In] int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.281 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 90

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}$$

```
[Out] EllipticE(x*b^(1/2)/a^(1/2),(a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(d*x
^2-c)^(1/2)/b^(1/2)/(b*x^2-a)^(1/2)/(1-d*x^2/c)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {438, 437, 435}

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx=\frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{dx^2-c}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{bx^2-a}\sqrt{1-\frac{dx^2}{c}}}$$

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[a]*Sqrt[1 - (b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[(Sqrt[b]*x)/
Sqrt[a]], (a*d)/(b*c)])/(Sqrt[b]*Sqrt[-a + b*x^2]*Sqrt[1 - (d*x^2)/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{-c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}} \\
&= \frac{\left(\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}\right)\int \frac{\sqrt{1-\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}} \\
&= \frac{\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{-c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{-a+bx^2}\sqrt{1-\frac{dx^2}{c}}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00

$$
\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \frac{\sqrt{\frac{a-bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{\frac{b}{a}}\sqrt{-a+bx^2}\sqrt{\frac{c-dx^2}{c}}}
$$

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a + b*x^2],x]
```

```
[Out] (Sqrt[(a - b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[b/a]*x], (a*d)/
(b*c)])/(Sqrt[b/a]*Sqrt[-a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(75) = 150$.

Time = 2.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

method	result
default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{b\,x^{2}-a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{-b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cF\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-adE\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)}{\left(bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{\left(-b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}}+\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1-\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-\right.\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^{4}-ad\,x^{2}-cb\,x^{2}+ac}\,b}}{\sqrt{b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(b*c
/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4-a*d*x
^2-b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$
$$= \frac{\sqrt{bd}a^2dx\sqrt{\frac{a}{b}}E(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad}) + \sqrt{bx^2-a}\sqrt{dx^2-c}abd + (b^2c-a^2d)\sqrt{bd}x\sqrt{\frac{a}{b}}F(\arcsin\left(\frac{\sqrt{\frac{a}{b}}}{x}\right) \mid \frac{bc}{ad})}{ab^2dx}$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*a^2*d*x*sqrt(a/b)*elliptic_e(arcsin(sqrt(a/b)/x), b*c/(a*d)) + s
qrt(b*x^2 - a)*sqrt(d*x^2 - c)*a*b*d + (b^2*c - a^2*d)*sqrt(b*d)*x*sqrt(a/b
)*elliptic_f(arcsin(sqrt(a/b)/x), b*c/(a*d)))/(a*b^2*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx$$

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2-a)**(1/2),x)
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2-a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2-a}}\,dx$$

```
[In] int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(b*x^2 - a)^(1/2), x)
```

3.282 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 204

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx=\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}+\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] d*x*(b*x^2+a)^(1/2)/b/(d*x^2+c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^
2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))
*(b*x^2+a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-
(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^
2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(c*(b*x^2+a
)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx=\frac{c^{3/2}\sqrt{a+bx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+\frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2
]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^(3/2)*Sqrt[a + b*x^2]*Ellipt
icF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= c\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx \\
&= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{(cd)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}
\end{aligned}
$$

$$= \frac{dx\sqrt{a+bx^2}}{b\sqrt{c+dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)/(-b/
a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{bx^2+a}\sqrt{dx^2+c}d}{bdx}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.283 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] -d*x*(b*x^2+a)^(1/2)/b/(-d*x^2-c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(b*x^2+a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/2)/b/(-d*x^2-
c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}$$

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((d*x*Sqrt[a + b*x^2])/(b*Sqrt[-c - d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[a + b
*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
 d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) - (c^(3/2)*Sqrt[a + b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(c\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx\right)-d\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{-c-dx^2}}\,dx\\ &= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}}-\frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{(cd)\int \frac{\sqrt{a+bx^2}}{\left(-c-dx^2\right)^{3/2}}\,dx}{b}\end{aligned}$$

$$
= -\frac{dx\sqrt{a+bx^2}}{b\sqrt{-c-dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.42

$$
\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c+dx^2}{c}}}
$$

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

method	result
default	$\frac{\left(-adF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)+cF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b+adE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)\sqrt{-d\,x^2-c}\,\sqrt{b\,x^2+a}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(b\,x^2+a)(d\,x^2+c)}\left(-\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}}+\frac{da\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{}\right)\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}\,b}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2-c}}$

```
[In] int((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-a*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))+c*EllipticF(x*(-d/c)^(1/2),
(b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))*(-d*x^2-c
)^(1/2)*(b*x^2+a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{-bd}cx\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-\sqrt{-bd}(c+d)x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-\sqrt{bx^2+a}\sqrt{-dx^2}}{bdx}$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(-b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - s
qrt(-b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c))
- sqrt(b*x^2 + a)*sqrt(-d*x^2 - c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((-d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c - d*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((- c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.284 $\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 214

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}+\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -d*x*(-b*x^2-a)^(1/2)/b/(d*x^2+c)^(1/2)-c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)+(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d
*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(c*(b*x
^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}}$$

```
[In] Int[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((d*x*Sqrt[-a - b*x^2])/(b*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[d]*Sqrt[-a -
b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*Sqrt[-a - b*x^2]*
EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = c\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx + d\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{c+dx^2}}\,dx$$

$$= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{(cd)\int \frac{\sqrt{-a-bx^2}}{(c+dx^2)^{3/2}}\,dx}{b}$$

$$
= -\frac{dx\sqrt{-a-bx^2}}{b\sqrt{c+dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} - \frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.42

$$
\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}
$$

```
[In] Integrate[Sqrt[c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d
)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.76

method	result
default	$\frac{\sqrt{d\,x^2+c}\,\sqrt{-b\,x^2-a}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,\left(adF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)-cF\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)b-adE\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)\right)}{(bd\,x^4+ad\,x^2+cb\,x^2+ac)\sqrt{-\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{-(b\,x^2+a)(d\,x^2+c)}\left(\frac{c\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}}-\frac{da\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)-E\left(x\sqrt{-\frac{d}{c}},\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2-cb\,x^2-ac}\,b}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{d\,x^2+c}}$

```
[In] int((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*((d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(a
*d*EllipticF(x*(-d/c)^(1/2),(b*c/a/d)^(1/2))-c*EllipticF(x*(-d/c)^(1/2),(b*
c/a/d)^(1/2))*b-a*d*EllipticE(x*(-d/c)^(1/2),(b*c/a/d)^(1/2)))/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)/(-d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{-bd}cx\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{-bd}(c+d)x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - \sqrt{-bx^2-a}\sqrt{dx^2+c}}{bdx}$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sq
rt(-b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
 sqrt(-b*x^2 - a)*sqrt(d*x^2 + c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/sqrt(-a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{-b\,x^2-a}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((c + d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.285 $\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 222

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[Out] d*x*(-b*x^2-a)^(1/2)/b/(-d*x^2-c)^(1/2)+c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))*(-b*x^2-a)^(1/2)/a/d^(1/2)/(-d*x^2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1
/2)-(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1/2)/b/(-d*x^
2-c)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{c^{3/2}\sqrt{-a-bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}}$$

```
[In] Int[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (d*x*Sqrt[-a - b*x^2])/(b*Sqrt[-c - d*x^2]) - (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[-c -
d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]) + (c^(3/2)*Sqrt[-a - b*x^2]*E
llipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[-c
- d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\left(c\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx\right) - d\int \frac{x^2}{\sqrt{-a-bx^2}\sqrt{-c-dx^2}}\,dx \\
&= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}} + \frac{c^{3/2}\sqrt{-a-bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{(cd)\int \frac{\sqrt{-a-bx^2}}{(-c-dx^2)^{3/2}}\,dx}{b}
\end{aligned}
$$

$$= \frac{dx\sqrt{-a-bx^2}}{b\sqrt{-c-dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{c^{3/2}\sqrt{-a-bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{-c-dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c+dx^2}{c}}}$$

```
[In] Integrate[Sqrt[-c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*
d)/(b*c)])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c + d*x^2)/c])
```

Maple [A] (verified)

Time = 2.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.50

method	result
default	$\frac{\sqrt{-d\,x^2-c}\,\sqrt{-b\,x^2-a}\,c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\left(-bd\,x^4-ad\,x^2-cb\,x^2-ac\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}+\frac{c\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ac}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}\right.}{\sqrt{-b\,x^2-a}\,\sqrt{-d\,x^2-c}}$

```
[In] int((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-b*d*x^4-a*d*x^2-b*c*x^2-a*c)/(-b/a)^(1/2)*(-d*x^2-c)^(1/2)*(-b*x^2-a)^(
1/2)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{bd}cx\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - \sqrt{bd}(c+d)x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - \sqrt{-bx^2-a}\sqrt{-dx^2-cd}}{bdx}$$

[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] (sqrt(b*d)*c*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(b*d)*(c + d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - sqrt(-b*x^2 - a)*sqrt(-d*x^2 - c)*d)/(b*d*x)

Sympy [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

[In] integrate((-d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)

[Out] Integral(sqrt(-c - d*x**2)/sqrt(-a - b*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)

Giac [F]

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

```
[In] int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((- c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.286 $\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] -EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1
/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}} - \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
) + (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sq
rt[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = -\frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}}$$

$$= -\frac{\left(d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

$$= -\frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[a + b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.85

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{b\,x^{2}+a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}\,\left(adF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)+cF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b-adE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{(b\,x^{2}+a)(-d\,x^{2}+c)}\left(\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}+\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-c}{}}\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}\,b}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{-d\,x^{2}+c}}$

```
[In] int((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*(
a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))+c*EllipticF(x*(d/c)^(1/2),(-b
*c/a/d)^(1/2))*b-a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))/(-b*d*x^4-a
*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) - \sqrt{-bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{bx^2+a}\sqrt{-dx^2+c}d}{bdx}$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sqr
t(-b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) + s
qrt(b*x^2 + a)*sqrt(-d*x^2 + c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((-d*x**2+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c - d*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-dx^2+c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c-d\,x^2}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.287 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(b*x^2+a)^(1/
2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Elliptic
F(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)
^(1/2)/b/d^(1/2)/(b*x^2+a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}} - \frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt
[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2])
- (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[-c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{\sqrt{a+bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{a+bx^2}\sqrt{-c+dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}}$$

$$= \frac{\left(d\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{a+bx^2}\sqrt{-c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{a+bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[a + b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[a + b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{d\,x^{2}-c}\,\sqrt{b\,x^{2}+a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right)}{\left(-b d\,x^{4}-a d\,x^{2}+c b\,x^{2}+a c\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-(b\,x^{2}+a)(-d\,x^{2}+c)}\,\left(-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}-c b\,x^{2}-a c}}+\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}-c b\,x^{2}-a c}}\right.}{\sqrt{b\,x^{2}+a}\,\sqrt{d\,x^{2}-c}}$

```
[In] int((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (d*x^2-c)^(1/2)*(b*x^2+a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2)*
EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(-b*d*x^4-a*d*x^2+b*c*x^2+a*c)/(
-b/a)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) - \sqrt{bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{bx^2+a}\sqrt{dx^2-c}d}{bdx}$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sqrt
(b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) + sqr
t(b*x^2 + a)*sqrt(d*x^2 - c)*d)/(b*d*x)
```

Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((d*x**2-c)**(1/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(-c + d*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(a + b*x^2)^(1/2), x)
```

3.288 $\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 194

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(1
/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}} + \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}}$$

```
[In] Int[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqr
t[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[c - d*x^2])
+ (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[Ar
cSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]*Sqr
t[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{c-dx^2}}\,dx}{b} + \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{c-dx^2}}\,dx}{b}$$

$$= \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{c-dx^2}}$$

$$= \frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

$$= \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{c-dx^2}}$$

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{c-dx^2}\,E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}$$

```
[In] Integrate[Sqrt[c - d*x^2]/Sqrt[-a - b*x^2],x]
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[c - d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((a
*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{-d\,x^{2}+c}\,\sqrt{-b\,x^{2}-a}\,c\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,E\left(x\sqrt{-\frac{b}{a}},\sqrt{-\frac{a d}{b c}}\right)}{\left(b d\,x^{4}+a d\,x^{2}-c b\,x^{2}-a c\right)\sqrt{-\frac{b}{a}}}$
elliptic	$\frac{\sqrt{-\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}-c b\,x^{2}-a c}}-\frac{c\sqrt{1+\frac{b\,x^{2}}{a}}\,\sqrt{1-\frac{d\,x^{2}}{c}}\,\left(F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\frac{a d-b c}{c b}}\right)-E\left(x\sqrt{-\frac{b}{a}},\sqrt{-1-\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d\,x^{4}+a d\,x^{2}-c b\,x^{2}-a c}}\right.}{\sqrt{-b\,x^{2}-a}\,\sqrt{-d\,x^{2}+c}}$

[In] int((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-d*x^2+c)^(1/2)*(-b*x^2-a)^(1/2)*c*((b*x^2+a)/a)^(1/2)*((-d*x^2+c)/c)^(1/2
)*EllipticE(x*(-b/a)^(1/2),(-a*d/b/c)^(1/2))/(b*d*x^4+a*d*x^2-b*c*x^2-a*c)/
(-b/a)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) - \sqrt{bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\mid -\frac{ad}{bc}) + \sqrt{-bx^2-a}\sqrt{-dx^2+c}d}{bdx}$$

[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sqr
t(b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) + sq
rt(-b*x^2 - a)*sqrt(-d*x^2 + c)*d)/(b*d*x)

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx$$

[In] integrate((-d*x**2+c)**(1/2)/(-b*x**2-a)**(1/2),x)

[Out] Integral(sqrt(c - d*x**2)/sqrt(-a - b*x**2), x)

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx=\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx=\int \frac{\sqrt{-dx^2+c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)/sqrt(-b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}}{\sqrt{-a-bx^2}}\,dx=\int \frac{\sqrt{c-d\,x^2}}{\sqrt{-b\,x^2-a}}\,dx$$

```
[In] int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((c - d*x^2)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.289 $\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 198

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}} - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{-c+dx^2}}$$

```
[Out] -EllipticE(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*d^(1/2)*(-b*x^2-a)^(
1/2)*(1-d*x^2/c)^(1/2)/b/(1+b*x^2/a)^(1/2)/(d*x^2-c)^(1/2)-(a*d+b*c)*Ellipt
icF(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*c^(1/2)*(1+b*x^2/a)^(1/2)*(1-d*x^2/
c)^(1/2)/b/d^(1/2)/(-b*x^2-a)^(1/2)/(d*x^2-c)^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {434, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = -\frac{\sqrt{c}\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}(ad+bc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{dx^2-c}} - \frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{\frac{bx^2}{a}+1}\sqrt{dx^2-c}}$$

```
[In] Int[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] -((Sqrt[c]*Sqrt[d]*Sqrt[-a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(S
qrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[1 + (b*x^2)/a]*Sqrt[-c + d*x^2
])) - (Sqrt[c]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*Elliptic
F[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(b*Sqrt[d]*Sqrt[-a - b*x^2]
*Sqrt[-c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{d\int \frac{\sqrt{-a-bx^2}}{\sqrt{-c+dx^2}}\,dx}{b} - \frac{(bc+ad)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{-c+dx^2}}\,dx}{b}\\
&= -\frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{-a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}} - \frac{\left((bc+ad)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-c+dx^2}}\\
&= -\frac{\left(d\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&\quad - \frac{\left((bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{b\sqrt{-a-bx^2}\sqrt{-c+dx^2}}\\
&= -\frac{\sqrt{c}\sqrt{d}\sqrt{-a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{1+\frac{bx^2}{a}}\sqrt{-c+dx^2}}\\
&\quad - \frac{\sqrt{c}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{-a-bx^2}\sqrt{-c+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.47

$$
\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}\sqrt{-c+dx^2}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|-\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{-a-bx^2}\sqrt{\frac{c-dx^2}{c}}}
$$

```
[In] Integrate[Sqrt[-c + d*x^2]/Sqrt[-a - b*x^2],x]
```

```
[Out] (Sqrt[(a + b*x^2)/a]*Sqrt[-c + d*x^2]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], -((
a*d)/(b*c))])/(Sqrt[-(b/a)]*Sqrt[-a - b*x^2]*Sqrt[(c - d*x^2)/c])
```

Maple [A] (verified)

Time = 2.45 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.84

method	result
default	$\frac{\left(-adF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)-cF\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)b+adE\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)\right)\sqrt{d\,x^{2}-c}\,\sqrt{-b\,x^{2}-a}\,\sqrt{\frac{-d\,x^{2}+c}{c}}\,\sqrt{\frac{b\,x^{2}+a}{a}}}{\left(-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac\right)\sqrt{\frac{d}{c}}\,b}$
elliptic	$\frac{\sqrt{\left(b\,x^{2}+a\right)\left(-d\,x^{2}+c\right)}\,\left(-\frac{c\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}}-\frac{da\sqrt{1-\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{b\,x^{2}}{a}}\,\left(F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)-E\left(x\sqrt{\frac{d}{c}},\sqrt{-1\right.\right.}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^{4}-ad\,x^{2}+cb\,x^{2}+ac}\,b}}{\sqrt{-b\,x^{2}-a}\,\sqrt{d\,x^{2}-c}}$

[In] int((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-a*d*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))-c*EllipticF(x*(d/c)^(1/2),(
-b*c/a/d)^(1/2))*b+a*d*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(1/2)))*(d*x^2-c)
^(1/2)*(-b*x^2-a)^(1/2)*((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)/(-b*d*x^4-
a*d*x^2+b*c*x^2+a*c)/(d/c)^(1/2)/b

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx =$$
$$-\frac{\sqrt{-bd}cx\sqrt{\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\,|-\frac{ad}{bc})-\sqrt{-bd}(c-d)x\sqrt{\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{\frac{c}{d}}}{x}\right)\,|-\frac{ad}{bc})+\sqrt{-bx^2-a}\sqrt{dx^2-}}{bdx}$$

[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(-b*d)*c*x*sqrt(c/d)*elliptic_e(arcsin(sqrt(c/d)/x), -a*d/(b*c)) - sq
rt(-b*d)*(c - d)*x*sqrt(c/d)*elliptic_f(arcsin(sqrt(c/d)/x), -a*d/(b*c)) +
sqrt(-b*x^2 - a)*sqrt(d*x^2 - c)*d)/(b*d*x)

Sympy [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx$$

```
[In] integrate((d*x**2-c)**(1/2)/(-b*x**2-a)**(1/2),x)
[Out] Integral(sqrt(-c + d*x**2)/sqrt(-a - b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Giac [F]

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{dx^2-c}}{\sqrt{-bx^2-a}}\,dx$$

```
[In] integrate((d*x^2-c)^(1/2)/(-b*x^2-a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 - c)/sqrt(-b*x^2 - a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-c+dx^2}}{\sqrt{-a-bx^2}}\,dx = \int \frac{\sqrt{d\,x^2-c}}{\sqrt{-b\,x^2-a}}\,dx$$

```
[In] int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2),x)
[Out] int((d*x^2 - c)^(1/2)/(- a - b*x^2)^(1/2), x)
```

3.290 $\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \frac{\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] 1/2*(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*
d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2
+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {429}

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \frac{\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}}$$

```
[In] Int[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
```

```
[Out] (Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(
Sqrt[2]*Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*Sqrt[3 + d*x^2])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{2}\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \frac{\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-b}x}{\sqrt{2}}\right),\frac{2d}{3b}\right)}{\sqrt{3}\sqrt{-b}}$$

```
[In] Integrate[1/(Sqrt[2 + b*x^2]*Sqrt[3 + d*x^2]),x]
[Out] EllipticF[ArcSin[(Sqrt[-b]*x)/Sqrt[2]], (2*d)/(3*b)]/(Sqrt[3]*Sqrt[-b])
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{F\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{2\sqrt{-d}}$	38
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\,\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{2\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}\,\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}$	112

```
[In] int(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticF(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(
1/2)/(-d)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = -\frac{\sqrt{6}\sqrt{2}\sqrt{-b}F\left(\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-b}x\right)\mid\frac{2d}{3b}\right)}{6b}$$

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")
[Out] -1/6*sqrt(6)*sqrt(2)*sqrt(-b)*elliptic_f(arcsin(1/2*sqrt(2)*sqrt(-b)*x), 2/
3*d/b)/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

```
[In] integrate(1/(b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
[Out] Integral(1/(sqrt(b*x**2 + 2)*sqrt(d*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+2}\sqrt{dx^2+3}}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}\,dx$$

```
[In] int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)),x)
[Out] int(1/((b*x^2 + 2)^(1/2)*(d*x^2 + 3)^(1/2)), x)
```

3.291 $\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 39

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}} \, dx = \frac{\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right), -\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[Out] EllipticF(1/2*x,2*(-d/c)^(1/2))*(1+d*x^2/c)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}} \, dx = \frac{\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right), -\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[In] Int[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[1 + (d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^2}{c}}\int\frac{1}{\sqrt{4-x^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{\sqrt{c+dx^2}}$$
$$= \frac{\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int\frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{x}{2}\right),-\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(Sqrt[4 - x^2]*Sqrt[c + d*x^2]),x]
[Out] (Sqrt[(c + d*x^2)/c]*EllipticF[ArcSin[x/2], (-4*d)/c])/Sqrt[c + d*x^2]
```

Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\,x^2+c}{c}}\,F\left(\frac{x}{2},2\sqrt{-\frac{d}{c}}\right)}{\sqrt{d\,x^2+c}}$	38
elliptic	$\frac{\sqrt{-(d\,x^2+c)(x^2-4)}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(\frac{x}{2},\sqrt{-1-\frac{-c+4d}{c}}\right)}{\sqrt{d\,x^2+c}\,\sqrt{-d\,x^4-c\,x^2+4d\,x^2+4c}}$	83

```
[In] int(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(1/2*x,2*(-d/c)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.38

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \frac{F\left(\arcsin\left(\frac{1}{2}\,x\right)\,|-\frac{4\,d}{c}\right)}{\sqrt{c}}$$

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] elliptic_f(arcsin(1/2*x), -4*d/c)/sqrt(c)
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \begin{cases} \frac{F\left(\operatorname{asin}\left(\frac{x}{2}\right)\middle|-\frac{4d}{c}\right)}{\sqrt{c}} & \text{for}\: x > -2 \wedge x < 2 \end{cases}$$

```
[In] integrate(1/(-x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Piecewise((elliptic_f(asin(x/2), -4*d/c)/sqrt(c), (x > -2) & (x < 2)))
```

Maxima [F]

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{-x^2+4}}\,dx$$

```
[In] integrate(1/(-x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(-x^2 + 4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4-x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{4-x^2}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((4 - x^2)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.292 $\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

```
[Out] (1/(x^2+4))^(1/2)*EllipticF(x/(x^2+4)^(1/2),(1-4*d/c)^(1/2))*(d*x^2+c)^(1/2
)/c/((d*x^2+c)/c/(x^2+4))^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {429}

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx=\frac{\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{x}{2}\right),1-\frac{4d}{c}\right)}{c\sqrt{x^2+4}\sqrt{\frac{c+dx^2}{c(x^2+4)}}}$$

```
[In] Int[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
```

```
[Out] (Sqrt[c + d*x^2]*EllipticF[ArcTan[x/2], 1 - (4*d)/c])/(c*Sqrt[4 + x^2]*Sqrt
[(c + d*x^2)/(c*(4 + x^2))])
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{x}{2}\right)|1-\frac{4d}{c}\right)}{c\sqrt{4+x^2}\sqrt{\frac{c+dx^2}{c(4+x^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{\frac{c+dx^2}{c}}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{2}\right),\frac{4d}{c}\right)}{\sqrt{c+dx^2}}$$

```
[In] Integrate[1/(Sqrt[4 + x^2]*Sqrt[c + d*x^2]),x]
[Out] ((-I)*Sqrt[(c + d*x^2)/c]*EllipticF[I*ArcSinh[x/2], (4*d)/c])/Sqrt[c + d*x^
2]
```

Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{\frac{c}{d}}}{2}\right)}{2\sqrt{d\,x^2+c}\,\sqrt{-\frac{d}{c}}}$	53
elliptic	$\frac{\sqrt{(d\,x^2+c)(x^2+4)}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{d}{c}},\frac{\sqrt{-4+\frac{c+4d}{d}}}{2}\right)}{2\sqrt{d\,x^2+c}\,\sqrt{-\frac{d}{c}}\,\sqrt{d\,x^4+c\,x^2+4d\,x^2+4c}}$	95

```
[In] int(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/(d*x^2+c)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-d/c)^(1/2),1/2*(c/d)^
(1/2))/(-d/c)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.26

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = -\frac{i\,F\left(\arcsin\left(\frac{1}{2}i\,x\right)\mid\frac{4\,d}{c}\right)}{\sqrt{c}}$$

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] -I*elliptic_f(arcsin(1/2*I*x), 4*d/c)/sqrt(c)
```

Sympy [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{c+dx^2}\sqrt{x^2+4}}\,dx$$

```
[In] integrate(1/(x**2+4)**(1/2)/(d*x**2+c)**(1/2),x)
[Out] Integral(1/(sqrt(c + d*x**2)*sqrt(x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{dx^2+c}\sqrt{x^2+4}}\,dx$$

```
[In] integrate(1/(x^2+4)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(x^2 + 4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{4+x^2}\sqrt{c+dx^2}}\,dx = \int \frac{1}{\sqrt{x^2+4}\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)),x)
[Out] int(1/((x^2 + 4)^(1/2)*(c + d*x^2)^(1/2)), x)
```

3.293 $\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 6

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -\operatorname{EllipticF}(\arccos(x),2)$$

```
[Out] -(x^2)^(1/2)/x*EllipticF((-x^2+1)^(1/2),2^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {431}

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -\operatorname{EllipticF}(\arccos(x),2)$$

```
[In] Int[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
```

```
[Out] -EllipticF[ArcCos[x], 2]
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\text{integral} = -F\left(\cos^{-1}(x)\middle|2\right)$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \frac{\sqrt{1-2x^2}\,\text{EllipticF}(\arcsin(x),2)}{\sqrt{-1+2x^2}}$$

```
[In] Integrate[1/(Sqrt[1 - x^2]*Sqrt[-1 + 2*x^2]),x]
[Out] (Sqrt[1 - 2*x^2]*EllipticF[ArcSin[x], 2])/Sqrt[-1 + 2*x^2]
```

Maple [A] (verified)

Time = 2.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 4.17

method	result	size
default	$\frac{F\left(x,\sqrt{2}\right)\sqrt{-2x^2+1}}{\sqrt{2x^2-1}}$	25
elliptic	$\frac{\sqrt{-(2x^2-1)(x^2-1)}\,\sqrt{-2x^2+1}\,F\left(x,\sqrt{2}\right)}{\sqrt{2x^2-1}\,\sqrt{-2x^4+3x^2-1}}$	55

```
[In] int(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x,method=_RETURNVERBOSE)
[Out] EllipticF(x,2^(1/2))*(-2*x^2+1)^(1/2)/(2*x^2-1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = -i\,F(\arcsin{(x)}\,|\,2)$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="fricas")
[Out] -I*elliptic_f(arcsin(x), 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{-(x-1)(x+1)}\sqrt{2x^2-1}}\,dx$$

```
[In] integrate(1/(-x**2+1)**(1/2)/(2*x**2-1)**(1/2),x)
[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*sqrt(2*x**2 - 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{2\,x^2-1}\sqrt{-x^2+1}}\,dx$$

```
[In] integrate(1/(-x^2+1)^(1/2)/(2*x^2-1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^2 - 1)*sqrt(-x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1-x^2}\sqrt{-1+2x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}\,\sqrt{2\,x^2-1}}\,dx$$

```
[In] int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)),x)
[Out] int(1/((1 - x^2)^(1/2)*(2*x^2 - 1)^(1/2)), x)
```

3.294 $\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 23

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = -\frac{E(\arcsin(cx)|-1)}{c} + \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c}$$

```
[Out] -EllipticE(c*x,I)/c+2*EllipticF(c*x,I)/c
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {434, 435, 254, 227}

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{2\,\text{EllipticF}(\arcsin(cx),-1)}{c} - \frac{E(\arcsin(cx)|-1)}{c}$$

```
[In] Int[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
```

```
[Out] -(EllipticE[ArcSin[c*x], -1]/c) + (2*EllipticF[ArcSin[c*x], -1])/c
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
```

```
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}\,dx - \int \frac{\sqrt{1+c^2x^2}}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + 2\int \frac{1}{\sqrt{1-c^4x^4}}\,dx \\
&= -\frac{E(\sin^{-1}(cx)|-1)}{c} + \frac{2F(\sin^{-1}(cx)|-1)}{c}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{E\left(\arcsin\left(\sqrt{-c^2}x\right)\middle|-1\right)}{\sqrt{-c^2}}$$

```
[In] Integrate[Sqrt[1 - c^2*x^2]/Sqrt[1 + c^2*x^2],x]
[Out] EllipticE[ArcSin[Sqrt[-c^2]*x], -1]/Sqrt[-c^2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{(2F(x\ \mathrm{csgn}(c)c,i)-E(x\ \mathrm{csgn}(c)c,i))\ \mathrm{csgn}(c)}{c}$	28
elliptic	$\frac{\sqrt{-c^4x^4+1}\left(\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,F\left(x\sqrt{c^2},i\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}+\frac{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}\,\left(F\left(x\sqrt{c^2},i\right)-E\left(x\sqrt{c^2},i\right)\right)}{\sqrt{c^2}\,\sqrt{-c^4x^4+1}}\right)}{\sqrt{-c^2x^2+1}\,\sqrt{c^2x^2+1}}$	153

```
[In] int((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2*EllipticF(x*csgn(c)*c,I)-EllipticE(x*csgn(c)*c,I))*csgn(c)/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(21) = 42$.

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.17

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \frac{\sqrt{c^2x^2+1}\sqrt{-c^2x^2+1}c^3+\sqrt{-c^4}\left((c^2-1)xF\left(\arcsin\left(\frac{1}{cx}\right)\,|-1\right)+xE\left(\arcsin\left(\frac{1}{cx}\right)\,|-1\right)\right)}{c^5x}$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)*c^3 + sqrt(-c^4)*((c^2 - 1)*x*ellipti
c_f(arcsin(1/(c*x)), -1) + x*elliptic_e(arcsin(1/(c*x)), -1)))/(c^5*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-(cx-1)(cx+1)}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c**2*x**2+1)**(1/2)/(c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/sqrt(c**2*x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{-c^2x^2+1}}{\sqrt{c^2x^2+1}}\,dx$$

```
[In] integrate((-c^2*x^2+1)^(1/2)/(c^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/sqrt(c^2*x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-c^2x^2}}{\sqrt{1+c^2x^2}}\,dx = \int \frac{\sqrt{1-c^2\,x^2}}{\sqrt{c^2\,x^2+1}}\,dx$$

```
[In] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2),x)
[Out] int((1 - c^2*x^2)^(1/2)/(c^2*x^2 + 1)^(1/2), x)
```

3.295 $\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}$$

```
[Out] x*(b*x^2+2)^(1/2)/(d*x^2+3)^(1/2)-(1/(3*d*x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*E
llipticE(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(
b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3))^(1/2)/(d*x^2+3)^(1/2)+(1/(3*d*
x^2+9))^(1/2)*(3*d*x^2+9)^(1/2)*EllipticF(x*d^(1/2)*3^(1/2)/(3*d*x^2+9)^(1/
2),1/2*(4-6*b/d)^(1/2))*2^(1/2)*(b*x^2+2)^(1/2)/d^(1/2)/((b*x^2+2)/(d*x^2+3
))^(1/2)/(d*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {433, 429, 506, 422}

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{\sqrt{2}\sqrt{bx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right),1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} - \frac{\sqrt{2}\sqrt{bx^2+2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{dx^2+3}\sqrt{\frac{bx^2+2}{dx^2+3}}} + \frac{x\sqrt{bx^2+2}}{\sqrt{dx^2+3}}$$

```
[In] Int[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]
```

```
[Out] (x*Sqrt[2 + b*x^2])/Sqrt[3 + d*x^2] - (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 +
 d*x^2)]*Sqrt[3 + d*x^2]) + (Sqrt[2]*Sqrt[2 + b*x^2]*EllipticF[ArcTan[(Sqrt
[d]*x)/Sqrt[3]], 1 - (3*b)/(2*d)])/(Sqrt[d]*Sqrt[(2 + b*x^2)/(3 + d*x^2)]*S
qrt[3 + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\int \frac{1}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+bx^2}\sqrt{3+dx^2}}\,dx \\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} - 3\int \frac{\sqrt{2+bx^2}}{\left(3+dx^2\right)^{3/2}}\,dx \\
&= \frac{x\sqrt{2+bx^2}}{\sqrt{3+dx^2}} - \frac{\sqrt{2}\sqrt{2+bx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}} + \frac{\sqrt{2}\sqrt{2+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{3}}\right)|1-\frac{3b}{2d}\right)}{\sqrt{d}\sqrt{\frac{2+bx^2}{3+dx^2}}\sqrt{3+dx^2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \frac{\sqrt{2}E\left(\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{3}}\right)|\frac{3b}{2d}\right)}{\sqrt{-d}}$$

[In] Integrate[Sqrt[2 + b*x^2]/Sqrt[3 + d*x^2],x]

[Out] (Sqrt[2]*EllipticE[ArcSin[(Sqrt[-d]*x)/Sqrt[3]], (3*b)/(2*d)])/Sqrt[-d]

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.20

method	result
default	$\frac{E\left(\frac{x\sqrt{3}\,\sqrt{-d}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{b}{d}}}{2}\right)\sqrt{2}}{\sqrt{-d}}$
elliptic	$\frac{\sqrt{(b\,x^2+2)(d\,x^2+3)}\left(\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\,F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}-\frac{\sqrt{3d\,x^2+9}\,\sqrt{2b\,x^2+4}\left(F\left(\frac{x\sqrt{-3d}}{3},\frac{\sqrt{-4+\frac{6b+4d}{d}}}{2}\right)-E\left(\frac{x\sqrt{-3d}}{3},\sqrt{-4\right.\right.}{\sqrt{-3d}\,\sqrt{bd\,x^4+3b\,x^2+2d\,x^2+6}}\right.}{\sqrt{b\,x^2+2}\,\sqrt{d\,x^2+3}}$

[In] int((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] EllipticE(1/3*x*3^(1/2)*(-d)^(1/2),1/2*2^(1/2)*3^(1/2)*(b/d)^(1/2))*2^(1/2)
/(-d)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = -\frac{9\sqrt{3}\sqrt{bd}bx\sqrt{-\frac{1}{d}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)|\frac{2d}{3b}) - \sqrt{3}\sqrt{bd}(2d^2+9b)x\sqrt{-\frac{1}{d}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{d}}}{x}\right)|\frac{2d}{3b}) - 3\sqrt{bx}}{3bd^2x}$$

[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="fricas")

[Out] -1/3*(9*sqrt(3)*sqrt(b*d)*b*x*sqrt(-1/d)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/
d)/x), 2/3*d/b) - sqrt(3)*sqrt(b*d)*(2*d^2 + 9*b)*x*sqrt(-1/d)*elliptic_f(a
rcsin(sqrt(3)*sqrt(-1/d)/x), 2/3*d/b) - 3*sqrt(b*x^2 + 2)*sqrt(d*x^2 + 3)*b
*d)/(b*d^2*x)

Sympy [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x**2+2)**(1/2)/(d*x**2+3)**(1/2),x)
[Out] Integral(sqrt(b*x**2 + 2)/sqrt(d*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Giac [F]

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{bx^2+2}}{\sqrt{dx^2+3}}\,dx$$

```
[In] integrate((b*x^2+2)^(1/2)/(d*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^2 + 2)/sqrt(d*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+bx^2}}{\sqrt{3+dx^2}}\,dx = \int \frac{\sqrt{b\,x^2+2}}{\sqrt{d\,x^2+3}}\,dx$$

```
[In] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2),x)
[Out] int((b*x^2 + 2)^(1/2)/(d*x^2 + 3)^(1/2), x)
```

3.296 $\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 19

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[Out] -1/3*(x^2)^(1/2)/x*EllipticE(1/2*(-6*x^2+4)^(1/2),2^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {436}

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = -\frac{E\left(\arccos\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

```
[In] Int[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
```

```
[Out] -(EllipticE[ArcCos[Sqrt[3/2]*x], 2]/Sqrt[3])
```

Rule 436

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(-Sqrt[a - b*(c/d)]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcCos[Rt[-d/c, 2]*x],
 b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0]
 && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\text{integral} = -\frac{E\left(\cos^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{\sqrt{-1+3x^2}E\left(\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3-9x^2}}$$

```
[In] Integrate[Sqrt[-1 + 3*x^2]/Sqrt[2 - 3*x^2],x]
[Out] (Sqrt[-1 + 3*x^2]*EllipticE[ArcSin[Sqrt[3/2]*x], 2])/Sqrt[3 - 9*x^2]
```

Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

method	result	size
default	$-\frac{E\left(\frac{x\sqrt{2}\,\sqrt{3}}{2},\sqrt{2}\right)\sqrt{-3x^2+1}\,\sqrt{3}}{3\sqrt{3x^2-1}}$	37
elliptic	$\frac{\sqrt{-(3x^2-2)(3x^2-1)}\,\left(-\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)}{6\sqrt{-9x^4+9x^2-2}}+\frac{\sqrt{6}\,\sqrt{-6x^2+4}\,\sqrt{-3x^2+1}\,\left(F\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)-E\left(\frac{x\sqrt{6}}{2},\sqrt{2}\right)\right)}{6\sqrt{-9x^4+9x^2-2}}\right)}{\sqrt{3x^2-1}\,\sqrt{-3x^2+2}}$	146

```
[In] int((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/3*EllipticE(1/2*x*2^(1/2)*3^(1/2),2^(1/2))*(-3*x^2+1)^(1/2)*3^(1/2)/(3*x
^2-1)^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.79

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \frac{-4i\,\sqrt{3}\sqrt{2}xE(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right) \mid \frac{1}{2}) + i\,\sqrt{3}\sqrt{2}xF(\arcsin\left(\frac{\sqrt{3}\sqrt{2}}{3\,x}\right) \mid \frac{1}{2}) - 6\,\sqrt{3\,x^2-1}\sqrt{-3\,x^2+2}}{18\,x}$$

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="fricas")
[Out] 1/18*(-4*I*sqrt(3)*sqrt(2)*x*elliptic_e(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2)
 + I*sqrt(3)*sqrt(2)*x*elliptic_f(arcsin(1/3*sqrt(3)*sqrt(2)/x), 1/2) - 6*s
qrt(3*x^2 - 1)*sqrt(-3*x^2 + 2))/x
```

Sympy [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3x^2-1}}{\sqrt{2-3x^2}}\,dx$$

```
[In] integrate((3*x**2-1)**(1/2)/(-3*x**2+2)**(1/2),x)
[Out] Integral(sqrt(3*x**2 - 1)/sqrt(2 - 3*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Giac [F]

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{-3\,x^2+2}}\,dx$$

```
[In] integrate((3*x^2-1)^(1/2)/(-3*x^2+2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(3*x^2 - 1)/sqrt(-3*x^2 + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1+3x^2}}{\sqrt{2-3x^2}}\,dx = \int \frac{\sqrt{3\,x^2-1}}{\sqrt{2-3\,x^2}}\,dx$$

```
[In] int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2),x)
[Out] int((3*x^2 - 1)^(1/2)/(2 - 3*x^2)^(1/2), x)
```

3.297 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 95

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((-b-(-4*a*c+b
^2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {435}

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{\sqrt{b^2-4ac}+b}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))])/(
Sqrt[2]*Sqrt[c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(80) = 160$.

Time = 2.89 (sec) , antiderivative size = 809, normalized size of antiderivative = 8.52

method	result
elliptic	$\frac{\sqrt{\frac{-2c\,x^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(-2c\,x^2+\sqrt{-4ac+b^2}-b\right)\left(-2c\,x^2+\sqrt{-4ac+b^2}+b\right)}{ac}}\left(\frac{\sqrt{2}\sqrt{1-\frac{2c\,x^2}{-b+\sqrt{-4ac+b^2}}}\sqrt{1-\frac{2c\,x^2}{b+\sqrt{-4ac+}}}}{2\sqrt{\frac{c}{-b+\sqrt{-4ac+b^2}}}}\right.}{}$

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c*
x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1
/2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)
^(1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2*2^(1/2)/(c/(-b+(-4*a*c+b^2)^(1/
2)))^(1/2)*(1-2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*Ell
ipticF(x*2^(1/2)*(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a
*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))
+2*c/(-b+(-4*a*c+b^2)^(1/2))*2^(1/2)/(c/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2
*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2)*(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1
/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^2/
(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(-2*c/(b+(-4*a*c+b
^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2))-b/a)*(EllipticF(x*2^(1/2)*(c/(-b+(-4*
a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*
c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x*2^(1/2)*(c/(-b+
(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4+2*(-2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-
4*a*c+b^2)^(1/2)))/c*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 318 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 318, normalized size of antiderivative = 3.35

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = -\frac{2\sqrt{\frac{1}{2}}\left(\sqrt{b^2-4\,ac}bx+(b^2-2\,ac)x\right)\sqrt{\frac{b+\sqrt{b^2-4\,ac}}{c}}\sqrt{-\frac{c}{a}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{b+\sqrt{b^2-4\,ac}}{c}}}{x}\right)\mid -\frac{b^2-2\,ac-\sqrt{b^2-4\,ac}b}{2\,ac}\right)-$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x + (b^2 - 2*a*c)*x)*sqrt((b + sqrt(
b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((b + sqrt(b^2
- 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)/(a*c)) - 2*sqrt(1
/2)*(sqrt(b^2 - 4*a*c)*(b - c)*x + (b^2 - (2*a - b)*c)*x)*sqrt((b + sqrt(b^
2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((b + sqrt(b^2 -
4*a*c))/c)/x), -1/2*(b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*b)/(a*c)) + (b*c + sqr
t(b^2 - 4*a*c)*c)*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt(-(b*x^
2 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

```
[In] int((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2),x)
```

```
[Out] int((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.298 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 94

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{b+\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] 1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2),((b+(-4*a*c+b^
2)^(1/2))/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.017, Rules used = {435}

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{\sqrt{b^2-4ac}+b}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqr
t[2]*Sqrt[c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{b+\sqrt{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 - (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]], -((b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c]))])/
(Sqrt[2]*Sqrt[c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. 2(76) = 152.

Time = 2.72 (sec) , antiderivative size = 1388, normalized size of antiderivative = 14.77

method	result	size
elliptic	Expression too large to display	1388

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((-2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/
2))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(
1/2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^
2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)
```

```
^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a
*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
)+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(
1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)
^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2-4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-2
*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4
*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a
*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4
*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(-2*c/(b+(-4*a*c+b^2)^(1
/2))-2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(EllipticF(1/2*x*(-2*
((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)^(1/2))-2*
c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*
c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(1
/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2
)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(-2*c/(b+(-4*a*c+b^2)^
(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b
^2-4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(76) = 152$.

Time = 0.10 (sec) , antiderivative size = 410, normalized size of antiderivative = 4.36

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2x+\sqrt{b^2-4\,ac}bx+\left(bcx+\sqrt{b^2-4\,ac}cx\right)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right)}{}$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*(b^2*x + sqrt(b^2 - 4*a*c)*b*x + (b*c*x + sqrt(b^2 - 4*a*c)
*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)*sqrt
(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x
```

```
), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) - sqrt(1/2)*(sqr
t(b^2 - 4*a*c)*b*x + (b^2 - 2*b*c)*x + (sqrt(b^2 - 4*a*c)*c*x + (b*c + 2*c^
2)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)*sqrt
(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) + b)/c)/x
), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)/(a*c)) + (b*c + sqrt(b^
2 - 4*a*c)*c)*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)*sqrt(-(b*x^2 -
 sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{-\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1-2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt(-(-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2)))
, x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(-2*c*x^2/(b + sqr
t(b^2 - 4*a*c)) + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

```
[In] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2),x)
```

```
[Out] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/(1 - (2*c*x^2)/(b + (b^
2 - 4*a*c)^(1/2)))^(1/2), x)
```

3.299 $\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 478

$$\begin{aligned}
&\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx \\
&= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&\quad - \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&\quad + \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

```
[Out] x*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)
))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticE(x*2^(1/
2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^
(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+
2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1
/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)
```

```
/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*
(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b
-(-4*a*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {433, 429, 506, 422}

$$\begin{aligned}
&\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx \\
&= \frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} \\
&\quad -\frac{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}} \\
&\quad +\frac{x\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}
\end{aligned}$$

```
[In] Int[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] (x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])] - (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*EllipticE[ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*
Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))])])*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}} + \int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx \\
&= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&\quad + \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)\middle|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&\quad - \int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}\,dx
\end{aligned}
$$

$$
= \frac{x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
+ \frac{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
$$

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.21

$$
\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\sqrt{-b-\sqrt{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}
$$

```
[In] Integrate[Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
- Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(S
qrt[2]*Sqrt[c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. 2(457) = 914.

Time = 2.69 (sec) , antiderivative size = 1388, normalized size of antiderivative = 2.90

method	result	size
elliptic	Expression too large to display	1388

```
[In] int((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x
^2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/
2))*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(
1/2)/(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/2/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b
^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2
)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*
a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/
```

```
a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF
(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b
^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)
^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))+2
*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4
*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a
*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*
b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1
+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4
*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(2*c/(b+(-4*a*c+b^2)^(1/
2))+2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(EllipticF(1/2*x*(-2*(
(-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/
(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)
/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(1/2
*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^
(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/
2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+
4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(b^2x+\sqrt{b^2-4\,ac}bx-\left(bcx+\sqrt{b^2-4\,ac}cx\right)\sqrt{\frac{b^2-4\,ac}{c^2}}\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\sqrt{\frac{c}{a}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right.\right.}{}$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*(b^2*x + sqrt(b^2 - 4*a*c)*b*x - (b*c*x + sqrt(b^2 - 4*a*c)
*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*sqrt
(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x
), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - sqrt(1/2)*(sqrt
(b^2 - 4*a*c)*b*x + (b^2 + 2*b*c)*x - (sqrt(b^2 - 4*a*c)*c*x + (b*c - 2*c^2
```

```
)*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)*sqrt(
c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x)
, 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c)) - (b*c + sqrt(b^2
- 4*a*c)*c)*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b*x^2 - sqr
t(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c^2*x)
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{b+2cx^2-\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

```
[In] integrate((1+2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
[Out] Integral(sqrt((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**2)
))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt(
b^2 - 4*a*c)) + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

```
[In] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2),x)
```

```
[Out] int(((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.300 $\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 59, antiderivative size = 215

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=-\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] b*EllipticF(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((-b+(-4*a*c+b^2
)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)-1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((-b
+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))*
2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {434, 435, 430}

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{2}b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right),-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\sqrt{b^2-4ac}+b\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[In] Int[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c])],x]
```

```
[Out] -(((b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c]))])/(Sq
rt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])) + (Sqrt[2]*b*EllipticF[ArcSin[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], -((b - Sqrt[b^2 - 4*a*c])/
(b + Sqrt[b^2 - 4*a*c]))])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(2b)\int \frac{1}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}} - \frac{\left(b+\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}\,dx}{b-\sqrt{b^2-4ac}}\\ &= -\frac{\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}\\ &\quad+\frac{\sqrt{2}bF\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|-\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\frac{\sqrt{-b-\sqrt{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-b-\sqrt{b^2-4ac}}}\right)|\frac{b+\sqrt{b^2-4ac}}{-b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Integrate[Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]/Sqrt[1 + (2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c])],x]
```

```
[Out] (Sqrt[-b - Sqrt[b^2 - 4*a*c]]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b
- Sqrt[b^2 - 4*a*c]]], (b + Sqrt[b^2 - 4*a*c])/(-b + Sqrt[b^2 - 4*a*c])])/(
Sqrt[2]*Sqrt[c])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(173) = 346$.

Time = 2.63 (sec) , antiderivative size = 872, normalized size of antiderivative = 4.06

method	result
elliptic	$\sqrt{\frac{2cx^2+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}}\left(-b+\sqrt{-4ac+b^2}\right)\sqrt{-\frac{\left(2cx^2+\sqrt{-4ac+b^2}-b\right)\left(2cx^2+\sqrt{-4ac+b^2}+b\right)}{ac}}\left(\frac{\sqrt{1+\frac{2cx^2}{b+\sqrt{-4ac+b^2}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{-4ac+b^2}}}F\left(x\ldots\right.}{\sqrt{-\frac{2c}{b+\sqrt{-4ac+b^2}}}\ldots}\right.$

```
[In] int((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^
2+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-b+(-4*a*c+b^2)^(1/2
))*(-(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/
2)/(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(1/(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1
+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^
(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a*c+b^2)^(1/2))-4*c^
2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(x*(-2*
c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(
b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(b+
(-4*a*c+b^2)^(1/2)))^(1/2))-4*c/(-b+(-4*a*c+b^2)^(1/2))/(-2*c/(b+(-4*a*c+b^
```

```
2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c/(-b+(-4*a*
c+b^2)^(1/2))*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2))-4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/
2)/(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2))-b/a)*(EllipticF(x
*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-
2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2)
)*(b+(-4*a*c+b^2)^(1/2)))^(1/2))-EllipticE(x*(-2*c/(b+(-4*a*c+b^2)^(1/2)))^
(1/2),1/2*(-4-2*(2*c/(b+(-4*a*c+b^2)^(1/2))-2*c/(b-(-4*a*c+b^2)^(1/2)))/c/(
-b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{4\sqrt{\frac{1}{2}}ax\sqrt{\frac{b-\sqrt{b^2-4\,ac}}{c}}\sqrt{-\frac{c}{a}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{b-\sqrt{b^2-4\,ac}}{c}}}{x}\right)\middle|-\frac{b^2-2\,ac+\sqrt{b^2-4\,ac}b}{2\,ac}\right)-2\sqrt{\frac{1}{2}}\left((2\,a-b)x-\sqrt{b^2-4\,ac}\right.}{}$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(1/2)*a*x*sqrt((b - sqrt(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_e(
arcsin(sqrt(1/2)*sqrt((b - sqrt(b^2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c + sq
rt(b^2 - 4*a*c)*b)/(a*c)) - 2*sqrt(1/2)*((2*a - b)*x - sqrt(b^2 - 4*a*c)*x)
*sqrt((b - sqrt(b^2 - 4*a*c))/c)*sqrt(-c/a)*elliptic_f(arcsin(sqrt(1/2)*sqr
t((b - sqrt(b^2 - 4*a*c))/c)/x), -1/2*(b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*b)/(
a*c)) + (b + sqrt(b^2 - 4*a*c))*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)
/a)*sqrt((b*x^2 - sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c*x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{b-\sqrt{-4ac+b^2}}}}{\sqrt{\frac{b+2cx^2+\sqrt{-4ac+b^2}}{b+\sqrt{-4ac+b^2}}}}\,dx$$

```
[In] integrate((1-2*c*x**2/(b-(-4*a*c+b**2)**(1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a
*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral(sqrt(-(-b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b - sqrt(-4*a*c + b**
2)))/sqrt((b + 2*c*x**2 + sqrt(-4*a*c + b**2))/(b + sqrt(-4*a*c + b**2))),
x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}{\sqrt{\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)/sqrt(2*c*x^2/(b + sqrt
(b^2 - 4*a*c)) + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((1-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}}{\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

```
[In] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2),x)
```

```
[Out] int((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2)))^(1/2)/((2*c*x^2)/(b + (b^2 -
4*a*c)^(1/2)) + 1)^(1/2), x)
```

3.301 $\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = -\frac{2^{-2-m}\sqrt{x^2}(2-4x^2)^{1+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},(1-2x^2)^2\right)}{(1+m)x}$$

```
[Out] -2^(-2-m)*(-4*x^2+2)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],(-2*x^2+1
)^2)*(x^2)^(1/2)/(1+m)/x
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.37, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {440}

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = x\,\text{AppellF1}\left(\frac{1}{2},-m,\frac{1}{2},\frac{3}{2},2x^2,x^2\right)$$

```
[In] Int[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
```

```
[Out] x*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2]
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\text{integral} = xF_1\left(\frac{1}{2}; -m, \frac{1}{2}; \frac{3}{2}; 2x^2, x^2\right)$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 1.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.97

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx$$
$$= \frac{3x(1-2x^2)^m \operatorname{AppellF1}\left(\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right)}{\sqrt{1-x^2}\left(3\operatorname{AppellF1}\left(\frac{1}{2}, -m, \frac{1}{2}, \frac{3}{2}, 2x^2, x^2\right) + x^2\left(-4m\operatorname{AppellF1}\left(\frac{3}{2}, 1-m, \frac{1}{2}, \frac{5}{2}, 2x^2, x^2\right) + \operatorname{AppellF1}\left(\frac{3}{2}, -\right.\right.\right.}$$

```
[In] Integrate[(1 - 2*x^2)^m/Sqrt[1 - x^2],x]
[Out] (3*x*(1 - 2*x^2)^m*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2])/(Sqrt[1 - x^2]*
(3*AppellF1[1/2, -m, 1/2, 3/2, 2*x^2, x^2] + x^2*(-4*m*AppellF1[3/2, 1 - m,
 1/2, 5/2, 2*x^2, x^2] + AppellF1[3/2, -m, 3/2, 5/2, 2*x^2, x^2])))
```

Maple [F]

$$\int \frac{(-2x^2+1)^m}{\sqrt{-x^2+1}}dx$$

```
[In] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
[Out] int((-2*x^2+1)^m/(-x^2+1)^(1/2),x)
```

Fricas [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-x^2 + 1)*(-2*x^2 + 1)^m/(x^2 - 1), x)
```

Sympy [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(1-2x^2)^m}{\sqrt{-(x-1)(x+1)}}\,dx$$

```
[In] integrate((-2*x**2+1)**m/(-x**2+1)**(1/2),x)
[Out] Integral((1 - 2*x**2)**m/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(-2\,x^2+1)^m}{\sqrt{-x^2+1}}\,dx$$

```
[In] integrate((-2*x^2+1)^m/(-x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((-2*x^2 + 1)^m/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1-2x^2)^m}{\sqrt{1-x^2}}\,dx = \int \frac{(1-2\,x^2)^m}{\sqrt{1-x^2}}\,dx$$

```
[In] int((1 - 2*x^2)^m/(1 - x^2)^(1/2),x)
[Out] int((1 - 2*x^2)^m/(1 - x^2)^(1/2), x)
```

3.302 $\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 46

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

```
[Out] EllipticF(x,I*3^(1/2)+2*I)*(-x^2+1)^(1/2)/(x^2-1)^(1/2)/(2-3^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {432, 430}

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{x^2-1}}$$

```
[In] Int[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], -7 - 4*Sqrt[3]])/(Sqrt[7 - 4*Sqrt[3]]*S
qrt[-1 + x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-x^2}\int \frac{1}{\sqrt{1-x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx}{\sqrt{-1+x^2}}\\ &= \frac{\sqrt{1-x^2}F\left(\sin^{-1}(x)\middle|-7-4\sqrt{3}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \frac{\sqrt{1-x^2}\,\text{EllipticF}\left(\arcsin(x),\frac{1}{-7+4\sqrt{3}}\right)}{\sqrt{7-4\sqrt{3}}\sqrt{-1+x^2}}$$

```
[In] Integrate[1/(Sqrt[-1 + x^2]*Sqrt[7 - 4*Sqrt[3] + x^2]),x]
```

```
[Out] (Sqrt[1 - x^2]*EllipticF[ArcSin[x], (-7 + 4*Sqrt[3])^(-1)])/(Sqrt[7 - 4*Sqr
t[3]]*Sqrt[-1 + x^2])
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(37) = 74.

Time = 2.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

method	result	size
default	$-\frac{iF\left(\frac{ix}{-2+\sqrt{3}},2i-i\sqrt{3}\right)\sqrt{-x^2+1}\sqrt{-\left(-x^2+4\sqrt{3}-7\right)\left(-4\sqrt{3}+7\right)}\left(-2+\sqrt{3}\right)\sqrt{x^2-1}\sqrt{7+x^2-4\sqrt{3}}}{\left(4\sqrt{3}-7\right)\left(-x^4+4x^2\sqrt{3}-6x^2-4\sqrt{3}+7\right)}$	117
elliptic	$-\frac{i\sqrt{-\left(x^2-1\right)\left(-x^2+4\sqrt{3}-7\right)}\sqrt{-4\sqrt{3}+7}\sqrt{1-\frac{x^2}{4\sqrt{3}-7}}\sqrt{-x^2+1}\,F\left(\frac{ix}{\sqrt{-4\sqrt{3}+7}},2i-i\sqrt{3}\right)}{\sqrt{x^2-1}\sqrt{7+x^2-4\sqrt{3}}\sqrt{6x^2-7+x^4-4x^2\sqrt{3}+4\sqrt{3}}}$	128

```
[In] int(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*EllipticF(I*x/(-2+3^(1/2)),2*I-I*3^(1/2))*(-x^2+1)^(1/2)*(-(-x^2+4*3^(1/
2)-7)*(-4*3^(1/2)+7))^(1/2)/(4*3^(1/2)-7)*(-2+3^(1/2))*(x^2-1)^(1/2)*(7+x^2
-4*3^(1/2))^(1/2)/(-x^4+4*x^2*3^(1/2)-6*x^2-4*3^(1/2)+7)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(34) = 68$.

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.57

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = -\frac{1}{2}\left(\left(2\sqrt{3}\sqrt{2}+3\sqrt{2}\right)\sqrt{4\sqrt{3}-7}+2\sqrt{2}\sqrt{4\sqrt{3}+7}\sqrt{4\sqrt{3}-7}\right)\sqrt{-4\sqrt{3}+4\sqrt{4\sqrt{3}+7}-6}F(\arcsin -4\sqrt{4\sqrt{3}+7}\left(2\sqrt{3}-3\right)-7)$$

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="fricas")
[Out] -1/2*((2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(4*sqrt(3) - 7) + 2*sqrt(2)*sqrt(
4*sqrt(3) + 7)*sqrt(4*sqrt(3) - 7))*sqrt(-4*sqrt(3) + 4*sqrt(4*sqrt(3) + 7)
 - 6)*elliptic_f(arcsin(1/2*sqrt(2)*x*sqrt(-4*sqrt(3) + 4*sqrt(4*sqrt(3) +
7) - 6)), -4*sqrt(4*sqrt(3) + 7)*(2*sqrt(3) - 3) - 7)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{(x-1)(x+1)}\sqrt{x^2-4\sqrt{3}+7}}\,dx$$

```
[In] integrate(1/(x**2-1)**(1/2)/(7+x**2-4*3**(1/2))**(1/2),x)
[Out] Integral(1/(sqrt((x - 1)*(x + 1))*sqrt(x**2 - 4*sqrt(3) + 7)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-4\sqrt{3}+7}\sqrt{x^2-1}}\,dx$$

```
[In] integrate(1/(x^2-1)^(1/2)/(7+x^2-4*3^(1/2))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(x^2 - 4*sqrt(3) + 7)*sqrt(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-1+x^2}\sqrt{7-4\sqrt{3}+x^2}}\,dx = \int \frac{1}{\sqrt{x^2-1}\,\sqrt{x^2-4\,\sqrt{3}+7}}\,dx$$

```
[In] int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)),x)
[Out] int(1/((x^2 - 1)^(1/2)*(x^2 - 4*3^(1/2) + 7)^(1/2)), x)
```

3.303 $\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+(-3+\sqrt{3})x^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 47

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = -\frac{1}{6}\sqrt{3+\sqrt{3}}\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

```
[Out] -1/6*(x^2*(9-3*3^(1/2)))^(1/2)/x/(9-3*3^(1/2))^(1/2)*EllipticF(1/3*(9-x^2*(
9-3*3^(1/2)))^(1/2),1/2*(2+2*3^(1/2))^(1/2))*(3+3^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.024$, Rules used = {431}

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = -\frac{1}{6}\sqrt{3+\sqrt{3}}\,\text{EllipticF}\left(\arccos\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right),\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

```
[In] Int[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]),x]
```

```
[Out] -1/6*(Sqrt[3 + Sqrt[3]]*EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqr
t[3])/2])
```

Rule 431

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(-(Sqrt[c]*Rt[-d/c, 2]*Sqrt[a - b*(c/d)])^(-1))*EllipticF[ArcCos[Rt[-d/
c, 2]*x], b*(c/(b*c - a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&
GtQ[c, 0] && GtQ[a - b*(c/d), 0]
```

Rubi steps

$$\text{integral} = -\frac{1}{6}\sqrt{3+\sqrt{3}}F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)$$

Mathematica [A] (warning: unable to verify)

Time = 1.98 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \frac{\sqrt{3+\sqrt{3}-2x^2}\sqrt{-3+3\sqrt{3}-2\sqrt{3}x^2}\,\text{EllipticF}\left(\arcsin\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right),2+\sqrt{3}\right)}{6\sqrt{\left(-2+\sqrt{3}\right)\left(3-6x^2+2x^4\right)}}$$

```
[In] Integrate[1/(Sqrt[3 - 3*Sqrt[3] + 2*Sqrt[3]*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^
2]),x]
```

```
[Out] (Sqrt[3 + Sqrt[3] - 2*x^2]*Sqrt[-3 + 3*Sqrt[3] - 2*Sqrt[3]*x^2]*EllipticF[A
rcSin[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3]])/(6*Sqrt[(-2 + Sqrt[3])*(3 - 6*x
^2 + 2*x^4)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(64) = 128$.

Time = 2.61 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.19

method	result
default	$\frac{\sqrt{x^2\sqrt{3}-3x^2+3}\,\sqrt{3-3\sqrt{3}+2x^2\sqrt{3}}\,\sqrt{2}\,\sqrt{-12x^2\sqrt{3}+18x^2+9\sqrt{3}-9}\,\sqrt{3}\,\sqrt{-\left(3-3\sqrt{3}+2x^2\sqrt{3}\right)\left(\sqrt{3}-1\right)}\,F\left(\frac{x\sqrt{2}\,\sqrt{3}\,\sqrt{\left(2\sqrt{3}-3\right)\left(\sqrt{3}-1\right)}}{3\sqrt{3}-3}\right.}{54\left(\sqrt{3}-1\right)^2\sqrt{2\sqrt{3}-3}\,\left(2x^4\sqrt{3}-2x^4-6x^2\sqrt{3}+6x^2+3\sqrt{3}-3\right)}$
elliptic	$\frac{\sqrt{\left(x^2\sqrt{3}-3x^2+3\right)\left(3-3\sqrt{3}+2x^2\sqrt{3}\right)}\,\sqrt{6}\,\sqrt{9-\frac{6\left(2\sqrt{3}-3\right)x^2}{\sqrt{3}-1}}\,\sqrt{9-\frac{6\sqrt{3}\,x^2}{\sqrt{3}-1}}\,F\left(\frac{x\sqrt{6}\,\sqrt{\frac{2\sqrt{3}-3}{\sqrt{3}-1}}}{3},\frac{\sqrt{-9-\frac{6\left(18\sqrt{3}-18\right)\sqrt{3}}{\left(\sqrt{3}-1\right)\left(6-6\sqrt{3}\right)}}}{3}\right)}{18\sqrt{x^2\sqrt{3}-3x^2+3}\,\sqrt{3-3\sqrt{3}+2x^2\sqrt{3}}\,\sqrt{\frac{2\sqrt{3}-3}{\sqrt{3}-1}}\,\sqrt{18x^2\sqrt{3}-18x^2+6x^4-6x^4\sqrt{3}+9-9\sqrt{3}}}$

```
[In] int(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,method
=_RETURNVERBOSE)
```

```
[Out] 1/54*(x^2*3^(1/2)-3*x^2+3)^(1/2)*(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2)*2^(1/2)*
(-12*x^2*3^(1/2)+18*x^2+9*3^(1/2)-9)^(1/2)/(3^(1/2)-1)^2*3^(1/2)*(-(3-3*3^(
1/2)+2*x^2*3^(1/2))*(3^(1/2)-1))^(1/2)*EllipticF(1/3*x*2^(1/2)*3^(1/2)/(3^(
1/2)-1)*((2*3^(1/2)-3)*(3^(1/2)-1))^(1/2),1/(3^(1/2)-1)*((3^(1/2)-1)*(1+3^(
1/2)))^(1/2))*(-3+3^(1/2))/(2*3^(1/2)-3)^(1/2)/(2*x^4*3^(1/2)-2*x^4-6*x^2*3
^(1/2)+6*x^2+3*3^(1/2)-3)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = -\frac{1}{18}\sqrt{\sqrt{3}+3}\sqrt{-9\sqrt{3}+9}F(\arcsin\left(\frac{1}{3}\sqrt{3}x\sqrt{\sqrt{3}+3}\right)|-\sqrt{3}+2)$$

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,
 algorithm="fricas")
```

```
[Out] -1/18*sqrt(sqrt(3) + 3)*sqrt(-9*sqrt(3) + 9)*elliptic_f(arcsin(1/3*sqrt(3)*
x*sqrt(sqrt(3) + 3)), -sqrt(3) + 2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{-3x^2+\sqrt{3}x^2+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

```
[In] integrate(1/(3+x**2*(-3+3**(1/2)))**(1/2)/(3-3*3**(1/2)+2*x**2*3**(1/2))**(
1/2),x)
```

```
[Out] Integral(1/(sqrt(-3*x**2 + sqrt(3)*x**2 + 3)*sqrt(2*sqrt(3)*x**2 - 3*sqrt(3
) + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,
 algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{x^2\left(\sqrt{3}-3\right)+3}\sqrt{2\sqrt{3}x^2-3\sqrt{3}+3}}\,dx$$

```
[In] integrate(1/(3+x^2*(-3+3^(1/2)))^(1/2)/(3-3*3^(1/2)+2*x^2*3^(1/2))^(1/2),x,
 algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2*(sqrt(3) - 3) + 3)*sqrt(2*sqrt(3)*x^2 - 3*sqrt(3) + 3
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{3-3\sqrt{3}+2\sqrt{3}x^2}\sqrt{3+\left(-3+\sqrt{3}\right)x^2}}\,dx = \int \frac{1}{\sqrt{\left(\sqrt{3}-3\right)\,x^2+3}\,\sqrt{2\,\sqrt{3}\,x^2-3\,\sqrt{3}+3}}\,dx$$

```
[In] int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2))
,x)
```

```
[Out] int(1/((x^2*(3^(1/2) - 3) + 3)^(1/2)*(2*3^(1/2)*x^2 - 3*3^(1/2) + 3)^(1/2))
, x)
```

3.304 $\int \frac{1}{\sqrt[4]{2+3x^2}(4+3x^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)} dx = -\frac{\arctan\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] -1/12*arctan(1/6*(2*2^(3/4)+2*2^(1/4)*(3*x^2+2)^(1/2))/x/(3*x^2+2)^(1/4)*3^
(1/2))*2^(1/4)*3^(1/2)-1/12*arctanh(1/6*(2*2^(3/4)-2*2^(1/4)*(3*x^2+2)^(1/2
))/x/(3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)} dx = -\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{3x^2+2}+2\ 2^{3/4}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}} - \frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{3x^2+2}}{2\sqrt{3}x\sqrt[4]{3x^2+2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[In] Int[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] -1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + 3*x^2])/(2*Sqrt[3]*x*(2 + 3*x^2
)^(1/4))]/(2^(3/4)*Sqrt[3]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + 3*x^2
])/(2*Sqrt[3]*x*(2 + 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
```

```
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+3x^2}}{2\sqrt{3}x\sqrt[4]{2+3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \frac{\arctan\left(\frac{3\sqrt{2}x^2-4\sqrt{2+3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2+3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2+3x^2}}{3\sqrt{2}x^2+4\sqrt{2+3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

```
[In] Integrate[1/((2 + 3*x^2)^(1/4)*(4 + 3*x^2)),x]
```

```
[Out] (ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 + 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 + 3*x^2)^(1/4))/(3*Sqrt[2]*x^2
+ 4*Sqrt[2 + 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.45 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.45

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^4+72\right)\ln\left(-\frac{6\left(3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)+\left(3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^4+72\right)^3-18\sqrt{3x^2+2}\,x-3\operatorname{RootOf}\left(\_Z^4+72\right)^2x}{3x^2+4}\right)}{24}$

```
[In] int(1/(3*x^2+2)^(1/4)/(3*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*RootOf(_Z^4+72)*ln(-(6*(3*x^2+2)^(3/4)*RootOf(_Z^4+72)+(3*x^2+2)^(1/4
)*RootOf(_Z^4+72)^3-18*(3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2+4))+
1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln((6*(3*x^2+2)^(3/4)*RootOf(_Z^2+RootO
f(_Z^4+72)^2)-(3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+72)^2)*RootOf(_Z^4+72
)^2+18*(3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2+4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \left(\frac{1}{288}i + \frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x - (3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x + 12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}} + 12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) - \left(\frac{1}{288}i - \frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x + (3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x - 12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}} + 12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) + \left(\frac{1}{288}i - \frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x - (3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x - 12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}} + 12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right) - \left(\frac{1}{288}i + \frac{1}{288}\right) \cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x + (3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{3\,x^2+2}x + 12i\,\sqrt{2}(3\,x^2+2)^{\frac{1}{4}} + 12\,(3\,x^2+2)^{\frac{3}{4}}}{3\,x^2+4}\right)$$

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="fricas")
```

```
[Out] (1/288*I + 1/288)*18^(3/4)*sqrt(2)*log(((I + 1)*18^(3/4)*sqrt(2)*x - (3*I -
 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x + 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 1
2*(3*x^2 + 2)^(3/4))/(3*x^2 + 4)) - (1/288*I - 1/288)*18^(3/4)*sqrt(2)*log(
(-(I - 1)*18^(3/4)*sqrt(2)*x + (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x
 - 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 12*(3*x^2 + 2)^(3/4))/(3*x^2 + 4)) + (1
/288*I - 1/288)*18^(3/4)*sqrt(2)*log(((I - 1)*18^(3/4)*sqrt(2)*x - (3*I + 3
)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x - 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 12*
(3*x^2 + 2)^(3/4))/(3*x^2 + 4)) - (1/288*I + 1/288)*18^(3/4)*sqrt(2)*log((-
(I + 1)*18^(3/4)*sqrt(2)*x + (3*I - 3)*18^(1/4)*sqrt(2)*sqrt(3*x^2 + 2)*x +
 12*I*sqrt(2)*(3*x^2 + 2)^(1/4) + 12*(3*x^2 + 2)^(3/4))/(3*x^2 + 4))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \int \frac{1}{\sqrt[4]{3x^2+2}\cdot(3x^2+4)}\,dx$$

```
[In] integrate(1/(3*x**2+2)**(1/4)/(3*x**2+4),x)
[Out] Integral(1/((3*x**2 + 2)**(1/4)*(3*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \int \frac{1}{(3\,x^2+4)(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(3*x^2+2)^(1/4)/(3*x^2+4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 4)*(3*x^2 + 2)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2+3x^2}\,(4+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2)^{1/4}\,(3\,x^2+4)}\,dx$$

```
[In] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)),x)
[Out] int(1/((3*x^2 + 2)^(1/4)*(3*x^2 + 4)), x)
```

3.305 $\int \frac{1}{\sqrt[4]{2-3x^2}(4-3x^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[Out] 1/12*arctan(1/6*(2-2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)/x/(-3*x^2+2)^(1/4)*3^(
1/2))*2^(1/4)*3^(1/2)+1/12*arctanh(1/6*(2+2^(1/2)*(-3*x^2+2)^(1/2))*2^(3/4)
/x/(-3*x^2+2)^(1/4)*3^(1/2))*2^(1/4)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx=\frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2-3x^2}+2}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

```
[In] Int[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[3]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - 3*x^2])/(2^(1/4)*Sqrt[
3]*x*(2 - 3*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[3])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
```

```
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}} + \frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-3x^2}}{\sqrt[4]{2}\sqrt{3}x\sqrt[4]{2-3x^2}}\right)}{2\ 2^{3/4}\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \frac{\arctan\left(\frac{3\sqrt{2}x^2-4\sqrt{2-3x^2}}{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}\right) + \operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{3}x\sqrt[4]{2-3x^2}}{3\sqrt{2}x^2+4\sqrt{2-3x^2}}\right)}{4\ 2^{3/4}\sqrt{3}}$$

```
[In] Integrate[1/((2 - 3*x^2)^(1/4)*(4 - 3*x^2)),x]
[Out] (ArcTan[(3*Sqrt[2]*x^2 - 4*Sqrt[2 - 3*x^2])/(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[3]*x*(2 - 3*x^2)^(1/4))/(3*Sqrt[2]*x^2
+ 4*Sqrt[2 - 3*x^2])])/(4*2^(3/4)*Sqrt[3])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.47 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.57

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+72\right)^2\right)\ln\left(-\frac{6\left(-3x^2+2\right)^{\frac{3}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{RootOf}\left(\_Z^4+72\right)^2\right)+\left(-3x^2+2\right)^{\frac{1}{4}}\operatorname{RootOf}\left(\_Z^2+\operatorname{Roo}\right.}{3x^2-4}\right.}{24}$

```
[In] int(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x,method=_RETURNVERBOSE)
[Out] -1/24*RootOf(_Z^2+RootOf(_Z^4+72)^2)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^2+Ro
otOf(_Z^4+72)^2)+(-3*x^2+2)^(1/4)*RootOf(_Z^2+RootOf(_Z^4+72)^2)*RootOf(_Z^
4+72)^2-18*(-3*x^2+2)^(1/2)*x-3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))-1/24*RootOf
(_Z^4+72)*ln(-(6*(-3*x^2+2)^(3/4)*RootOf(_Z^4+72)-(-3*x^2+2)^(1/4)*RootOf(_
Z^4+72)^3-18*(-3*x^2+2)^(1/2)*x+3*RootOf(_Z^4+72)^2*x)/(3*x^2-4))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right) + \left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right) - \left(\frac{1}{288}i-\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{(i-1)\cdot 18^{\frac{3}{4}}\sqrt{2}x+(3i+3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x+12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right) + \left(\frac{1}{288}i+\frac{1}{288}\right)\cdot 18^{\frac{3}{4}}\sqrt{2}\log\left(\frac{-(i+1)\cdot 18^{\frac{3}{4}}\sqrt{2}x-(3i-3)\cdot 18^{\frac{1}{4}}\sqrt{2}\sqrt{-3\,x^2+2}x-12i\,\sqrt{2}(-3\,x^2+2)^{\frac{1}{4}}+12\,(-3\,x^2+2)^{\frac{3}{4}}}{3\,x^2-4}\right)$$

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="fricas")
```

```
[Out] -(1/288*I + 1/288)*18^(3/4)*sqrt(2)*log(((I + 1)*18^(3/4)*sqrt(2)*x + (3*I
- 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4)
+ 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I - 1/288)*18^(3/4)*sqrt(2)*
log((-(I - 1)*18^(3/4)*sqrt(2)*x - (3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 +
 2)*x + 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4
)) - (1/288*I - 1/288)*18^(3/4)*sqrt(2)*log(((I - 1)*18^(3/4)*sqrt(2)*x + (
3*I + 3)*18^(1/4)*sqrt(2)*sqrt(-3*x^2 + 2)*x + 12*I*sqrt(2)*(-3*x^2 + 2)^(1
/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2 - 4)) + (1/288*I + 1/288)*18^(3/4)*sqrt
(2)*log((-(I + 1)*18^(3/4)*sqrt(2)*x - (3*I - 3)*18^(1/4)*sqrt(2)*sqrt(-3*x
^2 + 2)*x - 12*I*sqrt(2)*(-3*x^2 + 2)^(1/4) + 12*(-3*x^2 + 2)^(3/4))/(3*x^2
 - 4))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{3x^2\sqrt[4]{2-3x^2}-4\sqrt[4]{2-3x^2}}\,dx$$

```
[In] integrate(1/(-3*x**2+2)**(1/4)/(-3*x**2+4),x)
[Out] -Integral(1/(3*x**2*(2 - 3*x**2)**(1/4) - 4*(2 - 3*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-4)(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2+2)^(1/4)/(-3*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 4)*(-3*x^2 + 2)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-3x^2}\,(4-3x^2)}\,dx = -\int \frac{1}{(2-3\,x^2)^{1/4}\,(3\,x^2-4)}\,dx$$

```
[In] int(-1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)),x)
[Out] -int(1/((2 - 3*x^2)^(1/4)*(3*x^2 - 4)), x)
```

3.306 $\int \frac{1}{\sqrt[4]{2+bx^2}(4+bx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 129

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=-\frac{\arctan\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*(2*2^(3/4)+2*2^(1/4)*(b*x^2+2)^(1/2))/x/(b*x^2+2)^(1/4)/b^(
1/2))*2^(1/4)/b^(1/2)-1/4*arctanh(1/2*(2*2^(3/4)-2*2^(1/4)*(b*x^2+2)^(1/2))
/x/(b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx=-\frac{\arctan\left(\frac{2\sqrt[4]{2}\sqrt{bx^2+2}+2\ 2^{3/4}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{bx^2+2}}{2\sqrt{b}x\sqrt[4]{bx^2+2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

```
[In] Int[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(2*2^(3/4) + 2*2^(1/4)*Sqrt[2 + b*x^2])/(2*Sqrt[b]*x*(2 + b*x^2
)^(1/4))]/(2^(3/4)*Sqrt[b]) - ArcTanh[(2*2^(3/4) - 2*2^(1/4)*Sqrt[2 + b*x^2
])/(2*Sqrt[b]*x*(2 + b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
```

```
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{2\ 2^{3/4}+2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{2\ 2^{3/4}-2\sqrt[4]{2}\sqrt{2+bx^2}}{2\sqrt{b}x\sqrt[4]{2+bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \frac{\arctan\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2+bx^2}}{4\sqrt{b}x\sqrt[4]{2+bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{b}x\sqrt[4]{2+bx^2}}{\sqrt{2}bx^2+4\sqrt{2+bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((2 + b*x^2)^(1/4)*(4 + b*x^2)),x]
[Out] (ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 + b*x^2])/(4*Sqrt[b]*x*(2 + b*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 + b*x^2)^(1/4))/(Sqrt[2]*b*x^2
+ 4*Sqrt[2 + b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(b\,x^2+2)^{\frac{1}{4}}\,(b\,x^2+4)}dx$$

```
[In] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
[Out] int(1/(b*x^2+2)^(1/4)/(b*x^2+4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.93

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx =$$
$$-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(bx^2+}{bx^2+4}\right.$$
$$+\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}-(bx^2}{bx^2+4}\right.$$
$$+\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(bx^2}{bx^2+4}\right.$$
$$-\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(b}{bx^2+4}\right.$$

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="fricas")
```

```
[Out] -1/8*(1/2)^(1/4)*(-1/b^2)^(1/4)*log(((1/2)^(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1/
b^2)^(3/4) - (1/2)^(1/4)*b*x*(-1/b^2)^(1/4) + 2*sqrt(1/2)*(b*x^2 + 2)^(1/4)
*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/4))/(b*x^2 + 4)) + 1/8*(1/2)^(1/4)*(-1/b^2
)^(1/4)*log(-((1/2)^(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) - (1/2)^(1/4
)*b*x*(-1/b^2)^(1/4) - 2*sqrt(1/2)*(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) - (b*x^
2 + 2)^(3/4))/(b*x^2 + 4)) + 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((I*(1/2)^
(3/4)*sqrt(b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + I*(1/2)^(1/4)*b*x*(-1/b^2)^(1/
4) - 2*sqrt(1/2)*(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/4))/(b*x
^2 + 4)) - 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((-I*(1/2)^(3/4)*sqrt(b*x^2
+ 2)*b^2*x*(-1/b^2)^(3/4) - I*(1/2)^(1/4)*b*x*(-1/b^2)^(1/4) - 2*sqrt(1/2)*
(b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (b*x^2 + 2)^(3/4))/(b*x^2 + 4))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{\sqrt[4]{bx^2+2}\,(bx^2+4)}\,dx$$

```
[In] integrate(1/(b*x**2+2)**(1/4)/(b*x**2+4),x)
```

```
[Out] Integral(1/((b*x**2 + 2)**(1/4)*(b*x**2 + 4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{(bx^2+4)(bx^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(b*x^2+2)^(1/4)/(b*x^2+4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 4)*(b*x^2 + 2)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2+bx^2}\,(4+bx^2)}\,dx = \int \frac{1}{{\left(b\,x^2+2\right)}^{1/4}\,\left(b\,x^2+4\right)}\,dx$$

```
[In] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)),x)
[Out] int(1/((b*x^2 + 2)^(1/4)*(b*x^2 + 4)), x)
```

3.307 $\int \frac{1}{\sqrt[4]{2-bx^2}(4-bx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

```
[Out] 1/4*arctan(1/2*(2-2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x/(-b*x^2+2)^(1/4)/b^(1
/2))*2^(1/4)/b^(1/2)+1/4*arctanh(1/2*(2+2^(1/2)*(-b*x^2+2)^(1/2))*2^(3/4)/x
/(-b*x^2+2)^(1/4)/b^(1/2))*2^(1/4)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \frac{\arctan\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2-bx^2}+2}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\,2^{3/4}\sqrt{b}}$$

```
[In] Int[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
```

```
[Out] ArcTan[(2 - Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))]
/(2*2^(3/4)*Sqrt[b]) + ArcTanh[(2 + Sqrt[2]*Sqrt[2 - b*x^2])/(2^(1/4)*Sqrt[
b]*x*(2 - b*x^2)^(1/4))]/(2*2^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
```

```
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{2-\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{2+\sqrt{2}\sqrt{2-bx^2}}{\sqrt[4]{2}\sqrt{b}x\sqrt[4]{2-bx^2}}\right)}{2\ 2^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \frac{\arctan\left(\frac{2^{3/4}bx^2-4\sqrt[4]{2}\sqrt{2-bx^2}}{4\sqrt{b}x\sqrt[4]{2-bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\ 2^{3/4}\sqrt{b}x\sqrt[4]{2-bx^2}}{\sqrt{2}bx^2+4\sqrt{2-bx^2}}\right)}{4\ 2^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((2 - b*x^2)^(1/4)*(4 - b*x^2)),x]
[Out] (ArcTan[(2^(3/4)*b*x^2 - 4*2^(1/4)*Sqrt[2 - b*x^2])/(4*Sqrt[b]*x*(2 - b*x^2
)^(1/4))] + ArcTanh[(2*2^(3/4)*Sqrt[b]*x*(2 - b*x^2)^(1/4))/(Sqrt[2]*b*x^2
+ 4*Sqrt[2 - b*x^2])])/(4*2^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^2+2\right)^{\frac{1}{4}}\left(-b\,x^2+4\right)}dx$$

```
[In] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
[Out] int(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.13

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx$$
$$=\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(-\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}-(-bx^2+2)^{\frac{3}{4}}}{bx^2-4}\right)$$
$$-\frac{1}{8}\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}-2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(-bx^2+2)^{\frac{3}{4}}}{bx^2-4}\right)$$
$$+\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}-i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(-bx^2+2)^{\frac{3}{4}}}{bx^2-4}\right)$$
$$-\frac{1}{8}i\left(\frac{1}{2}\right)^{\frac{1}{4}}\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}\log\left(\frac{-i\left(\frac{1}{2}\right)^{\frac{3}{4}}\sqrt{-bx^2+2}b^2x\left(-\frac{1}{b^2}\right)^{\frac{3}{4}}+i\left(\frac{1}{2}\right)^{\frac{1}{4}}bx\left(-\frac{1}{b^2}\right)^{\frac{1}{4}}+2\sqrt{\frac{1}{2}}(-bx^2+2)^{\frac{1}{4}}b\sqrt{-\frac{1}{b^2}}+(-bx^2+2)^{\frac{3}{4}}}{bx^2-4}\right)$$

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/8*(1/2)^(1/4)*(-1/b^2)^(1/4)*log(-((1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1
/b^2)^(3/4) + (1/2)^(1/4)*b*x*(-1/b^2)^(1/4) + 2*sqrt(1/2)*(-b*x^2 + 2)^(1/
4)*b*sqrt(-1/b^2) - (-b*x^2 + 2)^(3/4))/(b*x^2 - 4)) - 1/8*(1/2)^(1/4)*(-1/
b^2)^(1/4)*log(((1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + (1/2)^(
1/4)*b*x*(-1/b^2)^(1/4) - 2*sqrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (
-b*x^2 + 2)^(3/4))/(b*x^2 - 4)) + 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((I*(
1/2)^(3/4)*sqrt(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) - I*(1/2)^(1/4)*b*x*(-1/b^
2)^(1/4) + 2*sqrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (-b*x^2 + 2)^(3/
4))/(b*x^2 - 4)) - 1/8*I*(1/2)^(1/4)*(-1/b^2)^(1/4)*log((-I*(1/2)^(3/4)*sqr
t(-b*x^2 + 2)*b^2*x*(-1/b^2)^(3/4) + I*(1/2)^(1/4)*b*x*(-1/b^2)^(1/4) + 2*s
qrt(1/2)*(-b*x^2 + 2)^(1/4)*b*sqrt(-1/b^2) + (-b*x^2 + 2)^(3/4))/(b*x^2 - 4
))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = -\int \frac{1}{bx^2\sqrt[4]{-bx^2+2}-4\sqrt[4]{-bx^2+2}}\,dx$$

```
[In] integrate(1/(-b*x**2+2)**(1/4)/(-b*x**2+4),x)
```

```
[Out] -Integral(1/(b*x**2*(-b*x**2 + 2)**(1/4) - 4*(-b*x**2 + 2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \int -\frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = \int -\frac{1}{(bx^2-4)(-bx^2+2)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2+2)^(1/4)/(-b*x^2+4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 - 4)*(-b*x^2 + 2)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{2-bx^2}\,(4-bx^2)}\,dx = -\int \frac{1}{\left(2-b\,x^2\right)^{1/4}\,\left(b\,x^2-4\right)}\,dx$$

```
[In] int(-1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)),x)
[Out] -int(1/((2 - b*x^2)^(1/4)*(b*x^2 - 4)), x)
```

3.308 $\int \frac{1}{\sqrt[4]{a+3x^2}(2a+3x^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] -1/6*arctan(1/3*a^(3/4)*(1+(3*x^2+a)^(1/2)/a^(1/2))/x/(3*x^2+a)^(1/4)*3^(1/
2))/a^(3/4)*3^(1/2)-1/6*arctanh(1/3*a^(3/4)*(1-(3*x^2+a)^(1/2)/a^(1/2))/x/(
3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[In] Int[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] -1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a + 3*x^2)^
(1/4))]/(Sqrt[3]*a^(3/4)) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + 3*x^2]/Sqrt[a]))
/(Sqrt[3]*x*(a + 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a+3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \frac{-\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a+3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a+3x^2}}{3x^2+2\sqrt{a}\sqrt{a+3x^2}}\right)}{4\sqrt{3}a^{3/4}}$$

```
[In] Integrate[1/((a + 3*x^2)^(1/4)*(2*a + 3*x^2)),x]
```

```
[Out] (-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a + 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a + 3*x
^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a + 3*x^2)^(1/4))/(3*x^2 + 2*Sq
rt[a]*Sqrt[a + 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

Maple [F]

$$\int \frac{1}{(3x^2+a)^{\frac{1}{4}}\,(3x^2+2a)}dx$$

```
[In] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

```
[Out] int(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.03 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.19

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx =$$
$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+(3}{3x^2+2a}\right.$$
$$+\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-}{3x^2+2a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+}{3x^2+2a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(3x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}}{3x^2+2a}\right.$$

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x
*(-1/a^3)^(3/4) + (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(
-1/a^3)^(1/4) + (3*x^2 + a)^(3/4))/(3*x^2 + 2*a)) + 1/4*(1/36)^(1/4)*(-1/a^
3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^
2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) - (3*x^2
+ a)^(3/4))/(3*x^2 + 2*a)) + 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((18*I*(1
/36)^(3/4)*sqrt(3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2*sqr
t(-1/a^3) + 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (3*x^2 + a)^(3/4))/(3*x^2
 + 2*a)) - 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((-18*I*(1/36)^(3/4)*sqrt(3
*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) - 3*I*(
1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (3*x^2 + a)^(3/4))/(3*x^2 + 2*a))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{\sqrt[4]{a+3x^2}\cdot(2a+3x^2)}\,dx$$

```
[In] integrate(1/(3*x**2+a)**(1/4)/(3*x**2+2*a),x)
```

```
[Out] Integral(1/((a + 3*x**2)**(1/4)*(2*a + 3*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)(3\,x^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(3*x^2+a)^(1/4)/(3*x^2+2*a),x, algorithm="giac")
[Out] integrate(1/((3*x^2 + 2*a)*(3*x^2 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+3x^2}\,(2a+3x^2)}\,dx = \int \frac{1}{(3\,x^2+2\,a)\,\left(3\,x^2+a\right)^{1/4}}\,dx$$

```
[In] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)),x)
[Out] int(1/((2*a + 3*x^2)*(a + 3*x^2)^(1/4)), x)
```

3.309 $\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)} \, dx = \frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}} + \frac{\text{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[Out] 1/6*arctan(1/3*a^(3/4)*(1-(-3*x^2+a)^(1/2)/a^(1/2))/x/(-3*x^2+a)^(1/4)*3^(1
/2))/a^(3/4)*3^(1/2)+1/6*arctanh(1/3*a^(3/4)*(1+(-3*x^2+a)^(1/2)/a^(1/2))/x
/(-3*x^2+a)^(1/4)*3^(1/2))/a^(3/4)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{a-3x^2}(2a-3x^2)} \, dx = \frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}} + \frac{\text{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-3x^2}}{\sqrt{a}}+1\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

```
[In] Int[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - 3*x^2]/Sqrt[a]))/(Sqrt[3]*x*(a - 3*x^2)^(1/4)
)]/(2*Sqrt[3]*a^(3/4)) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - 3*x^2]/Sqrt[a]))/(S
qrt[3]*x*(a - 3*x^2)^(1/4))]/(2*Sqrt[3]*a^(3/4))
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-3x^2}}{\sqrt{a}}\right)}{\sqrt{3}x\sqrt[4]{a-3x^2}}\right)}{2\sqrt{3}a^{3/4}}$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \frac{-\arctan\left(\frac{-3x^2+2\sqrt{a}\sqrt{a-3x^2}}{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt{3}\sqrt[4]{a}x\sqrt[4]{a-3x^2}}{3x^2+2\sqrt{a}\sqrt{a-3x^2}}\right)}{4\sqrt{3}a^{3/4}}$$

```
[In] Integrate[1/((a - 3*x^2)^(1/4)*(2*a - 3*x^2)),x]
```

```
[Out] (-ArcTan[(-3*x^2 + 2*Sqrt[a]*Sqrt[a - 3*x^2])/(2*Sqrt[3]*a^(1/4)*x*(a - 3*x
^2)^(1/4))] + ArcTanh[(2*Sqrt[3]*a^(1/4)*x*(a - 3*x^2)^(1/4))/(3*x^2 + 2*Sq
rt[a]*Sqrt[a - 3*x^2])])/(4*Sqrt[3]*a^(3/4))
```

Maple [F]

$$\int \frac{1}{(-3x^2+a)^{\frac{1}{4}}\,(-3x^2+2a)}dx$$

```
[In] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

```
[Out] int(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.05 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.18

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(-3\,x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}-}{3\,x^2-2\,a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}-\left(-3\,x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}+}{3\,x^2-2\,a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(-3\,x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a^3}\right)}{3\,x^2-2\,a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3}\right)^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2+a}a^2x\left(-\frac{1}{a^3}\right)^{\frac{3}{4}}+\left(-3\,x^2+a\right)^{\frac{1}{4}}a^2\sqrt{-\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}ax\left(-\frac{1}{a}\right.}{3\,x^2-2\,a}\right.$$

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(1/36)^(1/4)*(-1/a^3)^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*
x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x
*(-1/a^3)^(1/4) - (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a)) - 1/4*(1/36)^(1/4)*(-1
/a^3)^(1/4)*log((18*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) - (-
3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^3) + 3*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3
*x^2 + a)^(3/4))/(3*x^2 - 2*a)) + 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((18
*I*(1/36)^(3/4)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*
a^2*sqrt(-1/a^3) - 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4)
)/(3*x^2 - 2*a)) - 1/4*I*(1/36)^(1/4)*(-1/a^3)^(1/4)*log((-18*I*(1/36)^(3/4
)*sqrt(-3*x^2 + a)*a^2*x*(-1/a^3)^(3/4) + (-3*x^2 + a)^(1/4)*a^2*sqrt(-1/a^
3) + 3*I*(1/36)^(1/4)*a*x*(-1/a^3)^(1/4) + (-3*x^2 + a)^(3/4))/(3*x^2 - 2*a
))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = -\int \frac{1}{-2a\sqrt[4]{a-3x^2}+3x^2\sqrt[4]{a-3x^2}}\,dx$$

```
[In] integrate(1/(-3*x**2+a)**(1/4)/(-3*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - 3*x**2)**(1/4) + 3*x**2*(a - 3*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int -\frac{1}{(3\,x^2-2\,a)(-3\,x^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2+a)^(1/4)/(-3*x^2+2*a),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 - 2*a)*(-3*x^2 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-3x^2}\,(2a-3x^2)}\,dx = \int \frac{1}{(2\,a-3\,x^2)\,\left(a-3\,x^2\right)^{1/4}}\,dx$$

```
[In] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)),x)
[Out] int(1/((2*a - 3*x^2)*(a - 3*x^2)^(1/4)), x)
```

3.310 $\int \frac{1}{\sqrt[4]{a+bx^2}(2a+bx^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -1/2*arctan(a^(3/4)*(1+(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)^(1/4)/b^(1/2))/
a^(3/4)/b^(1/2)-1/2*arctanh(a^(3/4)*(1-(b*x^2+a)^(1/2)/a^(1/2))/x/(b*x^2+a)
^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx=-\frac{\arctan\left(\frac{a^{3/4}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[In] Int[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
```

```
[Out] -1/2*ArcTan[(a^(3/4)*(1 + Sqrt[a + b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a + b*x^2)^
(1/4))]/(a^(3/4)*Sqrt[b]) - ArcTanh[(a^(3/4)*(1 - Sqrt[a + b*x^2]/Sqrt[a]))
/(Sqrt[b]*x*(a + b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a+bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a+bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}\right) + \operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a+bx^2}}{bx^2+2\sqrt{a}\sqrt{a+bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x]
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a + b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2
)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a + b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a + b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,(b\,x^2+2a)}dx$$

```
[In] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
[Out] int(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.40 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.76

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}}{bx^2+2a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)}{bx^2+2a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)}{bx^2+2a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2+a}a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right)}{bx^2+2a}\right.$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] -1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2 + a)*a^
2*b^2*x*(-1/(a^3*b^2))^(3/4) + (b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2)) -
 (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(3/4))/(b*x^2 + 2*a))
 + 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2 + a)
*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2)
) - (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (b*x^2 + a)^(3/4))/(b*x^2 + 2*
a)) + 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt(b*x^
2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^
3*b^2)) + I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(3/4))/(b*
x^2 + 2*a)) - 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((-2*I*(1/4)^(3/4)*
sqrt(b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (b*x^2 + a)^(1/4)*a^2*b*sq
rt(-1/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (b*x^2 + a)^(
3/4))/(b*x^2 + 2*a))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\cdot(2a+bx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/4)/(b*x**2+2*a),x)
```

```
[Out] Integral(1/((a + b*x**2)**(1/4)*(2*a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{(bx^2+2\,a)(bx^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(b*x^2+2*a),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + 2*a)*(b*x^2 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(2a+bx^2)}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{1/4}\,\left(b\,x^2+2\,a\right)}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)),x)
[Out] int(1/((a + b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.311 $\int \frac{1}{\sqrt[4]{a-bx^2}(2a-bx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 124

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] 1/2*arctan(a^(3/4)*(1-(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2+a)^(1/4)/b^(1/2))
/a^(3/4)/b^(1/2)+1/2*arctanh(a^(3/4)*(1+(-b*x^2+a)^(1/2)/a^(1/2))/x/(-b*x^2
+a)^(1/4)/b^(1/2))/a^(3/4)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {406}

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx=\frac{\arctan\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{a^{3/4}\left(\frac{\sqrt{a-bx^2}}{\sqrt{a}}+1\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[In] Int[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] ArcTan[(a^(3/4)*(1 - Sqrt[a - b*x^2]/Sqrt[a]))/(Sqrt[b]*x*(a - b*x^2)^(1/4)
)]/(2*a^(3/4)*Sqrt[b]) + ArcTanh[(a^(3/4)*(1 + Sqrt[a - b*x^2]/Sqrt[a]))/(S
qrt[b]*x*(a - b*x^2)^(1/4))]/(2*a^(3/4)*Sqrt[b])
```

Rule 406

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[b^2/a, 4]}, Simp[(-b/(2*a*d*q))*ArcTan[(b + q^2*Sqrt[a + b*x^2])/
(q^3*x*(a + b*x^2)^(1/4))], x] - Simp[(b/(2*a*d*q))*ArcTanh[(b - q^2*Sqrt[a
 + b*x^2])/(q^3*x*(a + b*x^2)^(1/4))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ
[b*c - 2*a*d, 0] && PosQ[b^2/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{a^{3/4}\left(1-\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}} + \frac{\tanh^{-1}\left(\frac{a^{3/4}\left(1+\frac{\sqrt{a-bx^2}}{\sqrt{a}}\right)}{\sqrt{b}x\sqrt[4]{a-bx^2}}\right)}{2a^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \frac{\arctan\left(\frac{bx^2-2\sqrt{a}\sqrt{a-bx^2}}{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}\right)+\operatorname{arctanh}\left(\frac{2\sqrt[4]{a}\sqrt{b}x\sqrt[4]{a-bx^2}}{bx^2+2\sqrt{a}\sqrt{a-bx^2}}\right)}{4a^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x]
```

```
[Out] (ArcTan[(b*x^2 - 2*Sqrt[a]*Sqrt[a - b*x^2])/(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2
)^(1/4))] + ArcTanh[(2*a^(1/4)*Sqrt[b]*x*(a - b*x^2)^(1/4))/(b*x^2 + 2*Sqrt
[a]*Sqrt[a - b*x^2])])/(4*a^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,(-b\,x^2+2a)}dx$$

```
[In] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

```
[Out] int(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 25.30 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.70

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right.}{bx^2-2\,a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\frac{1}{a^3b^2}\right.}{bx^2-2\,a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\right.}{bx^2-2\,a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2+a}\,a^2b^2x\left(-\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2+a)^{\frac{1}{4}}a^2b\sqrt{-\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(-\right.}{bx^2-2\,a}\right.$$

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="fricas")
```

```
[Out] 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-b*x^2 + a)*a
^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*b^2))
 + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) - (-b*x^2 + a)^(3/4))/(b*x^2 - 2*
a)) - 1/4*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(-b*x^2 +
 a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) - (-b*x^2 + a)^(1/4)*a^2*b*sqrt(-1/(a^3*
b^2)) + (1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^2 + a)^(3/4))/(b*x^2
 - 2*a)) + 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt
(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4)*a^2*b*sqrt
(-1/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b*x^2 + a)^(3
/4))/(b*x^2 - 2*a)) - 1/4*I*(1/4)^(1/4)*(-1/(a^3*b^2))^(1/4)*log((-2*I*(1/4
)^(3/4)*sqrt(-b*x^2 + a)*a^2*b^2*x*(-1/(a^3*b^2))^(3/4) + (-b*x^2 + a)^(1/4
)*a^2*b*sqrt(-1/(a^3*b^2)) + I*(1/4)^(1/4)*a*b*x*(-1/(a^3*b^2))^(1/4) + (-b
*x^2 + a)^(3/4))/(b*x^2 - 2*a))
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = -\int \frac{1}{-2a\sqrt[4]{a-bx^2}+bx^2\sqrt[4]{a-bx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2+a)**(1/4)/(-b*x**2+2*a),x)
```

```
[Out] -Integral(1/(-2*a*(a - b*x**2)**(1/4) + b*x**2*(a - b*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int -\frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int -\frac{1}{(bx^2-2\,a)(-bx^2+a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2+a)^(1/4)/(-b*x^2+2*a),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 - 2*a)*(-b*x^2 + a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a-bx^2}\,(2a-bx^2)}\,dx = \int \frac{1}{(a-b\,x^2)^{1/4}\,(2\,a-b\,x^2)}\,dx$$

```
[In] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)),x)
[Out] int(1/((a - b*x^2)^(1/4)*(2*a - b*x^2)), x)
```

3.312 $\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/(3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(1
/2)/(3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {407}

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{3x^2-1}}\right)}{2\sqrt{6}}$$

```
[In] Int[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(-1 + 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]*
x)/(-1 + 3*x^2)^(1/4)]/(2*Sqrt[6])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

NegQ[b^2/a]

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1+3x^2}}{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1+3x^2}}\right)}{2\sqrt{6}}$$

```
[In] Integrate[1/((-2 + 3*x^2)*(-1 + 3*x^2)^(1/4)),x]
[Out] (ArcTan[(Sqrt[2/3]*(-1 + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[3/2]*x)/(-1 + 3*x
^2)^(1/4)])/(2*Sqrt[6])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{3}{4}}+3\sqrt{3x^2-1}\,x-\operatorname{RootOf}\left(\_Z^2+6\right)\left(3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2-2}\right)}{12}+\frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln\left(-\right.}{}$

```
[In] int(1/(3*x^2-2)/(3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
[Out] -1/12*RootOf(_Z^2+6)*ln((RootOf(_Z^2+6)*(3*x^2-1)^(3/4)+3*(3*x^2-1)^(1/2)*x
-RootOf(_Z^2+6)*(3*x^2-1)^(1/4)-3*x)/(3*x^2-2))+1/12*RootOf(_Z^2-6)*ln(-(Ro
otOf(_Z^2-6)*(3*x^2-1)^(3/4)-3*(3*x^2-1)^(1/2)*x+RootOf(_Z^2-6)*(3*x^2-1)^(
1/4)-3*x)/(3*x^2-2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(43) = 86$.

Time = 2.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.70

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}}{3\,x}\right) + \frac{1}{24}\sqrt{6}\log\left(-\frac{9\,x^4-6\,\sqrt{6}(3\,x^2-1)^{\frac{1}{4}}x^3+12\,\sqrt{3\,x^2-1}x^2-4\,\sqrt{6}(3\,x^2-1)^{\frac{3}{4}}x+12\,x^2-4}{9\,x^4-12\,x^2+4}\right)$$

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(6)*arctan(1/3*sqrt(6)*(3*x^2 - 1)^(1/4)/x) + 1/24*sqrt(6)*log(-(9
*x^4 - 6*sqrt(6)*(3*x^2 - 1)^(1/4)*x^3 + 12*sqrt(3*x^2 - 1)*x^2 - 4*sqrt(6)
*(3*x^2 - 1)^(3/4)*x + 12*x^2 - 4)/(9*x^4 - 12*x^2 + 4))
```

Sympy [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3x^2-2)\sqrt[4]{3x^2-1}}\,dx$$

```
[In] integrate(1/(3*x**2-2)/(3*x**2-1)**(1/4),x)
```

```
[Out] Integral(1/((3*x**2 - 2)*(3*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx = \int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Giac [F]

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx=\int \frac{1}{(3\,x^2-1)^{\frac{1}{4}}(3\,x^2-2)}\,dx$$

```
[In] integrate(1/(3*x^2-2)/(3*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+3x^2)\sqrt[4]{-1+3x^2}}\,dx=\int \frac{1}{(3\,x^2-1)^{1/4}\,(3\,x^2-2)}\,dx$$

```
[In] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)),x)
[Out] int(1/((3*x^2 - 1)^(1/4)*(3*x^2 - 2)), x)
```

3.313 $\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/(-3*x^2-1)^(1/4))*6^(1/2)-1/12*arctanh(1/2*x*6^(
1/2)/(-3*x^2-1)^(1/4))*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {407}

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-3x^2-1}}\right)}{2\sqrt{6}}$$

```
[In] Int[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(-1 - 3*x^2)^(1/4)]/Sqrt[6] - ArcTanh[(Sqrt[3/2]*
x)/(-1 - 3*x^2)^(1/4)]/(2*Sqrt[6])
```

<u>Rule 407</u>

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

NegQ[b^2/a]

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{-1-3x^2}}{x}\right) + \operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{-1-3x^2}}\right)}{2\sqrt{6}}$$

```
[In] Integrate[1/((-2 - 3*x^2)*(-1 - 3*x^2)^(1/4)),x]
[Out] -1/2*(-ArcTan[(Sqrt[2/3]*(-1 - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[3/2]*x)/(-1
 - 3*x^2)^(1/4)])/Sqrt[6]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.26

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{3}{4}}-3\sqrt{-3x^2-1}\,x+\operatorname{RootOf}\left(\_Z^2+6\right)\left(-3x^2-1\right)^{\frac{1}{4}}-3x}{3x^2+2}\right)}{12} - \frac{\operatorname{RootOf}\left(\_Z^2-6\right)\ln}{}$

```
[In] int(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x,method=_RETURNVERBOSE)
[Out] 1/12*RootOf(_Z^2+6)*ln(-(RootOf(_Z^2+6)*(-3*x^2-1)^(3/4)-3*(-3*x^2-1)^(1/2)
*x+RootOf(_Z^2+6)*(-3*x^2-1)^(1/4)-3*x)/(3*x^2+2))-1/12*RootOf(_Z^2-6)*ln((
RootOf(_Z^2-6)*(-3*x^2-1)^(3/4)+3*(-3*x^2-1)^(1/2)*x-RootOf(_Z^2-6)*(-3*x^2
-1)^(1/4)-3*x)/(3*x^2+2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.98

$$\begin{aligned}
&\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx\\
&= -\frac{1}{24}\sqrt{6}\log\left(\frac{\sqrt{6}\sqrt{-3\,x^2-1}x-\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}-2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\
&\quad+\frac{1}{24}\sqrt{6}\log\left(-\frac{\sqrt{6}\sqrt{-3\,x^2-1}x-\sqrt{6}x-2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\
&\quad+\frac{1}{24}i\sqrt{6}\log\left(\frac{i\sqrt{6}\sqrt{-3\,x^2-1}x+i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)\\
&\quad-\frac{1}{24}i\sqrt{6}\log\left(\frac{-i\sqrt{6}\sqrt{-3\,x^2-1}x-i\sqrt{6}x+2\,(-3\,x^2-1)^{\frac{3}{4}}+2\,(-3\,x^2-1)^{\frac{1}{4}}}{3\,(3\,x^2+2)}\right)
\end{aligned}$$

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/24*sqrt(6)*log(1/3*(sqrt(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x + 2*(-3*x^2 -
 1)^(3/4) - 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2)) + 1/24*sqrt(6)*log(-1/3*(sqr
t(6)*sqrt(-3*x^2 - 1)*x - sqrt(6)*x - 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)
^(1/4))/(3*x^2 + 2)) + 1/24*I*sqrt(6)*log(1/3*(I*sqrt(6)*sqrt(-3*x^2 - 1)*x
 + I*sqrt(6)*x + 2*(-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
- 1/24*I*sqrt(6)*log(1/3*(-I*sqrt(6)*sqrt(-3*x^2 - 1)*x - I*sqrt(6)*x + 2*(
-3*x^2 - 1)^(3/4) + 2*(-3*x^2 - 1)^(1/4))/(3*x^2 + 2))
```

Sympy [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\int \frac{1}{3x^2\sqrt[4]{-3x^2-1}+2\sqrt[4]{-3x^2-1}}\,dx$$

```
[In] integrate(1/(-3*x**2-2)/(-3*x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(3*x**2*(-3*x**2 - 1)**(1/4) + 2*(-3*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2)(-3\,x^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate(1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2)(-3\,x^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-3*x^2-2)/(-3*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((3*x^2 + 2)*(-3*x^2 - 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-3x^2)\sqrt[4]{-1-3x^2}}\,dx = -\int \frac{1}{(-3\,x^2-1)^{1/4}\,(3\,x^2+2)}\,dx$$

```
[In] int(-1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)),x)
[Out] -int(1/((- 3*x^2 - 1)^(1/4)*(3*x^2 + 2)), x)
```

3.314 $\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*arct
anh(1/2*x*b^(1/2)/(b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {407}

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[In] Int[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) - A
rcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 + b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

```
NegQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1+bx^2}}{\sqrt{b}x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1+bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[1/((-2 + b*x^2)*(-1 + b*x^2)^(1/4)),x]
[Out] (ArcTan[(Sqrt[2]*(-1 + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[b]*x)/(Sq
rt[2]*(-1 + b*x^2)^(1/4))])/(2*Sqrt[2]*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(b\,x^2-2)\,(b\,x^2-1)^{\frac{1}{4}}}dx$$

```
[In] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
[Out] int(1/(b*x^2-2)/(b*x^2-1)^(1/4),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(55) = 110.

Time = 5.88 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.56

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx = \left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right) + \sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4-2\sqrt{2}(bx^2-1)^{\frac{1}{4}}b^{\frac{3}{2}}x^3+4\sqrt{bx^2-1}bx^2+4bx^2-4\sqrt{2}(bx^2-1)^{\frac{3}{4}}\sqrt{b}x-4}{b^2x^4-4bx^2+4}\right)}{8b}, \right.$$

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sqr
t(2)*sqrt(b)*log(-(b^2*x^4 - 2*sqrt(2)*(b*x^2 - 1)^(1/4)*b^(3/2)*x^3 + 4*sq
rt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(2)*(b*x^2 - 1)^(3/4)*sqrt(b)*x - 4)/
(b^2*x^4 - 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(b*x^2
- 1)^(1/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 + 2*sqrt(2)*(b*
x^2 - 1)^(1/4)*sqrt(-b)*b*x^3 - 4*sqrt(b*x^2 - 1)*b*x^2 + 4*b*x^2 - 4*sqrt(
2)*(b*x^2 - 1)^(3/4)*sqrt(-b)*x - 4)/(b^2*x^4 - 4*b*x^2 + 4)))/b]
```

Sympy [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=\int \frac{1}{(bx^2-2)\sqrt[4]{bx^2-1}}\,dx$$

```
[In] integrate(1/(b*x**2-2)/(b*x**2-1)**(1/4),x)
[Out] Integral(1/((b*x**2 - 2)*(b*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=\int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Giac [F]

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=\int \frac{1}{(bx^2-1)^{\frac{1}{4}}(bx^2-2)}\,dx$$

```
[In] integrate(1/(b*x^2-2)/(b*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2+bx^2)\sqrt[4]{-1+bx^2}}\,dx=\int \frac{1}{\left(b\,x^2-1\right)^{1/4}\,\left(b\,x^2-2\right)}\,dx$$

```
[In] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)),x)
[Out] int(1/((b*x^2 - 1)^(1/4)*(b*x^2 - 2)), x)
```

3.315 $\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 79

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)-1/4*arc
tanh(1/2*x*b^(1/2)/(-b*x^2-1)^(1/4)*2^(1/2))*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {407}

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-bx^2-1}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[In] Int[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(Sqrt[2]*Sqrt[b]) - A
rcTanh[(Sqrt[b]*x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))]/(2*Sqrt[2]*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

```
NegQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1-bx^2}}{\sqrt{b}x}\right) + \operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{-1-bx^2}}\right)}{2\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[1/((-2 - b*x^2)*(-1 - b*x^2)^(1/4)),x]
[Out] -1/2*(-ArcTan[(Sqrt[2]*(-1 - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[(Sqrt[b]*
x)/(Sqrt[2]*(-1 - b*x^2)^(1/4))])/(Sqrt[2]*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(-b\,x^2-2)\,(-b\,x^2-1)^{\frac{1}{4}}}dx$$

```
[In] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
[Out] int(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(57) = 114$.

Time = 5.87 (sec) , antiderivative size = 273, normalized size of antiderivative = 3.46

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = \left[\frac{2\sqrt{2}\sqrt{b}\arctan\left(\frac{\sqrt{2}(-bx^2-1)^{\frac{1}{4}}}{\sqrt{b}x}\right) + \sqrt{2}\sqrt{b}\log\left(-\frac{b^2x^4+4\sqrt{-bx^2-1}bx^2-4\,bx^2-2\sqrt{2}\left((-bx^2-1)^{\frac{1}{4}}bx^3+2\,(-bx^2-1)^{\frac{3}{4}}x\right)\sqrt{b}-4}{b^2x^4+4\,bx^2+4}\right.}{8\,b}\right.$$

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(2)*sqrt(b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1/4)/(sqrt(b)*x)) + sq
rt(2)*sqrt(b)*log(-(b^2*x^4 + 4*sqrt(-b*x^2 - 1)*b*x^2 - 4*b*x^2 - 2*sqrt(2
)*((-b*x^2 - 1)^(1/4)*b*x^3 + 2*(-b*x^2 - 1)^(3/4)*x)*sqrt(b) - 4)/(b^2*x^4
 + 4*b*x^2 + 4)))/b, 1/8*(2*sqrt(2)*sqrt(-b)*arctan(sqrt(2)*(-b*x^2 - 1)^(1
/4)*sqrt(-b)/(b*x)) - sqrt(2)*sqrt(-b)*log(-(b^2*x^4 - 4*sqrt(-b*x^2 - 1)*b
*x^2 - 4*b*x^2 + 2*sqrt(2)*((-b*x^2 - 1)^(1/4)*b*x^3 - 2*(-b*x^2 - 1)^(3/4)
*x)*sqrt(-b) - 4)/(b^2*x^4 + 4*b*x^2 + 4)))/b]
```

Sympy [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\int \frac{1}{bx^2\sqrt[4]{-bx^2-1}+2\sqrt[4]{-bx^2-1}}\,dx$$

```
[In] integrate(1/(-b*x**2-2)/(-b*x**2-1)**(1/4),x)
[Out] -Integral(1/(b*x**2*(-b*x**2 - 1)**(1/4) + 2*(-b*x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = \int -\frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = \int -\frac{1}{(bx^2+2)(-bx^2-1)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2-2)/(-b*x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 2)*(-b*x^2 - 1)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2-bx^2)\sqrt[4]{-1-bx^2}}\,dx = -\int \frac{1}{{\left(-b\,x^2-1\right)}^{1/4}\,\left(b\,x^2+2\right)} \,dx$$

```
[In] int(-1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)),x)
[Out] -int(1/((- b*x^2 - 1)^(1/4)*(b*x^2 + 2)), x)
```

3.316 $\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*ar
ctanh(1/2*x*6^(1/2)/a^(1/4)/(3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {407}

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{3x^2-a}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[In] Int[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4)) -
 ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a + 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

NegQ[b^2/a]

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a+3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right) - \operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a+3x^2}}{x}\right)}{2\sqrt{6}a^{3/4}}$$

```
[In] Integrate[1/((-2*a + 3*x^2)*(-a + 3*x^2)^(1/4)),x]
[Out] (ArcTan[(Sqrt[2/3]*a^(1/4)*(-a + 3*x^2)^(1/4))/x] - ArcTanh[(Sqrt[2/3]*a^(1
/4)*(-a + 3*x^2)^(1/4))/x])/(2*Sqrt[6]*a^(3/4))
```

Maple [F]

$$\int \frac{1}{(3x^2-2a)(3x^2-a)^{\frac{1}{4}}}dx$$

```
[In] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
[Out] int(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.41

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = -\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x+(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right) + \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right) + \frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right) - \frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{3x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(3x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(3x^2-a)^{\frac{3}{4}}}{3x^2-2a}\right)$$

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(
a^(-3))^(3/4)*x + (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^
(-3))^(1/4)*x + (3*x^2 - a)^(3/4))/(3*x^2 - 2*a)) + 1/4*(1/36)^(1/4)*(a^(-3
))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^
2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x - (3*x^2
- a)^(3/4))/(3*x^2 - 2*a)) + 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((18*I*(1
/36)^(3/4)*sqrt(3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2*sqr
t(a^(-3)) - 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (3*x^2 - a)^(3/4))/(3*x^2
 - 2*a)) - 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((-18*I*(1/36)^(3/4)*sqrt(3
*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) + 3*I*(
1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (3*x^2 - a)^(3/4))/(3*x^2 - 2*a))
```

Sympy [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx$$

```
[In] integrate(1/(3*x**2-2*a)/(3*x**2-a)**(1/4),x)
```

```
[Out] Integral(1/((-2*a + 3*x**2)*(-a + 3*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Giac [F]

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = \int \frac{1}{(3\,x^2-a)^{\frac{1}{4}}(3\,x^2-2\,a)}\,dx$$

```
[In] integrate(1/(3*x^2-2*a)/(3*x^2-a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((3*x^2 - a)^(1/4)*(3*x^2 - 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a+3x^2)\sqrt[4]{-a+3x^2}}\,dx = -\int \frac{1}{(2\,a-3\,x^2)\,(3\,x^2-a)^{1/4}}\,dx$$

```
[In] int(-1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)),x)
[Out] -int(1/((2*a - 3*x^2)*(3*x^2 - a)^(1/4)), x)
```

3.317 $\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[Out] -1/12*arctan(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)-1/12*a
rctanh(1/2*x*6^(1/2)/a^(1/4)/(-3*x^2-a)^(1/4))/a^(3/4)*6^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {407}

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

```
[In] Int[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(Sqrt[6]*a^(3/4)) -
 ArcTanh[(Sqrt[3/2]*x)/(a^(1/4)*(-a - 3*x^2)^(1/4))]/(2*Sqrt[6]*a^(3/4))
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

NegQ[b^2/a]

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{3}{2}}x}{\sqrt[4]{a}\sqrt[4]{-a-3x^2}}\right)}{2\sqrt{6}a^{3/4}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)+\operatorname{arctanh}\left(\frac{\sqrt{\frac{2}{3}}\sqrt[4]{a}\sqrt[4]{-a-3x^2}}{x}\right)}{2\sqrt{6}a^{3/4}}$$

```
[In] Integrate[1/((-2*a - 3*x^2)*(-a - 3*x^2)^(1/4)),x]
[Out] -1/2*(-ArcTan[(Sqrt[2/3]*a^(1/4)*(-a - 3*x^2)^(1/4))/x] + ArcTanh[(Sqrt[2/3
]*a^(1/4)*(-a - 3*x^2)^(1/4))/x])/(Sqrt[6]*a^(3/4))
```

Maple [F]

$$\int \frac{1}{(-3x^2-2a)\,(-3x^2-a)^{\frac{1}{4}}}\,dx$$

```
[In] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
[Out] int(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.89 (sec) , antiderivative size = 373, normalized size of antiderivative = 4.39

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(-\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3\,x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x-(-3\,x^2-a)^{\frac{3}{4}}}{3\,x^2+2\,a}\right)$$
$$-\frac{1}{4}\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x-(-3\,x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3\,x^2-a)^{\frac{3}{4}}}{3\,x^2+2\,a}\right)$$
$$+\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3\,x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}+3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3\,x^2-a)}{3\,x^2+2\,a}\right)$$
$$-\frac{1}{4}i\left(\frac{1}{36}\right)^{\frac{1}{4}}\frac{1}{a^3}^{\frac{1}{4}}\log\left(\frac{-18i\left(\frac{1}{36}\right)^{\frac{3}{4}}\sqrt{-3\,x^2-a}a^2\frac{1}{a^3}^{\frac{3}{4}}x+(-3\,x^2-a)^{\frac{1}{4}}a^2\sqrt{\frac{1}{a^3}}-3i\left(\frac{1}{36}\right)^{\frac{1}{4}}a\frac{1}{a^3}^{\frac{1}{4}}x+(-3\,x^2-}{3\,x^2+2\,a}\right)$$

```
[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(1/36)^(1/4)*(a^(-3))^(1/4)*log(-(18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*
(a^(-3))^(3/4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(
a^(-3))^(1/4)*x - (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a)) - 1/4*(1/36)^(1/4)*(a^
(-3))^(1/4)*log((18*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x - (-
3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3)) - 3*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3
*x^2 - a)^(3/4))/(3*x^2 + 2*a)) + 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((18
*I*(1/36)^(3/4)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x + (-3*x^2 - a)^(1/4)*
a^2*sqrt(a^(-3)) + 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3/4)
)/(3*x^2 + 2*a)) - 1/4*I*(1/36)^(1/4)*(a^(-3))^(1/4)*log((-18*I*(1/36)^(3/4
)*sqrt(-3*x^2 - a)*a^2*(a^(-3))^(3/4)*x + (-3*x^2 - a)^(1/4)*a^2*sqrt(a^(-3
)) - 3*I*(1/36)^(1/4)*a*(a^(-3))^(1/4)*x + (-3*x^2 - a)^(3/4))/(3*x^2 + 2*a
))
```

Sympy [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\int \frac{1}{2a\sqrt[4]{-a-3x^2}+3x^2\sqrt[4]{-a-3x^2}}\,dx$$

```
[In] integrate(1/(-3*x**2-2*a)/(-3*x**2-a)**(1/4),x)
```

```
[Out] -Integral(1/(2*a*(-a - 3*x**2)**(1/4) + 3*x**2*(-a - 3*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="maxima")

[Out] -integrate(1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)

Giac [F]

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = \int -\frac{1}{(3\,x^2+2\,a)(-3\,x^2-a)^{\frac{1}{4}}}\,dx$$

[In] integrate(1/(-3*x^2-2*a)/(-3*x^2-a)^(1/4),x, algorithm="giac")

[Out] integrate(-1/((3*x^2 + 2*a)*(-3*x^2 - a)^(1/4)), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a-3x^2)\sqrt[4]{-a-3x^2}}\,dx = -\int \frac{1}{(3\,x^2+2\,a)\,\left(-3\,x^2-a\right)^{1/4}}\,dx$$

[In] int(-1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)),x)

[Out] -int(1/((2*a + 3*x^2)*(- a - 3*x^2)^(1/4)), x)

3.318 $\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 101

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2)/
b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*
2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {407}

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx^2-a}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[In] Int[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(Sqrt[2]*a^(3
/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))]/(2
*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}x}\right) - \text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a+bx^2}}{\sqrt{b}x}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((-2*a + b*x^2)*(-a + b*x^2)^(1/4)),x]
[Out] (ArcTan[(Sqrt[2]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)] - ArcTanh[(Sqrt[2
]*a^(1/4)*(-a + b*x^2)^(1/4))/(Sqrt[b]*x)])/(2*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(b\,x^2-2a)\,(b\,x^2-a)^{\frac{1}{4}}}dx$$

```
[In] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
[Out] int(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.58 (sec) , antiderivative size = 457, normalized size of antiderivative = 4.52

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx =$$

$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(bx^2-}{bx^2-2\,a}\right.$$

$$+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(bx^2}{bx^2-2\,a}\right.$$

$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(bx}{bx^2-2\,a}\right.$$

$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(}{bx^2-2\,a}\right.$$

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(b*x^2 - a)*a^2
*b^2*x*(1/(a^3*b^2))^(3/4) + (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) + (1
/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 - 2*a)) + 1
/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(b*x^2 - a)*a^2*
b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) + (1/
4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (b*x^2 - a)^(3/4))/(b*x^2 - 2*a)) + 1/
4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt(b*x^2 - a)*a^
2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) - I
*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 - 2*a))
- 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((-2*I*(1/4)^(3/4)*sqrt(b*x^2 -
a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) - (b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)
) + I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (b*x^2 - a)^(3/4))/(b*x^2 - 2
*a))
```

Sympy [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx$$

```
[In] integrate(1/(b*x**2-2*a)/(b*x**2-a)**(1/4),x)
[Out] Integral(1/((-2*a + b*x**2)*(-a + b*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{4}}(bx^2-2\,a)}\,dx$$

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Giac [F]

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{4}}(bx^2-2\,a)}\,dx$$

```
[In] integrate(1/(b*x^2-2*a)/(b*x^2-a)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x^2 - a)^(1/4)*(b*x^2 - 2*a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a+bx^2)\sqrt[4]{-a+bx^2}}\,dx = -\int \frac{1}{(b\,x^2-a)^{1/4}\,(2\,a-b\,x^2)}\,dx$$

```
[In] int(-1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)),x)
[Out] -int(1/((b*x^2 - a)^(1/4)*(2*a - b*x^2)), x)
```

3.319 $\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 103

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[Out] -1/4*arctan(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3/4)*2^(1/2)
/b^(1/2)-1/4*arctanh(1/2*x*b^(1/2)/a^(1/4)/(-b*x^2-a)^(1/4)*2^(1/2))/a^(3/4
)*2^(1/2)/b^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {407}

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[In] Int[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(Sqrt[2]*a^(3
/4)*Sqrt[b]) - ArcTanh[(Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))]/(2
*Sqrt[2]*a^(3/4)*Sqrt[b])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
NegQ[b^2/a]
```

Rubi steps

$$\text{integral} = -\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}x}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{-a-bx^2}}{\sqrt{b}x}\right)}{2\sqrt{2}a^{3/4}\sqrt{b}}$$

```
[In] Integrate[1/((-2*a - b*x^2)*(-a - b*x^2)^(1/4)),x]
[Out] -1/2*(-ArcTan[(Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)] + ArcTanh[(
Sqrt[2]*a^(1/4)*(-a - b*x^2)^(1/4))/(Sqrt[b]*x)])/(Sqrt[2]*a^(3/4)*Sqrt[b])
```

Maple [F]

$$\int \frac{1}{(-b\,x^2-2a)\,(-b\,x^2-a)^{\frac{1}{4}}}dx$$

```
[In] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
[Out] int(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.42 (sec) , antiderivative size = 469, normalized size of antiderivative = 4.55

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx$$
$$= \frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}-(-}{bx^2+2a}\right.$$
$$-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}-(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+(-}{bx^2+2a}\right.$$
$$+\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}+i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}+}{bx^2+2a}\right.$$
$$-\frac{1}{4}i\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}\log\left(\frac{-2i\left(\frac{1}{4}\right)^{\frac{3}{4}}\sqrt{-bx^2-a}a^2b^2x\left(\frac{1}{a^3b^2}\right)^{\frac{3}{4}}+(-bx^2-a)^{\frac{1}{4}}a^2b\sqrt{\frac{1}{a^3b^2}}-i\left(\frac{1}{4}\right)^{\frac{1}{4}}abx\left(\frac{1}{a^3b^2}\right)^{\frac{1}{4}}}{bx^2+2a}\right.$$

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log(-(2*(1/4)^(3/4)*sqrt(-b*x^2 - a)*a^
2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) -
(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) - (-b*x^2 - a)^(3/4))/(b*x^2 + 2*a))
- 1/4*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*(1/4)^(3/4)*sqrt(-b*x^2 - a)*a
^2*b^2*x*(1/(a^3*b^2))^(3/4) - (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^2)) -
 (1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))/(b*x^2 + 2*a))
 + 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((2*I*(1/4)^(3/4)*sqrt(-b*x^2 -
 a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1/(a^3*b^
2)) + I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))/(b*x^2
+ 2*a)) - 1/4*I*(1/4)^(1/4)*(1/(a^3*b^2))^(1/4)*log((-2*I*(1/4)^(3/4)*sqrt(
-b*x^2 - a)*a^2*b^2*x*(1/(a^3*b^2))^(3/4) + (-b*x^2 - a)^(1/4)*a^2*b*sqrt(1
/(a^3*b^2)) - I*(1/4)^(1/4)*a*b*x*(1/(a^3*b^2))^(1/4) + (-b*x^2 - a)^(3/4))
/(b*x^2 + 2*a))
```

Sympy [F]

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\int \frac{1}{2a\sqrt[4]{-a-bx^2}+bx^2\sqrt[4]{-a-bx^2}}\,dx$$

```
[In] integrate(1/(-b*x**2-2*a)/(-b*x**2-a)**(1/4),x)
[Out] -Integral(1/(2*a*(-a - b*x**2)**(1/4) + b*x**2*(-a - b*x**2)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = \int -\frac{1}{(bx^2+2\,a)(-bx^2-a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="maxima")
[Out] -integrate(1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = \int -\frac{1}{(bx^2+2\,a)(-bx^2-a)^{\frac{1}{4}}}\,dx$$

```
[In] integrate(1/(-b*x^2-2*a)/(-b*x^2-a)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((b*x^2 + 2*a)*(-b*x^2 - a)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-2a-bx^2)\sqrt[4]{-a-bx^2}}\,dx = -\int \frac{1}{(-b\,x^2-a)^{1/4}\,(b\,x^2+2\,a)}\,dx$$

```
[In] int(-1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)),x)
[Out] -int(1/((- a - b*x^2)^(1/4)*(2*a + b*x^2)), x)
```

3.320 $\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

```
[Out] 1/4*arctan(1/2*x/(x^2-1)^(1/4)*2^(1/2))*2^(1/2)+1/4*arctanh(1/2*x/(x^2-1)^(
1/4)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {407}

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{\arctan\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{x^2-1}}\right)}{2\sqrt{2}}$$

```
[In] Int[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
```

```
[Out] ArcTan[x/(Sqrt[2]*(-1 + x^2)^(1/4))]/(2*Sqrt[2]) + ArcTanh[x/(Sqrt[2]*(-1 +
 x^2)^(1/4))]/(2*Sqrt[2])
```

Rule 407

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Wit
h[{q = Rt[-b^2/a, 4]}, Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTan[q*(x/(Sqrt[2]*(a +
 b*x^2)^(1/4)))], x] + Simp[(b/(2*Sqrt[2]*a*d*q))*ArcTanh[q*(x/(Sqrt[2]*(a
+ b*x^2)^(1/4)))], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c - 2*a*d, 0] &&
```

```
NegQ[b^2/a]
```

Rubi steps

$$\text{integral} = \frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \frac{-\arctan\left(\frac{\sqrt{2}\sqrt[4]{-1+x^2}}{x}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{2}\sqrt[4]{-1+x^2}}\right)}{2\sqrt{2}}$$

```
[In] Integrate[1/((2 - x^2)*(-1 + x^2)^(1/4)),x]
[Out] (-ArcTan[(Sqrt[2]*(-1 + x^2)^(1/4))/x] + ArcTanh[x/(Sqrt[2]*(-1 + x^2)^(1/4
))])/(2*Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.21

method	result
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{3}{4}}+x\sqrt{x^2-1}+\operatorname{RootOf}\left(\_Z^2-2\right)\left(x^2-1\right)^{\frac{1}{4}}+x}{x^2-2}\right)}{4} - \frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(-\frac{\operatorname{RootOf}(}{}\right.}{}$

```
[In] int(1/(-x^2+2)/(x^2-1)^(1/4),x,method=_RETURNVERBOSE)
[Out] 1/4*RootOf(_Z^2-2)*ln((RootOf(_Z^2-2)*(x^2-1)^(3/4)+x*(x^2-1)^(1/2)+RootOf(
_Z^2-2)*(x^2-1)^(1/4)+x)/(x^2-2))-1/4*RootOf(_Z^2+2)*ln(-(RootOf(_Z^2+2)*(x
^2-1)^(3/4)-x*(x^2-1)^(1/2)-RootOf(_Z^2+2)*(x^2-1)^(1/4)+x)/(x^2-2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(39) = 78$.

Time = 1.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\begin{aligned}
&\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx \\
&= -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)^{\frac{1}{4}}}{x}\right) \\
&\quad + \frac{1}{8}\sqrt{2}\log\left(-\frac{x^4+2\sqrt{2}(x^2-1)^{\frac{1}{4}}x^3+4\sqrt{x^2-1}x^2+4\sqrt{2}(x^2-1)^{\frac{3}{4}}x+4x^2-4}{x^4-4x^2+4}\right)
\end{aligned}$$

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*(x^2 - 1)^(1/4)/x) + 1/8*sqrt(2)*log(-(x^4 + 2*
sqrt(2)*(x^2 - 1)^(1/4)*x^3 + 4*sqrt(x^2 - 1)*x^2 + 4*sqrt(2)*(x^2 - 1)^(3/
4)*x + 4*x^2 - 4)/(x^4 - 4*x^2 + 4))
```

Sympy [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{1}{x^2\sqrt[4]{x^2-1}-2\sqrt[4]{x^2-1}}\,dx$$

```
[In] integrate(1/(-x**2+2)/(x**2-1)**(1/4),x)
```

```
[Out] -Integral(1/(x**2*(x**2 - 1)**(1/4) - 2*(x**2 - 1)**(1/4)), x)
```

Maxima [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Giac [F]

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = \int -\frac{1}{(x^2-1)^{\frac{1}{4}}(x^2-2)}\,dx$$

```
[In] integrate(1/(-x^2+2)/(x^2-1)^(1/4),x, algorithm="giac")
[Out] integrate(-1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-x^2)\sqrt[4]{-1+x^2}}\,dx = -\int \frac{1}{(x^2-1)^{1/4}\,(x^2-2)}\,dx$$

```
[In] int(-1/((x^2 - 1)^(1/4)*(x^2 - 2)),x)
[Out] -int(1/((x^2 - 1)^(1/4)*(x^2 - 2)), x)
```

3.321 $\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 362

$$\begin{aligned}\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx &= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} \\ &+ \frac{2bx(a+bx^2)^{3/4}}{5d} - \frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}} \\ &+ \frac{2\sqrt{a}\sqrt{b}(bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}} \\ &+ \frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x} \\ &- \frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x}\end{aligned}$$

```
[Out] 6/5*a*b*x/d/(b*x^2+a)^(1/4)-2*b*(-a*d+b*c)*x/d^2/(b*x^2+a)^(1/4)+2/5*b*x*(b
*x^2+a)^(3/4)/d-6/5*a^(3/2)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(
1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan
(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(1/4)+2*(-a*d+b*c)*(1+b*x
^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1
/2)*b^(1/2)/d^2/(b*x^2+a)^(1/4)+a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((b*x^2+a
)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^(5/2
)/x-a^(1/4)*(a*d-b*c)^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1
/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^(5/2)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {411, 201, 235, 233, 202, 408, 504, 1232}

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = -\frac{6a^{3/2}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(ad-bc)^{3/2}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^{5/2}x} + \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(bc-ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}} - \frac{2bx(bc-ad)}{d^2\sqrt[4]{a+bx^2}} + \frac{6abx}{5d\sqrt[4]{a+bx^2}} + \frac{2bx(a+bx^2)^{3/4}}{5d}$$

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2),x]
```

```
[Out] (6*a*b*x)/(5*d*(a + b*x^2)^(1/4)) - (2*b*(b*c - a*d)*x)/(d^2*(a + b*x^2)^(1
/4)) + (2*b*x*(a + b*x^2)^(3/4))/(5*d) - (6*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)
^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*d*(a + b*x^2)^(1/4))
 + (2*Sqrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(1/4)) + (a^(1/4)*(-(b*c) + a*d)
^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]
), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x) - (a^(1/4)*(-(b*c) +
 a*d)^(3/2)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(5/2)*x)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{b\int \left(a+bx^2\right)^{3/4}\,dx}{d} - \frac{(bc-ad)\int \frac{\left(a+bx^2\right)^{3/4}}{c+dx^2}\,dx}{d}$$

$$= \frac{2bx(a+bx^2)^{3/4}}{5d} + \frac{(3ab)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5d} - \frac{(b(bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{d^2}$$

$$= \frac{2bx(a+bx^2)^{3/4}}{5d} + \frac{\left(2(bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^2x} + \frac{\left(3ab\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5d\sqrt[4]{a+bx^2}} - \frac{\left(b(bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{d^2\sqrt[4]{a+bx^2}}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx(a+bx^2)^{3/4}}{5d} - \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^{5/2}x} + \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^{5/2}x} - \frac{\left(3ab\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5d\sqrt[4]{a+bx^2}} + \frac{\left(b(bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{d^2\sqrt[4]{a+bx^2}}$$

$$= \frac{6abx}{5d\sqrt[4]{a+bx^2}} - \frac{2b(bc-ad)x}{d^2\sqrt[4]{a+bx^2}} + \frac{2bx(a+bx^2)^{3/4}}{5d} - \frac{6a^{3/2}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5d\sqrt[4]{a+bx^2}} + \frac{2\sqrt{a}\sqrt{b}(bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x} - \frac{\sqrt[4]{a}(-bc+ad)^{3/2}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{5/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.12 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \frac{x\left(\frac{b(-5bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\operatorname{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac\left(5a^2d+2abdx^2+2b^2x^2(c+dx^2)\right)\operatorname{Appell}\ldots}{(c+dx^2)\left(-6ac\operatorname{AppellF1}\ldots\right.}\right.}{15\ldots}$$

[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2),x]

[Out] (x*((b*(-5*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(5*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/
c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
 + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(15*d*(a
 + b*x^2)^(1/4))

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{4}}}{d\,x^2+c}dx$$

[In] int((b*x^2+a)^(7/4)/(d*x^2+c),x)

[Out] int((b*x^2+a)^(7/4)/(d*x^2+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \text{Timed out}$$

[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{7}{4}}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{7/4}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(7/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(7/4)/(c + d*x^2), x)
```

3.322 $\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 302

$$\begin{aligned}\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx &= \frac{2bx\sqrt[4]{a+bx^2}}{3d} \\ &+ \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3d(a+bx^2)^{3/4}} \\ &- \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d^2(a+bx^2)^{3/4}} \\ &+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^2x} \\ &+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^2x}\end{aligned}$$

```
[Out] 2/3*b*x*(b*x^2+a)^(1/4)/d+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/d/(b*x^2+a)^(3/4)-2*(-a*d+b
*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*
arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(
1/2))*a^(1/2)*b^(1/2)/d^2/(b*x^2+a)^(3/4)+a^(1/4)*(-a*d+b*c)*EllipticPi((b*
x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d
^2/x+a^(1/4)*(-a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/
(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d^2/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {411, 201, 239, 237, 410, 109, 418, 1232}

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \frac{2a^{3/2}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3d\,(a+bx^2)^{3/4}} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^2x} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^2x} - \frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(bc-ad)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d^2\,(a+bx^2)^{3/4}} + \frac{2bx\sqrt[4]{a+bx^2}}{3d}$$

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x*(a + b*x^2)^(1/4))/(3*d) + (2*a^(3/2)*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*
EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*d*(a + b*x^2)^(3/4)) - (2*S
qrt[a]*Sqrt[b]*(b*c - a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/2, 2])/(d^2*(a + b*x^2)^(3/4)) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b
*x^2)^(1/4)/a^(1/4)], -1])/(d^2*x) + (a^(1/4)*(b*c - a*d)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/(d^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \sqrt[4]{a+bx^2}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx}{d}\\ &= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{(ab)\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3d}\\ &\quad - \frac{(b(bc-ad))\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{d^2} + \frac{(bc-ad)^2\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{d^2}\end{aligned}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{\left((bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2d^2x}$$
$$+ \frac{\left(ab\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3d\left(a+bx^2\right)^{3/4}} - \frac{\left(b(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{d^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}$$
$$- \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}$$
$$- \frac{\left(2(bc-ad)^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^2x}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}$$
$$- \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}$$
$$+ \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^2x}$$
$$+ \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^2x}$$

$$= \frac{2bx\sqrt[4]{a+bx^2}}{3d} + \frac{2a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3d\left(a+bx^2\right)^{3/4}}$$
$$- \frac{2\sqrt{a}\sqrt{b}(bc-ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d^2\left(a+bx^2\right)^{3/4}}$$
$$+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}$$
$$+ \frac{\sqrt[4]{a}(bc-ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^2x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.19 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \frac{x\left(\frac{b(-3bc+4ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}+\frac{6\left(-3ac\left(3a^2d+2abdx^2+2b^2x^2(c+dx^2)\right)\operatorname{AppellF}\right.}{(c+dx^2)\left(-6ac\operatorname{AppellF1}\left(\frac{1}{2}\right.\right.}\right.}{9d\left(\right.}$$

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2),x]
```

```
[Out] (x*((b*(-3*b*c + 4*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(-3*a*c*(3*a^2*d + 2*a*b*d*x^2 + 2*b^
2*x^2*(c + d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
 b*x^2*(a + b*x^2)*(c + d*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/
a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2
)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(9*d
*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{d\,x^2+c}dx$$

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{4}}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(5/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(5/4)/(c + d*x^2), x)
```

3.323 $\int \frac{(a+bx^2)^{3/4}}{c+dx^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 244

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2} dx = \frac{2bx}{d\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x}$$

```
[Out] 2*b*x/d/(b*x^2+a)^(1/4)-2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/
2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x
*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(1/4)+a^(1/4)*Ellip
ticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)
^(1/2)*(-b*x^2/a)^(1/2)/d^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4
),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/d^(3/
2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {411, 235, 233, 202, 408, 504, 1232}

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{d^{3/2}x} - \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}} + \frac{2bx}{d\sqrt[4]{a+bx^2}}$$

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (2*b*x)/(d*(a + b*x^2)^(1/4)) - (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*El
lipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(d*(a + b*x^2)^(1/4)) + (a^(1/4)
*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x) - (a^(1
/4)*Sqrt[-(b*c) + a*d]*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt
[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
```

```
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{d} \\
&= -\frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx} \\
&\quad + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{d\sqrt[4]{a+bx^2}}
\end{aligned}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} + \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^{3/2}x}$$
$$- \frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{d^{3/2}x}$$
$$- \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{d\sqrt[4]{a+bx^2}}$$

$$= \frac{2bx}{d\sqrt[4]{a+bx^2}} - \frac{2\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\sqrt[4]{a+bx^2}}$$
$$+ \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}$$
$$- \frac{\sqrt[4]{a}\sqrt{-bc+ad}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{d^{3/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 9.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \frac{6acx(a+bx^2)^{3/4}\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a}\right.}{(c+dx^2)\left(6ac\,\text{AppellF1}\left(\frac{1}{2},-\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4ad\,\text{AppellF1}\left(\frac{3}{2},-\frac{3}{4},2,\frac{5}{2},-\right.\right.\right.}$$

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(3/4)*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -3/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + 3*b*c*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{d\,x^{2}+c}dx$$

[In] int((b*x^2+a)^(3/4)/(d*x^2+c),x)

[Out] int((b*x^2+a)^(3/4)/(d*x^2+c),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \text{Timed out}$$

[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{4}}}{c+dx^2}\,dx$$

[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c),x)

[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2), x)

Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{dx^2+c}\,dx$$

[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)

Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{3/4}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(3/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(3/4)/(c + d*x^2), x)
```

3.324 $\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2} \, dx = \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{dx} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{dx}$$

```
[Out] 2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*a^(1/2)*b^(1/2)/d/(b*x^2+a)^(3/4)-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^
(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/d/x-a^(1/4)*Elli
pticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a
)^(1/2)/d/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

$= \{411, 239, 237, 410, 109, 418, 1232\}$

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = -\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{dx} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{dx} + \frac{2\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{d\,(a+bx^2)^{3/4}}$$

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/2, 2])/(d*(a + b*x^2)^(3/4)) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(d*x) - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[
-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(d*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 411

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{d} \\
&= -\frac{\left((bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2dx} \\
&\quad + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{d\,(a+bx^2)^{3/4}} \\
&= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\,(a+bx^2)^{3/4}} \\
&\quad + \frac{\left(2(bc-ad)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx}
\end{aligned}
$$

$$= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt{-\frac{bx^2}{a}}\,\mathrm{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx} - \frac{\sqrt{-\frac{bx^2}{a}}\,\mathrm{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{dx}$$

$$= \frac{2\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{d\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{dx}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 8.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$

$$= \frac{6acx\sqrt[4]{a+bx^2}\,\mathrm{AppellF1}\left(\frac{1}{2},-\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)\left(6ac\,\mathrm{AppellF1}\left(\frac{1}{2},-\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(-4ad\,\mathrm{AppellF1}\left(\frac{3}{2},-\frac{1}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+bc\,\mathrm{App}\right.\right.}$$

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2),x]
```

```
[Out] (6*a*c*x*(a + b*x^2)^(1/4)*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)])/((c + d*x^2)*(6*a*c*AppellF1[1/2, -1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(-4*a*d*AppellF1[3/2, -1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)
/c)] + b*c*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{d\,x^2+c}dx$$

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx$$

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c),x)
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^(1/4)/(c + d*x^2),x)
[Out] int((a + b*x^2)^(1/4)/(c + d*x^2), x)
```

3.325 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}\sqrt{-bc+ad}x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}\sqrt{-bc+ad}x}$$

```
[Out] a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2)
,I)*(-b*x^2/a)^(1/2)/x/d^(1/2)/(a*d-b*c)^(1/2)-a^(1/4)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/x/d^(1/2
)/(a*d-b*c)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {408, 504, 1232}

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}x\sqrt{ad-bc}} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{d}x\sqrt{ad-bc}}$$

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
 - (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(Sqrt[d]*Sqrt[-(b*c) + a*d]*x)
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\,(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x} \\
&= -\frac{\sqrt{-\frac{bx^2}{a}}\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt{d}x} \\
&\quad+\frac{\sqrt{-\frac{bx^2}{a}}\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt{d}x} \\
&= \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\sqrt{-bc+ad}x} \\
&\quad-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{d}\sqrt{-bc+ad}x}
\end{aligned}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.77 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx =$$

$$-\frac{6acx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\sqrt[4]{a+bx^2}\,(c+dx^2)\left(-6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4ad\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\right.\right.}$$

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/((a + b*x
^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d
*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}\left(d\,x^2+c\right)}dx$$

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{1/4}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)), x)
```

3.326 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} \, dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)x} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)x}$$

```
[Out] a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2)
,I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x+a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4
),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)/x
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)} \, dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*
d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x) + (a^(1/4)*Sqr
t[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a
+ b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2x} \\
&= -\frac{\left(2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{x} \\
&= \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} \\
&\quad + \frac{\sqrt{-\frac{bx^2}{a}}\,\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x}
\end{aligned}$$

$$= \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x} + \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)x}$$

Mathematica [A] (verified)

Time = 8.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\left(\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)+\text{EllipticPi}\left(\frac{\sqrt{}}{}\right.\right.}{(bc-ad)x}$$

```
[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)),x]
```

```
[Out] (a^(1/4)*Sqrt[-((b*x^2)/a)]*(EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a
*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1] + EllipticPi[(Sqrt[a]*Sqrt[d])
/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]))/((b*c - a*d)*
x)
```

Maple [F]

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}}\left(d\,x^2+c\right)}dx$$

```
[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{4}}(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{3/4}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)), x)
```

3.327 $\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 233

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{3/2}x}$$

```
[Out] 2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*ar
ctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/
2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(1/4)/a^(1/2)+a^(1/4)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/2)
/(a*d-b*c)^(3/2)/x-a^(1/4)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(3/2)/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {412, 203, 202, 408, 504, 1232}

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(ad-bc)^{3/2}} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(ad-bc)^{3/2}} + \frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (2*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2
])/(Sqrt[a]*(b*c - a*d)*(a + b*x^2)^(1/4)) + (a^(1/4)*Sqrt[d]*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x) - (a^(1/4)*Sqrt[d]*Sqrt[-((
b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/((-(b*c) + a*d)^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4)), Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
x] && PosQ[a] && PosQ[b/a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{(a+bx^2)^{5/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{bc-ad} \\
&= -\frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{a(bc-ad)\sqrt[4]{a+bx^2}} \\
&= \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} \\
&\quad + \frac{\left(\sqrt{d}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} \\
&\quad - \frac{\left(\sqrt{d}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x}
\end{aligned}$$

$$= \frac{2\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\Big|2\right)}{\sqrt{a}(bc-ad)\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\Big|-1\right)}{(-bc+ad)^{3/2}x} - \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\Big|-1\right)}{(-bc+ad)^{3/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.49 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \frac{x\left(\frac{bdx^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} + \frac{6\left(3ac\left(ad-b(c+2dx^2)\right)\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2}\right.}{(c+dx^2)\left(6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2}\right.}\right.}{3a(-bc+}$$

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c + (6*(3*a*c*(a*d - b*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*AppellF1[3/2,
1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -
((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*
x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*
x^2)/c)])))))/(3*a*(-(b*c) + a*d)*(a + b*x^2)^(1/4))
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{4}}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx=\int \frac{1}{\left(a+bx^2\right)^{\frac{5}{4}}\left(c+dx^2\right)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(dx^2+c\right)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}\left(dx^2+c\right)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{5/4}\,\left(d\,x^2+c\right)}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)), x)
```

3.328 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 254

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \frac{2bx}{3a(bc-ad)(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}(bc-ad)(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^2x} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^2x}$$

```
[Out] 2/3*b*x/a/(-a*d+b*c)/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(
x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/(-a*d+b*c)/(b*x^2+a)^(3/
4)/a^(1/2)-a^(1/4)*d*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a
*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^2/x-a^(1/4)*d*EllipticPi((b*x^
2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*
d+b*c)^2/x
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {412, 205, 239, 237, 410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = -\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)^2} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)^2} + \frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\,(a+bx^2)^{3/4}\,(bc-ad)} + \frac{2bx}{3a\,(a+bx^2)^{3/4}\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(3*a*(b*c - a*d)*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(1 + (b*x^2)/a)^(3
/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*(b*c - a*d)*(a
+ b*x^2)^(3/4)) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*
d)^2*x) - (a^(1/4)*d*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-
(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*c - a*d)^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
```

```
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/(
b*c - a*d), Int[(a + b*x^2)^p, x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^2)
^(p + 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
 && LtQ[p, -1] && EqQ[Denominator[p], 4] && (EqQ[p, -5/4] || EqQ[p, -7/4])
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{(a+bx^2)^{7/4}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{bc-ad} \\
&= \frac{2bx}{3a(bc-ad)\,(a+bx^2)^{3/4}} + \frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{3a(bc-ad)} \\
&\quad - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2(bc-ad)x}
\end{aligned}
$$

$$= \frac{2bx}{3a(bc-ad)(a+bx^2)^{3/4}} + \frac{\left(2d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)x} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{3a(bc-ad)(a+bx^2)^{3/4}}$$

$$= \frac{2bx}{3a(bc-ad)(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\Big|2\right)}{3\sqrt{a}(bc-ad)(a+bx^2)^{3/4}} - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x} - \frac{\left(d\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x}$$

$$= \frac{2bx}{3a(bc-ad)(a+bx^2)^{3/4}} + \frac{2\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\Big|2\right)}{3\sqrt{a}(bc-ad)(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\Big|-1\right)}{(bc-ad)^2x} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\Big|-1\right)}{(bc-ad)^2x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.30

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \frac{x\left(-\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} + \frac{6\left(3ac(-3bc+3ad-2bdx^2)\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{(c+dx^2)\left(6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\right.\right.}\right.}{9a(-bc+}$$

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)),x]
```

```
[Out] (x*(-((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])/c) + (6*(3*a*c*(-3*b*c + 3*a*d - 2*b*d*x^2)*AppellF1[1/2,
```

```
 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(4*a*d*Appell
F1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 3
/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2,
5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^
2)/a), -((d*x^2)/c)])))))/(9*a*(-(b*c) + a*d)*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{4}}\left(d\,x^{2}+c\right)}d x$$

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx=\int \frac{1}{(a+bx^2)^{\frac{7}{4}}(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{7/4}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)), x)
```

3.329 $\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 274

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \frac{2bx}{5a(bc-ad)(a+bx^2)^{5/4}} + \frac{2\sqrt{b}(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{5/2}x} - \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(-bc+ad)^{5/2}x}$$

```
[Out] 2/5*b*x/a/(-a*d+b*c)/(b*x^2+a)^(5/4)+2/5*(-8*a*d+3*b*c)*(1+b*x^2/a)^(1/4)*(
cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2
)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(
-a*d+b*c)^2/(b*x^2+a)^(1/4)+a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1
/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/2)/x-
a^(1/4)*d^(3/2)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c
)^(1/2),I)*(-b*x^2/a)^(1/2)/(a*d-b*c)^(5/2)/x
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \frac{2\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(3bc-8ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}\sqrt[4]{a+bx^2}(bc-ad)^2}$$
$$+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(ad-bc)^{5/2}}$$
$$-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(ad-bc)^{5/2}}$$
$$+\frac{2bx}{5a(a+bx^2)^{5/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(5*a*(b*c - a*d)*(a + b*x^2)^(5/4)) + (2*Sqrt[b]*(3*b*c - 8*a*d)*(1
 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*a^(3/2)
*(b*c - a*d)^2*(a + b*x^2)^(1/4)) + (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x) - (a^(1/4)*d^(3/2)*Sqrt[-((b*x^2)/a)
]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)
/a^(1/4)], -1])/((-(b*c) + a*d)^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} - \frac{2\int \frac{\frac{1}{2}(-3bc+5ad)-\frac{3}{2}bdx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx}{5a(bc-ad)}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{4\int \frac{\frac{1}{4}(-3b^2c^2+8abcd+5a^2d^2)-\frac{1}{4}bd(3bc-8ad)x^2}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{5a^2(bc-ad)^2}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{(bc-ad)^2} - \frac{(b(3bc-8ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{5a^2(bc-ad)^2}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2b(3bc-8ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}} + \frac{\left(2d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x} - \frac{\left(b(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} - \frac{\left(d^{3/2}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x} + \frac{\left(d^{3/2}\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x} + \frac{\left(b(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{5a^2(bc-ad)^2\sqrt[4]{a+bx^2}}$$

$$= \frac{2bx}{5a(bc-ad)\left(a+bx^2\right)^{5/4}} + \frac{2\sqrt{b}(3bc-8ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}(bc-ad)^2\sqrt[4]{a+bx^2}}$$
$$+ \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(-bc+ad)^{5/2}x}$$
$$- \frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(-bc+ad)^{5/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}\,dx = \frac{x\left(\frac{bd(-3bc+8ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} - \frac{6\left(3ac\left(5a^3d^2+3b^3cx^2\left(c+2dx^2\right)-\right.\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)),x]
```

```
[Out] (x*((b*d*(-3*b*c + 8*a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5
/2, -((b*x^2)/a), -((d*x^2)/c)])/c - (6*(3*a*c*(5*a^3*d^2 + 3*b^3*c*x^2*(c
+ 2*d*x^2) - a^2*b*d*(10*c + 13*d*x^2) + a*b^2*(5*c^2 - 16*d^2*x^4))*Appell
F1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + b*x^2*(c + d*x^2)*(9*a^2
*d - 3*b^2*c*x^2 - 4*a*b*(c - 2*d*x^2))*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x
^2)/c)])))))/(15*a^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^2+a\right)^{\frac{9}{4}}\left(d\,x^2+c\right)}dx$$

```
[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

```
[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(a+bx^2)^{\frac{9}{4}}(c+dx^2)}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c),x)
[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{9/4}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)),x)
```

```
[Out] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)), x)
```

3.330 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)} dx$

Optimal result

Integrand size = 21, antiderivative size = 304

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)} dx &= \frac{2bx}{7a(bc-ad)(a+bx^2)^{7/4}} \\
&+ \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2(a+bx^2)^{3/4}} \\
&+ \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}(bc-ad)^2(a+bx^2)^{3/4}} \\
&+ \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^3x} \\
&+ \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{(bc-ad)^3x}
\end{aligned}$$

```
[Out] 2/7*b*x/a/(-a*d+b*c)/(b*x^2+a)^(7/4)+2/21*b*(-12*a*d+5*b*c)*x/a^2/(-a*d+b*c
)^2/(b*x^2+a)^(3/4)+2/21*(-12*a*d+5*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(
x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(s
in(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/a^(3/2)/(-a*d+b*c)^2/(b*
x^2+a)^(3/4)+a^(1/4)*d^2*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2
)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/(-a*d+b*c)^3/x+a^(1/4)*d^2*EllipticPi
((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2
)/(-a*d+b*c)^3/x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}dx = \frac{2\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(5bc-12ad)\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}(a+bx^2)^{3/4}(bc-ad)^2} + \frac{2bx(5bc-12ad)}{21a^2(a+bx^2)^{3/4}(bc-ad)^2} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{x(bc-ad)^3} + \frac{2bx}{7a(a+bx^2)^{7/4}(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] (2*b*x)/(7*a*(b*c - a*d)*(a + b*x^2)^(7/4)) + (2*b*(5*b*c - 12*a*d)*x)/(21*
a^2*(b*c - a*d)^2*(a + b*x^2)^(3/4)) + (2*Sqrt[b]*(5*b*c - 12*a*d)*(1 + (b*
x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(21*a^(3/2)*(b*c
 - a*d)^2*(a + b*x^2)^(3/4)) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[-
((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/((b*c - a*d)^3*x) + (a^(1/4)*d^2*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a
]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/((b*
c - a*d)^3*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} - \frac{2\int \frac{\frac{1}{2}(-5bc+7ad)-\frac{5}{2}bdx^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{7a(bc-ad)}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{4\int \frac{\frac{1}{4}\left(5b^2c^2-12abcd+21a^2d^2\right)+\frac{1}{4}bd(5bc-12ad)x^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{21a^2(bc-ad)^2}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{d^2\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{(bc-ad)^2} + \frac{(b(5bc-12ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{21a^2(bc-ad)^2}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{\left(d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{2(bc-ad)^2x} + \frac{\left(b(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}(bc-ad)^2\left(a+bx^2\right)^{3/4}} - \frac{\left(2d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^2x}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{\left(d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^3x} + \frac{\left(d^2\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{(bc-ad)^3x}$$

$$= \frac{2bx}{7a(bc-ad)\left(a+bx^2\right)^{7/4}} + \frac{2b(5bc-12ad)x}{21a^2(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{2\sqrt{b}(5bc-12ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{21a^{3/2}(bc-ad)^2\left(a+bx^2\right)^{3/4}} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)^3x} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{(bc-ad)^3x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.83 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx = -\frac{x\left(\frac{bd(-5bc+12ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c} + \frac{6\left(3ac\left(21a^3d^2+5b^3cx^2\left(3c+2dx^2\right)-3a^2bd\left(14c+3dx^2\right)+ab^2\left(21c^2-20cd\right.\right.\right.}{\left(a+bx^2\right)\left(c+dx^2\right)\left(\right.}\right.}{6}$$

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)),x]
```

```
[Out] -1/63*(x*((b*d*(-5*b*c + 12*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/
4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/c + (6*(3*a*c*(21*a^3*d^2 + 5*b^3*c
*x^2*(3*c + 2*d*x^2) - 3*a^2*b*d*(14*c + 3*d*x^2) + a*b^2*(21*c^2 - 20*c*d*
x^2 - 24*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] +
```

```
b*x^2*(c + d*x^2)*(15*a^2*d - 5*b^2*c*x^2 + a*b*(-8*c + 12*d*x^2))*(4*a*d*
AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2
, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(-6*
a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*App
ellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7
/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(a^2*(b*c - a*d)^2*(a + b*x^2)
^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{11}{4}} \left(d\,x^{2}+c\right)}d x$$

[In] `int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)`

[Out] `int(1/(b*x^2+a)^(11/4)/(d*x^2+c),x)`

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx=\text{Timed out}$$

[In] `integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="fricas")`

[Out] `Timed out`

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx=\int \frac{1}{\left(a+bx^{2}\right)^{\frac{11}{4}}\left(c+dx^{2}\right)}\,dx$$

[In] `integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c),x)`

[Out] `Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)), x)`

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)}\,dx = \int \frac{1}{(b\,x^2+a)^{11/4}\,(d\,x^2+c)}\,dx$$

```
[In] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)),x)
[Out] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)), x)
```

3.331 $\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}
\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx &= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} \\
&- \frac{\sqrt{a}\sqrt{b}(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} \\
&+ \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x} \\
&- \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x}
\end{aligned}$$

```
[Out] 1/2*b*(-a*d+5*b*c)*x/c/d^2/(b*x^2+a)^(1/4)-1/2*(-a*d+b*c)*x*(b*x^2+a)^(3/4)
/c/d/(d*x^2+c)-1/2*(-a*d+5*b*c)*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)
/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*ar
ctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d^2/(b*x^2+a)^(1/4)+1/4
*a^(1/4)*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/
(a*d-b*c)^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x-1/4*a^(1/4)
*(2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)
^(1/2),I)*(a*d-b*c)^(1/2)*(-b*x^2/a)^(1/2)/c/d^(5/2)/x
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {424, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x}$$
$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}\sqrt{ad-bc}(2ad+5bc)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{5/2}x}$$
$$-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(5bc-ad)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}}$$
$$+\frac{bx(5bc-ad)}{2cd^2\sqrt[4]{a+bx^2}}-\frac{x(a+bx^2)^{3/4}(bc-ad)}{2cd(c+dx^2)}$$

```
[In] Int[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]
```

```
[Out] (b*(5*b*c - a*d)*x)/(2*c*d^2*(a + b*x^2)^(1/4)) - ((b*c - a*d)*x*(a + b*x^2
)^(3/4))/(2*c*d*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(5*b*c - a*d)*(1 + (b*x^2)/
a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d^2*(a + b*x^2)^
(1/4)) + (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)*Sqrt[-((b*x^2)/a)]*Ell
ipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a
^(1/4)], -1])/(4*c*d^(5/2)*x) - (a^(1/4)*Sqrt[-(b*c) + a*d]*(5*b*c + 2*a*d)
*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin
[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{\int \frac{a(bc+ad)+\frac{1}{2}b(5bc-ad)x^2}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{2cd}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{(b(5bc-ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd^2} - \frac{((bc-ad)(5bc+2ad))\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} - \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2cd^2x} + \frac{\left(b(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4cd^2\sqrt[4]{a+bx^2}}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} + \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4cd^{5/2}x} - \frac{\left((bc-ad)(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4cd^{5/2}x} - \frac{\left(b(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4cd^2\sqrt[4]{a+bx^2}}$$

$$= \frac{b(5bc-ad)x}{2cd^2\sqrt[4]{a+bx^2}} - \frac{(bc-ad)x(a+bx^2)^{3/4}}{2cd\,(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}(5bc-ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\sqrt[4]{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x} - \frac{\sqrt[4]{a}\sqrt{-bc+ad}(5bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{5/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.35 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \frac{x\left(-b(-5bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac(2a^2d-b^2cx^2+abd\right.}{(c+dx^2)\left(\right.}\right.}{}$$

[In] Integrate[(a + b*x^2)^(7/4)/(c + d*x^2)^2,x]

[Out] (x*(-(b*(-5*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]) + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x^2)*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)*x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a + b*x^2)^(1/4))

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{4}}}{(d\,x^2+c)^2}dx$$

[In] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)

[Out] int((b*x^2+a)^(7/4)/(d*x^2+c)^2,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{7}{4}}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(7/4)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(7/4)/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{7/4}}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^(7/4)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(7/4)/(c + d*x^2)^2, x)
```

3.332 $\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 279

$$
\begin{aligned}
\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = &-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\,(c+dx^2)} \\
&+\frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd^2\,(a+bx^2)^{3/4}} \\
&-\frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x} \\
&-\frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x}
\end{aligned}
$$

```
[Out] -1/2*(-a*d+b*c)*x*(b*x^2+a)^(1/4)/c/d/(d*x^2+c)+1/2*(a*d+3*b*c)*(1+b*x^2/a)
^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b
^(1/2)/c/d^2/(b*x^2+a)^(3/4)-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^2+a)
^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^2/x
-1/4*a^(1/4)*(2*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^2/x
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {424, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = -\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+3bc)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^2x} + \frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(ad+3bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd^2\left(a+bx^2\right)^{3/4}} - \frac{x\sqrt[4]{a+bx^2}(bc-ad)}{2cd\left(c+dx^2\right)}$$

```
[In] Int[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*((b*c - a*d)*x*(a + b*x^2)^(1/4))/(c*d*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]
*(3*b*c + a*d)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/
2, 2])/(2*c*d^2*(a + b*x^2)^(3/4)) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*d^2*x) - (a^(1/4)*(3*b*c + 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(
1/4)/a^(1/4)], -1])/(4*c*d^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 424

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a*d - c*b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(a*b*n*(p +
1))), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\int \frac{a(bc+ad)+\frac{1}{2}b(3bc+ad)x^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2cd}\\
&= -\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{(b(3bc+ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{4cd^2}\\
&\quad -\frac{((bc-ad)(3bc+2ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{4cd^2}
\end{aligned}$$

$$=-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}-\frac{\left((bc-ad)(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}dx,x,x^2\right)}{8cd^2x}+\frac{\left(b(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}dx}{4cd^2\left(a+bx^2\right)^{3/4}}$$

$$=-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}+\frac{\left((bc-ad)(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}dx,x,\sqrt[4]{a+bx^2}\right)}{2cd^2x}$$

$$=-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\left((3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}dx,x,\sqrt[4]{a+bx^2}\right)}{4cd^2x}-\frac{\left((3bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}dx,x,\sqrt[4]{a+bx^2}\right)}{4cd^2x}$$

$$=-\frac{(bc-ad)x\sqrt[4]{a+bx^2}}{2cd\left(c+dx^2\right)}+\frac{\sqrt{a}\sqrt{b}(3bc+ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd^2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}-\frac{\sqrt[4]{a}(3bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \frac{x\left(b(3bc+ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{6c\left(-6ac(2a^2d-b^2cx^2+abdx^2\right.}{(c+dx^2)\left(-6\right.}\right.}{12}$$

```
[In] Integrate[(a + b*x^2)^(5/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(b*(3*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -(
(b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*(2*a^2*d - b^2*c*x^2 + a*b*d*x^2)*
AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + (-(b*c) + a*d)*x^2
*(a + b*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)]
+ 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x
^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4
*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF
1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*d*(a + b*x^2)^
(3/4))
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{(d\,x^2+c)^2}dx$$

```
[In] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{5}{4}}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(5/4)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(5/4)/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^(5/4)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(5/4)/(c + d*x^2)^2, x)
```

3.333 $\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 309

$$\begin{aligned}\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx &= -\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x(a+bx^2)^{3/4}}{2c\,(c+dx^2)} \\ &+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}} \\ &+\frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{3/2}\sqrt{-bc+ad}x} \\ &-\frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{3/2}\sqrt{-bc+ad}x}\end{aligned}$$

```
[Out] -1/2*b*x/c/d/(b*x^2+a)^(1/4)+1/2*x*(b*x^2+a)^(3/4)/c/(d*x^2+c)+1/2*(1+b*x^2
/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(
1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2
)*b^(1/2)/c/d/(b*x^2+a)^(1/4)+1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2+a)^
(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^(3/2
)/x/(a*d-b*c)^(1/2)-1/4*a^(1/4)*(2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1
/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d^(3/2)/x/(a*d-b*
c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {423, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}$$
$$-\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(2ad+bc)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cd^{3/2}x\sqrt{ad-bc}}$$
$$+\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}-\frac{bx}{2cd\sqrt[4]{a+bx^2}}+\frac{x(a+bx^2)^{3/4}}{2c\,(c+dx^2)}$$

```
[In] Int[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] -1/2*(b*x)/(c*d*(a + b*x^2)^(1/4)) + (x*(a + b*x^2)^(3/4))/(2*c*(c + d*x^2)
) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqr
t[a]]/2, 2])/(2*c*d*(a + b*x^2)^(1/4)) + (a^(1/4)*(b*c + 2*a*d)*Sqrt[-((b*x
^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x
^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*x) - (a^(1/4)*(b*c
 + 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d
], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d^(3/2)*Sqrt[-(b*c) + a*d]*
x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x(a+bx^2)^{3/4}}{2c\,(c+dx^2)} - \frac{\int \frac{-a+\frac{bx^2}{2}}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{2c}\\ &= \frac{x(a+bx^2)^{3/4}}{2c\,(c+dx^2)} - \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4cd} + \frac{(bc+2ad)\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{4cd}\end{aligned}$$

$$= \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} + \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}dx, x, \sqrt[4]{a+bx^2}\right)}{2cdx}$$
$$- \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}dx}{4cd\sqrt[4]{a+bx^2}}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)}$$
$$- \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}dx, x, \sqrt[4]{a+bx^2}\right)}{4cd^{3/2}x}$$
$$+ \frac{\left((bc+2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}dx, x, \sqrt[4]{a+bx^2}\right)}{4cd^{3/2}x}$$
$$+ \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}dx}{4cd\sqrt[4]{a+bx^2}}$$

$$= -\frac{bx}{2cd\sqrt[4]{a+bx^2}} + \frac{x(a+bx^2)^{3/4}}{2c(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\sqrt[4]{a+bx^2}}$$
$$+ \frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}\sqrt{-bc+ad}x}$$
$$- \frac{\sqrt[4]{a}(bc+2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^{3/2}\sqrt{-bc+ad}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}dx = \frac{x\left(-\frac{bx^2\sqrt[4]{1+\frac{bx^2}{a}}\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2} + \frac{6\left(\frac{a+bx^2}{c} - \frac{6a^2 \cdots}{-6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4 \cdots\right.}\right.}{\cdots}\right.}{12\sqrt[4]{a+bx^2}\cdots}$$

```
[In] Integrate[(a + b*x^2)^(3/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(-((b*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a),
 -((d*x^2)/c)])/c^2) + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 1/4, 1, 3/2
, -((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)
/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
)/(c + d*x^2)))/(12*(a + b*x^2)^(1/4))
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a \right)^{\frac{3}{4}}}{\left(d\,x^{2}+c \right)^{2}}d x$$

```
[In] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(3/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{4}}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
```

```
[Out] Integral((a + b*x**2)**(3/4)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{(dx^2+c)^2}\,dx$$

[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")

[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)

Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{(dx^2+c)^2}\,dx$$

[In] integrate((b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^(3/4)/(d*x^2 + c)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^{3/4}}{(d\,x^2+c)^2}\,dx$$

[In] int((a + b*x^2)^(3/4)/(c + d*x^2)^2,x)

[Out] int((a + b*x^2)^(3/4)/(c + d*x^2)^2, x)

3.334 $\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 278

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \frac{x\sqrt[4]{a+bx^2}}{2c(c+dx^2)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd(bc-ad)x} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4cd(bc-ad)x}$$

```
[Out] 1/2*x*(b*x^2+a)^(1/4)/c/(d*x^2+c)+1/2*(1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b
^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticF(sin(
1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/d/(b*x^2+a)^(3/4)
-1/4*a^(1/4)*(-2*a*d+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/
2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x-1/4*a^(1/4)*(-2*a*d
+b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)
*(-b*x^2/a)^(1/2)/c/d/(-a*d+b*c)/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {423, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = -\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cdx(bc-ad)} - \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cdx(bc-ad)} + \frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2cd\,(a+bx^2)^{3/4}} + \frac{x\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)}$$

```
[In] Int[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^(1/4))/(2*c*(c + d*x^2)) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^
(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*d*(a + b*x^2)^(3/4)
) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*(b*c
- a*d)*x) - (a^(1/4)*(b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*S
qrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*d*
(b*c - a*d)*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 423

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-x)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(a*n*(p + 1))), x] + Dist[1
/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p +
1) + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x
] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} - \frac{\int \frac{-a-\frac{bx^2}{2}}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2c} \\
&= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{b\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{4cd} - \frac{\left(\frac{bc}{2}-ad\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{2cd}
\end{aligned}
$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{4cdx} + \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int\frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{4cd\left(a+bx^2\right)^{3/4}}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} + \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int\frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx,x,\sqrt[4]{a+bx^2}\right)}{cdx}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int\frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2cd(bc-ad)x} - \frac{\left(\left(\frac{bc}{2}-ad\right)\sqrt{-\frac{bx^2}{a}}\right)\mathrm{Subst}\left(\int\frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2cd(bc-ad)x}$$

$$= \frac{x\sqrt[4]{a+bx^2}}{2c\left(c+dx^2\right)} + \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2cd\left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd(bc-ad)x} - \frac{\sqrt[4]{a}(bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd(bc-ad)x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$$
$$= \frac{x\left(\frac{bx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}+\frac{6\left(\frac{a+bx^2}{c}-\frac{6a^2\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-6ac\operatorname{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+x^2\left(4ad\operatorname{AppellF1}\left(\frac{3}{2},\frac{3}{4},2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+3bc\operatorname{AppellF1}\left(\frac{3}{2},\frac{7}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\right)}\right)}{c+dx^2}\right)}{12\left(a+bx^2\right)^{3/4}}$$

```
[In] Integrate[(a + b*x^2)^(1/4)/(c + d*x^2)^2,x]
```

```
[Out] (x*((b*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])/c^2 + (6*((a + b*x^2)/c - (6*a^2*AppellF1[1/2, 3/4, 1, 3/2, -
((b*x^2)/a), -((d*x^2)/c)])/(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a)
, -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*
x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))
/(c + d*x^2)))/(12*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(d\,x^2+c)^2}dx$$

```
[In] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx=\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx$$

```
[In] integrate((b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
[Out] Integral((a + b*x**2)**(1/4)/(c + d*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(1/4)/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(c+dx^2)^2}\,dx=\int \frac{(b\,x^2+a)^{1/4}}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^(1/4)/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^(1/4)/(c + d*x^2)^2, x)
```

3.335 $\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 336

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx &= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\sqrt[4]{a+bx^2}}\\
&\quad - \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\ \text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x}\\
&\quad + \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\ \text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x}\end{aligned}$$

```
[Out] 1/2*b*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/2*d*x*(b*x^2+a)^(3/4)/c/(-a*d+b*c)/(
d*x^2+c)-1/2*(1+b*x^2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)
/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1
/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/(-a*d+b*c)/(b*x^2+a)^(1/4)-1/4*a^(1/4)*(-2
*a*d+3*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(
1/2),I)*(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)+1/4*a^(1/4)*(-2*a*d+3*
b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*
(-b*x^2/a)^(1/2)/c/(a*d-b*c)^(3/2)/x/d^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {425, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$$
$$= -\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}$$
$$+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(3bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{d}x(ad-bc)^{3/2}}$$
$$-\frac{\sqrt{a}\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c\sqrt[4]{a+bx^2}(bc-ad)}$$
$$+\frac{bx}{2c\sqrt[4]{a+bx^2}(bc-ad)}-\frac{dx(a+bx^2)^{3/4}}{2c\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (d*x*(a + b*x^2)^(3/4))/(2*c*(b
*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b*c - a*d)*(a + b*x^2)^(1/4)) - (a^
(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqr
t[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*Sqrt[d]*(-(b
*c) + a*d)^(3/2)*x) + (a^(1/4)*(3*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticP
i[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)],
-1])/(4*c*Sqrt[d]*(-(b*c) + a*d)^(3/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\,(c+dx^2)} + \frac{\int \frac{2bc-ad+\frac{1}{2}bdx^2}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{b\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4c(bc-ad)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} + \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2c(bc-ad)x} + \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4c(bc-ad)\sqrt[4]{a+bx^2}}$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c\sqrt{d}(bc-ad)x} + \frac{\left((3bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c\sqrt{d}(bc-ad)x} - \frac{\left(b\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4c(bc-ad)\sqrt[4]{a+bx^2}}$$

$$= \frac{bx}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{dx(a+bx^2)^{3/4}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\sqrt[4]{a+bx^2}} - \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x} + \frac{\sqrt[4]{a}(3bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{d}(-bc+ad)^{3/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.18 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$$
$$= \frac{-6acx\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{4},1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\left(-6c(-2bc+2ad+bdx^2)+bdx^2\sqrt[4]{1+\frac{bx^2}{a}}(c+dx^2)\,\text{AppellF1}\left(\right.\right.}{12c^2(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)\left(-6ac\right.}$$

```
[In] Integrate[1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x]
```

```
[Out] (-6*a*c*x*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)]*(-6*c*(-2*
b*c + 2*a*d + b*d*x^2) + b*d*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1
[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]) - d*x^3*(6*c*(a + b*x^2) -
b*x^2*(1 + (b*x^2)/a)^(1/4)*(c + d*x^2)*AppellF1[3/2, 1/4, 1, 5/2, -((b*x^2
)/a), -((d*x^2)/c)])*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x
^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 1/4, 1,
 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/2, -
((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a), -
((d*x^2)/c)])))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(d\,x^{2}+c\right)^{2}}dx$$

```
[In] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(1/4)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(1/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{a+bx^2}\,(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{1/4}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(1/4)*(c + d*x^2)^2), x)
```

3.336 $\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 292

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx &= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)(c+dx^2)} \\
&\quad - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2c(bc-ad)(a+bx^2)^{3/4}} \\
&\quad + \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^2x} \\
&\quad + \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^2x}
\end{aligned}
$$

```
[Out] -1/2*d*x*(b*x^2+a)^(1/4)/c/(-a*d+b*c)/(d*x^2+c)-1/2*(1+b*x^2/a)^(3/4)*(cos(
1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*
EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*a^(1/2)*b^(1/2)/c/(-a
*d+b*c)/(b*x^2+a)^(3/4)+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/
4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c
)^2/x+1/4*a^(1/4)*(-2*a*d+5*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)
*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^2/x
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {425, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^2}$$
$$+\frac{\sqrt[4]{a}\sqrt{-\frac{bx^2}{a}}(5bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^2}$$
$$-\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2c\,(a+bx^2)^{3/4}\,(bc-ad)}-\frac{dx\sqrt[4]{a+bx^2}}{2c\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x*(a + b*x^2)^(1/4))/(c*(b*c - a*d)*(c + d*x^2)) - (Sqrt[a]*Sqrt[b]
*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(2*c*(b
*c - a*d)*(a + b*x^2)^(3/4)) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*
EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4
)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x) + (a^(1/4)*(5*b*c - 2*a*d)*Sqrt[-((b
*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^
2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^2*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-\frac{1}{2}bdx^2}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{2c(bc-ad)}\\
&= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)}-\frac{b\int \frac{1}{(a+bx^2)^{3/4}}\,dx}{4c(bc-ad)}+\frac{(5bc-2ad)\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)}\,dx}{4c(bc-ad)}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} \\
&\quad + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx, x, x^2\right)}{8c(bc-ad)x} \\
&\quad - \frac{\left(b\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{4c(bc-ad)\left(a+bx^2\right)^{3/4}} \\
&= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} \\
&\quad - \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2c(bc-ad)x} \\
&= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} \\
&\quad + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^2x} \\
&\quad + \frac{\left((5bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^2x} \\
&= -\frac{dx\sqrt[4]{a+bx^2}}{2c(bc-ad)\left(c+dx^2\right)} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2c(bc-ad)\left(a+bx^2\right)^{3/4}} \\
&\quad + \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^2x} \\
&\quad + \frac{\sqrt[4]{a}(5bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^2x}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \frac{x\left(\frac{bdx^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{-bc+ad}+\frac{c\left(36ac(-2bc+2ad+bdx^2)\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4},1,\frac{3}{2}\right.\right.}{(bc-ad)(c+dx^2)\left(-6ac\,\text{AppellF1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}\right.}{12c^2\,(}$$

[In] Integrate[1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x]

[Out] (x*((b*d*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-(b*c) + a*d) + (c*(36*a*c*(-2*b*c + 2*a*d + b*d*x^2)*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - 6*d*x^2*(a + b*x^2)*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((b*c - a*d)*(c + d*x^2)*(-6*a*c*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))))/(12*c^2*(a + b*x^2)^(3/4))

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}(d\,x^2+c)^2}dx$$

[In] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)

[Out] int(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{3}{4}}(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(3/4)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(3/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{3/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{3/4}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(3/4)*(c + d*x^2)^2), x)
```

3.337 $\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 314

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx &= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)}\\
&+\frac{\sqrt{b}(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c(bc-ad)^2\sqrt[4]{a+bx^2}}\\
&-\frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{5/2}x}\\
&+\frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{5/2}x}\end{aligned}$$

```
[Out] -1/2*d*x/c/(-a*d+b*c)/(b*x^2+a)^(1/4)/(d*x^2+c)+1/2*(a*d+4*b*c)*(1+b*x^2/a)
^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2
)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/c
/(-a*d+b*c)^2/(b*x^2+a)^(1/4)/a^(1/2)-1/4*a^(1/4)*(-2*a*d+7*b*c)*EllipticPi
((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^
2/a)^(1/2)/c/(a*d-b*c)^(5/2)/x+1/4*a^(1/4)*(-2*a*d+7*b*c)*EllipticPi((b*x^2
+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*d^(1/2)*(-b*x^2/a)^(1/
2)/c/(a*d-b*c)^(5/2)/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = -\frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(ad-bc)^{5/2}} + \frac{\sqrt[4]{a}\sqrt{d}\sqrt{-\frac{bx^2}{a}}(7bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(ad-bc)^{5/2}} + \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(ad+4bc)E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c\sqrt[4]{a+bx^2}(bc-ad)^2} - \frac{dx}{2c\sqrt[4]{a+bx^2}\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] -1/2*(d*x)/(c*(b*c - a*d)*(a + b*x^2)^(1/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c
+ a*d)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
2*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^2)^(1/4)) - (a^(1/4)*Sqrt[d]*(7*b*c - 2*
a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]),
 ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(5/2)*x) + (a^
(1/4)*Sqrt[d]*(7*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d
])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c)
 + a*d)^(5/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\int \frac{2bc-ad-\frac{3}{2}bdx^2}{(a+bx^2)^{5/4}(c+dx^2)}\,dx}{2c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{\int \frac{\frac{1}{2}(2b^2c^2+4abcd-a^2d^2)+\frac{1}{4}bd(4bc+ad)x^2}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{(d(7bc-2ad))\int \frac{1}{\sqrt[4]{a+bx^2}(c+dx^2)}\,dx}{4c(bc-ad)^2} - \frac{(b(4bc+ad))\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{2ac(bc-ad)^2\sqrt[4]{a+bx^2}} - \frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} - \frac{\left(d(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}(bc-ad+dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2c(bc-ad)^2x} - \frac{\left(b(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{4ac(bc-ad)^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\left(\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^2x} - \frac{\left(\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^2x} + \frac{\left(b(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{4ac(bc-ad)^2\sqrt[4]{a+bx^2}}$$

$$= -\frac{dx}{2c(bc-ad)\sqrt[4]{a+bx^2}\,(c+dx^2)} + \frac{\sqrt{b}(4bc+ad)\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}c(bc-ad)^2\sqrt[4]{a+bx^2}}$$
$$- \frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(-bc+ad)^{5/2}x}$$
$$+ \frac{\sqrt[4]{a}\sqrt{d}(7bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(-bc+ad)^{5/2}x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.33 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \frac{x\left(-bd(4bc+ad)x^2\sqrt[4]{1+\frac{bx^2}{a}}\,\mathrm{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c\left(36ac(2a^2d^2\right.}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(-(b*d*(4*b*c + a*d)*x^2*(1 + (b*x^2)/a)^(1/4)*AppellF1[3/2, 1/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)]) + (c*(36*a*c*(2*a^2*d^2 + a*b*d*(-4*c + d*x^
2) + 2*b^2*c*(c + 2*d*x^2))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x
^2)/c)] - 6*x^2*(a^2*d^2 + a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d*Appell
F1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1
, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1[1/2, 1/4
, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 1/4, 2, 5/
2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4, 1, 5/2, -((b*x^2)/a
), -((d*x^2)/c)])))))/(12*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(1/4))
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{4}}\,(d\,x^2+c)^2}dx$$

```
[In] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{5}{4}}(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(5/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(5/4)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(5/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(5/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{5/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{5/4}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(5/4)*(c + d*x^2)^2), x)
```

3.338 $\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 345

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx &= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/4}} \\ &\quad - \frac{dx}{2c(bc-ad)(a+bx^2)^{3/4}(c+dx^2)} \\ &\quad + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}c(bc-ad)^2(a+bx^2)^{3/4}} \\ &\quad - \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^3x} \\ &\quad - \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^3x}\end{aligned}$$

```
[Out] 1/6*b*(3*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(3/4)-1/2*d*x/c/(-a*d+b*c)
/(b*x^2+a)^(3/4)/(d*x^2+c)+1/6*(3*a*d+4*b*c)*(1+b*x^2/a)^(3/4)*(cos(1/2*arc
tan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^(1/2)/a^(1/2)))*Ellipti
cF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/2)/c/(-a*d+b*c)^2/(b*x^
2+a)^(3/4)/a^(1/2)-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(1/4)/
a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^3
/x-1/4*a^(1/4)*d*(-2*a*d+9*b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*
d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^3/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = -\frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^3} - \frac{\sqrt[4]{a}d\sqrt{-\frac{bx^2}{a}}(9bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^3} + \frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(3ad+4bc)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}c\,(a+bx^2)^{3/4}\,(bc-ad)^2} + \frac{bx(3ad+4bc)}{6ac\,(a+bx^2)^{3/4}\,(bc-ad)^2} - \frac{dx}{2c\,(a+bx^2)^{3/4}\,(c+dx^2)\,(bc-ad)}$$

```
[In] Int[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (d*x)/(2*c*
(b*c - a*d)*(a + b*x^2)^(3/4)*(c + d*x^2)) + (Sqrt[b]*(4*b*c + 3*a*d)*(1 +
(b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(6*Sqrt[a]*c*
(b*c - a*d)^2*(a + b*x^2)^(3/4)) - (a^(1/4)*d*(9*b*c - 2*a*d)*Sqrt[-((b*x^2
)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2
)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x) - (a^(1/4)*d*(9*b*c - 2*a*d)*S
qrt[-((b*x^2)/a)]*EllipticPi[(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(
a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c - a*d)^3*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-\frac{5}{2}bdx^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\
&= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\\
&\quad -\frac{\int \frac{\frac{1}{2}\left(-2b^2c^2+12abcd-3a^2d^2\right)-\frac{1}{4}bd(4bc+3ad)x^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{3ac(bc-ad)^2}\\
&= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\\
&\quad -\frac{(d(9bc-2ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{4c(bc-ad)^2}+\frac{(b(4bc+3ad))\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{12ac(bc-ad)^2}\\
&= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\\
&\quad -\frac{\left(d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{8c(bc-ad)^2x}\\
&\quad +\frac{\left(b(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{12ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}\\
&= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2\left(a+bx^2\right)^{3/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\\
&\quad +\frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4}F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c(bc-ad)^2\left(a+bx^2\right)^{3/4}}\\
&\quad +\frac{\left(d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}\left(-bc+ad-dx^4\right)}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2c(bc-ad)^2x}
\end{aligned}
$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{3/4}(c+dx^2)} + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c(bc-ad)^2(a+bx^2)^{3/4}} - \frac{\left(d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^3x} - \frac{\left(d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^3x}$$

$$= \frac{b(4bc+3ad)x}{6ac(bc-ad)^2(a+bx^2)^{3/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{3/4}(c+dx^2)} + \frac{\sqrt{b}(4bc+3ad)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{6\sqrt{a}c(bc-ad)^2(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^3x} - \frac{\sqrt[4]{a}d(9bc-2ad)\sqrt{-\frac{bx^2}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^3x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.34 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \frac{x\left(bd(4bc+3ad)x^2\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\frac{c(36ac(6c}{}\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x]
```

```
[Out] (x*(b*d*(4*b*c + 3*a*d)*x^2*(1 + (b*x^2)/a)^(3/4)*AppellF1[3/2, 3/4, 1, 5/2
, -((b*x^2)/a), -((d*x^2)/c)] + (c*(36*a*c*(6*a^2*d^2 + 3*a*b*d*(-4*c + d*x
^2) + 2*b^2*c*(3*c + 2*d*x^2))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((
d*x^2)/c)] - 6*x^2*(3*a^2*d^2 + 3*a*b*d^2*x^2 + 4*b^2*c*(c + d*x^2))*(4*a*d
*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/
2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((c + d*x^2)*(6*a*c*AppellF1
[1/2, 3/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3
/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2,
-((b*x^2)/a), -((d*x^2)/c)])))))/(36*a*c^2*(b*c - a*d)^2*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{4}} \left(d\,x^{2}+c\right)^{2}}d x$$

```
[In] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
[Out] int(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{7/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{\left(a+bx^{2}\right)^{\frac{7}{4}} \left(c+dx^{2}\right)^{2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(7/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(7/4)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{{\left(bx^{2}+a\right)}^{\frac{7}{4}} {\left(dx^{2}+c\right)}^{2}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(7/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(7/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{7/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{7/4}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(7/4)*(c + d*x^2)^2), x)
```

3.339 $\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 371

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx &= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2(a+bx^2)^{5/4}} \\
&- \frac{dx}{2c(bc-ad)(a+bx^2)^{5/4}(c+dx^2)} \\
&+ \frac{\sqrt{b}(12b^2c^2-52abcd-5a^2d^2)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c(bc-ad)^3\sqrt[4]{a+bx^2}} \\
&- \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{7/2}x} \\
&+ \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(-bc+ad)^{7/2}x}
\end{aligned}$$

```
[Out] 1/10*b*(5*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(5/4)-1/2*d*x/c/(-a*d+b*c
)/(b*x^2+a)^(5/4)/(d*x^2+c)+1/10*(-5*a^2*d^2-52*a*b*c*d+12*b^2*c^2)*(1+b*x^
2/a)^(1/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arctan(x*b^
(1/2)/a^(1/2)))*EllipticE(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))*b^(1/
2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(1/4)-1/4*a^(1/4)*d^(3/2)*(-2*a*d+11*b*
c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(
-b*x^2/a)^(1/2)/c/(a*d-b*c)^(7/2)/x+1/4*a^(1/4)*d^(3/2)*(-2*a*d+11*b*c)*Ell
ipticPi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/
a)^(1/2)/c/(a*d-b*c)^(7/2)/x
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 235, 233, 202, 408, 504, 1232}

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2}\,dx = \frac{\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}(-5a^2d^2-52abcd+12b^2c^2)\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c\sqrt[4]{a+bx^2}(bc-ad)^3}$$
$$-\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(ad-bc)^{7/2}}$$
$$+\frac{\sqrt[4]{a}d^{3/2}\sqrt{-\frac{bx^2}{a}}(11bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(ad-bc)^{7/2}}$$
$$-\frac{dx}{2c(a+bx^2)^{5/4}(c+dx^2)(bc-ad)}+\frac{bx(5ad+4bc)}{10ac(a+bx^2)^{5/4}(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(10*a*c*(b*c - a*d)^2*(a + b*x^2)^(5/4)) - (d*x)/(2*c
*(b*c - a*d)*(a + b*x^2)^(5/4)*(c + d*x^2)) + (Sqrt[b]*(12*b^2*c^2 - 52*a*b
*c*d - 5*a^2*d^2)*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a
]]/2, 2])/(10*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(1/4)) - (a^(1/4)*d^(3/2)
*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt[d])/Sqrt[-(
b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(-(b*c) + a*d)^(
7/2)*x) + (a^(1/4)*d^(3/2)*(11*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[(
Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1]
)/(4*c*(-(b*c) + a*d)^(7/2)*x)
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Dist[a, Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4), Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x] &
```

```
& PosQ[a]
```

Rule 408

```
Int[1/(((a_) + (b_.)*(x_)^2)^(1/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[2*(Sqrt[(-b)*(x^2/a)]/x), Subst[Int[x^2/(Sqrt[1 - x^4/a]*(b*c - a*d + d*x
^4)), x], x, (a + b*x^2)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 504

```
Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] :>
With[{r = Numerator[Rt[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*
b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r -
 s*x^2)*Sqrt[c + d*x^4]), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*
d, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
```

```
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\text{integral} = -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-\frac{7}{2}bdx^2}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}$$

$$=\frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}-\frac{\int \frac{\frac{1}{2}\left(-6b^2c^2+20abcd-5a^2d^2\right)-\frac{3}{4}bd(4bc+5ad)x^2}{\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}\,dx}{5ac(bc-ad)^2}$$

$$=\frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}}+\frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}+\frac{2\int \frac{\frac{1}{4}\left(-6b^3c^3+26ab^2c^2d+30a^2bcd^2-5a^3d^3\right)-\frac{1}{8}bd\left(12b^2c^2-52abcd-5a^2d^2\right)x^2}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{5a^2c(bc-ad)^3}$$

$$=\frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}}+\frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}+\frac{\left(d^2(11bc-2ad)\right)\int \frac{1}{\sqrt[4]{a+bx^2}\left(c+dx^2\right)}\,dx}{4c(bc-ad)^3}-\frac{\left(b\left(12b^2c^2-52abcd-5a^2d^2\right)\right)\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx}{20a^2c(bc-ad)^3}$$

$$=\frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}}+\frac{b\left(12b^2c^2-52abcd-5a^2d^2\right)x}{10a^2c(bc-ad)^3\sqrt[4]{a+bx^2}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)}+\frac{\left(d^2(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{x^4}{a}}\left(bc-ad+dx^4\right)}\,dx,x,\sqrt[4]{a+bx^2}\right)}{2c(bc-ad)^3x}-\frac{\left(b\left(12b^2c^2-52abcd-5a^2d^2\right)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\sqrt[4]{1+\frac{bx^2}{a}}}\,dx}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^2}}$$

$$
\begin{aligned}
&= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} \\
&\quad - \frac{\left(d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}-\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^3x} \\
&\quad + \frac{\left(d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(\sqrt{-bc+ad}+\sqrt{d}x^2\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^3x} \\
&\quad + \frac{\left(b(12b^2c^2-52abcd-5a^2d^2)\sqrt[4]{1+\frac{bx^2}{a}}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{5/4}}\,dx}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^2}} \\
&= \frac{b(4bc+5ad)x}{10ac(bc-ad)^2\left(a+bx^2\right)^{5/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{5/4}\left(c+dx^2\right)} \\
&\quad + \frac{\sqrt{b}(12b^2c^2-52abcd-5a^2d^2)\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{10a^{3/2}c(bc-ad)^3\sqrt[4]{a+bx^2}} \\
&\quad - \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(-bc+ad)^{7/2}x} \\
&\quad + \frac{\sqrt[4]{a}d^{3/2}(11bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(-bc+ad)^{7/2}x}
\end{aligned}
$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.70 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.44

$$
\int \frac{1}{\left(a+bx^2\right)^{9/4}\left(c+dx^2\right)^2}\,dx = \frac{bd(-12b^2c^2+52abcd+5a^2d^2)\,x^3\sqrt[4]{1+\frac{bx^2}{a}}\,\text{AppellF1}\left(\frac{3}{2},\frac{1}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{}
$$

```
[In] Integrate[1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*d*(-12*b^2*c^2 + 52*a*b*c*d + 5*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(1/4)*Appel
lF1[3/2, 1/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + (6*c*(-6*a*c*x*(10*a^4*
d^3 + 15*a^3*b*d^2*(-2*c + d*x^2) - 6*b^4*c^2*x^2*(c + 2*d*x^2) + a^2*b^2*d
*(30*c^2 + 26*c*d*x^2 + 5*d^2*x^4) + 2*a*b^3*c*(-5*c^2 + 5*c*d*x^2 + 26*d^2
*x^4))*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] + x^3*(5*a^4*
d^3 + 10*a^3*b*d^3*x^2 - 12*b^4*c^2*x^2*(c + d*x^2) + a^2*b^2*d*(56*c^2 + 5
```

```
6*c*d*x^2 + 5*d^2*x^4) + 4*a*b^3*c*(-4*c^2 + 9*c*d*x^2 + 13*d^2*x^4))*(4*a*
d*AppellF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2
, 5/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))/((a + b*x^2)*(c + d*x^2)*(6*a
*c*AppellF1[1/2, 1/4, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*Appe
llF1[3/2, 1/4, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)] + b*c*AppellF1[3/2, 5/4,
 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))))/(60*a^2*c^2*(b*c - a*d)^3*(a + b*x
^2)^(1/4))
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{4}} \left(d\,x^{2}+c\right)^{2}}d x$$

[In] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)

[Out] int(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}\,(c+dx^2)^2}\,dx = \text{Timed out}$$

[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{9/4}\,(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{9}{4}}\,(c+dx^2)^2}\,dx$$

[In] integrate(1/(b*x**2+a)**(9/4)/(d*x**2+c)**2,x)

[Out] Integral(1/((a + b*x**2)**(9/4)*(c + d*x**2)**2), x)

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^2} \, dx$$

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2} \, dx = \int \frac{1}{(bx^2+a)^{\frac{9}{4}}(dx^2+c)^2} \, dx$$

```
[In] integrate(1/(b*x^2+a)^(9/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(9/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{9/4}(c+dx^2)^2} \, dx = \int \frac{1}{(b\,x^2+a)^{9/4}\,(d\,x^2+c)^2} \, dx$$

```
[In] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(9/4)*(c + d*x^2)^2), x)
```

3.340 $\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 419

$$\begin{aligned}
\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx &= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2(a+bx^2)^{7/4}} \\
&+ \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3(a+bx^2)^{3/4}} - \frac{dx}{2c(bc-ad)(a+bx^2)^{7/4}(c+dx^2)} \\
&+ \frac{\sqrt{b}(20b^2c^2-76abcd-21a^2d^2)\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{42a^{3/2}c(bc-ad)^3(a+bx^2)^{3/4}} \\
&+ \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^4x} \\
&+ \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}},\arcsin\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right),-1\right)}{4c(bc-ad)^4x}
\end{aligned}$$

```
[Out] 1/14*b*(7*a*d+4*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^2+a)^(7/4)+1/42*b*(-21*a^2*d^2
-76*a*b*c*d+20*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)-1/2*d*x/c/(-a*
d+b*c)/(b*x^2+a)^(7/4)/(d*x^2+c)+1/42*(-21*a^2*d^2-76*a*b*c*d+20*b^2*c^2)*(
1+b*x^2/a)^(3/4)*(cos(1/2*arctan(x*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2*arcta
n(x*b^(1/2)/a^(1/2)))*EllipticF(sin(1/2*arctan(x*b^(1/2)/a^(1/2))),2^(1/2))
*b^(1/2)/a^(3/2)/c/(-a*d+b*c)^3/(b*x^2+a)^(3/4)+1/4*a^(1/4)*d^2*(-2*a*d+13*
b*c)*EllipticPi((b*x^2+a)^(1/4)/a^(1/4),-a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)
*(-b*x^2/a)^(1/2)/c/(-a*d+b*c)^4/x+1/4*a^(1/4)*d^2*(-2*a*d+13*b*c)*Elliptic
Pi((b*x^2+a)^(1/4)/a^(1/4),a^(1/2)*d^(1/2)/(a*d-b*c)^(1/2),I)*(-b*x^2/a)^(1
/2)/c/(-a*d+b*c)^4/x
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {425, 541, 544, 239, 237, 410, 109, 418, 1232}

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \frac{bx(-21a^2d^2-76abcd+20b^2c^2)}{42a^2c(a+bx^2)^{3/4}(bc-ad)^3} + \frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}(-21a^2d^2-76abcd+20b^2c^2)\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{42a^{3/2}c(a+bx^2)^{3/4}(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^4} + \frac{\sqrt[4]{a}d^2\sqrt{-\frac{bx^2}{a}}(13bc-2ad)\,\text{EllipticPi}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{ad-bc}},\arcsin\left(\frac{\sqrt[4]{bx^2+a}}{\sqrt[4]{a}}\right),-1\right)}{4cx(bc-ad)^4} - \frac{dx}{2c(a+bx^2)^{7/4}(c+dx^2)(bc-ad)} + \frac{bx(7ad+4bc)}{14ac(a+bx^2)^{7/4}(bc-ad)^2}$$

```
[In] Int[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] (b*(4*b*c + 7*a*d)*x)/(14*a*c*(b*c - a*d)^2*(a + b*x^2)^(7/4)) + (b*(20*b^2
*c^2 - 76*a*b*c*d - 21*a^2*d^2)*x)/(42*a^2*c*(b*c - a*d)^3*(a + b*x^2)^(3/4
)) - (d*x)/(2*c*(b*c - a*d)*(a + b*x^2)^(7/4)*(c + d*x^2)) + (Sqrt[b]*(20*b
^2*c^2 - 76*a*b*c*d - 21*a^2*d^2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(42*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^2)^(3/4)) +
(a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[-((Sqrt[a]*Sqrt
[d])/Sqrt[-(b*c) + a*d]), ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1])/(4*c*(b*c
 - a*d)^4*x) + (a^(1/4)*d^2*(13*b*c - 2*a*d)*Sqrt[-((b*x^2)/a)]*EllipticPi[
(Sqrt[a]*Sqrt[d])/Sqrt[-(b*c) + a*d], ArcSin[(a + b*x^2)^(1/4)/a^(1/4)], -1
])/(4*c*(b*c - a*d)^4*x)
```

Rule 109

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(
3/4)), x_Symbol] :> Dist[-4, Subst[Int[1/((b*e - a*f - b*x^4)*Sqrt[c - d*(e
/f) + d*(x^4/f)]), x], x, (e + f*x)^(1/4)], x] /; FreeQ[{a, b, c, d, e, f},
 x] && GtQ[-f/(d*e - c*f), 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Dist[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4), Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^2)^(3/4)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Dis
t[Sqrt[(-b)*(x^2/a)]/(2*x), Subst[Int[1/(Sqrt[(-b)*(x/a)]*(a + b*x)^(3/4)*(
c + d*x)), x], x, x^2], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-d/c, 2]*x^2)), x], x] + Dist[1/(2*
c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-d/c, 2]*x^2)), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 425

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c
+ d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n,
x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b,
c, d, n, p, q, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 544

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 1232

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}+\frac{\int \frac{2bc-ad-\frac{9}{2}bdx^2}{\left(a+bx^2\right)^{11/4}\left(c+dx^2\right)}\,dx}{2c(bc-ad)}\\
&= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}}-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\\
&\quad-\frac{\int \frac{\frac{1}{2}\left(-10b^2c^2+28abcd-7a^2d^2\right)-\frac{5}{4}bd(4bc+7ad)x^2}{\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\,dx}{7ac(bc-ad)^2}\\
&= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}}+\frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}}\\
&\quad-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\\
&\quad+\frac{2\int \frac{\frac{1}{4}\left(10b^3c^3-38ab^2c^2d+126a^2bcd^2-21a^3d^3\right)+\frac{1}{8}bd\left(20b^2c^2-76abcd-21a^2d^2\right)x^2}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{21a^2c(bc-ad)^3}\\
&= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}}+\frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}}\\
&\quad-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}+\frac{\left(d^2(13bc-2ad)\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}\left(c+dx^2\right)}\,dx}{4c(bc-ad)^3}\\
&\quad+\frac{\left(b\left(20b^2c^2-76abcd-21a^2d^2\right)\right)\int \frac{1}{\left(a+bx^2\right)^{3/4}}\,dx}{84a^2c(bc-ad)^3}\\
&= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}}+\frac{b\left(20b^2c^2-76abcd-21a^2d^2\right)x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}}\\
&\quad-\frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)}\\
&\quad+\frac{\left(d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-\frac{bx}{a}}(a+bx)^{3/4}(c+dx)}\,dx,x,x^2\right)}{8c(bc-ad)^3x}\\
&\quad+\frac{\left(b\left(20b^2c^2-76abcd-21a^2d^2\right)\left(1+\frac{bx^2}{a}\right)^{3/4}\right)\int \frac{1}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\,dx}{84a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}}
\end{aligned}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(20b^2c^2-76abcd-21a^2d^2)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\,2\right)}{42a^{3/2}c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{\left(d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^4}{a}}(-bc+ad-dx^4)}\,dx, x, \sqrt[4]{a+bx^2}\right)}{2c(bc-ad)^3x}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(20b^2c^2-76abcd-21a^2d^2)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\,2\right)}{42a^{3/2}c(bc-ad)^3\left(a+bx^2\right)^{3/4}} + \frac{\left(d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^4x} + \frac{\left(d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-bc+ad}}\right)\sqrt{1-\frac{x^4}{a}}}\,dx, x, \sqrt[4]{a+bx^2}\right)}{4c(bc-ad)^4x}$$

$$= \frac{b(4bc+7ad)x}{14ac(bc-ad)^2\left(a+bx^2\right)^{7/4}} + \frac{b(20b^2c^2-76abcd-21a^2d^2)\,x}{42a^2c(bc-ad)^3\left(a+bx^2\right)^{3/4}} - \frac{dx}{2c(bc-ad)\left(a+bx^2\right)^{7/4}\left(c+dx^2\right)} + \frac{\sqrt{b}(20b^2c^2-76abcd-21a^2d^2)\left(1+\frac{bx^2}{a}\right)^{3/4} F\left(\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|\,2\right)}{42a^{3/2}c(bc-ad)^3\left(a+bx^2\right)^{3/4}} + \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^4x} + \frac{\sqrt[4]{a}d^2(13bc-2ad)\sqrt{-\frac{bx^2}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-bc+ad}};\sin^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)^4x}$$

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 10.65 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.31

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \frac{\frac{bd(-20b^2c^2+76abcd+21a^2d^2)x^3\left(1+\frac{bx^2}{a}\right)^{3/4}\text{AppellF1}\left(\frac{3}{2},\frac{3}{4},1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(-bc+ad)^3}+\frac{6c\left(-6acx(42a^4d^3+\right.}{}}{}$$

```
[In] Integrate[1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x]
```

```
[Out] ((b*d*(-20*b^2*c^2 + 76*a*b*c*d + 21*a^2*d^2)*x^3*(1 + (b*x^2)/a)^(3/4)*App
ellF1[3/2, 3/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])/(-(b*c) + a*d)^3 + (6*
c*(-6*a*c*x*(42*a^4*d^3 + 63*a^3*b*d^2*(-2*c + d*x^2) - 10*b^4*c^2*x^2*(3*c
 + 2*d*x^2) + a^2*b^2*d*(126*c^2 - 38*c*d*x^2 + 21*d^2*x^4) + 2*a*b^3*c*(-2
1*c^2 + 41*c*d*x^2 + 38*d^2*x^4))*AppellF1[1/2, 3/4, 1, 3/2, -((b*x^2)/a),
-((d*x^2)/c)] + x^3*(21*a^4*d^3 + 42*a^3*b*d^3*x^2 - 20*b^4*c^2*x^2*(c + d*
x^2) + 4*a*b^3*c*(-8*c^2 + 11*c*d*x^2 + 19*d^2*x^4) + a^2*b^2*d*(88*c^2 + 8
8*c*d*x^2 + 21*d^2*x^4))*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x^2)/a), -(
(d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c)])
))/((b*c - a*d)^3*(a + b*x^2)*(c + d*x^2)*(6*a*c*AppellF1[1/2, 3/4, 1, 3/2,
 -((b*x^2)/a), -((d*x^2)/c)] - x^2*(4*a*d*AppellF1[3/2, 3/4, 2, 5/2, -((b*x
^2)/a), -((d*x^2)/c)] + 3*b*c*AppellF1[3/2, 7/4, 1, 5/2, -((b*x^2)/a), -((d
*x^2)/c)]))))/(252*a^2*c^2*(a + b*x^2)^(3/4))
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{11}{4}}\,(d\,x^2+c)^2}dx$$

```
[In] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

```
[Out] int(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(a+bx^2)^{\frac{11}{4}}(c+dx^2)^2}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(11/4)/(d*x**2+c)**2,x)
[Out] Integral(1/((a + b*x**2)**(11/4)*(c + d*x**2)**2), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{4}}(dx^2+c)^2}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(11/4)/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)^(11/4)*(d*x^2 + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^{11/4}(c+dx^2)^2}\,dx = \int \frac{1}{(b\,x^2+a)^{11/4}\,(d\,x^2+c)^2}\,dx$$

```
[In] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2),x)
[Out] int(1/((a + b*x^2)^(11/4)*(c + d*x^2)^2), x)
```

3.341 $\int (a+bx^2)^p (c+dx^2)^q \, dx$

Optimal result

Integrand size = 19, antiderivative size = 79

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} (c+dx^2)^q \left(1 + \frac{dx^2}{c}\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}, -p, -q, \frac{3}{2}, -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)$$

```
[Out] x*(b*x^2+a)^p*(d*x^2+c)^q*AppellF1(1/2,-p,-q,3/2,-b*x^2/a,-d*x^2/c)/((1+b*x
^2/a)^p)/((1+d*x^2/c)^q)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int (a+bx^2)^p (c+dx^2)^q \, dx = x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (c+dx^2)^q \left(\frac{dx^2}{c} + 1\right)^{-q} \operatorname{AppellF1}\left(\frac{1}{2}, -p, -q, \frac{3}{2}, -\frac{bx^2}{a}, -\frac{dx^2}{c}\right)$$

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((
d*x^2)/c)])/((1 + (b*x^2)/a)^p*(1 + (d*x^2)/c)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\left(c+dx^2\right)^q\,dx\\&= \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}\right)\int\left(1+\frac{bx^2}{a}\right)^p\left(1+\frac{dx^2}{c}\right)^q\,dx\\&= x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(c+dx^2\right)^q\left(1+\frac{dx^2}{c}\right)^{-q}F_1\left(\frac{1}{2};-p,-q;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 172 vs. $2(79) = 158$.

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.18

$$\int\left(a+bx^2\right)^p\left(c+dx^2\right)^q\,dx$$
$$= \frac{3acx(a+bx^2)^p\left(c+dx^2\right)^q\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{3ac\,\text{AppellF1}\left(\frac{1}{2},-p,-q,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,-q,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+adq\,\text{AppellF}\right.}$$

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^q,x]
```

```
[Out] (3*a*c*x*(a + b*x^2)^p*(c + d*x^2)^q*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a
), -((d*x^2)/c)])/(3*a*c*AppellF1[1/2, -p, -q, 3/2, -((b*x^2)/a), -((d*x^2)
/c)] + 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, -q, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] + a*d*q*AppellF1[3/2, -p, 1 - q, 5/2, -((b*x^2)/a), -((d*x^2)/c)]))
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{q}dx$$

```
[In] int((b*x^2+a)^p*(d*x^2+c)^q,x)
[Out] int((b*x^2+a)^p*(d*x^2+c)^q,x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**q,x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^q \, dx = \int (bx^2+a)^p (dx^2+c)^q \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)
```

Giac [F]

$$\int (a + bx^2)^p (c + dx^2)^q dx = \int (bx^2 + a)^p (dx^2 + c)^q dx$$

[In] integrate((b*x^2+a)^p*(d*x^2+c)^q,x, algorithm="giac")

[Out] integrate((b*x^2 + a)^p*(d*x^2 + c)^q, x)

Mupad [F(-1)]

Timed out.

$$\int (a + bx^2)^p (c + dx^2)^q dx = \int (b x^2 + a)^p (d x^2 + c)^q dx$$

[In] int((a + b*x^2)^p*(c + d*x^2)^q,x)

[Out] int((a + b*x^2)^p*(c + d*x^2)^q, x)

3.342 $\int (a+bx^2)^p (c+dx^2)^3 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))\,x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}(c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} - \frac{(15a^3d^3-9a^2bcd^2(7+2p)+3ab^2c^2d(35+24p+4p^2)-b^3c^3(105+142p+60p^2+8p^3))\,x(a+bx^2)^p\left(1 \right.}{b^3(3+2p)(5+2p)(7+2p)}$$

```
[Out] d*(15*a^2*d^2-8*a*b*c*d*(6+p)+b^2*c^2*(4*p^2+28*p+57))*x*(b*x^2+a)^(p+1)/b^
3/(8*p^3+60*p^2+142*p+105)-d*(5*a*d-b*c*(11+2*p))*x*(b*x^2+a)^(p+1)*(d*x^2+
c)/b^2/(4*p^2+24*p+35)+d*x*(b*x^2+a)^(p+1)*(d*x^2+c)^2/b/(7+2*p)-(15*a^3*d^
3-9*a^2*b*c*d^2*(7+2*p)+3*a*b^2*c^2*d*(4*p^2+24*p+35)-b^3*c^3*(8*p^3+60*p^2
+142*p+105))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^3/(8*p^3+6
0*p^2+142*p+105)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {427, 542, 396, 252, 251}

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \frac{dx(a+bx^2)^{p+1}(15a^2d^2-8abcd(p+6)+b^2c^2(4p^2+28p+57))}{b^3(2p+3)(2p+5)(2p+7)} - \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(15a^3d^3-9a^2bcd^2(2p+7)+3ab^2c^2d(4p^2+24p+35)-b^3c^3(8p^3+60p^2+142p}{b^3(2p+3)(2p+5)(2p+7)} - \frac{dx(c+dx^2)(a+bx^2)^{p+1}(5ad-bc(2p+11))}{b^2(2p+5)(2p+7)} + \frac{dx(c+dx^2)^2(a+bx^2)^{p+1}}{b(2p+7)}$$

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (d*(15*a^2*d^2 - 8*a*b*c*d*(6 + p) + b^2*c^2*(57 + 28*p + 4*p^2))*x*(a + b*
x^2)^(1 + p))/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)) - (d*(5*a*d - b*c*(11 + 2
*p))*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b^2*(5 + 2*p)*(7 + 2*p)) + (d*x*(a
 + b*x^2)^(1 + p)*(c + d*x^2)^2)/(b*(7 + 2*p)) - ((15*a^3*d^3 - 9*a^2*b*c*d
^2*(7 + 2*p) + 3*a*b^2*c^2*d*(35 + 24*p + 4*p^2) - b^3*c^3*(105 + 142*p + 6
0*p^2 + 8*p^3))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a
)])/(b^3*(3 + 2*p)*(5 + 2*p)*(7 + 2*p)*(1 + (b*x^2)/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} + \frac{\int (a+bx^2)^p(c+dx^2)\left(-c(ad-bc(7+2p))-d(5ad-bc(11+2p))x^2\right)dx}{b(7+2p)}$$

$$= -\frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}(c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} + \frac{\int (a+bx^2)^p\left(c(5a^2d^2-4abcd(4+p)+b^2c^2(35+24p+4p^2))+d(15a^2d^2-8abcd(6+p)+b^2c^2(\right.}{b^2(5+2p)(7+2p)}$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}(c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} - \frac{(15a^3d^3-9a^2bcd^2(7+2p)+3ab^2c^2d(35+24p+4p^2)-b^3c^3(105+142p+60p^2+8p^3))\int (a+b}{b^3(3+2p)(5+2p)(7+2p)}$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}(c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} - \frac{\left((15a^3d^3-9a^2bcd^2(7+2p)+3ab^2c^2d(35+24p+4p^2)-b^3c^3(105+142p+60p^2+8p^3))(a+b\right.}{b^3(3+2p)(5+2p)(7+2p)}$$

$$= \frac{d(15a^2d^2-8abcd(6+p)+b^2c^2(57+28p+4p^2))x(a+bx^2)^{1+p}}{b^3(3+2p)(5+2p)(7+2p)} - \frac{d(5ad-bc(11+2p))x(a+bx^2)^{1+p}(c+dx^2)}{b^2(5+2p)(7+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)^2}{b(7+2p)} - \frac{(15a^3d^3-9a^2bcd^2(7+2p)+3ab^2c^2d(35+24p+4p^2)-b^3c^3(105+142p+60p^2+8p^3))x(a+b}{b^3(3+2p)(5+2p)(7+2p)}$$

Mathematica [A] (verified)

Time = 7.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.46

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx$$
$$= \frac{1}{35}x\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \left(35c^3 \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + dx^2\left(35c^2 \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) + dx^2\left(21c\,\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)+5dx^2 \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)\right)\right)\right)$$

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*(35*c^3*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(35*c^2*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + d*x^2*(21*c*Hyp
ergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)] + 5*d*x^2*Hypergeometric2F1[7/2,
 -p, 9/2, -((b*x^2)/a)]))))/(35*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^3 dx$$

```
[In] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^3 \, dx = \int \left(dx^2+c\right)^3 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int (a+bx^2)^p (c+dx^2)^3\,dx = a^p c^3 x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right) + a^p c^2 d x^3\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right) + \frac{3a^p c d^2 x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{5}{2},-p\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5} + \frac{a^p d^3 x^7\,{}_2F_1\left(\left.\begin{matrix}\frac{7}{2},-p\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{7}$$

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**3,x)
```

```
[Out] a**p*c**3*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*c**2*
d*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a) + 3*a**p*c*d**2*x
**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5 + a**p*d**3*x**7*h
yper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int (a+bx^2)^p (c+dx^2)^3\,dx = \int (dx^2+c)^3 (bx^2+a)^p\,dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int (a+bx^2)^p (c+dx^2)^3\,dx = \int (dx^2+c)^3 (bx^2+a)^p\,dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^3*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)^p (c+dx^2)^3 \, dx = \int (b\,x^2+a)^p \, (d\,x^2+c)^3 \, dx$$

```
[In] int((a + b*x^2)^p*(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^p*(c + d*x^2)^3, x)
```

3.343 $\int (a+bx^2)^p (c+dx^2)^2 \, dx$

Optimal result

Integrand size = 19, antiderivative size = 176

$$\int (a+bx^2)^p (c+dx^2)^2 \, dx = -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)}$$
$$+\frac{(3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\right.}{b^2(3+2p)(5+2p)}$$

```
[Out] -d*(3*a*d-b*c*(7+2*p))*x*(b*x^2+a)^(p+1)/b^2/(4*p^2+16*p+15)+d*x*(b*x^2+a)^
(p+1)*(d*x^2+c)/b/(5+2*p)+(3*a^2*d^2-2*a*b*c*d*(5+2*p)+b^2*c^2*(4*p^2+16*p+
15))*x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/b^2/(4*p^2+16*p+15)/
((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {427, 396, 252, 251}

$$\int (a+bx^2)^p (c+dx^2)^2 \, dx$$
$$= \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}(3a^2d^2-2abcd(2p+5)+b^2c^2(4p^2+16p+15))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\right.}{b^2(2p+3)(2p+5)}$$
$$-\frac{dx(a+bx^2)^{p+1}(3ad-bc(2p+7))}{b^2(2p+3)(2p+5)}+\frac{dx(c+dx^2)(a+bx^2)^{p+1}}{b(2p+5)}$$

```
[In] Int[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] -((d*(3*a*d - b*c*(7 + 2*p))*x*(a + b*x^2)^(1 + p))/(b^2*(3 + 2*p)*(5 + 2*p
))) + (d*x*(a + b*x^2)^(1 + p)*(c + d*x^2))/(b*(5 + 2*p)) + ((3*a^2*d^2 - 2
*a*b*c*d*(5 + 2*p) + b^2*c^2*(15 + 16*p + 4*p^2))*x*(a + b*x^2)^p*Hypergeom
etric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(b^2*(3 + 2*p)*(5 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{1+p}\left(c+dx^2\right)}{b(5+2p)} \\
&\quad + \frac{\int \left(a+bx^2\right)^p\left(-c(ad-bc(5+2p))-d(3ad-bc(7+2p))x^2\right)\,dx}{b(5+2p)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} \\
&\quad + \frac{(3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4p^2))\int (a+bx^2)^p\,dx}{b^2(3+2p)(5+2p)} \\
&= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} \\
&\quad + \frac{\left((3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4p^2))(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p dx}{b^2(3+2p)(5+2p)} \\
&= -\frac{d(3ad-bc(7+2p))x(a+bx^2)^{1+p}}{b^2(3+2p)(5+2p)} + \frac{dx(a+bx^2)^{1+p}(c+dx^2)}{b(5+2p)} \\
&\quad + \frac{(3a^2d^2-2abcd(5+2p)+b^2c^2(15+16p+4p^2))\,x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)}{b^2(3+2p)(5+2p)}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 5.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

$$
\begin{aligned}
\int (a+bx^2)^p (c+dx^2)^2\,dx &= \frac{1}{15}x(a+bx^2)^p\Bigg(1 \\
&\quad + \frac{bx^2}{a}\Bigg)^{-p}\Bigg(15c^2\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) \\
&\quad + dx^2\Bigg(10c\,\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right) \\
&\quad + 3dx^2\,\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)\Bigg)\Bigg)
\end{aligned}
$$

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*(15*c^2*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)] + d*
x^2*(10*c*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)] + 3*d*x^2*Hypergeom
etric2F1[5/2, -p, 7/2, -((b*x^2)/a)])))/(15*(1 + (b*x^2)/a)^p)
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)^{2}d x$$

```
[In] int((b*x^2+a)^p*(d*x^2+c)^2,x)
[Out] int((b*x^2+a)^p*(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(dx^{2}+c\right)^{2} \left(bx^{2}+a\right)^{p} \,dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="fricas")
[Out] integral((d^2*x^4 + 2*c*d*x^2 + c^2)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = a^{p}c^{2}x{}_{2}F_{1}\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right) + \frac{2a^{p}cdx^{3}{}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3} + \frac{a^{p}d^{2}x^{5}{}_{2}F_{1}\left(\begin{matrix} \frac{5}{2}, -p \\ \frac{7}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{5}$$

```
[In] integrate((b*x**2+a)**p*(d*x**2+c)**2,x)
[Out] a**p*c**2*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + 2*a**p*c*d
*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3 + a**p*d**2*x**5
*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(dx^2+c\right)^2 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(dx^2+c\right)^2 \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((d*x^2 + c)^2*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)^2 \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right)^2 \, dx$$

```
[In] int((a + b*x^2)^p*(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^p*(c + d*x^2)^2, x)
```

3.344 $\int (a+bx^2)^p (c+dx^2)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int (a+bx^2)^p (c+dx^2)\, dx = \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \frac{(ad-bc(3+2p))x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)}{b(3+2p)}$$

```
[Out] d*x*(b*x^2+a)^(p+1)/b/(3+2*p)-(a*d-b*c*(3+2*p))*x*(b*x^2+a)^p*hypergeom([1/
2, -p],[3/2],-b*x^2/a)/b/(3+2*p)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {396, 252, 251}

$$\int (a+bx^2)^p (c+dx^2)\, dx = x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \left(c - \frac{ad}{2bp+3b}\right) \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right) + \frac{dx(a+bx^2)^{p+1}}{b(2p+3)}$$

```
[In] Int[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (d*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + ((c - (a*d)/(3*b + 2*b*p))*x*(a +
 b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \left(-c+\frac{ad}{3b+2bp}\right)\int \left(a+bx^2\right)^p\,dx \\
&= \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} - \left(\left(-c+\frac{ad}{3b+2bp}\right)\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int \left(1+\frac{bx^2}{a}\right)^p\,dx \\
&= \frac{dx(a+bx^2)^{1+p}}{b(3+2p)} + \left(c-\frac{ad}{3b+2bp}\right)x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.97

$$\int \left(a+bx^2\right)^p\left(c+dx^2\right)\,dx = \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\left(d(a+bx^2)\left(1+\frac{bx^2}{a}\right)^p+(-ad+bc(3+2p))\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right.\right.}{b(3+2p)}$$

```
[In] Integrate[(a + b*x^2)^p*(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*(d*(a + b*x^2)*(1 + (b*x^2)/a)^p + (-(a*d) + b*c*(3 + 2*p)
)*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)]))/(b*(3 + 2*p)*(1 + (b*x^2)
/a)^p)
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} \left(d\,x^{2}+c\right)dx$$

```
[In] int((b*x^2+a)^p*(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p*(d*x^2+c),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx=\int \left(dx^2+c\right)\left(bx^2+a\right)^p\,dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.76 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right)\,dx = a^p c x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)+\frac{a^p d x^3\,{}_2F_1\left(\left.\begin{matrix}\frac{3}{2},-p\\ \frac{5}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

```
[In] integrate((b*x**2+a)**p*(d*x**2+c),x)
```

```
[Out] a**p*c*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a) + a**p*d*x**3*h
yper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right) \, dx = \int \left(dx^2+c\right)\left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right) \, dx = \int \left(dx^2+c\right)\left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p*(d*x^2+c),x, algorithm="giac")
[Out] integrate((d*x^2 + c)*(b*x^2 + a)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^p \left(c+dx^2\right) \, dx = \int \left(b\,x^2+a\right)^p \left(d\,x^2+c\right) \, dx$$

```
[In] int((a + b*x^2)^p*(c + d*x^2),x)
[Out] int((a + b*x^2)^p*(c + d*x^2), x)
```

3.345 $\int (a+bx^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

```
[Out] x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {252, 251}

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

```
[In] Int[(a + b*x^2)^p,x]
```

```
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\left(1+\frac{bx^2}{a}\right)^p\,dx\\ &= x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{bx^2}{a}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^p\,dx = x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

```
[In] Integrate[(a + b*x^2)^p,x]
[Out] (x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^2
)/a)^p
```

Maple [F]

$$\int (b\,x^2+a)^p\,dx$$

```
[In] int((b*x^2+a)^p,x)
[Out] int((b*x^2+a)^p,x)
```

Fricas [F]

$$\int (a+bx^2)^p\,dx = \int (bx^2+a)^p\,dx$$

```
[In] integrate((b*x^2+a)^p,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^p \, dx = a^p x_2 F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

```
[In] integrate((b*x**2+a)**p,x)
[Out] a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p \, dx$$

```
[In] integrate((b*x^2+a)^p,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)^p \, dx = \frac{x\,\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^p}$$

```
[In] int((a + b*x^2)^p,x)
[Out] (x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

3.346 $\int \frac{(a+bx^2)^p}{c+dx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,1,3/2,-b*x^2/a,-d*x^2/c)/c/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \frac{x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

```
[In] Int[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c*
(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left(\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{c+dx^2}\,dx$$
$$= \frac{x\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. 2(57) = 114.

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{\left(a+bx^2\right)^p}{c+dx^2}\,dx = -\frac{3acx\left(a+bx^2\right)^p\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{\left(c+dx^2\right)\left(-3ac\operatorname{AppellF1}\left(\frac{1}{2},-p,1,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+2x^2\left(-bcp\operatorname{AppellF1}\left(\frac{3}{2},1-p,1,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)+\right.\right.}$$

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2),x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)*(-3*a*c*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), -((d*x^2)/
c)] + 2*x^2*(-(b*c*p*AppellF1[3/2, 1 - p, 1, 5/2, -((b*x^2)/a), -((d*x^2)/c
)]) + a*d*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int\frac{\left(b\,x^2+a\right)^p}{d\,x^2+c}dx$$

```
[In] int((b*x^2+a)^p/(d*x^2+c),x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(d*x^2 + c), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/(d*x**2+c),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \int \frac{(bx^2+a)^p}{dx^2+c}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{c+dx^2}\,dx = \int \frac{(b\,x^2+a)^p}{d\,x^2+c}\,dx$$

```
[In] int((a + b*x^2)^p/(c + d*x^2),x)
[Out] int((a + b*x^2)^p/(c + d*x^2), x)
```

3.347 $\int \frac{(a+bx^2)^p}{(c+dx^2)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2} dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,2,3/2,-b*x^2/a,-d*x^2/c)/c^2/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {441, 440}

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2} dx = \frac{x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
2*(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\text{integral} = \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(c+dx^2)^2}\,dx$$
$$= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^2}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx =$$
$$-\frac{3acx(a+bx^2)^p\,\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^2\left(-3ac\,\text{AppellF1}\left(\frac{1}{2},-p,2,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,2,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-\right.\right.}$$

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^2,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^2*(-3*a*c*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 2, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 2*a*d*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int\frac{(b\,x^2+a)^p}{(d\,x^2+c)^2}dx$$

```
[In] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(d^2*x^4 + 2*c*d*x^2 + c^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**2,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^2}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^2,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^2}\,dx = \int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^2}\,dx$$

```
[In] int((a + b*x^2)^p/(c + d*x^2)^2,x)
[Out] int((a + b*x^2)^p/(c + d*x^2)^2, x)
```

3.348 $\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx = \frac{x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

```
[Out] x*(b*x^2+a)^p*AppellF1(1/2,-p,3,3/2,-b*x^2/a,-d*x^2/c)/c^3/((1+b*x^2/a)^p)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {441, 440}

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3} dx = \frac{x(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}$$

```
[In] Int[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c)])/(c^
3*(1 + (b*x^2)/a)^p)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left((a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\right)\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(c+dx^2)^3}\,dx\\ &= \frac{x(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 162 vs. $2(57) = 114$.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.84

$$\int\frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = -\frac{3acx(a+bx^2)^p\,\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)}{(c+dx^2)^3\left(-3ac\,\text{AppellF1}\left(\frac{1}{2},-p,3,\frac{3}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-2x^2\left(bcp\,\text{AppellF1}\left(\frac{3}{2},1-p,3,\frac{5}{2},-\frac{bx^2}{a},-\frac{dx^2}{c}\right)-\right.\right.}$$

```
[In] Integrate[(a + b*x^2)^p/(c + d*x^2)^3,x]
```

```
[Out] (-3*a*c*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2)/c
)])/((c + d*x^2)^3*(-3*a*c*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), -((d*x^2
)/c)] - 2*x^2*(b*c*p*AppellF1[3/2, 1 - p, 3, 5/2, -((b*x^2)/a), -((d*x^2)/c
)] - 3*a*d*AppellF1[3/2, -p, 4, 5/2, -((b*x^2)/a), -((d*x^2)/c)])))
```

Maple [F]

$$\int\frac{(b\,x^2+a)^p}{(d\,x^2+c)^3}dx$$

```
[In] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

```
[Out] int((b*x^2+a)^p/(d*x^2+c)^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="fricas")
[Out] integral((b*x^2 + a)^p/(d^3*x^6 + 3*c*d^2*x^4 + 3*c^2*d*x^2 + c^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)**p/(d*x**2+c)**3,x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(bx^2+a)^p}{(dx^2+c)^3}\,dx$$

```
[In] integrate((b*x^2+a)^p/(d*x^2+c)^3,x, algorithm="giac")
[Out] integrate((b*x^2 + a)^p/(d*x^2 + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{(c+dx^2)^3}\,dx = \int \frac{(b\,x^2+a)^p}{(d\,x^2+c)^3}\,dx$$

```
[In] int((a + b*x^2)^p/(c + d*x^2)^3,x)
[Out] int((a + b*x^2)^p/(c + d*x^2)^3, x)
```

3.349 $\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx$

Optimal result

Integrand size = 50, antiderivative size = 53

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx = \frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[Out] x*(d*x^2+c)^(a*d/(-2*a*d+2*b*c))/a/c/((b*x^2+a)^(b*c/(-2*a*d+2*b*c)))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {389}

$$\int (a+bx^2)^{-1-\frac{bc}{2bc-2ad}} (c+dx^2)^{-1+\frac{ad}{2bc-2ad}} \, dx = \frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[In] Int[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/(2*b*c
 - 2*a*d)),x]
```

```
[Out] (x*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d)))/(a*c*(a + b*x^2)^((b*c)/(2*b*c - 2*
a*d)))
```

Rule 389

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c)), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\text{integral} = \frac{x(a+bx^2)^{-\frac{bc}{2bc-2ad}} (c+dx^2)^{\frac{ad}{2bc-2ad}}}{ac}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} \, dx = \frac{x\left(a+bx^2\right)^{\frac{bc}{-2bc+2ad}} \left(c+dx^2\right)^{\frac{ad}{2bc-2ad}}}{ac}$$

```
[In] Integrate[(a + b*x^2)^(-1 - (b*c)/(2*b*c - 2*a*d))*(c + d*x^2)^(-1 + (a*d)/
(2*b*c - 2*a*d)),x]
```

```
[Out] (x*(a + b*x^2)^((b*c)/(-2*b*c + 2*a*d))*(c + d*x^2)^((a*d)/(2*b*c - 2*a*d))
)/(a*c)
```

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

method	result	size
gosper	$\frac{x\left(b\,x^{2}+a\right)^{1-\frac{2ad-3bc}{2(ad-bc)}} \left(d\,x^{2}+c\right)^{1-\frac{3ad-2bc}{2(ad-bc)}}}{ac}$	71

```
[In] int((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c)),x,m
ethod=_RETURNVERBOSE)
```

```
[Out] x/a/c*(b*x^2+a)^(1-1/2*(2*a*d-3*b*c)/(a*d-b*c))*(d*x^2+c)^(1-1/2*(3*a*d-2*b
*c)/(a*d-b*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.72

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}} \left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}} \, dx = \frac{bdx^5+(bc+ad)x^3+acx}{\left(bx^2+a\right)^{\frac{3\,bc-2\,ad}{2\,(bc-ad)}} \left(dx^2+c\right)^{\frac{2\,bc-3\,ad}{2\,(bc-ad)}} ac}$$

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^5 + (b*c + a*d)*x^3 + a*c*x)/((b*x^2 + a)^(1/2*(3*b*c - 2*a*d)/(b*c
- a*d))*(d*x^2 + c)^(1/2*(2*b*c - 3*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx=\text{Timed out}$$

```
[In] integrate((b*x**2+a)**(-1-b*c/(-2*a*d+2*b*c))*(d*x**2+c)**(-1+a*d/(-2*a*d+2
*b*c)),x)
[Out] Timed out
```

Maxima [F]

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx=\int \left(bx^2+a\right)^{-\frac{bc}{2(bc-ad)}-1}\left(dx^2+c\right)^{\frac{ad}{2(bc-ad)}-1}\,dx$$

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx=\int \left(bx^2+a\right)^{-\frac{bc}{2(bc-ad)}-1}\left(dx^2+c\right)^{\frac{ad}{2(bc-ad)}-1}\,dx$$

```
[In] integrate((b*x^2+a)^(-1-b*c/(-2*a*d+2*b*c))*(d*x^2+c)^(-1+a*d/(-2*a*d+2*b*c
)),x, algorithm="giac")
[Out] integrate((b*x^2 + a)^(-1/2*b*c/(b*c - a*d) - 1)*(d*x^2 + c)^(1/2*a*d/(b*c
- a*d) - 1), x)
```

Mupad [B] (verification not implemented)

Time = 5.93 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \left(a+bx^2\right)^{-1-\frac{bc}{2bc-2ad}}\left(c+dx^2\right)^{-1+\frac{ad}{2bc-2ad}}\,dx =\frac{x\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}+\frac{x^3\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}\,(a\,d+b\,c)}{a\,c}+\frac{b\,d\,x^5\left(b\,x^2+a\right)^{\frac{b\,c}{2\,a\,d-2\,b\,c}-1}}{a\,c}}{\left(d\,x^2+c\right)^{\frac{a\,d}{2\,a\,d-2\,b\,c}+1}}$$

```
[In] int((a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1)/(c + d*x^2)^((a*d)/(2*a*d - 2*b
*c) + 1),x)
```

```
[Out] (x*(a + b*x^2)^((b*c)/(2*a*d - 2*b*c) - 1) + (x^3*(a + b*x^2)^((b*c)/(2*a*d
 - 2*b*c) - 1)*(a*d + b*c))/(a*c) + (b*d*x^5*(a + b*x^2)^((b*c)/(2*a*d - 2*
b*c) - 1))/(a*c))/(c + d*x^2)^((a*d)/(2*a*d - 2*b*c) + 1)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count i
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                    convert(leaf_count_result,string),"$ vs. $2(",
                                    convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```python
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```